$$\left.\begin{aligned} \min_{w} \quad & f(w,p) \\ & g(w,p) = 0 \\ & h(w,p) \geq 0 \end{aligned}\right\} \quad f(w^*, p) \rightarrow \max_{p}$$

value function

$$\frac{df}{dp_j} = ?$$

$$\mathcal{L} = f(w,p) - \lambda^T g(w,p) - \mu^T h(w,p)$$

$$\frac{\partial \mathcal{L}}{\partial w} = 0 = \frac{\partial f}{\partial w} - \lambda^T \frac{\partial g(w,p)}{\partial w} - \mu^T \frac{\partial h(w,p)}{\partial w}$$

$$g(w,p) = 0$$

(without inequalities)

$$\frac{dg}{dp_j} = \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial g}{\partial p_j} = 0$$

$$\frac{df}{dp_j} = \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial f}{\partial p_j}$$

$$\frac{\partial \mathcal{L}}{\partial w} \cdot \frac{\partial w}{\partial p_j} = \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial p_j} - \lambda^T \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j} = 0$$

$$\Rightarrow \quad \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial p_j} = \lambda^T \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j} = -\lambda^T \frac{\partial g}{\partial p_j}$$

$$\Rightarrow \frac{df}{dp_j} = \frac{\partial f}{\partial p_j} - \lambda^T \frac{\partial g}{\partial p_j}$$

$$\frac{\partial \mathcal{L}}{\partial w} = \boxed{0 = \frac{\partial f}{\partial w} - \lambda^T \frac{\partial g}{\partial w} \quad g(w,p) = 0}$$

IFT

$$F(w,\lambda,p) = \begin{pmatrix} \nabla_w \mathcal{L}(w,\lambda,p) \\ g(w,p) \end{pmatrix} = 0$$

$$\frac{\partial F}{\partial w} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial F}{\partial \lambda} \cdot \frac{\partial \lambda}{\partial p_j} + \frac{\partial F}{\partial p_j} = 0$$

$$\underbrace{\begin{pmatrix} \frac{\partial F}{\partial w} & \frac{\partial F}{\partial \lambda} \end{pmatrix}}_{\text{KKT matrix invertierbar}} \begin{pmatrix} \frac{\partial w}{\partial p_j} \\ \frac{\partial \lambda}{\partial p_j} \end{pmatrix} = - \underbrace{\frac{\partial F}{\partial p_j}}_{\begin{pmatrix} \frac{\partial^2 f}{\partial p_j \partial w} - \lambda^T \frac{\partial^2 g}{\partial p_j \partial w} \\ \frac{\partial g}{\partial p_j} \end{pmatrix}}$$

2nd order derivatives:

$$\frac{dg}{dp_j} = \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial g}{\partial p_j} = 0$$

$$\frac{d^2 g}{dp_j^2} = \left(\frac{\partial^2 g}{\partial w^2} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial^2 g}{\partial p_j \partial w} \right) \cdot \frac{\partial w}{\partial p_j}$$

$$+ \frac{\partial g}{\partial w} \quad \frac{\partial^2 w}{\partial^2 p_j} + \frac{\partial^2 g}{\partial w \partial p_j} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial^2 g}{\partial p_j^2} = 0$$

$$\frac{df}{dp_j} = \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial f}{\partial p_j}$$

$$\frac{d^2 f}{dp_j^2} = \left(\frac{\partial^2 f}{\partial w^2} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial^2 f}{\partial p_j \partial w} \right) \frac{\partial w}{\partial p_j} + \frac{\partial f}{\partial w} \cdot \frac{\partial^2 w}{\partial p_j^2}$$

$$+ \frac{\partial^2 f}{\partial w \cdot \partial p_j} \cdot \frac{\partial w}{\partial p_j} + \frac{\partial^2 f}{\partial p_j^2}$$

$$\frac{d^2 f}{dp_j^2} - \lambda^T \overbrace{\frac{d^2 g}{dp_j^2}}^{=0} = \frac{\partial^2 f}{\partial p_j^2} - \lambda^T \frac{\partial^2 g}{\partial p_j^2}$$

$$+ \left(\frac{\partial^2 f}{\partial w^2} - \lambda^T \frac{\partial^2 g}{\partial w^2} \right) \frac{\partial w}{\partial p_j} \frac{\partial w}{\partial p_j}$$

$$= \frac{\partial^T \lambda}{\partial p_j} \cdot \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j} - \left(\frac{\partial^2 f}{\partial p_j \partial w} - \lambda^T \frac{\partial^2 g}{\partial p_j \partial w} \right) \cdot \frac{\partial w}{\partial p_j}$$

$$+ \left(\frac{\partial^2 f}{\partial p_j \partial w} - \lambda^T \frac{\partial^2 g}{\partial p_j \partial w} \right) \frac{\partial w}{\partial p_j}$$

$$+ \overbrace{\left(\frac{\partial f}{\partial w} - \lambda^T \frac{\partial g}{\partial w} \right)}^{=0} \frac{\partial^2 w}{\partial p_j^2}$$

$$+ \left(\frac{\partial^2 f}{\partial w \partial p_j} - \lambda^T \frac{\partial g}{\partial w \partial p_j} \right) \frac{\partial w}{\partial p_j}$$

$$\Rightarrow \frac{d^2 f}{dp_j^2} = \frac{\partial^2 f}{\partial p_j^2} - \lambda^T \frac{\partial^2 g}{\partial p_j^2} + \left(\frac{\partial^2 f}{\partial w \partial p_j} - \lambda^T \frac{\partial g}{\partial w \partial p_j} \right) \frac{\partial w}{\partial p_j}$$

$$+ \frac{\partial \lambda^T}{\partial p_j} \cdot \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial p_j}$$