Evaluation of Strategic Asset Allocation

ISE 441

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2016

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Executive Summary

This paper replicates the approach provided by Riccardo Cesari and Davide Cremonini (2002) with a few extensions being made. As the original paper, we first perform an in-depth analysis and comparison of various popular dynamic strategies of asset allocation. Each strategy is elaborated both intuitively and mathematically. A Monte Carlo Simulation is carried out to test each strategy under different market trends. The performance is measured by alternative methodologies including risk calculation, return and risk-adjusted performance such as Sharpe Ratio, Sortino Ratio and Return at Risk (RaR). We successfully achieve the same results as the authors did, showing a dominant role of constant proportion portfolio insurance (CPPI) strategy under bear and no-trend markets and a preference for benchmarking strategy under bull markets, which suggests our implementation work out very well. Then, we further compare the performance of CPPI and benchmarking strategies under bull markets but with the existence of price jumps to make it much closer to the real world. We find that the average performances of the two strategies are almost identical. Overall, we conclude that CPPI strategy is a more robust investing strategy in terms of unknown markets trends. The conclusion is also independent of volatility and risk-adjusted measure adopted.

1. Literature Review

1.1 What is dynamic asset allocation

Dynamic asset allocation is a portfolio management strategy that involves rebalancing a portfolio so as to bring the asset mix back to its long-term target. Such rebalancing would generally involve reducing positions in the best-performing asset class while adding to positions in underperforming assets. The general premise of dynamic asset allocation is to reduce the fluctuation risks and achieve returns that exceed the target benchmark.

1.2 Previous research on dynamic strategies

Portfolio strategies are summarized in the class of portfolio insurance strategies. They are designed to limit downside risk and at the same time to profit from rising markets, Among others, Grossman and Villa (1989) and Basak (2002) define a portfolio insurance trading strategy as a strategy which guarantees a minimum level of wealth at a specified time horizon, but also participates in the potential gains of a reference portfolio. The most prominent examples of dynamic versions are the constant proportion portfolio insurance (CPPI) strategies and option–based portfolio insurance (OBPI) strategies with synthetic puts. Here, synthetic is understood in the sense of a trading strategy in basic (traded) assets, which creates the put. Sven Balder, Michael Brandl and Antje Mahayni (2005) analyze the effectiveness of the constant proportion portfolio insurance (CPPI) method under trading restrictions. If the CPPI method is applied in continuous time, the CPPI strategies provide a value above a floor level unless the price dynamic of the risky asset permits jumps. The risk of violating the floor protection is called gap risk. In practice, it is caused by liquidity constraints and price jumps. Both can be modeled in a setup in which a continuous–time stochastic process describes the price dynamic of the risky asset but trading is restricted to discrete time. We propose a discrete–time version of the continuous–time CPPI strategies, which satisfies three conditions. The resulting strategies are self–financing, the asset exposure is non–negative and the value process converges. We determine risk measures such as the shortfall probability and the expected shortfall and discuss criteria, which ensure that the gap risk does not increase to a level, which contradicts the original intention of portfolio insurance.

Technical analysts, also known as “chartists”, attempt to forecast future prices by the study of patterns of past prices and a few other related summary statistics about security trading. Basically, it uses past prices and perhaps other past statistics to make investment decisions. Academics, however, have long been skeptical about the usefulness of technical analysis, despite its wide- spread acceptance and adoption by practitioners. There are perhaps three reasons. The first reason is that no theoretical basis exists for it, which this paper attempts to provide. The second reason is that earlier theoretical studies often assume a random walk model for the stock price, which completely rules out any profitability from technical trading. The third reason is that earlier empirical findings, such as Cowles (1933) and Fama and Blume (1966), are mixed and inconclusive. However, Brock, Lakonishok, and LeBaron (1992), and especially Lo, Mamaysky, and Wang (2000) find strong evidence of profitability in technical trading based on more data and more elaborate strategies. Yingzi Zhu and Guofu Zhou (2008) analyze the usefulness of technical analysis, specifically the widely employed moving average trading rule from an asset allocation perspective. They show that, when stock returns are predictable, technical analysis adds value to commonly used allocation rules that invest fixed proportions of wealth in stocks.

These studies stimulated much subsequent academic research on technical analysis. There also have been various studies of the use and profitability of technical analysis. Taylor and Allen (1992) document the enduring popularity of the trading rules in their survey of currency traders in London. Of the respondents, 90% replied that technical trading rules are an important component of short-term investment strategies. Allen and Taylor (1990) suggest that this is an important finding given the apparent ability of exchange rates to move far from fundamentals over protracted periods of time, as documented by Frankel and Froot (1986, 1990). Pesaran and Timmermann (1994, 1995) present evidence on the predictability of excess returns on common stocks for the S&P 500 and Dow Jones Industrial portfolios, and examine the robustness of the evidence on the predictability of U.S. stock returns. Brock et al. (1992) investigate the sources of the predictability by applying the bootstrap technique to two of the simplest and most popular trading rules, the moving average (MA) and the trading range break rules. They document that buy signals generate higher returns than sell signals and the returns following buy signals are less volatile than returns following sell signals. Recent studies, such as Lo et al. (2000), Boswijk et al. (2000) and Goldbaum (2003), have also examined explicitly the profitability of technical trading rules and the implications for market efficiency. Griffioen (2003) contains extensive statistical testing of the profitability of technical trading rules, after correcting for transaction costs and data snooping, of many stock market indices including the Dow Jones index. Most of the cited research has focused on empirical studies. Carl Chiarella, Xue-Zhong He, Cars Hommes (2007) propose a dynamic financial market model in which demand for traded assets has both a fundamentalist and a technical analysis component. By increasing the intensity of choice to switching strategies, they examine various rational routes to randomness for different MA rules. The price dynamics of the MA rule are also examined and one of their main findings is that an increase of the window length of the MA rule can destabilize an otherwise stable system, leading to more complicated, even chaotic behavior.

One of the more popular strategies of portfolio insurance is the Option Based Portfolio Insurance (OBPI), introduced in Leland and Rubinstein (1976). It consists basically in buying simultaneously the stock (generally a financial index) and a put written on it. The value of this portfolio at maturity is always greater than the strike of the put, whatever the market fluctuations. Thus, this strike is the insured amount, which is often equal to a given percentage of the initial investment. Yuan-Hung Hsuku (2008) shows that an inter-temporal investment-consumption technique can be applied to investigate the optimal consumption and dynamic option-based portfolio insurance strategy when there is predictable variation in return volatility. An optimal dynamic option-based portfolio insurance strategy can be separated into a myopic component and an inter-temporal hedging component. The myopic component is simply linked to the risk-and-return tradeoff associated with price risk of the portfolio value under the option-based portfolio insurance. The inter-temporal hedging component of the optimal dynamic option-based portfolio insurance is an affine function of the reciprocal of the time-varying volatility. The inter-temporal hedging demand is further separated into three effects. The correlation effect results in a conservative investor having a negative position on the inter-temporal hedging demand of the option-based portfolio insurance strategy. The negative instantaneous correlation between unexpected return on the risky stock and its stochastic volatility implies the investor will have negative inter-temporal hedging demand due to changes solely in the volatility of the risky stock, because of its lack of hedging ability against an increase in volatility. The two other positive effects in the inter-temporal hedging component for the option-based portfolio insurance are the Delta effect and the Vega effect. From these two effects, a conservative investor will have a positive position on the inter-temporal hedging demand of the option-based portfolio insurance strategy. Incorporating options' considerations in portfolio decisions to create a dynamic option-based portfolio insurance strategy improves the hedging ability in the inter-temporal hedging component, especially in down markets.

P. Bertrand and J.L. Prigent (2006) analyze the performance of portfolio insurance methods: especially the OBPI and CPPI, during which they use the Omega performance measure, which takes the entire return distribution into account. The Omega performance measure takes potentially into account all the moments of the returns distribution. Thus, it can be used to study asset with non-normally distributed returns, such as hedge funds, equity in illiquid markets. They have shown in particular that for the Omega performance measure, the CPPI method is better than the OBPI one for "rational" thresholds when assuming log-normality of the stock price. Further studies can extend this analysis when jumps may occur or may be based on generalized downside risk-adjusted performance measures such as the Kappa.

1.3 Paper we choose for replication

The paper we choose is *Benchmarking, portfolio insurance and technical analysis: a Monte Carlo comparison of dynamic strategies of asset allocation* by Riccardo Cesari and Davide Cremonini (2002).

2. Paper Replication

2.1 Introduction

The purpose of this section is to give readers a succinct overview of the original Monte Carlo simulation approach adopted by Riccardo Cesari and Davide Cremonini (2002).

Strategic asset allocation is defined as the process used to identify the optimal portfolio for a given investor over his or her investment horizon. In simple terms, such a portfolio will be a combination of risky assets (stocks, bonds) and risk-free components (cash, money market instruments). In our case, we use MSCI world index as the risky asset and cash as the riskless component. The asset allocation strategies are then described mathematically and their different risk-return attributes are profiled accordingly. According to their paper (2002), they introduced a simple static strategy called *buy and hold*: a passive investment strategy where a portfolio is constructed for the whole investment horizon, regardless of the market fluctuations. Besides, they also discussed other dynamic strategies that imply a periodic rebalancing process among the assets. Overall, these strategies fall into three broad categories: *benchmarking*, *proportion portfolio insurance*, and *technical strategies*. At any rate, the strategies considered are not discretionary in nature, in the sense that they can be expressed in terms of precise quantitative rules of action. Moreover, all these strategies are currently implemented in actual markets.

Technical analysis is perhaps the oldest device designed to beat the market. It has been discussed in a lot of academic and non-academic publications. Portfolio insurance arose as a by-product of the option pricing theory. Finally, benchmarking has in recent years become the truly risk-free choice. In fact, given the general rule of contractually specifying a single or composite market index as the reference benchmark for both passive and active management, it is soon apparent that tracking the index and minimizing the so-called “active risk” is, in a sense, a risk-free policy and a meaningful standard of comparison for any alternative.

Moreover, they considered three different market situations, namely bear market, bull market and no-trend market. By their definition, a market with -30% to -5% average returns was regarded as a bear market; a no-trend market had a -5% to 5% average return; a bull market had a 5% to 30% average return. They also had assigned different market volatility ranges for each market situation.

The 8 different portfolio strategies are described and analyzed in Section 2 and Section 3 contains our extension and conclusion.

2.2 Description of the strategy

Dynamic asset allocation strategies (Perold and Sharpe, 1988) consist of precise rules of variation of asset quantities (trading rules). A distinguishing feature of each strategy consists of the payoff function, i.e. the relation between the price of the risky component and the total portfolio value. Linear, concave and convex payoffs may be used to identify three different classes of strategies. In the following, we consider four different strategies: (1) buy and hold (BH), (2) constant mix (CM), (3) constant proportion portfolio insurance (CPPI), (4) option based and (5) technical strategies, with three variations of the option based approach (BCDT, NL PS) and two kinds of technical or stop-loss strategies (MA and MA2). In total, 8 strategies are implemented. In this section, we give a brief summary of their properties.

The following is the basic setup for the mathematical description:

A_t is the value of the portfolio at time t, E_t is the equity value, B_t is the cash account value, Q_0 is the initial number of share, and Q_0^B is the initial cash amount.

2.2.1 Buy and Hold (BH)

The simplest strategy is to keep the initial quantities constant. In practice, a portfolio is constructed for the whole investment horizon without any rebalancing, regardless of performance. Note investors who employ a buy-and-hold strategy actively select stocks, but once in a position, is not concerned with short-term price movements and technical indicators.

From a quantitative standpoint:

$$A_t = E_t + B_t = Q_0 S_t + Q_0^B e^{rt},$$

where the payoff is linear in this case. In particular, the payoff function is linear in the stock price. Moreover, the potential gain is unlimited. In practice, for single-index benchmarking, i.e. the practice of replicating a single market price index is almost always a BH strategy because the index weights are normally changed only a few times a year.

2.2.2 Constant mix (CM)

CM implies keeping the initial portfolio proportions constant, not the initial quantities. To be more detailed, the objective of constant mix strategy is to maintain a ratio of, for example, 60% stocks and 40% cash by rebalancing. Stocks are bought when prices are falling and are sold while rising, as a result of keeping a constant ratio relative to the whole portfolio. Constant-mix strategy takes a contrarian view to maintain the desired mix of risky assets and riskless assets, regardless of the size of the portfolio.

Constant mix implies that the initial portfolio share, instead of the initial quantities, is kept constant.

$$a_t = \frac{E_t}{A_t} = a_0, 1 - a_t = \frac{B_t}{A_t} = 1 - a_0,$$

which yields

$$A_t = E_t + B_t = a_0 A_t + (1 - a_0) A_t = Q_t S_t + Q_t^B e^{rt}.$$

Hence,

$$Q_t = \frac{a_0 A_t}{S_t}$$

and

$$Q_t^B = \frac{(1 - a_0) A_t}{S_t}.$$

The payoff function is obtained by considering the total variation of A_t, given by

$$dA = [S\,dQ + P^B dQ^B] + QdS + Q^B dP^B.$$

Because of the self-financing assumption for each strategy, the net investment must be zero,

$$SdQ + P^B dQ^B = 0.$$

Therefore, the general solution is (payoff function)

$$A_t = kS_t^{a_0} e^{(1-a_0)rt}.$$

The payoff is concave and this implies that price and asset demand are counter-varying: as we mention before the price goes up the quantity goes down and vice versa, implying a “buy low and sell high” rule. It is apparent that if the market has frequent oscillations (reversals), CM should be better than BH because after a drop in price it implies a buy order, which (on average) will be followed by a rise and therefore a capital gain.

2.2.3 Constant proportion portfolio insurance (CPPI)

CPPI is a simple portfolio insurance strategy suggested by Black and Jones (1987) and Black and Perold (1992) with the risky asset kept as a multiple of the cushion, i.e. the difference between the portfolio value and a protective floor. This strategy assumes that as investors' wealth increases, so does their risk tolerance. The basic premise of this technique stems from having a preference for maintaining a minimum safety reserve held in cash. When the value of the portfolio increases, more funds are invested in equities whereas a fall in portfolio worth results in a smaller position toward risky assets.

Let K be the granted level at time T and let F_0 (initial floor) be its actual value such that

$$F_T = F_0 e^{rT} = K,$$

i.e.

$$F_0 = F_T e^{-rT} = Ke^{-rT}$$

and

$$F_t = F_0 e^{rt}.$$

Constant proportion is defined as the following rule for the stock amount:

$$E_t = \begin{cases} \min\{A_t, m(A_t - F_t) & if\ A_t > F_t \\ 0 & if\ A_t \leq F_t' \end{cases}$$

where $m > 1$ is called the multiplier.

The payoff function is obtained as before as the solution of

$$dA = \frac{\min\{A_t, m(A_t - F_t)}{S_t} dS + \frac{A_t - \min\{A_t, m(A_t - F_t)}{e^{rt}} e^{rt} r dt.$$

Giving, for r=0, the general solution will be

$$A_t = F_0 + hS_t^m.$$

The payoff function is convex, implying a “buy high and sell low” rule: if the market has decreased you have to sell a fraction of stocks and switch into money if the market has increased you have to reduce cash, buy stock and rally with the market (momentum strategy).

Clearly, in bull or bear markets, i.e. markets with trends, CPPI is expected to dominate both BH and CM because you buy stocks that will eventually appreciate and sell stocks which will depreciate.

2.2.4 Option based strategies

The following strategies are based on the replicating portfolio in option pricing theory. A so-called insured portfolio is built up so that it can guarantee a minimum profit K. Knowing that a put option can be replicated with a long position in the interest-bearing cash and a short position in the stock, naturally a portfolio that consists of stocks and cash to replicate the performance of an insured portfolio is set up to guarantee a minimum profit at the maturity of the put option.

Let S_0 denotes the initial unit value of a stock index, which is used to represent a stock portfolio here. Let $P(S,K)$ and $C(S,K)$ denote the price of a put option and a call option with strike price K on the same underlying asset, namely the stock index.

The floor value of the insured portfolio is taken as K, same as the strike price. Given a floor K, the insured portfolio$S_0 + P_0$ does not generally match the initial investment A_0, so that the initial balance constraint can be satisfied in different ways. The followings are the different strategies designed to match the initial investment.

To begin with, considering the Black-Sholes formula, the price of a put option is given by

$$p = KN\big(-d_2(0)\big)e^{-rT} - S_0N\big(-d_1(0)\big),$$

where

$$d_1(0) = \frac{ln\frac{S0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2(0) = d_1(0) - \sigma\sqrt{T},$$

and $N(\cdot)$ is the cumulative distribution function of the standard normal distribution.

a) BCDT strategy

Only a fraction $0 \leq g \leq 1$ of the initial capital is invested in the insured portfolio, the remaining amount B_0 being invested in the risk-free cash. Thus, at time 0 we have the following equation:

$$g[S_0 + P(S_0, K)] + B_0 = A_0,$$

with parameters g and B_0 to be found.

Substituting $P(S_0, K)$ into the previous equation we can get

$$g\left[S_0 + KN\left(-d_2(0)\right)e^{-rT} - S_0N\left(-d_1(0)\right)\right] + B_0 = A_0,$$

which is equivalent to

$$g\left[S_0N\left(d_1(0)\right) + KN\left(-d_2(0)\right)e^{-rT}\right] + B_0 = A_0.$$

Since the minimum value must be guaranteed at maturity if $S_T < K$, we have

$$gK + B_0e^{rT} = K,$$

which yields

$$B_0 = (1-g)Ke^{-rT},$$

$$g = \frac{A_0 - Ke^{-rT}}{Call_0}.$$

In this strategy the initial capital invested in stock index is $gS_0N\left(d_1(0)\right)$ and the initial capital invested in cash is

$$gN\left(-d_2(0)\right)e^{-rT} + B_0 = A_0 - S_0N\left(d_1(0)\right).$$

The payoff function of this portfolio can be easily calculated at any time t, which is

$$A_t = g\left[S_0 N\big(d_1(t)\big) + KN\big(-d_2(t)\big)e^{-r(T-t)}\right] + B_0 e^{rt},$$

where

$$d_1(t) = \frac{ln\frac{St}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \quad d_2(t) = d_1(t) - \sigma\sqrt{T-t}$$

At maturity T, if $S_T > K$, we have

$$A_T = gS_T + B_T = gS_T + (1-g)K.$$

If $S_T < K$, then we have

$$A_T = gK + B_T = K.$$

b) Non-linear Strategy (NL)

Find h and H such that:

$$h[S_0 + \mathrm{Put}(S_0, H)] = A_0,$$

$$hH = K.$$

Now define $S_0^0 = hS_0$, we have

$$S_0^0 + \mathrm{Put}(S_0^0, K) = \mathrm{Call}(S_0^0, K) + Ke^{-rT} = K,$$

in which the value of S_0^0 can be obtained numerically, given A_0 and K. And we get the value of h by $h = \frac{S_0^0}{S_0}$.

Here the initial amount invested in stock index is $S_0^0 N\big(d_1^0(0)\big)$ and the amount invested in cash is $Ke^{-rT}N\big(-d_2^0(0)\big)$, where $d_1^0(0) = \frac{ln\frac{S_0^0}{K} + \left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $d_2^0(0) = d_1^0(0) - \sigma\sqrt{T}$.

At time t the value invested in stock is given by

$$S_t^0 N\big(d_1^0(t)\big) = S_t h N\big(d_1^0(t)\big).$$

At maturity T, if $S_T > K$, $A_T = S_T^0$,

if $S_T < K$, $A_T = K$.

c) PS Strategy

In this strategy an initial amount of A_0 is used to buy a default-free zero-coupon bond and n call options, thus yielding the following equation:

$$n\text{Call}(S_0, K) + B_0 = A_0.$$

Imposing a constraint that cash at maturity equals to the floor K when the calls expire out of the money, then $B_0 = Ke^{-rT}$and in this case PS strategy is equivalent to BCDT strategy with $n = g$.

Vice versa, imposing a liquidity constraint in T to ensure the total delivery payment if the calls expire in the money gives

$$B_0 = nKe^{-rT},$$

$$n = \frac{A_0}{Call_0 + Ke^{-rT}}.$$

Using the call function,

$$A_0 = nS_0N\big(d_1(0)\big) + nKN\big(-d_2(0)\big)e^{-rT},$$

which means the amount invested in stock index is $nS_0N\big(d_1(0)\big)$ while the amount invested in risk-free cash is $nKN\big(-d2(0)\big)e^{-rT}$

The payoff function of the portfolio at time t is given by

$$A_t = nS_tN\big(d_1(t)\big) + nKN\big(-d_2(t)\big)e^{-r(T-t)}.$$

Specifically, at maturity T if $S_T > K$, $A_T = nS_T$,

if $S_T < K$, $A_T = nK$.

2.2.5 Technical strategies

Technical analysis is a well-known device used by practitioners to implement momentum strategies of tactical asset allocation. The main aim of this analysis is to get a signal to buy or to sell the risky asset from market data. Many different signal mechanisms (graphical or statistical) have been elaborated in the effort to anticipate the market movements and profit from correct market timing. A moving average of prices is certainly the simplest and most popular signaling device: if a short memory moving average (or just the current price level) crosses a long memory moving average from below the trader receives a “buy” suggestion; if the former crosses the latter from above the signal is to “sell”. So defined, the strategy uses the long memory moving average as a sort of (variable) floor: above the floor the portfolio is fully invested in the market; below the floor the portfolio is kept in risk-free cash. Such a strategy, therefore, is of the convex type, providing a sort of “technical insurance” for the portfolio value. Note that in our simulations the long memory is a 30-week moving average and the short memory is either the current price (MA strategy) or a 10-week moving average (MA2 strategy).

2.3 Implementation

2.3.1 Preliminary tests

To implement the Monte-Carlo simulation, two important assumptions must be satisfied namely the normality and independence of the time series data. To justify these assumptions, as a result, the normality test and independence test must be carried out with actual market data in order to verify if the simulation setup can be used as an approximation for real situations applying the data generating mechanism.

We follow the procedure in the replicated paper and consider one of the well-known stock market indices in US dollars, namely MSCI World, which covers all major stock markets in industrial as well as emerging countries. Weekly prices and log-returns are depicted in Fig. 1 and Fig 2, respectively.

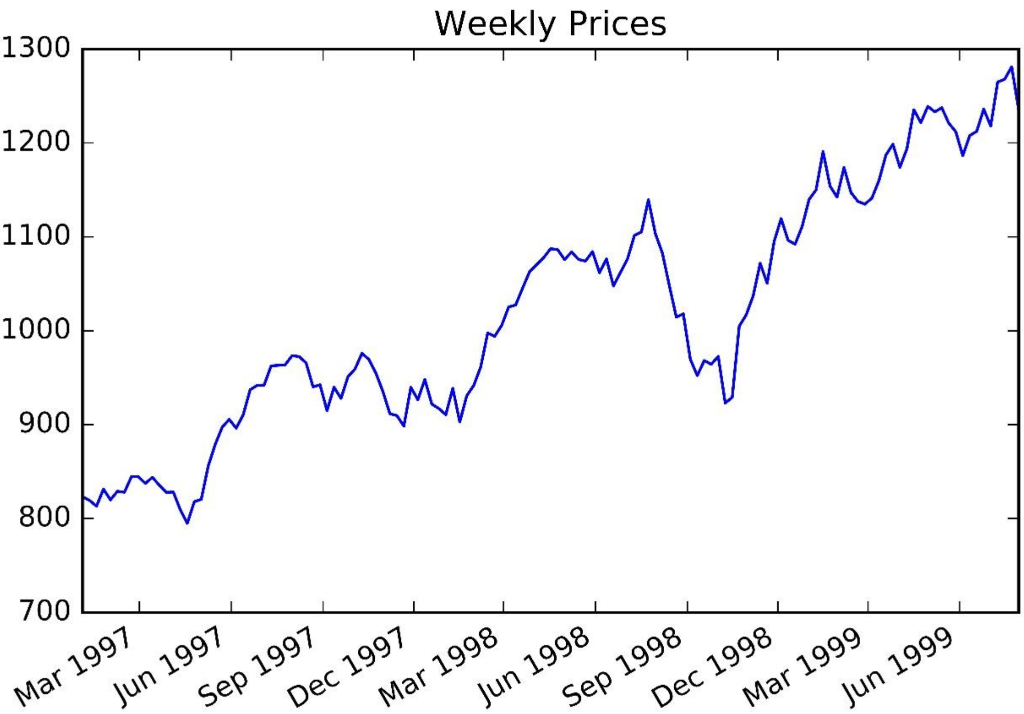


Fig. 1

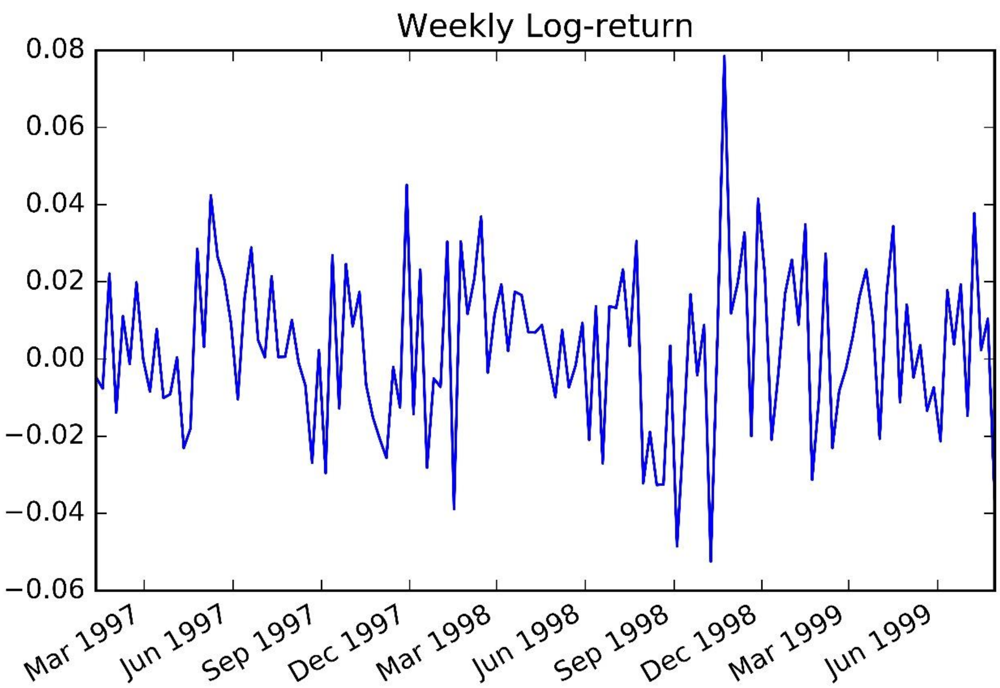


Fig. 2

The statistical hypothesis tests chosen for testing normality and independence are the Shapiro-Wilk test and the Ljung-Box test, respectively. The Shapiro–Wilk test follows the null hypothesis principle, which checks whether a sample comes from a normally distributed population while the Ljung-Box test checks if any of group of autocorrelations of a time series are different from zero. Technical details of these test are omitted given they are not our focus, however, what is important to know is that the null hypothesis of the Shapiro-Wilk test is that the population is normally distributed while the null hypothesis of the Ljung-Box test is that the data are independently distributed.

From January 1997 to July 1999 the Shapiro –Wilk test shows the normality of weekly returns at high significance levels for the World index.

Table 1 Test of normality of weekly returns: 1997–1999

Test statistic	p-value
0.9919	0. 6412

Moreover, the Ljung–Box test of autocorrelation shows a large degree of randomness in the weekly returns up to 24 lags (120 working days). Details of the test and the plot of autocorrelation function are shown in Table2 and Fig. 3.

Table 2 Test of autocorrelation of weekly returns: 1997–1999

Lags	p-value	Lags	p-value
1	0.5913	13	0.2645
2	0.1708	14	0.1614

3	0.3098	15	0.1938
4	0.4572	16	0.2400
5	0.5926	17	0.2906
6	0.6794	18	0.3460
7	0.1589	19	0.4072
8	0.2276	20	0.4678
9	0.2710	21	0.4514
10	0.3450	22	0.5096
11	0.4049	23	0.5103
12	0.2065	24	0.5661

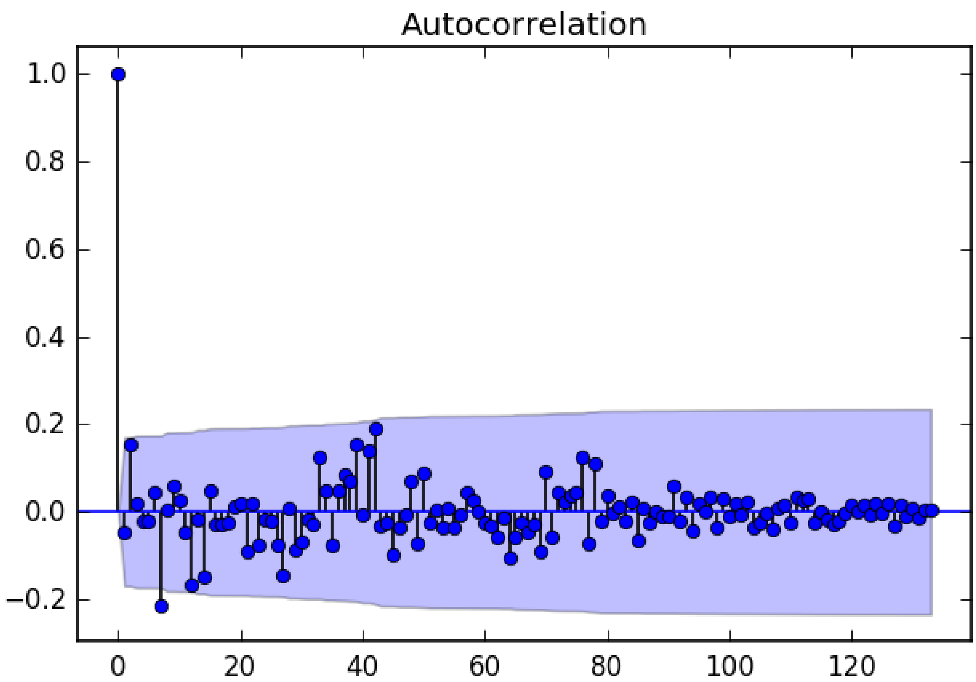


Fig. 3

In conclusion, we are confident in assuming normality and independence in our simulation exercise.

2.3.2 Simulation process

The simulation process is designed to generate as many different equity market scenarios as possible, in which we analyze the behavior of portfolio strategies. From the average returns standpoint, it is natural to identify negative, zero and positive market moments; from the risk perspective, the three typologies are that of low, medium and high volatility periods. Specifically, for market average returns we define the ranges (−30%, −5%); (−5%, +5%); (+5%, +30%). For market volatility we have the ranges (10%, 15%); (15%, 25%); (25%, 30%). A couple (average return, volatility) identifies a market scenario and it is used to generate a 5-year pseudo-time series of weekly equity returns. In particular, the Monte Carlo simulation process, with a time interval equal to 1 week (1/52 years), follows four steps:

a) From two uniform distribution $U(-30\%, +30\%)$ and $U(10\%, 30\%)$ an annual mean return μ and a volatility σ are generated (scenario);

b) Use Python's NumPy package to generate 360 weekly data from a normal distribution $N\left(\frac{\mu}{52}, \frac{\sigma}{\sqrt{52}}\right)$. Since the volatility for option-based strategy is estimated using the latest 100 observations, the purpose of using the extra 100 data is to guarantee the rolling window is available at the first simulation day.

c) The 8 strategies are implemented over $T = 260$ observations (5-year horizon) and summarized statistics are calculated in terms of performance and risk. Note that leverage has not been allowed, the interest rate for cash has been set to 3% per annum and portfolio insurance has a floor (strike price) equal to the initial portfolio value (minimum return equal to 0%);

d) A new run is repeated from step a) on, for a total of N = 10,000 scenarios.

The rebalance discipline adopted here is the weekly rebalance ($\Delta t = \frac{1}{52}$). Moreover, transaction costs are taken into account in two ways: in the form of a cost proportional to the value traded c = 0:3% and as a correction to the option volatility according to Leland (1985) formula:

$$\sigma_{adj}^2 = \sigma^2 \left(1 + c\,\frac{\sqrt{2/\pi}}{\sigma\sqrt{\Delta t}}\right).$$

In order to compare the performance of different strategies, we have assumed the same starting portfolio mix, given on average (over all scenarios) by a 50-50 stock-cash mix. For each scenario we have calculated 8 annualized risk and return measures: mean return net of transaction costs, standard deviation, asymmetry, kurtosis, downside deviation, Sharpe ratio, Sortino ratio and return at risk. Downside deviation is calculated as a shortfall risk, i.e. the risk of a result below the zero threshold return:

$$\text{Downside deviation} = \sqrt{\sum_{r_t<0} \frac{r_t^2}{T}}.$$

It is particularly useful in the case where we analyze the portfolio performance, given that the aim is to reshape the return distribution toward right-hand side asymmetry.

The Sharpe ratio is the portfolio excess return over the risk-free rate r_F, divided by portfolio standard deviation σ:

$$\text{Sharpe ratio} = \frac{\text{mean return} - r_F}{\sigma}.$$

Similarly, the Sortino ratio uses the threshold (=0) instead of the risk-free rate and downside deviation instead of volatility:

$$\text{Sortino ratio} = \frac{\text{mean return}}{\text{downside deviation}}.$$

Finally, note that return at risk, as Sharpe and Sortino ratios, is a risk-adjusted performance measure, defined as

$$\text{RaR} = \text{mean return} - \lambda \cdot \text{volatility}.$$

It resembles the classical mean-variance utility function with risk aversion parameter λ as well as the basic formula of the Value at Risk approach (Morgan, 1996) in relative terms, with λ reflecting the probability level. RaR can be defined as the minimum return with $(1-h)\%$ probability so that we have $P(r > \text{RaR}) = 1 - h$, in this case, h is set to be 1% and λ is therefore 2.33.

We have, finally, calculated the mean and the standard deviation of the 8 statistics over all scenarios.

As noted above, the trading strategies are expected to perform differently in each market situation. We therefore analyze the strategies in all scenarios with different market return (bull, bear, no-trend), ranging (-30%, -5%), (-5%, 5%), (5%, 30%) respectively, and market volatility (high, medium, low) ranging (10%, 15%), (15%, 25%), (25%, 30%), respectively.

2.4 Results

A preliminary historical simulation using the aforementioned stock market indexes for the period between 1997 and 1999 has been carried out with weekly rebalance. The result is given in Fig. 4

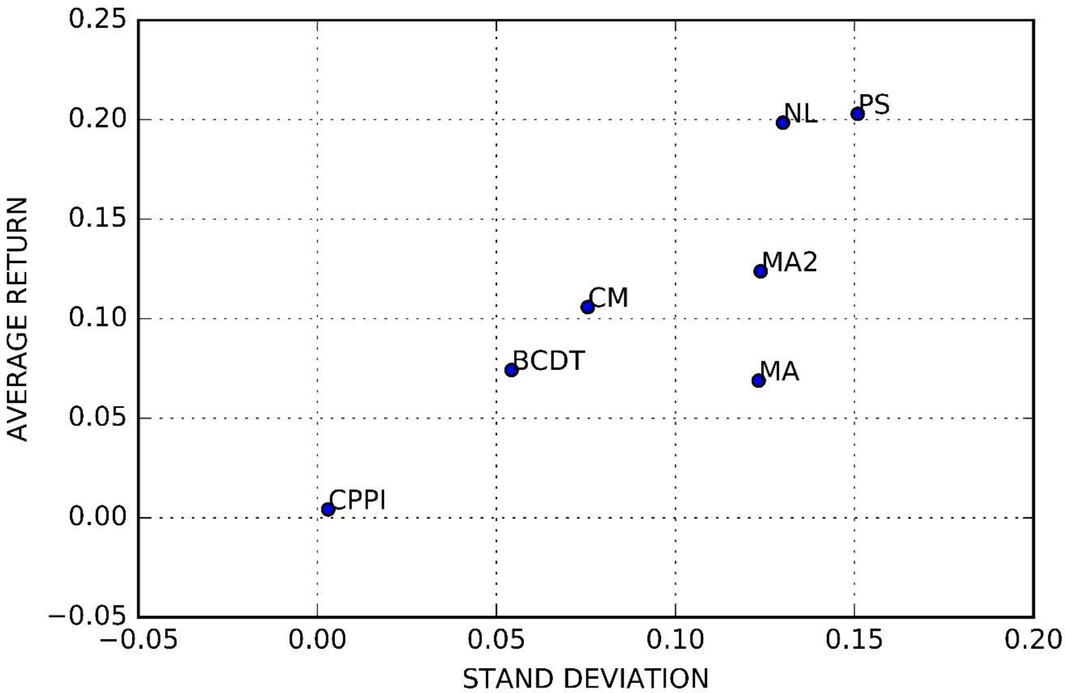


Fig. 4 Risk and return of trading strategies

The simulation shows that PS and NL strategies reach higher returns but also show greater standard deviations. CPPI strategy, vice versa, has low returns and low risk. With the exception of technical rules, the fundamental law of finance seems to be confirmed by the considered strategies applied to the World equity market.

Historical simulations, however, do not allow us to properly compare the considered strategies and test the significance of the performance differences. For this purpose, we run the Monte Carlo simulation process described in Section 3. Summarized statistics for the 8 strategies and the stock market index are reported in Figs. 5-8.

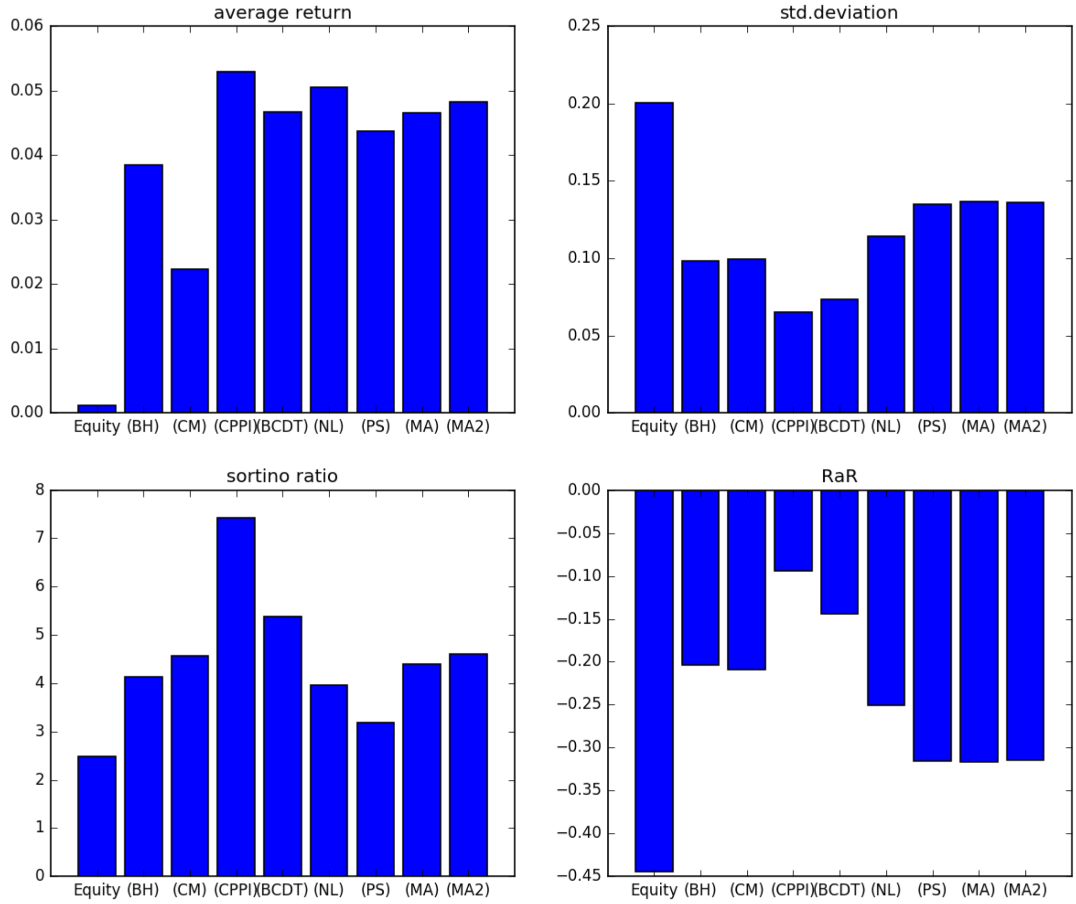


Fig. 5 Monte Carlo simulations: all markets

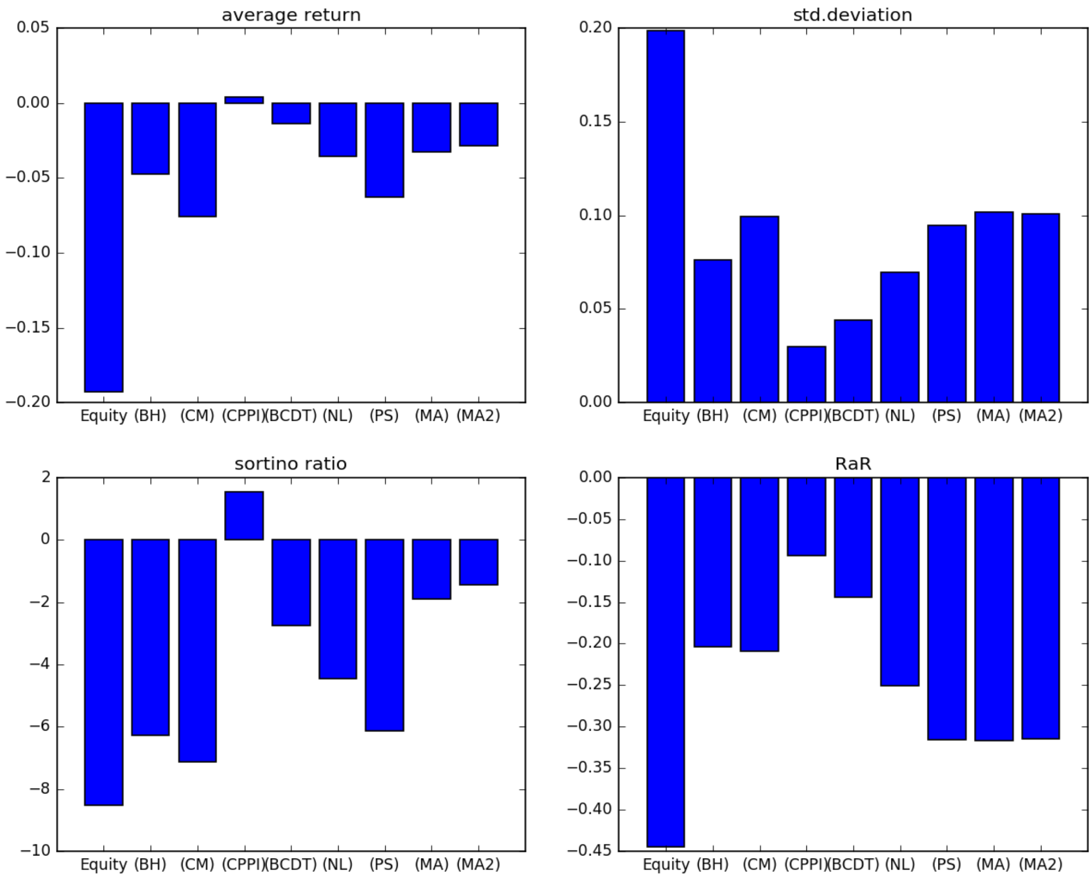


Fig. 6 Monte Carlo simulations: bear markets

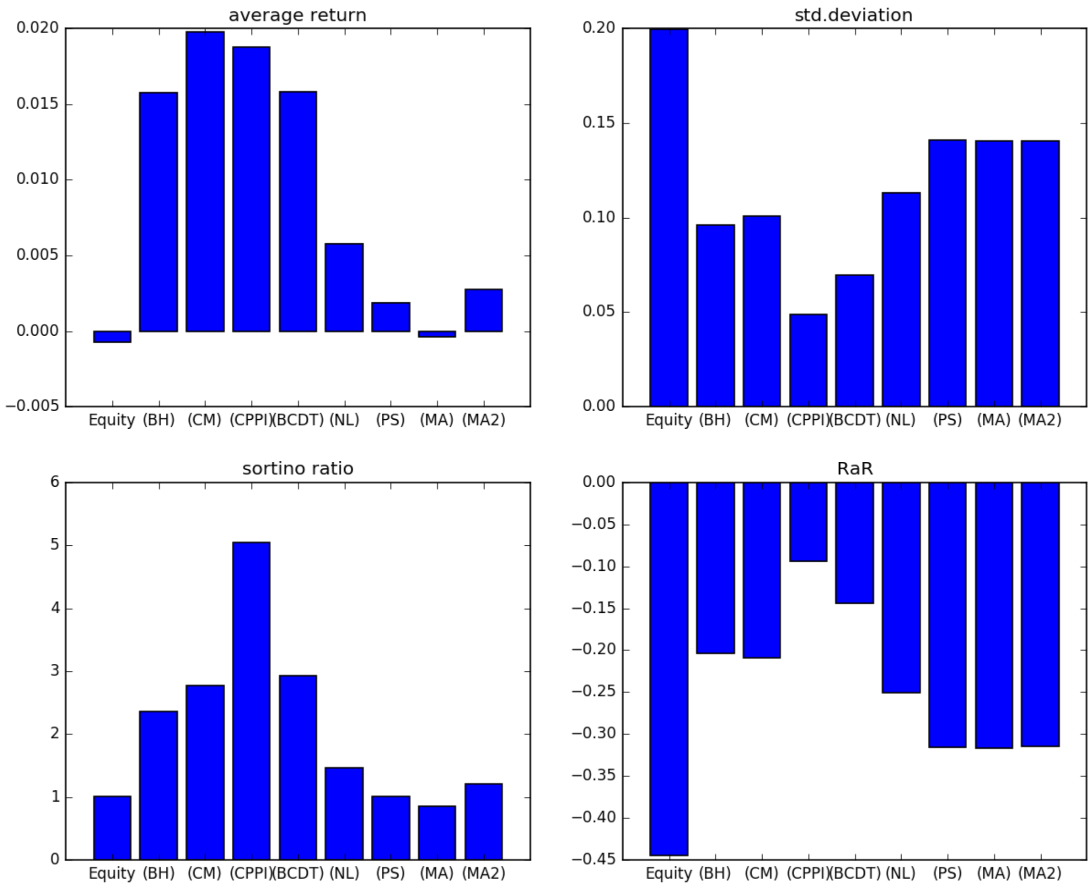


Fig. 7 Monte Carlo simulations: no trend markets

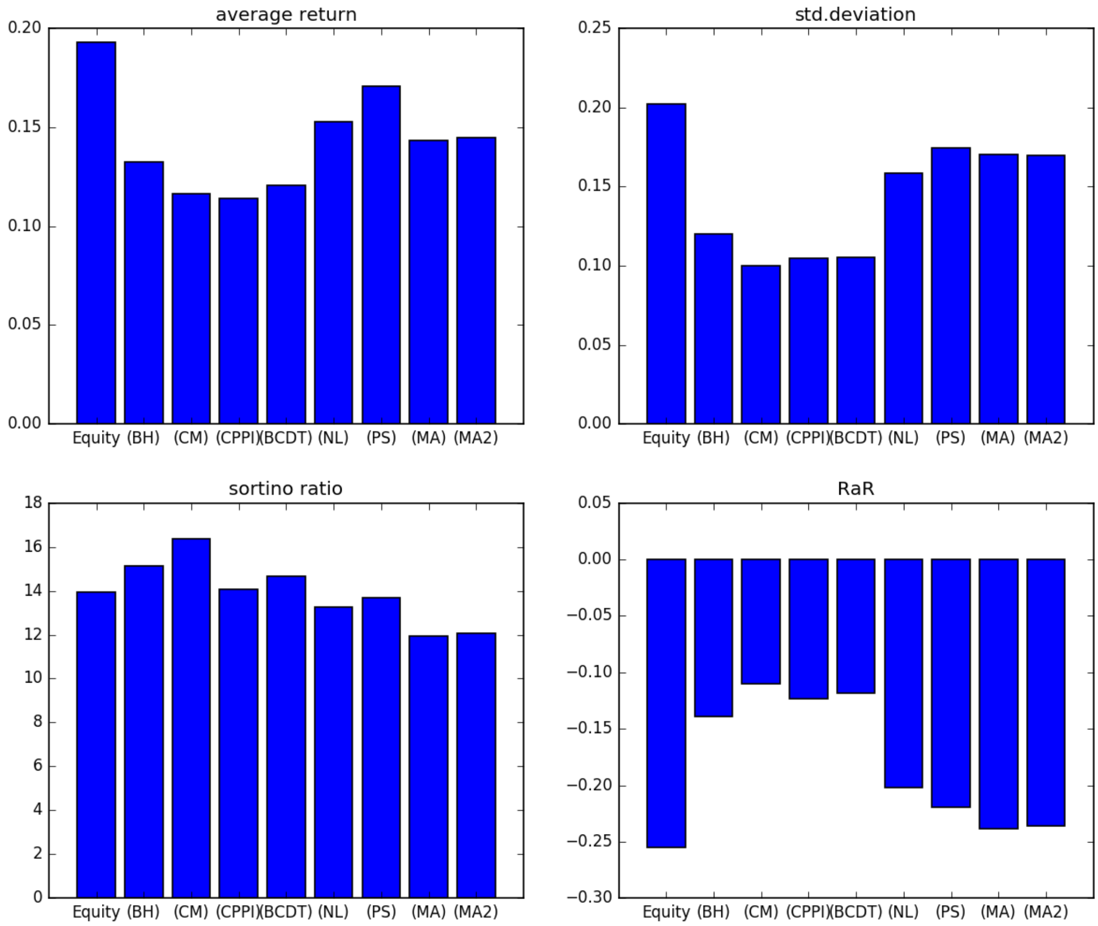


Fig. 8 Monte Carlo simulations: bull markets

Possible return curves under each market scenarios are depicted in Figs. 9-11

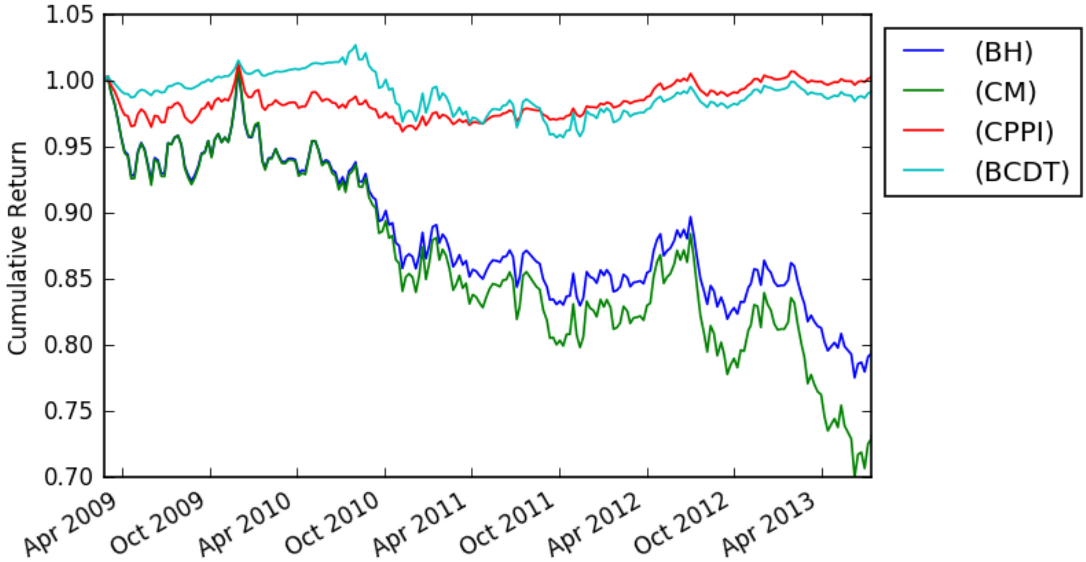


Fig. 9 Cumulative returns under bear markets

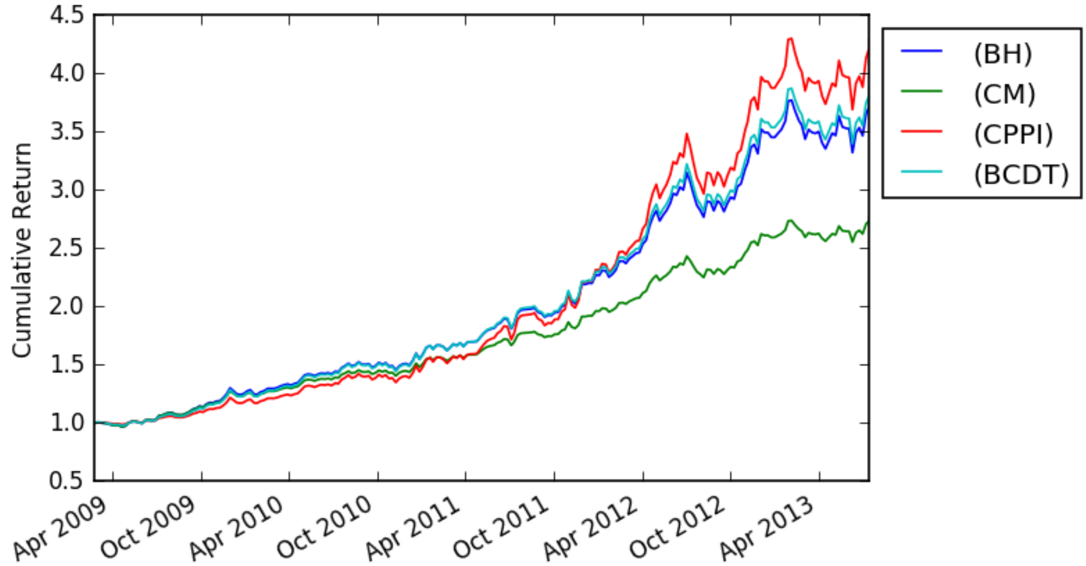


Fig. 10 Cumulative returns under bull markets

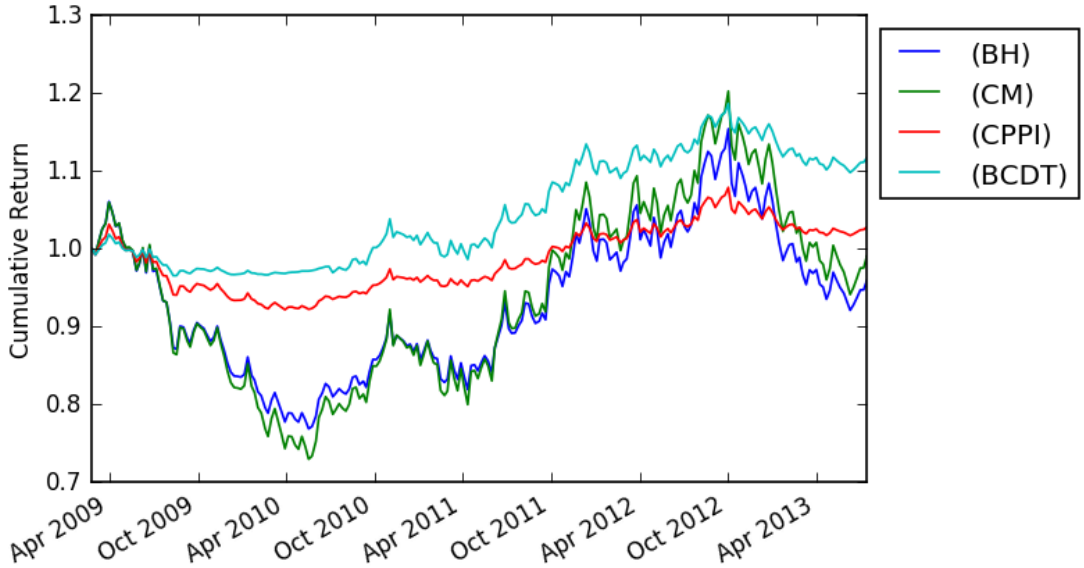


Fig. 11 Cumulative returns under no-trend markets

2.5 Conclusions

The main findings, with weekly rebalance, are concluded below:

a) As expected, the performance of the strategies is dependent on the market phase, bear market, no-trend and bull market, defined according to the level of the average rate of return. In a bear market as well as in a market without trend CPPI strategies appear to be the best choice, both in terms of Sortino ratio and RaR. In a market with no trend but high volatility CM strategy (benchmarking) is preferable, according to Sortino ratio. CM is also recommended in bull markets (Table 4).

b) If the market situation is unknown (uniform prior probability for bear, no-trend and bull market) CPPI and BCDT are the best performing strategies.

c) The best strategy is almost independent of the volatility level prevailing in the market, at least in the considered range 10–30% suggested by the historical data. This is especially true using the RaR criterion.

d) In contrast with the historical simulation, in the Monte Carlo experiment, the best strategy is largely independent of the adopted performance measure.

3. Further Research

In the original paper, the author claims that CPPI is dominated in both bear and no trend markets while CM is preferable in the bull markets in terms of Sortino ratio, which we already verified.

3.1 Jump diffusion model

Notice that in the replication process we stuck to the author's assumption that the index prices follows a geometric Brownian motion, which implies the continuity nature of the index evolution. However, in the real markets, as shown in Fig. 12, there are often jumps during some periods, leading to discontinuities where regular Brownian motion model is no longer applicable. This may be due to some macroeconomic factors or crisis, which cannot be depicted by the original model. Therefore, the original conclusions are not robust under real markets conditions.

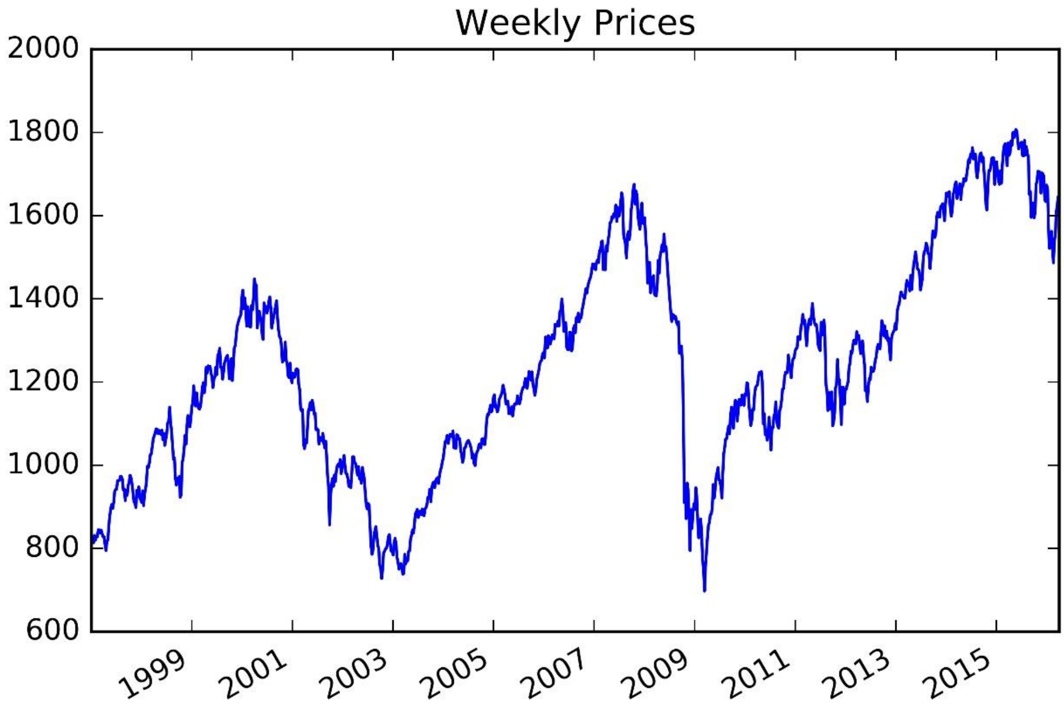


Fig. 12

It’s natural to incorporate the jump diffusion into the index price movement, and this is where the jump diffusion model, first introduced by Robert C. Merton (1976), comes into play. The jump diffusion model uses Poisson process to depict the jump events, and mathematically it can be written as follows

$$S_t = S_0 exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t + \sum_{i=1}^{N_t} Y_i t\right),$$

where N_t is a Poisson process and Y_i is the size of jumps on an annual basis.

3.2 Results

Since we already confirmed the benchmarking strategy is less attractive than CPPI under bear markets and preferable under bull markets, it’s tempting to investigate whether it’s still preferable to CPPI under bull markets with the existence of jumps. We don't investigate the bear markets with jumps because of the insurance nature of the CPPI, which makes it immune to any kinds of downward trend.

Specifically, let $\mu \sim U(0.3,0.5)$, $\sigma \sim U(0.1,0.3)$, $\lambda \sim U(3,5)$, where λ is the expected number of jumps during one year. Moreover, $Y_i \sim N(-4,1.5)$, which is the jump size on the annual basis. The simulation results are depicted in Fig. 13.

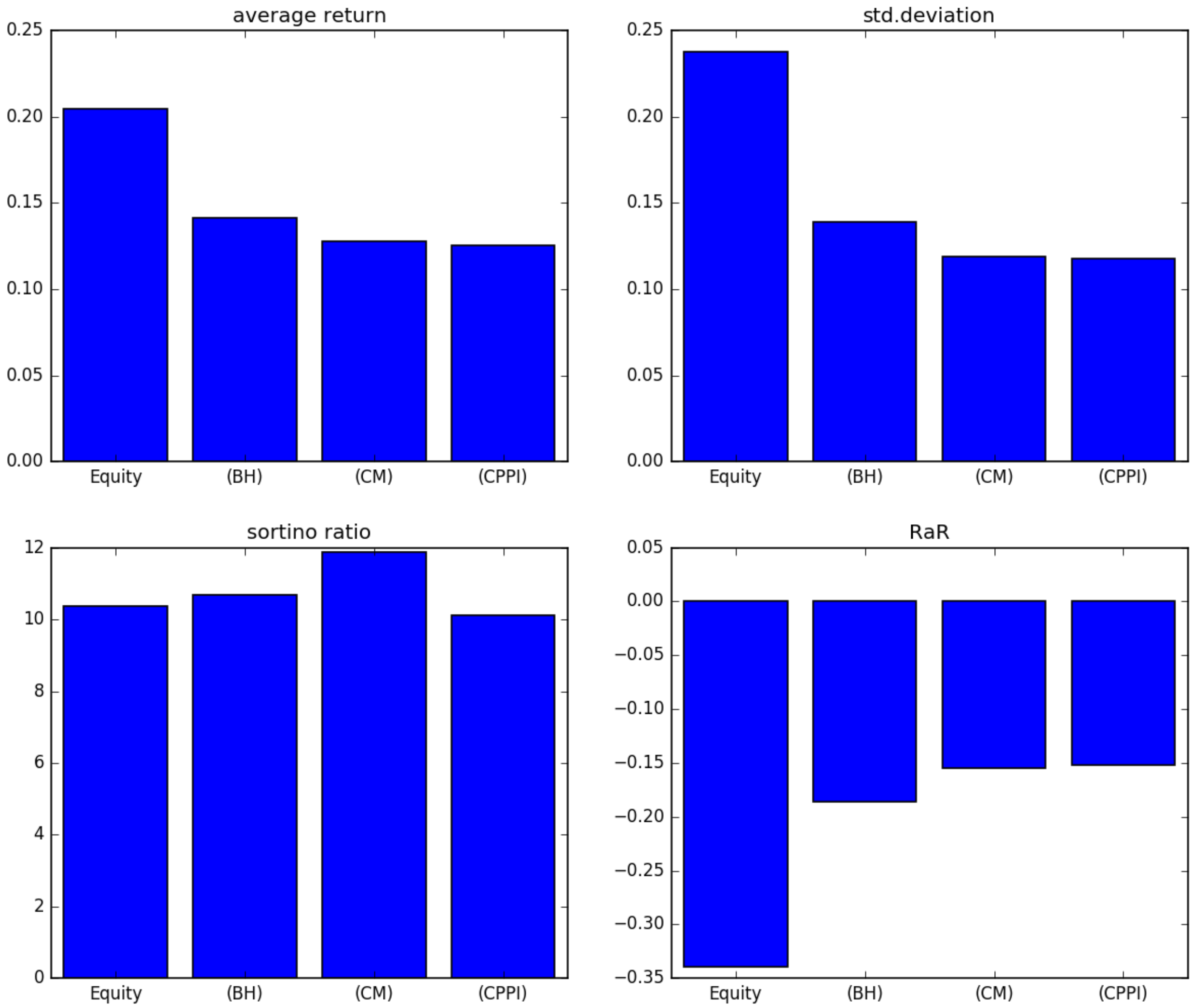
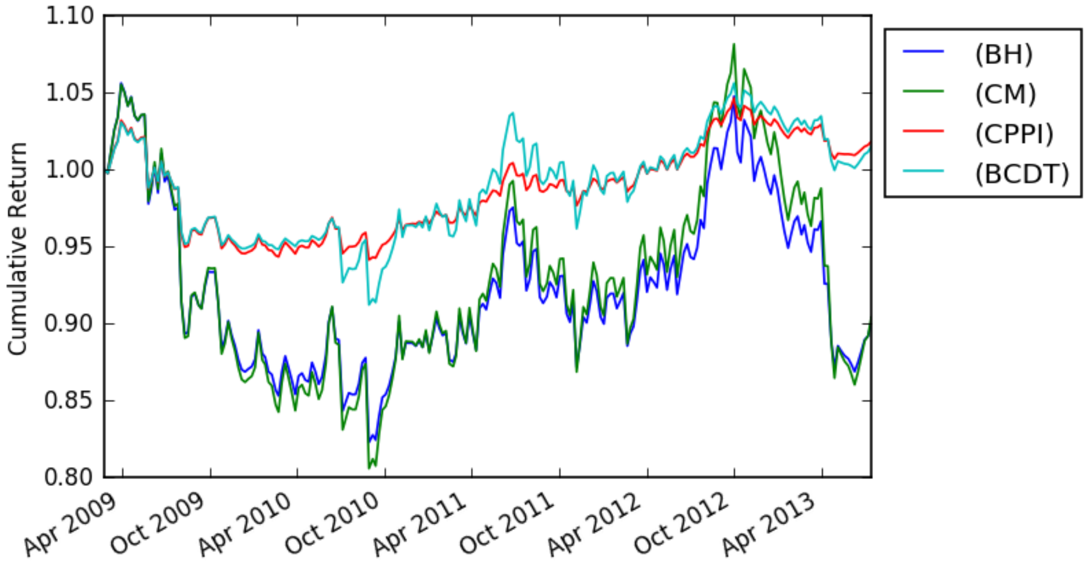


Fig. 13

Remember in the previous plot, CM strategy outperformed CPPI in the bull market with respect to every performance measure. However, after incorporating the effect of the jump diffusion, the performance of CM strategy is almost identical to the performance of CPPI as shown in Fig. 10 except that the Sortino ratio of CM is still higher than CPPI by a small amount. Therefore, we can conclude that CPPI strategy is also a relatively robust strategy under bull markets for risk-averse investors. Possible return curves are depicted in Fig. 14.



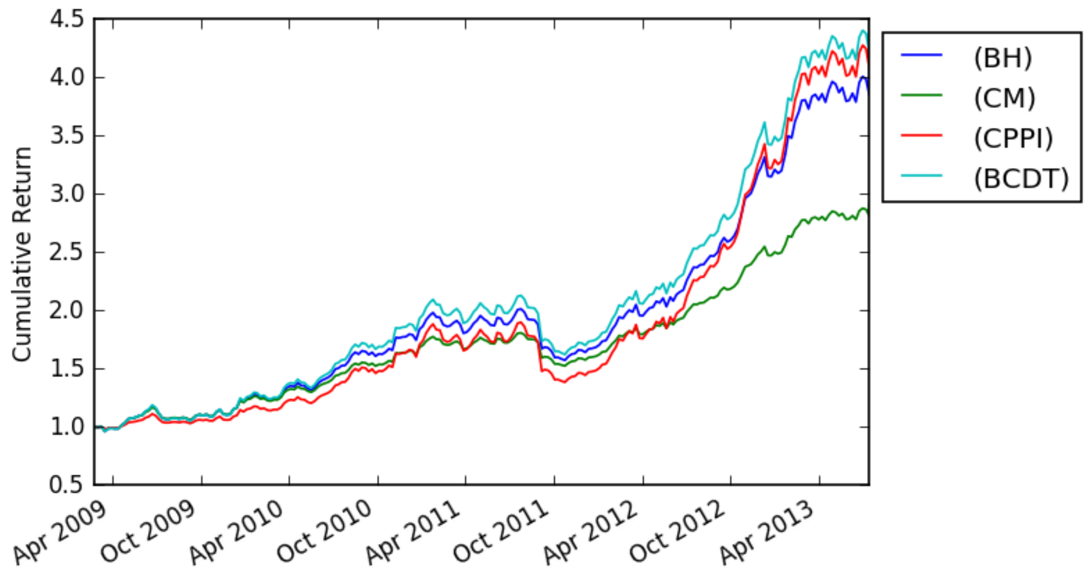


Fig. 14 Cumulative returns under bull markets with jumps

4. Conclusions

In this report, we have considered a typical problem in the practice of asset management. The asset manager, after agreeing upon an investor's strategic asset allocation (strategic benchmark), has to implement a tactical strategy in order to get the best performance results for the managed portfolio. A number of different alternatives are available: benchmarking, portfolio insurance and technical analysis are certainly the most popular ones. We followed the procedures adopted by Riccardo Cesari and David Cremonini (2002) to consider 8 dynamic strategies, 2 of benchmarking type, 4 of portfolio insurance and 2 of technical analysis. They have been analyzed and compared both in an historical simulation exercise and in a Monte Carlo experiment, taking into account discrete rebalancing and transaction costs. Preliminary tests of normality and autocorrelation of returns have been carried out in order to verify whether the simulation setup can be approximately compared with the data generating mechanism of real situations. Using data from the major world stock markets since 1997 the null hypotheses are rejected in the case of daily data but not in the case of weekly returns. The strategies are compared in terms of different risk, return and risk-adjusted performance measures (Sharpe ratio, Sortino ratio and return at risk).

Both the historical and the Monte Carlo simulations show that no strategy is dominant in all market situations. The Monte Carlo experiment, however, allows us to discriminate between the considered strategies according to the prevailing market phase and to test the statistical significance of the performance differences. The results show a dominant role of constant proportion portfolio insurance strategy (CPPI) in bear and no-trend markets and a preference for a constant mix strategy (CM) in bull markets. In the case of total ignorance about the market phase (uniform prior distribution for bear, no-trend and bull hypotheses) CPPI and BCDT are the best strategies. Moreover, these results seem to be independent of the volatility prevailing in the market and the risk-adjusted performance measure adopted.

We further investigate in impact of unexpected market jumps in the bull market on the performances of the benchmarking and portfolio insurance strategies. The results imply almost identical performances between each strategy. Overall, CPPI can be considered a safe investment strategy and it might be more appealing to the risk-averse investors.

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6. Appendix

The Monte Carlo routine is shown as follows and the full Python implementation can be accessed at https://github.com/xybhust/asset_allocation.

```
import pprint
import Queue
import datetime
import time
import cPickle
import matplotlib.pyplot as plt
import pandas_datareader.data as web


from data_handler import YahooDataHandler, MonteCarloDataHandler, CSVDataHandler
from position_handler import PositionHandler
from order_handler import OrderHandler
from strategy.strategy import BuyHold, ConstantMix, ConstantProportion, \
                              BCDT, NL, PS, MovingAverage, MovingAverage2
from performance import *


class Backtest(object):
    """
    Encapsulates the settings and components for carrying out
    an event-driven backtest.
    """
    def __init__(
            self, symbols, initial_capital,
            start, end, data_handler_cls,
            position_handler_cls, order_handler_cls, strategy_cls, interval, index=None
        ):
        """
        Initialises the backtest.
        Parameters:

        symbols - The list of symbol strings.
        intial_capital - The starting capital for the portfolio.
        start - The start datetime of the strategy.
```

```
        end - The end datetime of the strategy.
        data_handler_cls - (Class) Handles the market data feed.
        order_handler_cls - (Class) Keeps track of portfolio current
                            and prior positions.
        strategy_cls - (Class) Generates signals based on market data.
        """
        self.symbols = symbols
        self.start = start
        self.end = end
        self.events = Queue.Queue()
        self.interval = interval

        self.data_handler = data_handler_cls(self.events, symbols, start, end,
                                             interval, index)

        self.position_handler = position_handler_cls(self.data_handler,
                                                     initial_capital)

        self.order_handler = order_handler_cls(self.events)

        self.strategy = strategy_cls(self.data_handler,
                                     self.position_handler,
                                     self.events)

        self.orders = 0


    def simulate_trading(self):
        """
        Executes the backtest.
        """
        while True:
            # Update the market bars
            if self.data_handler.continue_backtest == True:
                self.data_handler.update_bars() # Produce market event
            else:
                break

            # Handle the events
            # If events queue is not empty, do the loop
            while True:
                try:
                    event = self.events.get(False)
                except Queue.Empty:
```

```
                    break
                else:
                    if event is not None:
                        if event.type == 'MARKET':
                            # Produce signal event
                            if self.data_handler.elapsed >= 30:
                                self.position_handler.update_from_new(event)
                                self.strategy.generate_signals(event)
                        elif event.type == 'SIGNAL':
                            # Produce order
                            self.order_handler.generate_order(event)
                        elif event.type == 'ORDER':
                            self.orders += 1
                            # Produce fill
                            self.position_handler.update_from_order(event)

        position_df = pd.DataFrame(
            self.position_handler.historical_position
            )
        position_df.set_index('datetime', inplace=True)
        position_df['returns'] = position_df['total'].pct_change()
        # Cumulative production, first element NaN
        position_df['cumulative_returns'] = \
            (1.0 + position_df['returns']).cumprod()
        position_df = position_df.dropna()
        position_df = position_df.loc[:, ('returns', 'cumulative_returns')]

        # The following only valid for Monte Carlo datahandler
        self.record = {'mu': self.data_handler.mu[0], # markeet
                       'sigma': self.data_handler.sigma[0], # market
                       'performance': position_df,
                       'orders': self.orders,
                       'interval': self.interval
                       }


#        with open('%s results.pkl' % self.strategy.id, 'wb') as f:
#            cPickle.dump(self.record, f)
#            print('Simulation results have been pickled into file')


if __name__ == "__main__":
    # Get the time index
```

```
    price_df = web.get_data_yahoo(symbols[0], start, end, interval='w')
    assert len(price_df.index) >= 260
    index = price_df.index[:260]
#
#     # Strategy list
    strategy = [BuyHold, ConstantMix, ConstantProportion,
                BCDT, NL, PS, MovingAverage, MovingAverage2]
    ##############################################
    # Loop over all strategies, each 10000 times #
    ##############################################
    for s in strategy:
        for i in range(0, 10000):
            backtest = Backtest(
                    symbols, initial_capital, start,
                    end, MonteCarloDataHandler, PositionHandler,
                    OrderHandler, s, 'w', index
                    )
            backtest.simulate_trading()
            p = backtest.record
            repo = backtest.strategy.id.strip().split(' ')[-1]
            with open('%s/%d.pkl' % (repo, i+1), 'wb') as f:
                cPickle.dump(backtest.record, f)
            print('Simulation %d finished' % (i+1,))
            #output_performance('%s/%d.pkl' % (repo, i+1), repo)
```