EECS 349: Machine Learning

Bryan Pardo

Topic: Concept Learning

Concept Learning

- Much of learning involves acquiring general concepts from specific training examples
- *Concept*: subset of objects from some space
- Concept learning: Defining a function that specifies which elements are in the concept set.

A Concept

- Let there be a set of unique instances ***X.***

  ***X* = {**duck, rabbit, chicken, pig, planet, bag-of-money, cow, dog**}**

- Let there be a set of unique labels ***L***

  ***L* = {0, 1}**

- *A concept* **C** is…

  A subset of ***X***

  e.g. **C = mammals= {**rabbit, pig, cow, dog**}**

  A function that returns ***1*** (or "true") only for elements in the concept

  e.g. **C**(rabbit) = 1, **C**(duck) = 0

Some more definitions

Call the set of unique examples ***X*** the *instance space*.

The set of unique labels ***L*** is the *label space.*

A hypothesis ***h***(***x***) is, like a concept ***c***(***x***), a function whose domain is ***X*** and whose range is ***L***.

The set of unique concepts is the *concept space* ***C.***

The set of unique hypotheses a learner will consider is called the *hypothesis space* ***H***.

It is usually not true that ***H=C*** (we'll see why in a moment)

Concept Learning Task

GIVEN:

- A space of instances X
- Target concept function c:
  E.g., Mammal: X → {0,1}
- Hypothesis space H
- Training data D
  positive and negative examples: $<x_1,c(x_1)>,\ldots, <x_n,c(x_n)>$

FIND:

- A hypothesis h in H such that h(x)=c(x) for all x in D.

Telling hypotheses/concepts apart

- **Definition:** Two functions $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$ are *distinguishable*, given the data $\boldsymbol{D}$, if they differ in their labeling of at least one of the examples in $\boldsymbol{D}$.

- **Definition:** A **set** of hypotheses is distinguishable, given $\boldsymbol{D}$, iff ALL pairs of hypotheses in the set are distinguishable given $\boldsymbol{D}$. Call $\boldsymbol{H_D}$ a largest set of distinguishable hypotheses, given $\boldsymbol{D}$.

Inductive Learning Hypothesis

- Any hypothesis found to approximate the target function well over the training examples, will also approximate the target function well over the unobserved examples.
- This might not be true. When it it isn’t the hypothesis does not generalize well.
- In fact, the target concept may not even be in the hypothesis space.
- …but maybe we can find a hypothesis that is good enough for our purposes

Version Spaces

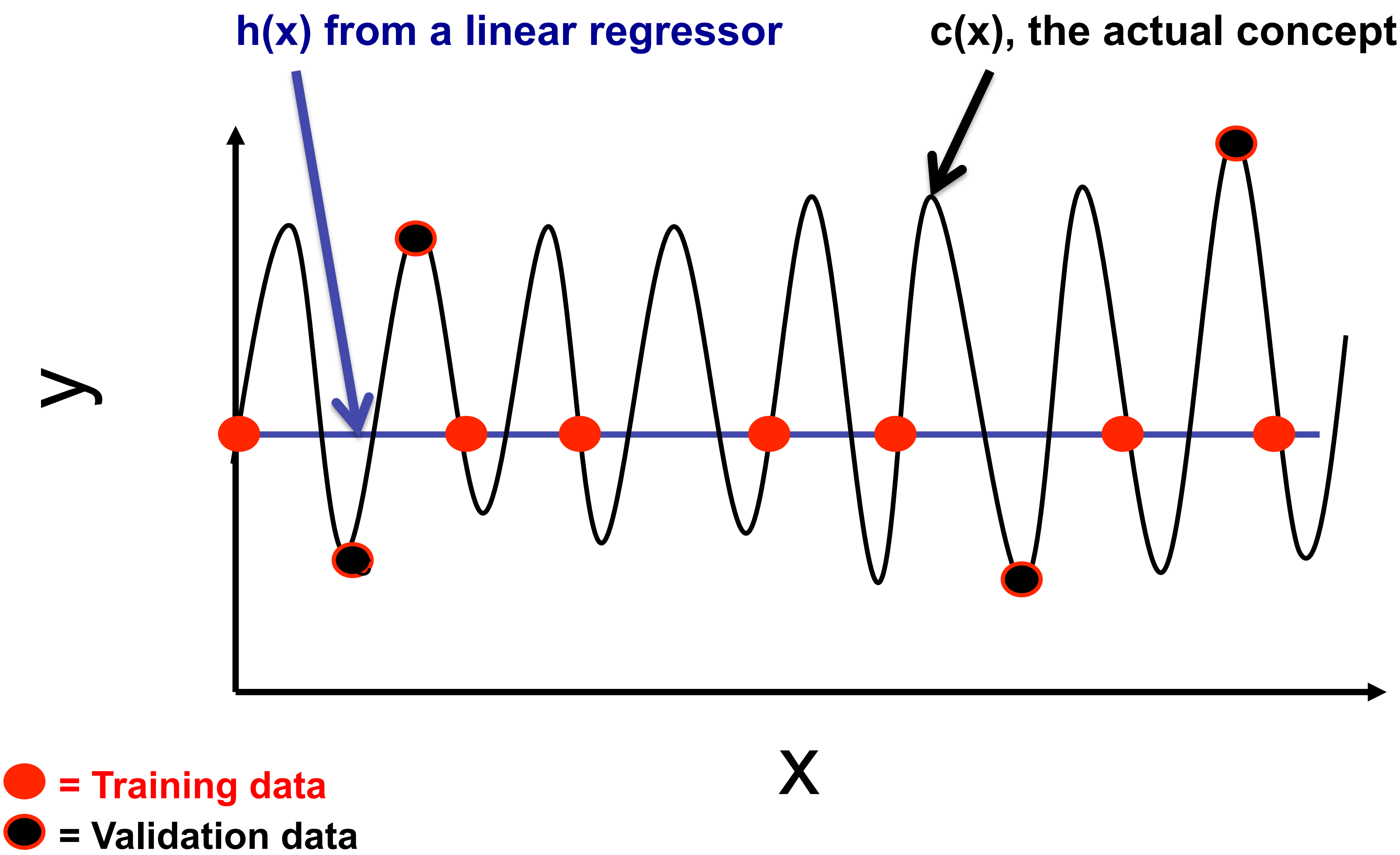
- Hypothesis **h** is ***consistent*** with a set of training examples **D** of the target concept **c** iff **h(x)=c(x)** for each training example **<x,c(x)>** in **D**.

$$Consistent(h,D) \equiv \left(\forall \langle x, c(x)\rangle \in D\right) h(x) = c(x)$$

- A **version space** : all the hypotheses that are consistent with the training examples.

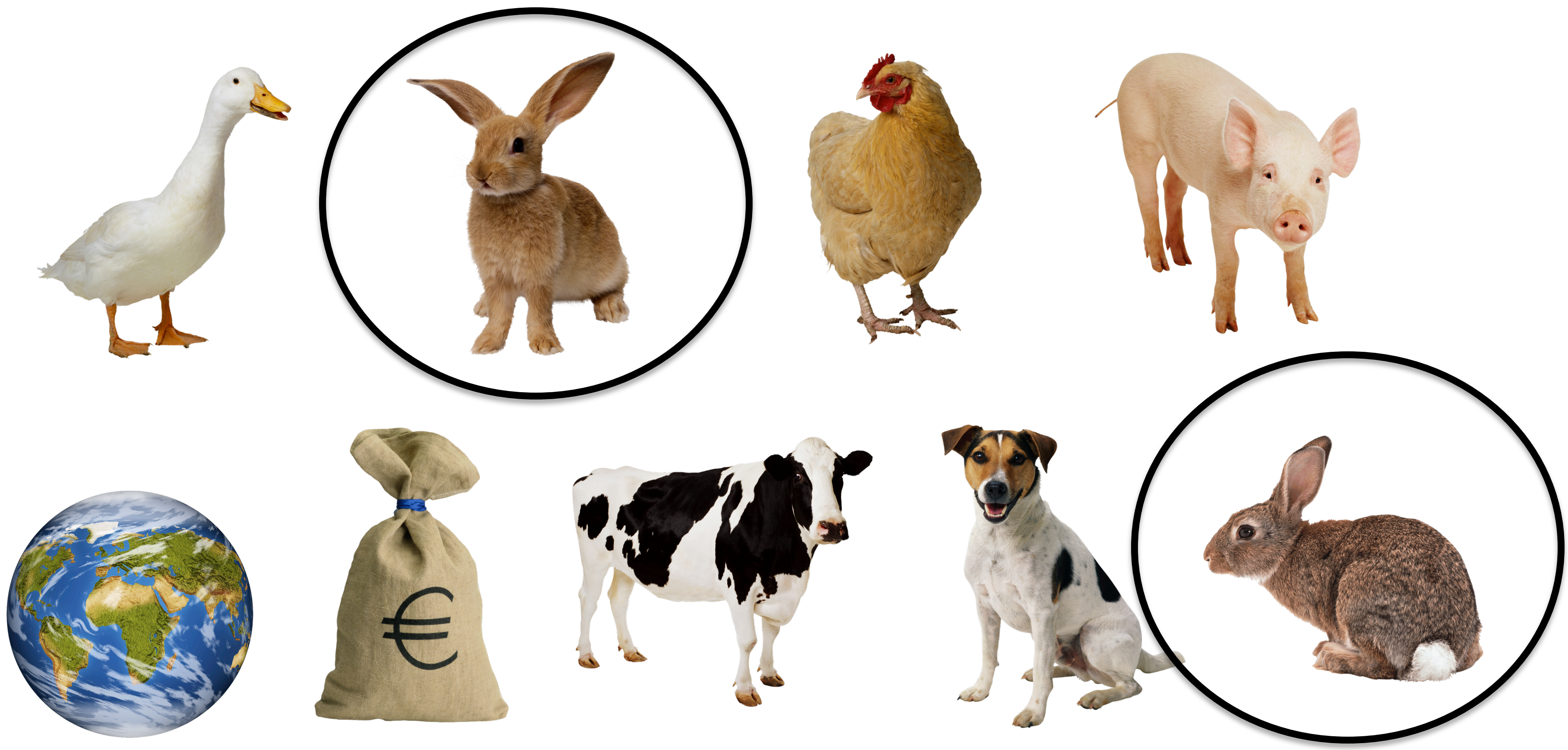
$$VS_{H,D} \equiv \{h \in H \mid Consistent(h,D)\}$$

A visualization



Encoding Matters: Image file

As images, the two rabbits are unique instances



Encoding Matters: Feature vector

- This feature encoding makes the rabbits identical to each other…

Number of Feet	Fur	Size	Has wings	Warm Blood
2	No	S	Yes	Yes
4	Yes	S	No	Yes
2	No	S	Yes	Yes
4	No	M	No	Yes
0	No	XXL	No	No
0	No	M	No	No
4	Yes	L	No	Yes
4	Yes	S	No	Yes
4	Yes	S	No	Yes

...and to the dog

Moral: pick the right encoding!

How many unique instances?

Number of Feet	Fur	Size	Has wings	Warm Blood
Integers 0 to 99	Yes,No	S,M,L,XL,XXL	Yes,No	Yes,No

100 * 2 * 5 * 2 * 2 = 4000 instances

How many unique concepts?

Number of Feet	Fur	Size	Has wings	Warm Blood
Integers 0 to 99	Yes,No	S,M,L,XL,XXL	Yes,No	Yes,No

100 * 2 * 5 * 2 * 2 = 4000 instances

2^4000 concepts

How big is the version space for this data?

Number of Feet	Fur	Size	Has wings	Warm Blood	C(x)
2	No	S	Yes	Yes	0
4	Yes	S	No	Yes	1
2	No	S	Yes	Yes	0
4	No	M	No	Yes	1
0	No	XXL	No	Yes	0
0	No	M	No	No	0
4	Yes	L	No	Yes	1
4	Yes	S	No	Yes	1
4	Yes	S	No	Yes	1

TRICK QUESTION! Until you know what set of hypothesis a learner can consider, you can't say how big the version space is.

Example: MC2 Hypothesis Space

- MC2 (Mitchell, Chapter 2) hypothesis space

  Hypothesis *h* is a conjunction of constraints on attributes

- Each constraint can be:

  A specific value : e.g. *Number of Feet = 4*

  A don't care value : e.g. *Fur = ?*

  No value allowed: e.g. *Size =Ø*

- Instances x that satisfy *h* have h(x) = 1, else h(x) = 0
- Example hypotheses:

Number of Feet	Fur	Size	Has wings	Warm Blood
4	?	?	?	?
?	Yes	?	?	?
2	Ø	S	Yes	Yes

How many unique hypotheses?

- Given this encoding of hypotheses how many hypotheses are possible?

Number of Feet	Fur	Size	Has wings	Warm Blood
Integers 0 to 99	Yes,No	S,M,L,XL,XXL	Yes,No	Yes,No

(102) * 4 * 7 * 4 * 4 = 45,696 hypotheses

Compare that to 2^4000 concepts

Questions

- Does the MC2 hypothesis space contain the concept "has either 2 feet or 4 feet"?

不能表示 有值的子集的属性

- NO! It cannot represent concepts that accept subsets of values in an attribute

- Why would we use such a limited hypothesis space?

Now how big is the version space?

Number of Feet	Fur	Size	Has wings	Warm Blood	C(x)
2	No	S	Yes	Yes	0
4	Yes	S	No	Yes	1
2	No	S	Yes	Yes	0
4	No	M	No	Yes	1
0	No	XXL	No	Yes	0
0	No	M	No	No	0
4	Yes	L	No	Yes	1
4	Yes	S	No	Yes	1
4	Yes	S	No	Yes	1

Why not just use the concept space?

How big is the number 2^4000?

线性回归

- Many approaches (e.g. Linear regression) limit the range of possible hypotheses.
- If you know something about the structure of a problem, you can limit the set of hypotheses you consider to be some tractable subset.
- Of course, if you're wrong about the structure….

Deductive reasoning 演绎推理

- Tries to show a conclusion MUST follow from a set of premises 假定 (axioms 公理)
- What we typically think of as “Logic”

  (1st order, 2nd order, etc.)
- Covered in EECS 348.
- Example

  All men are mortal.

  Socrates is a man.

  Therefore, Socrates is mortal.

归纳推理

Inductive reasoning

- The premises of an inductive argument indicate support (often probabilistic support) but do not ensure the conclusions are true.
- Example
  - 93% of students are right-handed.
  - Will is a student.
  - Therefore, Will is right-handed.

偏差

Inductive Bias

- NOT the same as bias in a statistical estimator

- DEFINITION: The set of axioms that would need to be added to the knowledge of the system so that a deductive reasoner would make the same inference as the inductive reasoner.

  – Example: Will does whatever the majority does.

Unbiased Learner

- Idea: Choose H that expresses every teachable concept, that means H is the set of all possible subsets of X
- $|X|=96$, therefore $|H|=2^{96} \sim 10^{28}$ concepts
- H surely contains the target concept
- But there are too many concepts to pick them randomly to try
- Why not try them in some helpful order?

Unbiased Learner

Assume positive examples (x_1, x_2, x_3) and negative examples (x_4, x_5)

How would we classify some new instance x_6?

For any instance not in the training examples
half of the version space says +
the other half says –

* To learn the target concept one would have to present *every* single instance in X as a training example (Rote learning)

What kinds of biases are there?

- Choice of data set

  e.g. Training an image classifier on photos from a foodie website means it won't work well on car photos
- Data representation

  How you code & represent the data has huge impact
- Hypothesis space

  e.g. Linear regression only does straight lines and can't fit a curve
- Order in which we select hypotheses to test

  If your hypothesis space has 10^10 hypotheses, you can't try them all
- Choice of performance measure

  Mean squared error? Maximum Margin? It makes a big difference

Summary

- Concept learning can be thought of as search through a space of hpyotheses to find one (or more) that match the data.

- An unbiased learner cannot make inductive leaps to classify unseen examples.

- Inductive learning algorithms can classify unseen examples only because of inductive bias

- There are biases in the learning algorithm, data representation, hypothesis space