L6-2: GAN and distribution divergence

Generative adversarial network (GAN)

The GAN loss, Wasserstein GAN, f-GAN (f-divergence)

- The standard GAN loss recovers JSD assuming global optimal
- General min-max formula for f-GAN

- GAN loss

$$\min_{G} \max_{D} V(G,D) = \mathbb{E}_{x \sim p_{data}} \log D(x) + \mathbb{E}_{z \sim p_z} \log(1 - D(G(z)))$$

let $p = p_{data}$, $q = G \# p_z = p_{model}$

$$\mathbb{E}_{x \sim p} \log D(x) + \mathbb{E}_{x \sim q} \log(1 - D(x)) \quad \text{logistic loss.}$$

$$L[D] = \int_{\mathcal{X}} p \log D + \int_{\mathcal{X}} q \log(1-D)$$

$$\frac{\delta L}{\delta D(x)} = p(x) \frac{1}{D(x)} + q(x) \frac{-1}{1-D(x)} = 0$$

$$\Rightarrow \frac{q}{1-D} = \frac{p}{D} \Rightarrow \frac{p}{q} = \frac{D}{1-D}$$

$$\Rightarrow D^* = \frac{p}{p+q}$$

$$\Rightarrow L[D^*] = \int_{\mathcal{X}} p \log \frac{p}{p+q} + \int_{\mathcal{X}} q \log \frac{q}{p+q}$$

$$\frac{1}{2} \left(\int p \log \frac{p}{p+q} + \int q \log \frac{q}{p+q} \right) = JSD(p, q)$$

Thus, $L[D^*]$ equals $JSD(p, q)$ up to a const.