L4 notes: Example comparison of GAD, PPM, EGM

$$\min_x \max_y L(x,y) = xy$$

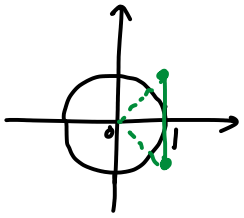
- GAD

$$\nabla L(x,y) = \begin{bmatrix} \partial_x L \\ -\partial_y L \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix}$$

$$\begin{cases} x_{k+1} = x_k - \eta y_k \\ y_{k+1} = y_k + \eta x_k \end{cases}$$

$$z_{k+1} = \begin{bmatrix} 1 & -\eta \\ \eta & 1 \end{bmatrix} z_k,$$

$$\lambda = 1 \pm i\eta$$



rotate and unstable

- PPM

$$L_S(x,y) = \min_u \max_v uv + \frac{1}{2s}\|u-x\|^2 - \frac{1}{2s}\|v-y\|^2$$

$$\begin{cases} v + \frac{1}{s}(u-x) = 0 \\ u - \frac{1}{s}(v-y) = 0 \end{cases} \Rightarrow \begin{bmatrix} 1 & s \\ s & -1 \end{bmatrix} \begin{bmatrix} u^* \\ v^* \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

$$\begin{bmatrix} x_+ \\ y_+ \end{bmatrix} = \underbrace{\frac{1}{1+s^2}\begin{bmatrix} 1 & s \\ -s & 1 \end{bmatrix}}_{\lambda = \frac{1 \pm is}{1+s^2},\ |\lambda| = \frac{1}{\sqrt{1+s^2}} < 1} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\Rightarrow$ converge to $(0,0)$

- EGM

$$\begin{cases} z' = z_k - \eta \bar{\partial} L(z_k) \\ \hat{z} = z_k - \eta \bar{\partial} L(z') \quad (\text{or } z_k) \\ z_{k+1} = (1-\lambda) z_k + \lambda \hat{z} \end{cases}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -\eta \\ \eta & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \bar{\partial} L(z') = \begin{bmatrix} y' \\ -x' \end{bmatrix} = \begin{bmatrix} \eta & 1 \\ -1 & \eta \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \eta \begin{bmatrix} \eta & 1 \\ -1 & \eta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 1-\eta^2 & -\eta \\ \eta & 1-\eta^2 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\lambda = 1 - \eta^2 \pm i\eta$$

$$|\lambda| = \sqrt{1 - \eta^2 + \eta^4}$$

$|\lambda| < 1$ for $0 < \eta < 1$.

Rk. $|z'| > |z|$, but $|\hat{z}|$ corrects it back.