Université Paul Sabatier
SRI 3 - **IA Avancee - incertitude** - 2020-2021

With documents (written or printed)

Theoretical questions (4 points)

- Is it true that $\forall PL(A) + PL(\bar{A}) = 1$? if yes, give a proof, otherwise give a counter example
- Is it true that$\forall A, B,$ t.q $A \cap B = \emptyset, Pl(A \cup B) = Pl(A) + Pl(B)$? if yes, give a prof, otherwise give a counter example
- For which kind of problem shall one use the optimistic Choquet integral ? In which case is it equal to the expected utility ?

Exercise 1 (8 points)

An investor wishes to invest his capital in a rental property investment. He targeted a neighborhood undergoing renovation whose future population may include *students*, *employees*, and *middle range managers*. Housing in this area includes *apartments* or *villas*.

The profitability of the investment depends on the rents received over the next 10 years period.

- If students (S) live in the area, the only profitable investment is apartments .
- If employees (E) live in the area, investment in apartments will be more profitable than in villas
- If middle range managers (M) live in the area, the apartments will be unprofitable, and the villas remunerative.

The gains predicted are given in the table below:

$Population \setminus Investment :$	*apartments*	*villas*
S	200	0
E	300	220
M	30	400

He contacts a consulting firm in order to have forecasts about the future population of the area and to anticipate the chances that he has of finding S, E or M tenants. The reliability of the advice given by the consultant about the category of the future population depend on the prediction made: the consultant is more reliable when it advises C_M="'*invest for M*"' or C_S="'*invest for S*"'. In details:

- When the firm advises C_S, there is a 3/4 probability that students will actually move in, and a 1/4 probability that they will be employees or middle managers
- When the firm advises C_M, there is a 3/4 probability that middle managers will actually move in, and a 1/4 probability that they will be employees or students
- When the firm advises C_E, there is a 1/2 probability that employees will move in, and a probability 1/2 that students or managers will actually move in.

The investor's utility is assumed to be neutral with respect to the gain

1. What will be the shape of the investor's utility function? give an example
2. If he does not want to make any additional assumptions, what theory can the investor use to model the information he can deduce from the consultant's advice? model the case where he receives the information C_E
3. If the firm advises C_E, what is the plausibility of the following events: $\{M\}$, $\{E, M\}$, $\{E, M, S\}$? what is the belief in these events ?
4. Using pessimistic Choquet utility or the multiple prior utility, determine the investor's choice when the advice is C_E

Exercise 2 (8 points)

Consider a geostationary satellite the mission of which is to acquire information about the zone it is in charge. It can point the zones it has to observe (action "Mission"), or point the earth station, upload the information acquire and empty its memory (action "Upload"), or orient itself in order to have its solar panels charging (action "Charge"). Some of these actions may fail when there in not enough charge or because the memory is full.

A simple model is to consider states: the 3 positions "Acquire", " Earth Station" and "Solar" ; and the two states of error, "Memory Full" and "Battery low".

The effect of the actions are the following:

- Action "Charge" succeeds in any state - the satellite moves to the "Solar" state

- Action "Upload" may fail (with probability e) when the battery satellite becomes low:
  - When executed in position "Acquire", "Memory Full" or in position "Earth Station", it leads to state "Low Battery" with probability e, and to state "Earth Station" with probability 1 - e.
  - When executed in state "Low Battery" nothing happens and the action lets the satellite in state "Low Battery"
  - When executed in state "Solar", action "Upload" succeeds for sure and leads to state "Earth Station"

- Action "Mission" may fail when the battery satellite is low (probability e) or too many observations are already recorded in the memory (probability p) :
  - When executed in position "Acquire", it leads to state "Low Battery" with probability e, to state "Memory full " with probability p, and to state "Acquire" otherwise.
  - When executed in position "Earth Station", it leads to state "Low Battery" with probability e, and to state "Acquire" with probability 1 - e.
  - When executed in state "Solar", action "Mission" leads to state "Memory full" with a probability p and to state Acquire with probability 1 - p
  - When executed in state "Memory full", action "Mission" is rejected and the satellite stays in state "Memory full"
  - When executed in state "Low Battery" action "Mission" is rejected the action lets the satellite in state "Low Battery"

The rewards are the following:

- The reward for being in state Acquisition is denoted ra
- The reward for being in state Earth station is denoted re
- The reward for being in states Low Battery or Solar are rl and rs respectively
- When in state Memory full, actions "charge" and "mission" have a reward of rlost (the memory is automatically cleared)

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Question a - Model the problem by a (infinite horizon) MDP - draw the MDP and provide the reward and transition matrices
Question b - Propose values for the rewards which are coherent with the intuition
Question c - Propose a policy that seems ok (no need to compute the value of the policy)