

# 费用流

## Min-cost Max-flow

Idea: 将 EK 算法中的 bfs 改为 spfa, 每次寻找费用最少的增广路。

Complexity:  $O(VE^2)$

ATT: 链式前向星存储时, edgeNum 初始化为1; 建图时建流为 0、费用为  $-f$  的反向边。

Code:

```
1  int minCost[N], minFlow[N], pre[N];
2  bool inq[N];
3  int spfa(){
4      for(int i = 1; i <= n; i++){
5          minCost[i] = minFlow[i] = INF;
6          pre[i] = 0;
7          inq[i] = 0;
8      }
9      queue<int> q;
10     q.push(src);
11     inq[src] = 1;
12     minCost[src] = 0;
13     while(!q.empty()){
14         int cur = q.front(); q.pop();
15         inq[cur] = 0;
16         for(int i = head[cur]; i; i = edge[i].nxt){
17             if(edge[i].flow && minCost[edge[i].to] > minCost[cur] + edge[i].cost){
18                 minCost[edge[i].to] = minCost[cur] + edge[i].cost;
19                 minFlow[edge[i].to] = min(minFlow[cur], edge[i].flow);
20                 pre[edge[i].to] = i;
21                 if(!inq[edge[i].to]){
22                     q.push(edge[i].to);
23                     inq[edge[i].to] = 1;
24                 }
25             }
26         }
27     }
28     if(pre[dst] == 0) return -1;
29     return minFlow[dst];
30 }
31
32 void EK(int &maxflow, int &mincost){
33     maxflow = mincost = 0;
34     int flow = 0;
35     while((flow = spfa()) != -1){
36         int t = dst;
37         while(t != src){
38             edge[pre[t]].flow -= flow;
39             edge[pre[t]^1].flow += flow;
40             t = edge[pre[t]^1].to;
41         }
42         maxflow += flow;
43         mincost += flow * minCost[dst];
44     }
45 }
```