

# 半平面交

## Halfplane Intersection

**Idea:** 用双端队列维护，将直线（即代表半平面）按极角排序后，每加入一个新的半平面，弹出队首队尾无用的半平面（前一个交点在当前直线的右边则无用）

**Application:** 求多边形的核，凸多边形最大内切圆，线性规划区域

**Complexity:**  $O(n \lg n)$

**ATT:** 我的代码中，如果半平面交为一个点，并不会视之为空，尽管其面积为零。

**Code:**

```
1 // Attention: In my code, if the intersection is a point, it will not be seen as empty, though its area is 0.
2 // if m >= 3, then the intersection exists (including the situation where the intersection is a point).
3 // if a point is not regarded valid in a particular problem, you should calculate the area.
4 Point P[N]; // p[i] is the intersection of line q[i] and q[i+1]
5 Line Q[N]; // deque
6 void HalfplaneIntersection(Line L[], int n, Point res[], int &m){
7     // L[] are the lines, n is the number of lines, res[] stores the result of the intersection (a polygon)
8     // m is the number of points of the intersection (which is a polygon)
9     sort(L + 1, L + n + 1);
10    int head, tail;
11    Q[head = tail = 0] = L[1];
12    for(int i = 2; i <= n; i++){
13        while(head < tail && PointOnRight(P[tail - 1], L[i])) tail--;
14        while(head < tail && PointOnRight(P[head], L[i])) head++;
15        Q[++tail] = L[i];
16        if(sgn(Q[tail].v ^ Q[tail - 1].v) == 0){ // parallel, the inner one remains
17            tail--;
18            if(!PointOnRight(L[i].p, Q[tail])) Q[tail] = L[i];
19        }
20        if(head < tail) P[tail - 1] = GetLineIntersection(Q[tail - 1], Q[tail]);
21    }
22    while(head < tail && PointOnRight(P[tail - 1], Q[head])) tail--; // delete useless plane
23    P[tail] = GetLineIntersection(Q[tail], Q[head]);
24
25    m = 0;
26    for(int i = head; i <= tail; i++) res[++m] = P[i];
27 }
```