逆序对

Property: 逆序对对数等于序列通过交换相邻两数变成全排列的交换次数。

Idea:

- method 1:在归并排序的 merge 中,每次取出右边元素时统计左边未取出的元素数量, 累和。
- method 2: 离散化之后,开值域树状数组,按**逆序**插入树状数组时,统计已插入的元素中 比当前元素小的数量。

Complexity:

• method 1: $O(n \lg n)$ • method 2: $O(n \lg n)$

Code (Mergesort):

```
void mergesort(int l, int r){
 2
        if(l >= r) return;
        int mid = (l + r) \gg 1;
 3
 4
        mergesort(l, mid);
        mergesort(mid+1, r);
        id = 0;
 6
 7
        int lpt = l, rpt = mid+1;
 8
        while(lpt <= mid && rpt <= r){</pre>
             if(a[lpt] <= a[rpt])</pre>
 9
                 t[++id] = a[lpt++];
10
11
             else{
12
                 t[++id] = a[rpt++];
                 cnt += 1ll * mid - lpt + 1; // cnt is the number of
13
    inversions
14
             }
15
        }
        while(lpt <= mid)</pre>
16
17
            t[++id] = a[lpt++];
18
        while(rpt <= r)</pre>
            t[++id] = a[rpt++];
19
        for(int i = l; i <= r; i++)
20
             a[i] = t[i - l + 1];
21
22
   }
```

Code (BIT):

```
int c[N];
    inline int lowbit(int x){
 3
        return x & -x;
 4
    void add(int x, int val){
 5
       while(x \le 'z'){
 6
7
            c[x] += val;
            x += lowbit(x);
8
9
        }
10
11
    inline int sum(int x){
       int res = 0;
12
        while(x){
13
14
           res += c[x];
            x -= lowbit(x);
15
16
        }
17
        return res;
18
    int cntInverse(int a[]){ // a[] here is already discretized
19
       int res = 0;
20
21
        memset(c, 0, sizeof c);
        for(int i = n; i >= 1; i--){
22
            res += sum(a[i]-1);
23
24
            add(a[i], 1);
        }
25
26
        return res;
27 }
```