高斯消元

Gauss-Jordan Elimination

Complexity: $O(n^3)$ Code (浮点):

```
namespace LA{
 2
        int n;
        double a[N][N], b[N];
 3
 4
        void init(int nn){
            n = nn;
 6
            for(int i = 1; i \le n; i++){
 7
 8
                b[i] = 0;
                for(int j = 0; j \le n; j++) a[i][j] = 0;
 9
            }
10
11
        }
12
        bool Gauss(){
13
        // false: no solution or multiple solutions; true: a[][n+1] is
    the only solution
14
        /* a[1,1]x1 + a[1,2]x2 + ... + a[1,n]xn = b[1]
15
            a[2,1]x1 + a[2,2]x2 + ... + a[2,n]xn = b[2]
16
17
            a[n,1]x1 + a[n,2]x2 + ... + a[n,n]xn = b[3] */
18
            for(int i = 1; i <= n; i++) a[i][n+1] = b[i];
19
            for(int j = 1; j \le n; j++){
20
21
                int r = j;
                for(int i = j + 1; i \le n; i++)
22
23
                    if(a[i][j] > a[j][j])
24
                         r = i;
                if(r != j) swap(a[r], a[j]);
25
                if(a[j][j] == 0) return false;
26
                for(int i = 1; i \le n; i++){
27
                    if(i == j) continue;
28
29
                    double div = a[i][j] / a[j][j];
                    for(int k = j; k \le n + 1; k++)
30
                         a[i][k] = div * a[j][k];
31
32
                }
```

```
33
            for(int i = 1; i <= n; i++) a[i][n+1] /= a[i][i];
34
35
            return true;
36
        }
37
        double det(){ // get determinant
38
            double res = 1;
            int flag = 1;
39
            for(int j = 1; j \le n; j++){
40
                int r = j;
41
                for(int i = j + 1; i \le n; i++)
42
                     if(a[i][j] > a[j][j])
43
                         r = i;
44
                if(r != j) swap(a[r], a[j]), flag = -flag;
45
                if(a[j][j] == 0) return 0;
46
                for(int i = 1; i \le n; i++){
47
48
                    if(i == j) continue;
                     double div = a[i][j] / a[j][j];
49
                    for(int k = j; k \le n; k++)
50
                         a[i][k] = div * a[j][k];
51
52
                }
            }
53
54
            for(int i = 1; i <= n; i++) res *= a[i][i];
            return flag ? res : -res;
55
        }
56
57 }
```

Code (取模):

```
namespace LA{
 1
 2
        int n;
 3
        LL a[N][N], b[N];
 4
 5
        void init(int nn){
 6
            n = nn;
 7
            for(int i = 1; i \le n; i++){
                b[i] = 0;
 8
                for(int j = 0; j \le n; j++) a[i][j] = 0;
 9
10
            }
11
        }
12
        bool Gauss(){
        // false: no solution or multiple solutions; true: a[][n+1] is
13
    the only solution
14
        /* a[1,1]x1 + a[1,2]x2 + ... + a[1,n]xn = b[1]
            a[2,1]x1 + a[2,2]x2 + ... + a[2,n]xn = b[2]
15
16
17
            a[n,1]x1 + a[n,2]x2 + ... + a[n,n]xn = b[3] */
```

```
18
            for(int i = 1; i <= n; i++) a[i][n+1] = b[i];
19
            for(int j = 1; j \le n; j++){
20
                int r = j;
21
22
                for(int i = j + 1; i \le n; i++)
                     if(a[i][j] > a[j][j])
23
24
                         r = i;
25
                if(r != j) swap(a[r], a[j]);
                if(a[j][j] == 0) return false;
26
                for(int i = 1; i <= n; i++){
27
                     if(i == j) continue;
28
                     LL div = a[i][j] * fpow(a[j][j], MOD-2) % MOD;
29
                     for(int k = j; k \le n + 1; k++){
30
31
                         a[i][k] \stackrel{=}{-} div * a[j][k];
32
                         ((a[i][k] \% = MOD) += MOD) \% = MOD;
33
                     }
                }
34
35
            }
            for(int i = 1; i \le n; i++) (a[i][n+1] *= fpow(a[i][i], MOD-
36
    2)) %= MOD;
37
            return true;
38
        }
        LL det(){ // get determinant
39
            LL res = 1;
40
            int flag = 1;
41
            for(int j = 1; j \le n; j++){
42
                int r = j;
43
                for(int i = j + 1; i \le n; i++)
44
45
                     if(a[i][j] > a[j][j])
                         r = i;
46
                if(r != j) swap(a[r], a[j]), flag = -flag;
47
                if(a[j][j] == 0) return 0;
48
                for(int i = 1; i <= n; i++){
49
                     if(i == j) continue;
50
                     LL div = a[i][j] * fpow(a[j][j], MOD-2) % MOD;
51
                     for(int k = j; k \le n; k++){
52
53
                         a[i][k] = div * a[j][k] % MOD;
                         ((a[i][k] %= MOD) += MOD) %= MOD;
54
55
                     }
                }
56
57
58
            for(int i = 1; i <= n; i++) (res *= a[i][i]) %= MOD;
            return flag > 0 ? res : MOD - res;
59
60
        }
61
   }
```