

# 欧拉数

## Eulerian Numbers

### 定义

欧拉数： $\left\langle n \atop k \right\rangle$  表示  $\{1, 2, \dots, n\}$  的有  $k$  个升高的排列  $\pi_1 \pi_2 \cdots \pi_n$  的个数。也即，有  $k$  个地方  $\pi_j < \pi_{j+1}$ 。

### 递归式

$$\left\langle n \atop k \right\rangle = (k+1) \left\langle n-1 \atop k \right\rangle + (n-k) \left\langle n-1 \atop k-1 \right\rangle$$

组合证明即可。

$n$	$\left\langle n \atop 0 \right\rangle$	$\left\langle n \atop 1 \right\rangle$	$\left\langle n \atop 2 \right\rangle$	$\left\langle n \atop 3 \right\rangle$	$\left\langle n \atop 4 \right\rangle$	$\left\langle n \atop 5 \right\rangle$	$\left\langle n \atop 6 \right\rangle$	$\left\langle n \atop 7 \right\rangle$	$\left\langle n \atop 8 \right\rangle$	$\left\langle n \atop 9 \right\rangle$
0	1									
1	1	0								
2	1	1	0							
3	1	4	1	0						
4	1	11	11	1	0					
5	1	26	66	26	1	0				
6	1	57	302	302	57	1	0			
7	1	120	1191	2416	1191	120	1	0		
8	1	247	4293	15619	15619	4293	247	1	0	
9	1	502	14608	88234	156190	88234	14608	502	1	0

### 恒等式

$$\left\langle n \atop k \right\rangle = \left\langle n-1-k \atop n \right\rangle$$

对称性

$$x^n = \sum_k \left\langle n \atop k \right\rangle \binom{x+k}{n}$$

Worpitzky 恒等式

$$\left\langle n \atop m \right\rangle = \sum_{k=0}^m \binom{n+1}{k} (m+1-k)^n (-1)^k$$
$$m! \left\{ n \atop m \right\} = \sum_k \left\langle n \atop k \right\rangle \binom{k}{n-m}$$
$$\left\langle n \atop m \right\rangle = \sum_k \left\{ n \atop k \right\} \binom{n-k}{m} (-1)^{n-k-m} k!$$

