Channel Estimation for RIS assisted mmWave Systems via OMP with Optimization

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Classification for compressive sensing

- **On-grid schemes** (E.g. orthogonal matching pursuit (OMP)):
  - low complexity
  - less desirable performance
- **Off-grid schemes** (E.g. sparse Bayesian learning (SBL)):
  - high complexity
  - superior to on-grid schemes
- **Gridless schemes** (E.g. atomic norm minimization)
  - prohibitive complexity
  - excellent performance

Off-grid errors

Table: Different OGD settings

Condition	Distance (D)
OFF0	$D = 0$
OFF1	$D = \frac{1}{8M_{G,x}}, \frac{1}{8M_{G,y}}, \frac{1}{8N_G}$
OFF2	$D = \frac{1}{4M_{G,x}}, \frac{1}{4M_{G,y}}, \frac{1}{4N_G}$
OFF3	$D = \frac{1}{2M_{G,x}}, \frac{1}{2M_{G,y}}, \frac{1}{2N_G}$
OFF4	$D = \frac{1}{M_{G,x}}, \frac{1}{M_{G,y}}, \frac{1}{N_G}$

- It is based on finite-size dictionary.
- Off-grid errors will seriously deteriorate the performance channel estimation.

- What is the off-grid error?

- The off-grid error is the distance between a real continuous angle and its nearest grid point.

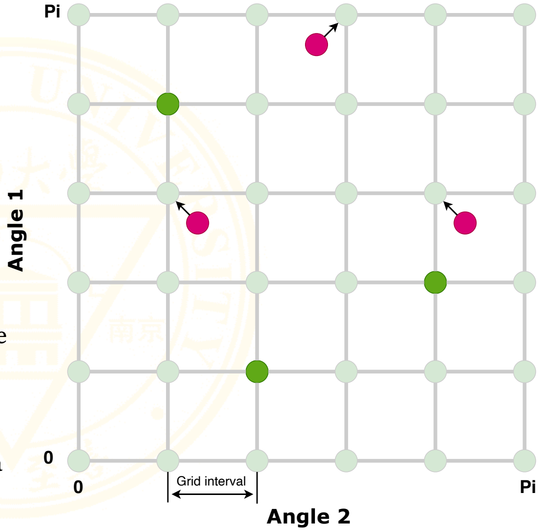


Figure: An illustration of the grid and off-grid angles

Drawbacks of OMP

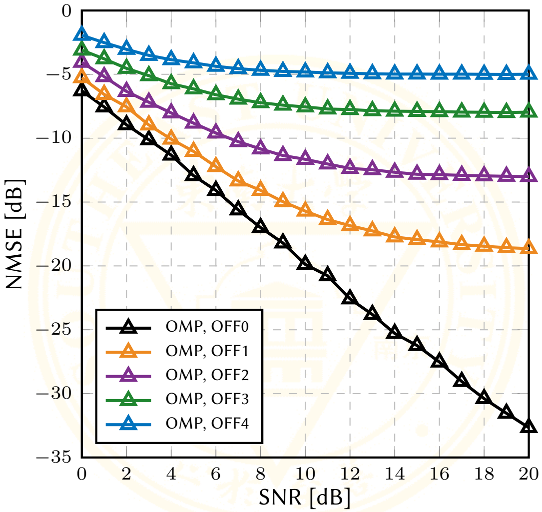


Figure: Impact off-grid errors have on performance

RIS-MISO systems

BS-to-RIS channel:

$$\mathbf{G} = \sqrt{\frac{MN}{L_{\mathrm{G}}}} \sum_{l_1=1}^{L_{\mathrm{G}}} \delta_{l_1} \mathbf{a}_{\mathrm{r}}(\vartheta'_{l_1}, \gamma'_{l_1}) \mathbf{a}_{\mathrm{t}}^{\mathrm{H}}(\phi'_{l_1}) \qquad (1)$$

RIS-to-user channel:

$$\mathbf{h} = \sqrt{\frac{M}{L_{\mathrm{h}}}} \sum_{l_2=1}^{\mathrm{L_h}} \xi_{l_2} \mathbf{a}_{\mathrm{r}}(\vartheta'_{l_2}, \gamma'_{l_2}) \qquad (2)$$

Sparse representation:

$$\mathbf{G} = (\mathbf{D}_{\mathrm{x}} \otimes \mathbf{D}_{\mathrm{y}})\mathbf{U}\mathbf{D}_{\mathrm{h}}^{\mathrm{H}} = \mathbf{D}_{\mathrm{g}}\mathbf{U}\mathbf{D}_{\mathrm{h}}^{\mathrm{H}} \qquad (3)$$

$$\mathbf{h} = \mathbf{D}_{\mathrm{g}}\mathbf{V}, \qquad (4)$$

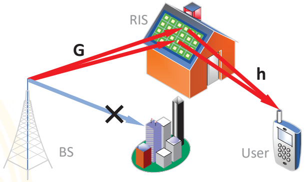


Figure: Application scenarios of RIS

Cascaded channel:

$$\mathbf{H} = \mathbf{U}_{\mathrm{s}}\mathbf{H}_{\mathrm{CS}}\mathbf{D}_{\mathrm{h}}^{\mathrm{H}} \qquad (5)$$

RIS-MISO systems considering off-grid errors

Considering the off-grid errors, $\mathbf{G}$ and $\mathbf{h}$ can be presented as $\hat{\mathbf{G}}$ and $\hat{\mathbf{h}}$ as

$$\hat{\mathbf{G}} = \mathbf{G} + \mathbf{E}_{\mathrm{G}} \tag{6}$$

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{E}_{\mathrm{h}} \tag{7}$$

Note: $\mathbf{E}_{\mathrm{G}}$ and $\mathbf{E}_{\mathrm{h}}$ are the off-grid errors matrices in the BS-to-RIS channel and RIS-to-user channel showing before.
The received signal:

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{N} + \mathbf{E}, \tag{8}$$

Note: $\mathbf{A}$ is the sensing matrix, $\mathbf{N}$ is the noise matrix and $\mathbf{E}$ is error matrix inducted by off-grid angles.

- How to mitigate off-grid errors?

* A simple way: increasing the grid size
* Limitation: increasing complexity, not satisfying RIP which leads to even worse performance.

Protocol

For single-user:

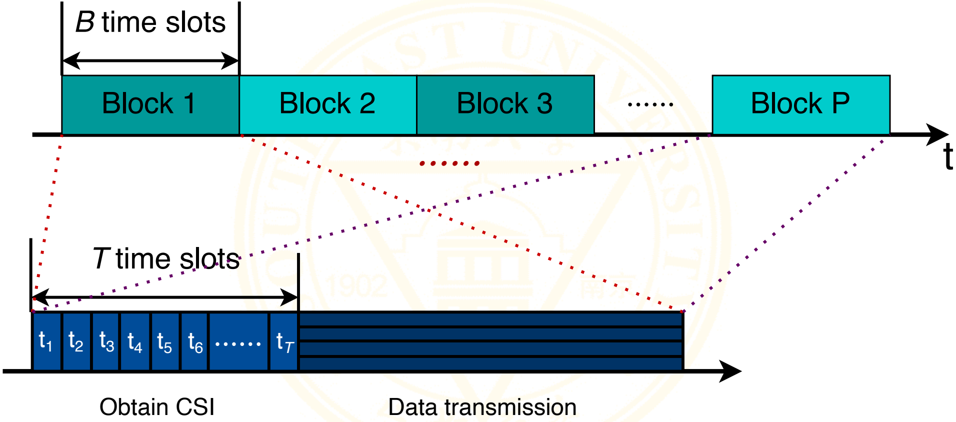


Figure: Downlink channel estimation protocol for single user

Protocol

Extend to multi-user:

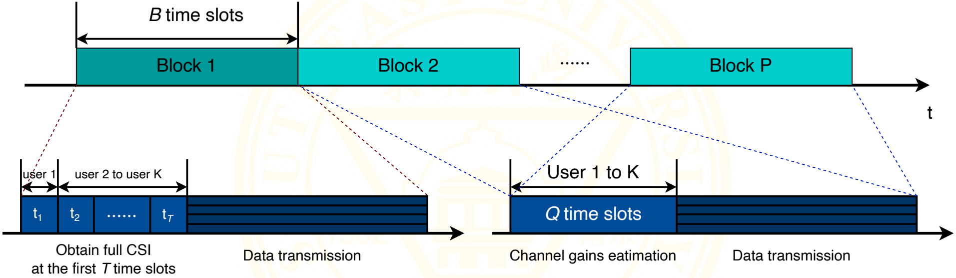


Figure: Downlink channel estimation protocol for multi-user

Discrete to continuous optimization aided OMP

- Idea of DC-OMP: combination of on-grid and gridless techniques
  - Initialization: down by OMP with a coarse grid
  - Optimization: formulated into a nonlinear constrained optimization problem

Objective function:

$$f\left(\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right) = \left| < \mathbf{k}_{v_s}\left(\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right), \mathbf{u}_{s-1} > \right| \quad (9)$$

Nonlinear constrained continuous optimization problem:

$$\left\{\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right\} = \arg \max_{\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s} f\left(\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right)$$

$$s.t. \begin{cases} |\phi'_s - \tilde{\phi}'_s| \leqslant \frac{1}{2N_{\mathrm{G}}} \\ |\vartheta'_s - \tilde{\vartheta}'_s| \leqslant \frac{1}{2M_{\mathrm{G,x}}} \\ |\gamma'_s - \tilde{\gamma}'_s| \leqslant \frac{1}{2M_{\mathrm{G,y}}} \end{cases} \quad (10)$$

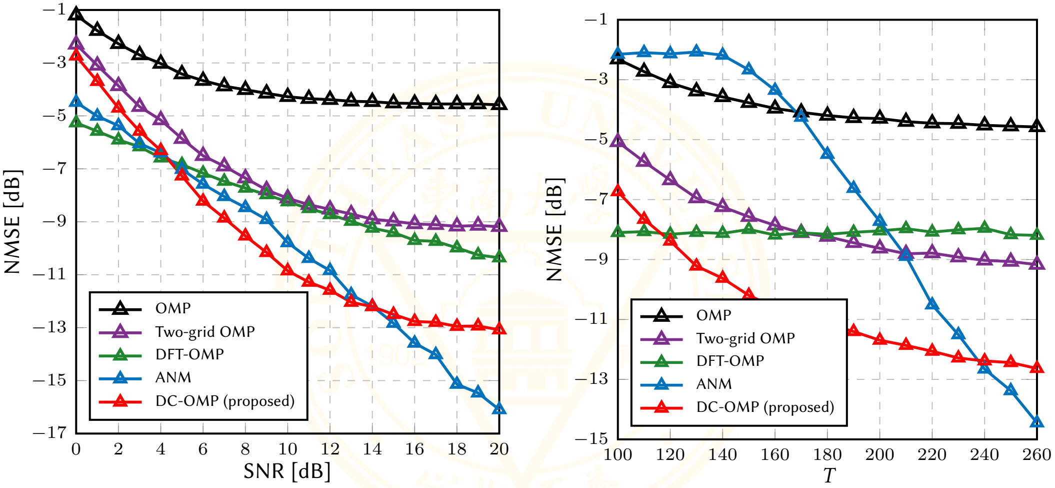
Algorithm 1 DC-OMP

Input: $\mathbf{y}$, $\mathbf{\Psi}$ and sparsity S

Output: estimated channel $\tilde{\mathbf{H}}$

Initialization: residual $\mathbf{u}_0 = \mathbf{y}$ and index set $\mathbf{\Upsilon} = \varnothing$
for $s = 1, 2, \ldots, S$ **do**
  $v_s = \arg \max_{j=1,\ldots,N} | < \mathbf{u}_{s-1}, \mathbf{k}_j > |$
  Calculate $\{\phi'_s, \vartheta'_s, \gamma'_s\}$ by Algorithm 2.
  $\left\{\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right\} = \arg \max_{\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s} f\left(\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right)$
  $\tilde{\mathbf{k}}_{v_s} = \tilde{\mathbf{k}}_{v_s}\left(\tilde{\phi}'_s, \tilde{\vartheta}'_s, \tilde{\gamma}'_s\right)$
  $\mathbf{\Omega}_s = \left[\mathbf{\Omega}_{s-1}, \tilde{\mathbf{k}}_{v_s}\right]$
  $\mathbf{\Upsilon}_s = [\mathbf{\Upsilon}_{s-1}, v_s]$
  $\tilde{\mathbf{D}}_{\mathrm{h}}(:, A, s) = \mathbf{a}'_{\mathrm{t}}(\tilde{\phi}'_s)$
  $\tilde{\mathbf{U}}_{\mathrm{s}}(:, B, s) = \mathtt{diag}\left(\mathbf{D}_{\mathrm{g}}(:, 1)^{\mathsf{T}}\right) \mathbf{a}'_{\mathrm{r}}(\tilde{\vartheta}'_s, \tilde{\gamma}'_s)$
  $\mathbf{x}_s = \arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{\Omega}_s \mathbf{x}\|_2$
  $\mathbf{u}_s = \mathbf{y} - \mathbf{\Omega}_s \mathbf{x}_s$
end for
for $s = 1, 2, \ldots, S$ **do**
  $\mathbf{X}(1, \mathbf{\Upsilon}_s(s), s) = \mathbf{x}_s(s)$
  $\tilde{\mathbf{H}}_{\mathrm{CS}}(:, :, s) = \mathtt{reshape}(\mathbf{X}(:, :, s), M_{\mathrm{G}}, N_{\mathrm{G}})$
  if $s = 1$ **then**
    $\tilde{\mathbf{H}} = \tilde{\mathbf{U}}_{\mathrm{s}}(:, :, s)\tilde{\mathbf{H}}_{\mathrm{CS}}(:, :, s)\tilde{\mathbf{D}}_{\mathrm{h}}^{\mathrm{H}}(:, :, s)$
  end if
  $\tilde{\mathbf{H}} = \tilde{\mathbf{H}} + \tilde{\mathbf{U}}_{\mathrm{s}}(:, :, s)\tilde{\mathbf{H}}_{\mathrm{CS}}(:, :, s)\tilde{\mathbf{D}}_{\mathrm{h}}^{\mathrm{H}}(:, :, s)$
end for
return $\tilde{\mathbf{H}}$

NMSE performance



(a) NMSE vs. SNR (b) NMSE vs. Pilot Number

Figure: Simulation results of the proposed DC-OMP.

Complexity analysis

Table: Complexity of the proposed method and benchmarks

Algorithm	Complexity	Runtime
OMP	$\mathcal{O}(TN_{\mathrm{G}}M_{\mathrm{G}})$	1.0
Two-grid OMP	$\mathcal{O}(TNM + TN_{\mathrm{G}}M_{\mathrm{G}})$	8.5
DFT-OMP	$\mathcal{O}(N^2 + 8DL_G)$	1.0
Atomic norm minimization	$\mathcal{O}\left((N + T_1)^{3.5}\right)$	45.0
DC-OMP (proposed)	slightly higher than $\mathcal{O}(TN_{\mathrm{G}}M_{\mathrm{G}})$	11.7

- Conclusion: our proposed DC-OMP achieves a trade-off between the CE performance and complexity.
- Limitation:

* is the correlation function optimal?
* how to design the dictionary?
* … …

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Thank you!