

1. **Data structures used for Q and visited. Your rationale behind the choice of data structures.**
 - a) Use a LinkedList for Q because it supports push and pop operation in $O(1)$ time.
 - b) Use a HashSet for visited because it supports add and search operation in $O(\log(n))$ time where n is the number of vertices.
2. **Number of edges and vertices in the graph WikiCS.txt.**
 - a) Number of edges: 23869
 - b) Number of vertices: 500
3. **Number of strongly connected components in WikiCS.txt**
9
4. **Size of the largest component in WikiCS.txt.**
492
5. **The data structures that you built/used in GraphProcessor.**
 LinkedHashMap<String, LinkedList<String>> G;
 For every entry $(u, [v_1, v_2, \dots, v_n])$ in G, $\langle u, v_i \rangle$ is an edge in the graph.
6. **Analyze and report the asymptotic run time for each of the class GraphProcessor.**

Let N be the number of vertices in the graph, and

E be the number of edges in the graph, and

C be the number of SCCs in the graph.

SCCAlgo:

$$\begin{aligned}
 T = O(& \\
 & E * \log(N) + N * \log(N) \quad // \text{computeOrderAlgo} \\
 & + N * \log(N) \quad // \text{sort V by finish time} \\
 & + N * \log(N) \quad // \text{sccDFS} \\
 &) = O((N+E) * \log(N))
 \end{aligned}$$

outDegree:

$T = O(\log(N))$ // find key in a map.

sameComponent:

$T = O ($
 $(N + E) * \log(N)$ // SCC Algo
 $+ C$ // loop for components
 $* \log(N)$ // find v in the component set
 $) = O((N+E+C) * \log(N))$

componentVertices:

$T = O ($
 $(N + E) * \log(N)$ // SCC Algo
 $+ C$ // loop for components
 $* (\log(N)$ // find v in the component set
 $+ N)$ // create list from a set
 $) = O((N+E) * \log(N) + CN)$

largestComponent:

$T = O ($
 $(N + E) * \log(N)$ // SCC Algo
 $+ C$ // loop for components
 $) = O((N+E) * \log(N) + C)$

numComponents:

$T = O((N+E) * \log(N) + C)$ // same as SCC Algo

bfsPath:

$T = O ($
 N // traverse through n vertices
 $* \log(N)$ // find node in the visited set
 $+ N$ // build the path
 $) = O(N * \log(N))$