

計算方法設計

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I. Pseudo code

```
(i). Randomize-Selection(arr, l, r, i) {  
    if(l==r) return l;  
  
    q = Partition(arr, l, r);  
  
    k = q-l+1;  
  
    if(i== k)      return q;  
  
    else if(i<k)   return Randomize-Selection(arr, l, q-1, i);  
  
    else           return Randomize-Selection(arr, q+1, r, i);  
  
}
```

```
Partition(arr, l, r) {  
  
    pivot_position = rand(l,r);    //choose a random  
                                    integer between l and r  
  
    pivot = arr[pivot_position];  //do insertion sort  
  
    int i = l-1;  
  
    for j = l  $\rightarrow$  r-1:  
  
        if(arr[j] < pivot):  
  
            i++;  
  
            swap(arr[i], arr[j]);  
  
    i++;  
  
    swap(arr[i], arr[r]);  
  
}
```

I. Pseudo code

[illegible]

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```
(ii). Median(arr, l, r, g) {  
    group_num = ceil(r-l+1 / g);  
    for i=0 → group_num-1:  
        subl = l+i*g;  
        subr = subl+(g-1);    //every g element choose  
                               //a median  
        m_idx = Select(arr, subl, subr, ceil(sub_length),g);  
        swap(arr[l+i], arr[m_idx]);  
    return Select(arr, l, l+group_numceil(group_num/2), g);  
    //find median of median  
}
```

II. Running Time

$N=20000000$, $K=150$ (但在分為3個一組($g=3$)時，會發生 segment fault，因此採用其他測資表示)

```
20000000 150
Start test
finish generate testcase
(random) 1: Running Time(clock per second) = 0.157495 Ans: 81
(g=5) 1: Running Time(clock per second) = 1.15655 Ans: 81
(g=7) 1: Running Time(clock per second) = 1.01362 Ans: 81
(g=9) 1: Running Time(clock per second) = 1.00446 Ans: 81

delete all testcase
average running time (random) : 0.266554
average running time (g = 3) : 0
average running time (g = 5) : 1.18723
average running time (g = 7) : 1.0597
average running time (g = 9) : 1.06624
logout
```

$N=3000000$, $K=150$ (目前測試 $N>5000000$, $g=3$ 會發生 segment fault，因此使用此數值當作測資)

```
3000000 150
Start test
finish generate testcase
(random) 1: Running Time(clock per second) = 0.026535 Ans: 492
(g=3) 1: Running Time(clock per second) = 0.296787 Ans: 492

average running time (random) : 0.0392575
average running time (g = 3) : 0.292892
average running time (g = 5) : 0
average running time (g = 7) : 0
average running time (g = 9) : 0
logout
```

III. Time Complexity

$g=3$:

$$T(n) = T(\lceil \frac{n}{3} \rceil) + \Theta(n) + T(\frac{4n}{6} + 4)$$

$$\text{show } T(n) \geq cn \lg n$$

$$T(n) \geq \frac{cn}{3} \lg \frac{n}{3} + \frac{cn}{6} \lg \frac{4n}{6} + \Theta(n)$$

$$\geq cn \lg n + \Theta(n)$$

$$\Rightarrow T(n) = \Omega(n \lg n) = \Omega(n)$$

$g=5$:

$$T(n) = T(\lceil \frac{n}{5} \rceil) + \Theta(n) + T(\frac{7n}{10} + 6)$$

$$\leq \frac{9cn}{10} + c + 6c + a_n$$

$$\leq cn \quad \text{if } c \geq 10a + 70$$

$$\Rightarrow T(n) = O(n)$$

$g=7$:

$$T(n) = T(\lceil \frac{n}{7} \rceil) + \Theta(n) + T(\frac{10n}{14} + 8)$$

$$\leq \frac{13}{14}cn + 7c + 8a_n \leq cn \quad \text{if } c \geq 7a + 63$$

$$\Rightarrow T(n) = O(n)$$

$g=9$

$$T(n) = T(\lceil \frac{n}{9} \rceil) + \Theta(n) + T(\frac{13n}{18} + 10)$$

$$\leq \frac{15cn}{18} + 11c + 10a_n \leq cn \quad \text{if } c \geq 6a + 66$$

$$\Rightarrow T(n) = O(n)$$