

1. $A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix}$, choose the first column to calculate determine.

$$\begin{aligned}
 \det(A) &= 1 \cdot (-1)^{2+1} \det(A_{11}) + 1 \cdot (-1)^{3+1} \det(A_{21}) + 1 \cdot (-1)^{4+1} \det(A_{31}) \\
 &= (x_2 x_3^2 - x_3 x_2^2) - (x_1 x_3^2 - x_3 x_1^2) + (x_1 x_2^2 - x_2 x_1^2) \\
 &= x_1 x_2 (x_2 - x_1) + x_1 x_3 (x_1 - x_3) + x_2 x_3 (x_3 - x_2) \\
 &= x_3 (x_1^2 - x_1 x_3 + x_2 x_3 - x_2^2) + x_1 x_2 (x_2 - x_1) \\
 &= x_3 [(x_1 + x_2)(x_1 - x_2) + x_3(x_2 - x_1)] + x_1 x_2 (x_2 - x_1) \\
 &= x_3 (x_2 - x_1)(x_3 - x_2 - x_1) + x_1 x_2 (x_2 - x_1) \\
 &= (x_2 - x_1)(x_3^2 - x_3 x_2 - x_3 x_1 + x_1 x_2) \\
 &= (x_2 - x_1)(x_3(x_3 - x_2) + x_1(x_2 - x_3)) \\
 &= (x_2 - x_1)(x_3 - x_2)(x_3 - x_1)
 \end{aligned}$$

If any two of x_1, x_2, x_3 are equal, $\det(A) = 0$. Under these conditions, A is singular.

2. $A1 = \begin{bmatrix} y_1 & x_1 & x_1^2 \\ y_2 & x_2 & x_2^2 \\ y_3 & x_3 & x_3^2 \end{bmatrix}$, $A2 = \begin{bmatrix} 1 & y_1 & y_1^2 \\ 1 & y_2 & y_2^2 \\ 1 & y_3 & y_3^2 \end{bmatrix}$, $A3 = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix}$

$$\det(A1) = y_1 x_2 x_3 (x_3 - x_2) - y_2 x_1 x_3 (x_3 - x_1) + y_3 x_1 x_2 (x_2 - x_1)$$

$$\det(A2) = y_1 (x_2 - x_3)(x_2 + x_3) + y_2 (x_3 - x_1)(x_3 + x_1) + y_3 (x_1 - x_2)(x_1 + x_2)$$

$$\det(A3) = (x_1 - x_2)(y_1 - y_3) - (x_1 - x_3)(y_1 - y_2)$$

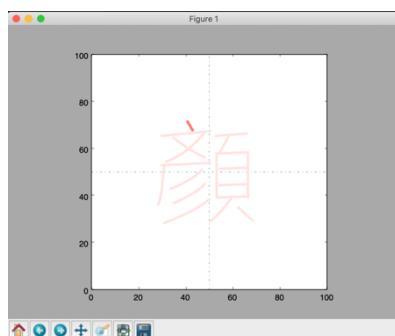
$$a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$a_0 = \frac{y_1 x_2 x_3}{(x_2 - x_1)(x_3 - x_1)} - \frac{y_2 x_1 x_3}{(x_2 - x_1)(x_3 - x_2)} + \frac{y_3 x_1 x_2}{(x_3 - x_2)(x_3 - x_1)}$$

$$a_1 = -\frac{y_1(x_2 + x_3)}{(x_2 - x_1)(x_3 - x_1)} + \frac{y_2(x_1 + x_3)}{(x_2 - x_1)(x_3 - x_2)} - \frac{y_3(x_1 + x_2)}{(x_3 - x_2)(x_3 - x_1)}$$

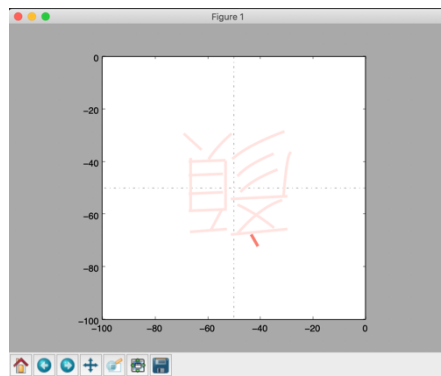
$$a_2 = \frac{y_1 - y_2}{(x_2 - x_1)(x_3 - x_2)} - \frac{y_1 - y_3}{(x_3 - x_2)(x_3 - x_1)}$$

3. Image

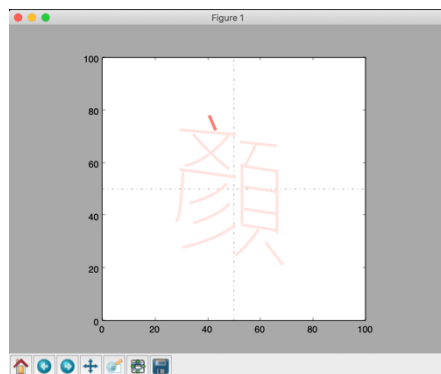


4. Transformation images :

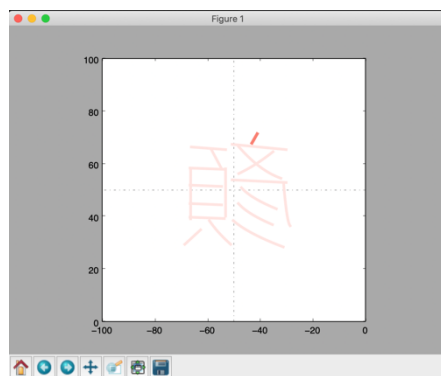
$A = [[-1,0], [0,-1]]$, $B = [[-1,0], [0,-1]]$ (B for middle adjusting line)



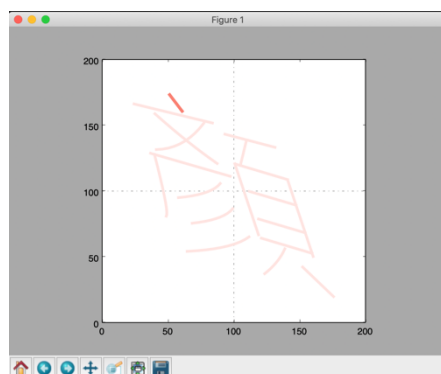
$A = [[1,0], [-0.2,1.2]]$, $B = [[1,0], [0,1]]$ (B for middle adjusting line)



$A = [[-1,0], [0,1]]$, $B = [[-1,0], [0,1]]$ (B for middle adjusting line)



$A = [[3, -1], [-1, 3]]$, $B = [[2,0], [0,2]]$ (B for middle adjusting line)



5. Motivation and process:

一開始認為要做「字」的創作時，讓我想到描字本的框線架構。因此我將圖的中央添加了兩條虛線仿作生字簿的分區，並將每一筆畫的顏色、寬度改變。然後為了要讓字的結構好看，我到網路上尋找我要的字於生字簿上的結構（如下附圖）。而讓我遭遇到最大困難的是中間那兩條虛線，因為在做 linear transformation 時，那兩條線也會跟著變動，但這樣與原本想做成生字本的概念背離，因此我另外新增了 matrix B，去控制那兩條線的變化。