Assignment 2: It Builds Character

2020 Fall EECS205002 Linear Algebra

Due: 2020/11/18

There are two types of fonts: Bitmap and TrueType (TTF) [1], as shown in Figure 1. Their difference is when the font size is changed, the TTF can keep the smooth shape while the bitmap font may cause zigzag boundary. The secret is that TTF uses vectors to store the font shapes, and the vectors are rasterized when the fonts are displayed.

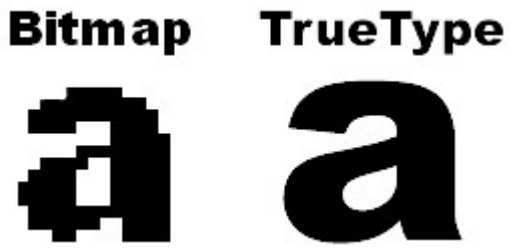


Figure 1: Two types of fonts.

Most TTFs use Bezier curves to store the shape. In this assignment, we use polynomials to represent fonts, which is named *PolyFont*. More specifically, we only use the polynomial of degree 2, which are

$$y = a_2x^2 + a_1x + a_0 \text{ (U-type) or } x = b_2y^2 + b_1y + b_0 \text{ (C-type).}$$

Figure 2 shows an example, in which the green, purple, and yellow curves are created using degree-2 polynomials, and other lines are degree-0 or degree-1 polynomials. The green and purple curves are in U-type, and the yellow one uses C-type.

For a curve, to find the coefficients of the polynomial, you need to solve a linear system. This process is called *interpolation*. We use U-type to explain the idea.

1. Select three points on the curve you want to fit, say (x_1, y_1), (x_2, y_2), and (x_3, y_3). Note for U-type, x_1, x_2, x_3 need to be different; for C-type, y_1, y_2, y_3 should be distinct.

[1] `https://codertw.com/\%E7\%A8\%8B\%E5\%BC\%8F\%E8\%AA\%9E\%E8\%A8\%80/645868/`

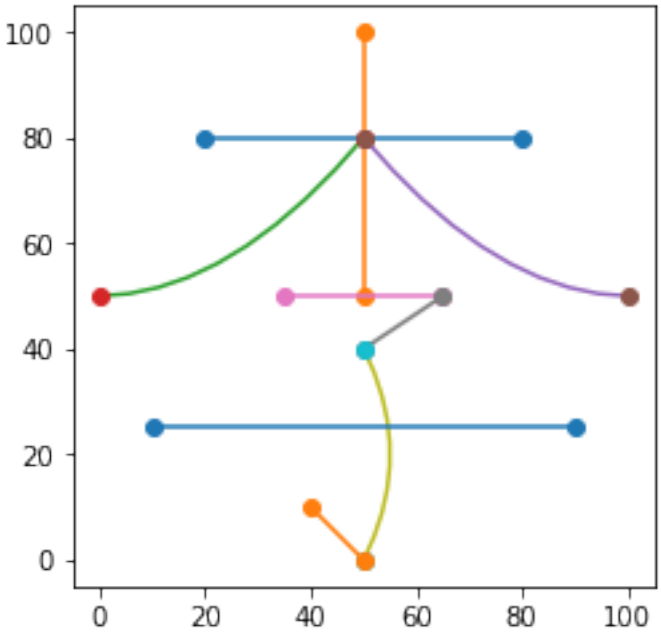


Figure 2: A Chinese character in PolyFont.

2. Construct the Vandermonde matrix for those three points

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{bmatrix} \tag{1}$$

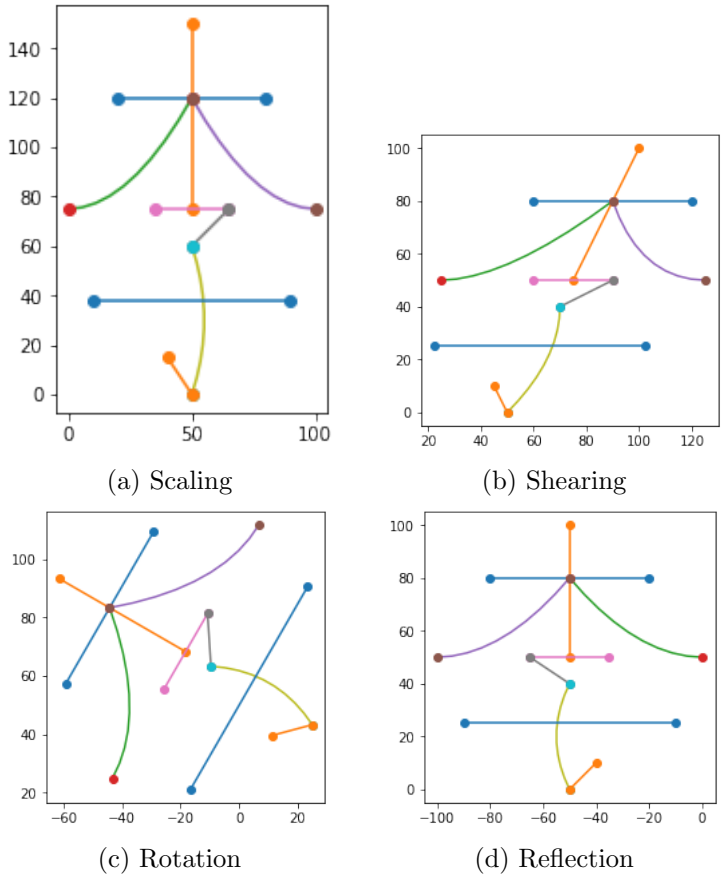
3. Solve the linear system $Xa = y$, where

$$a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \text{ and } y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

With the vectorized representation, we can perform some geometrically linear transformation to the fonts. Figure 3 shows four examples: scaling, shearing, rotation, and reflection. Those transformations can be performed by premultiplying the following matrices to the original points.

$$A_{\text{scale}} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1 \end{bmatrix}, A_{\text{shear}} = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix},$$

$$A_{\text{rotate}} = \begin{bmatrix} \cos(\pi/3) & \sin(\pi/3) \\ -\sin(\pi/3) & \cos(\pi/3) \end{bmatrix}, A_{\text{reflect}} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



(a) Scaling (b) Shearing

(c) Rotation (d) Reflection

Figure 3: Geometrically linear transformations.

1 Assignment

1. (20%) Derive the determinant of the Vandermonde matrix A in (1), and determine under what constraints will A be singular.

2. (20%) Let $\vec{y} = [y_1\ y_2\ y_3]^T$ and A be defined in (1). Use Cramer's rule to express the solution a of the linear solution a of the linear system $A\vec{a} = \vec{y}$. You may referencce Sec 2.3.

3. (20%) Design a PolyFont character, which needs to contain at least 1 U-type curve, 1 C-type curve, and 3 lines. If two submissions are the same or *similar*, both submissions will get 0 for the entire assignment. Please use functions provided by matplotlib package to output your character as an image file.

4. (20%) Design four different linear transformations for your PolyFont char-

acter. Write down your linear transformations in matrix form. Please include all four transformed images and matrices in your report.

5. (20%) We will have a competition for the artistic performance. The more beautiful and special design of PolyFont gets a better score. 請以一段300字內的短文描述你的創作動機和過程。

2 Submission

1. Write a report in a PDF file that includes (1) and (2). You will get 0 if no derivation of the answer or the derivation is copied from somewhere else.
2. Python code of your implementation of (3) and (4).
3. Images of the designed PolyFont in PDF files run from your code.
4. Zip them and submit.