1. $X_0 = \frac{\pi}{8}$, $X_1 = \frac{3\pi}{8}$, $X_2 = \frac{5\pi}{8}$, $X_3 = \frac{7\pi}{8}$.

$\cos(x0) = -\cos(x3),\ \cos(x2) = -\cos(x1),\ \cos(2x0) = -\cos(2x1),$
$\cos(2x2) = -\cos(2x3),\ \cos(3x0) = -\cos(3x3),\ \cos(3x1) = -\cos(3x2)$
Because cos(x) = sin(90-x), $\cos(3\pi/8)=\sin(\pi/8)$. And $\sin^2(x) + \cos^2(x) = 1$.
$\cos(2x0) = \cos(2x3),\ \cos(2x1) = \cos(2x2)$
$\cos(3x0) = \cos(x1), \cos(3x1) = -\cos(x0), \cos(3x2) = \cos(x3), \cos(3x3) = \cos(x2)$.

將下列矩陣相乘，並依上述各式化簡可得到結果。

$$X = \begin{bmatrix} 1/\sqrt{2} & \cos(\frac{\pi}{8}) & \cos(\frac{2\pi}{8}) & \cos(\frac{3\pi}{8}) \\ 1/\sqrt{2} & \cos(\frac{3\pi}{8}) & \cos(\frac{6\pi}{8}) & \cos(\frac{9\pi}{8}) \\ 1/\sqrt{2} & \cos(\frac{5\pi}{8}) & \cos(\frac{10\pi}{8}) & \cos(\frac{15\pi}{8}) \\ 1/\sqrt{2} & \cos(\frac{7\pi}{8}) & \cos(\frac{14\pi}{8}) & \cos(\frac{21\pi}{8}) \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos(\frac{\pi}{8}) & \cos(\frac{3\pi}{8}) & \cos(\frac{5\pi}{8}) & \cos(\frac{7\pi}{8}) \\ \cos(\frac{2\pi}{8}) & \cos(\frac{6\pi}{8}) & \cos(\frac{10\pi}{8}) & \cos(\frac{14\pi}{8}) \\ \cos(\frac{3\pi}{8}) & \cos(\frac{9\pi}{8}) & \cos(\frac{15\pi}{8}) & \cos(\frac{21\pi}{8}) \end{bmatrix}$$

$$X^TX = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} = 2I$$

2. By $a = 1/2 * X^TY$,

$a_1 = 1/2 *(\cos\left(\frac{\pi}{8}\right)y0 + \cos\left(\frac{3\pi}{8}\right)y1 + \cos\left(\frac{5\pi}{8}\right)y2 + \cos\left(\frac{7\pi}{8}\right)y3)$

$a_2 = 1/2 *(\cos(\frac{2\pi}{8})y0 + \cos(\frac{6\pi}{8})y1 + \cos(\frac{10\pi}{8})y2 + \cos(\frac{14\pi}{8})y3)$

By (6),

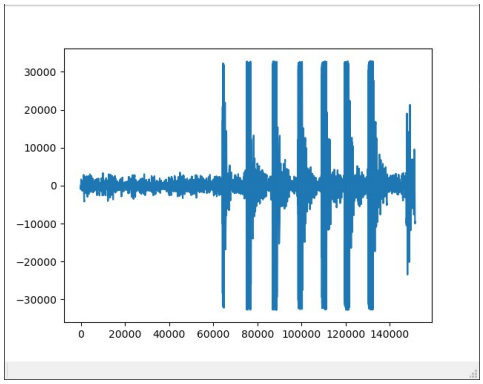
取 X 的第 2, 3 column 做 transpose，會得到以下兩式，

$a_1 = 1/2 *(\cos\left(\frac{\pi}{8}\right)y0 + \cos\left(\frac{3\pi}{8}\right)y1 + \cos\left(\frac{5\pi}{8}\right)y2 + \cos\left(\frac{7\pi}{8}\right)y3)$

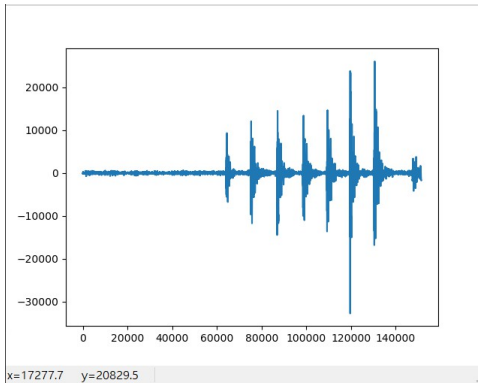
$a_2 = 1/2 *(\cos(\frac{2\pi}{8})y0 + \cos(\frac{6\pi}{8})y1 + \cos(\frac{10\pi}{8})y2 + \cos(\frac{14\pi}{8})y3)$

由兩式比較可知結果會相等。

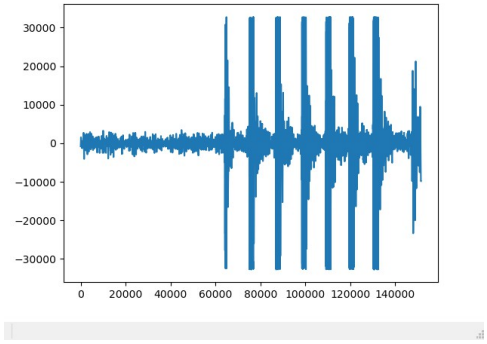
3. 處理前



濾低音



濾高音



因為錄製聲音音頻較低，可以明顯看出對高頻作過濾的效果並不顯著，但在濾低音的時候卻能明顯看出不同。

4. 在處理低頻音後，聲音大小明顯降低，甚至會有聲音消失的狀況出現。而處

理高音後，聲音有撕裂、破裂的感覺，有些類似音響壞掉的感覺。

5. 對音檔不做太多處理，但運用型態縮小檔案大小。