Lagman, Dinah

0B21757

COSC 3340/6309 Examination 1

Wednesday, June 10, 2009, 2 - 3:45 pm

Open Book and Notes

1. Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c	
1	2	2	3	1
2	3	2	3,4	1
→ 3	4	1	2	1
4	1	4	/	1

2. Consider the class $\mathcal{L}_A$ of all regular languages that contain only words of even length, over the fixed two-letter alphabet A={a,b}.

(a) Is $\mathcal{L}_A$ countable?

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a^2 \cup a^3)^* (a^3 \cup a^2)$ over the alphabet {a}

(b) $(01 \cup 10)^* ((01)^* \cup (10)^*)0^*$ over the alphabet {0,1}

4. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

	a	b	
→ A	B	C	1
B	A	/	0
C	/	B,C	0

Points: **1: 12** **2: 22** **3: 44** **4: 22**

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#1)

	a	b	c	
→ 3	4	1	2	1
4	1	4	1	1
1	2	2	3	1
2	3	2	3,4	1
3,4	1,4	1,4	2	1
1,4	1,2	2,4	3	1
1,2	2,3	2	3,4	1
2,4	1,3	2,4	3,4	1
2,3	3,4	1,2	234	1
1,3	2,4	1,2	2,3	1
234	1,34	124	234	1
134	124	124	2,3	1
124	123	24	34	1
123	~~1,2,3~~ 2,3,4	1,2	2,3,4	1

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✓ ✓ ✓ ✓ ϕ -2

#2 a) Since L_A is a regular language, then we know that there must be a DFA that accepts L_A. And we know that DFA is a finite automaton. Therefore it is ✓countable.

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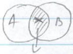
b.) We know that L_A is regular, and countable. Since M_A is NOT L_A, we dont know if M_A is regular or not. So we cannot ✓account for the languages represented by M_A. So it is not countable.

c.) Consider the following:

Intersection states that an element 'x' is an element of $A \cap B$ iff:

1.) it is an element of A and

2.) it is an element of B



$A \cap B$

But since M_A consists of all languages over A that is NOT in L_A, means that no element can be in both L_A AND M_A.

$L_A \cap M_A$ will yield an empty set, which is considered finite with cardinality zero, and therefore, countable.

— end of #2 —

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#3a) $(a_1a_2 \cup a_3a_4a_5)^*(a_6a_7a_8 \cup a_9a_{10})$ over $\{a\}$

	a	
→0	1	0
1	/	1

	a	
0	2	0
2	/	1

	a	
0	3	0
3	/	1

	a	
0	4	0
4	/	1

	a	
0	5	0
5	/	1

	a	
→0	1	0
1	2	0
2	/	1

	a	
→0	3	0
3	4	0
4	5	0
5	/	1

∪ =>

	a	
→0	1,3	0
1	2	0
2	/	1
3	4	0
4	5	0
5	/	1

* =>

	a	
→0	1,3	1
1	2	0
2	1,3	1
3	4	0
4	5	0
5	1,3	1

$(a_1a_2 \cup a_3a_4a_5)^*$

	a	
0	6	0
6	/	1

	a	
0	7	0
7	/	1

	a	
0	8	0
8	/	1

	a	
→0	6	0
6	7	0
7	8	0
8	/	1

	a	
0	9	0
9	/	1

	a	
0	10	0
10	/	1

	a	
0	9	0
9	10	0
10	/	1

	a	
→0	6, 9	0
6	7	0
7	8	0
8	/	1
9	10	0
10	/	1

$(a_6a_7a_8 \cup a_9a_{10})$

=> continued on next page.

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#3a cont.

NFA:

	a	
→0	1369	0
1	2	0
2	1369	0
3	4	0
4	5	0
5	1369	0
6	7	0
7	8	0
8	/	1
9	10	0
10	/	1

DFA:

		a	
A	→0	1369	0
B	1369	24710	0
C	24710	135689	1
D	135689	12346910	1
E	123467910	12345678910	1
F	12345678910	12345678910	1

	a	
→A	B	0
B	C	0
C	D	1
D	E	1
E	F	1
F	F	1

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Reduction:

accept | rej

$\boxed{CDEF}$ $\boxed{AB}$

$\boxed{CDEF}$ $\boxed{A}$ $\boxed{B}$

$\boxed{CDEF}$ $\boxed{A}$ $\boxed{B}$

1 2 3

	a	
1	1	1
→2	3	0
3	1	0

— end of #3a —

#3b) $(0_1 1_2 \cup 1_3 0_4)^* ((0_5 1_6)^* \cup (1_7 0_8)^*) 0_9^*$ over {0,1}

	0	1	
→0	1	/	0
1	/	/	1

	0	1	
0	/	2	0
2	/	/	1

	0	1	
0	/	3	0
3	/	/	1

	0	1	
0	4	/	0
4	/	/	1

	0	1	
→0	1	/	0
1	/	2	0
2	/	/	1

	0	1	
→0	/	3	0
3	4	/	0
4	/	/	1

∪ ⇒

	0	1	
→0	1	3	0
1	/	2	0
2	/	/	1
3	4	/	0
4	/	/	1

*

	0	1	
→0	1	3	1
1	/	2	0
2	1	3	1
3	4	/	0
4	1	3	1

$(0_1 1_2 \cup 1_3 0_4)^*$

	0	1	
→0	5	/	0
5	/	/	1

	0	1	
0	/	6	0
6	/	/	1

	0	1	
→0	5	/	0
5	/	6	0
6	/	/	1

$\overset{*}{\Rightarrow}$

	0	1	
→0	5	/	1
5	/	6	0
6	5	/	1

	0	1	
→0	/	7	0
7	/	/	1

	0	1	
0	8	/	0
8	/	/	1

$\overset{\cdot}{\Rightarrow}$

	0	1	
→0	/	7	0
7	8	/	0
8	/	/	1

$\overset{*}{\Rightarrow}$

	0	1	
→0	/	7	1
7	8	/	0
8	/	7	1

∪

	0	1	
→0	5	7	1
5	/	6	0
6	5	/	1
7	8	/	0
8	/	7	1

$((0_5 1_6)^* \cup (1_7 0_8)^*)$

	0	1	
→0	9	/	0
9	/	/	1

$\overset{*}{\Rightarrow}$

	0	1	
→0	9	/	1
9	9	/	1

$\overset{\cdot}{\Rightarrow}$

	0	1	
→0	59	7	1
5	/	6	0
6	59	$\cancel{7}$ /	1
7	8	/	0
8	9	7	1
9	9	/	1

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#36 cont...

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NFA:

	0	1	
→ 0	159	37	1
1	/	2	0
2	159	37	1
3	4	/	0
4	159	37	1
5	/	6	0
6	59	7	1
7	8	/	0
8	9	7	1
9	9	/	1

DFA:

		0	1	
A	→ 0	159	37	1
B	159	9	26	1
C	37	48	/	0
D	9	9	/	1
E	26	159	37	1
F	48	159	37	1
G	/	/	/	0

	0	1	
→ A	B	C	1
B	D	E	1
C	F	G	0
D	D	G	1
E	B	C	1
F	B	C	1
G	G	G	0

Reduction:

accept rej

$\boxed{ABDEF}$ $\boxed{CG}$

$\boxed{ADEF}$ $\boxed{B}$ $\boxed{C}$ $\boxed{G}$

$\boxed{AEF}$ $\boxed{D}$ $\boxed{B}$ $\boxed{C}$ $\boxed{G}$

$\boxed{AEF}$ $\boxed{D}$ $\boxed{B}$ $\boxed{C}$ $\boxed{G}$

1 2 3 4 5

	0	1	
→ 1	3	4	1
2	2	5	1
3	2	1	1
4	1	5	0
5	5	5	0

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— end of 36 —

#4

$L_A = aL_B \cup bL_C \cup \varepsilon$

$L_B = aL_A$

$L_C = bL_B \cup bL_C$

① plug in L_B in L_C

$L_C = b(aL_A) \cup bL_C$ then apply lemma

$L_C = \underbrace{baL_A}_{M} \cup \underbrace{b}_{L}\underbrace{L_C}_{X}$

$L_C = b^* \cdot baL_A$

② plug in L_B and L_C in L_A

$L_A = a(aL_A) \cup b(b^* baL_A) \cup \varepsilon$

$L_A = aaL_A \cup bb^*baL_A \cup \varepsilon$

③ factor out L_A

$L_A = \underbrace{(aa \cup bb^*ba)}_{L}\underbrace{L_A}_{X} \cup \underbrace{\varepsilon}_{M}$

apply lemma:

$L_A = (aa \cup bb^*ba)^* \varepsilon$

$L_A = (aa \cup bb^*ba)^*$

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