Math 3339

Written Homework 5 (Sections 4.6 - 4.8)

Name:________________________________ PeopleSoft ID:______________

Instructions:

- Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
- Print out this file and complete the problems.
- Use blue or black ink or a dark pencil.
- Write your solutions in the space provided. You must show all work for full credit.
- Submit this assignment at http://www.casa.uh.edu under "Assignments" and choose **HW5**
- Total points: **15.**

1. A geologist has collected 10 specimens of basaltic rock and 10 specimens of granite.* The geologist instructs a laboratory assistant to randomly select 15 of the specimens for analysis.
   a. What is the probability function of the number of granite specimens selected for analysis?

   P(X) = (10C20 * 10C) / (20C15)

   dhyper(X, 10, 10, 15)

   b. What is the probability that all specimens of one of the two types of rock are selected for analysis?

   Dhyper(10, 10, 10, 15) = 0.01625387

   c. What is the probability that the number of granite specimens selected for analysis is within 1 standard deviation of its mean value?

2. An urn contains 3 green balls, 2 blue balls, and 4 red balls. In a random sample of 5 balls, find the probability that at least one blue balls is selected.

P(x >= 1) = 1 – dhyper(0, 2, 7, 5)

= 0.8333333

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

3. The number of requests for assistance received by a towing service is a Poisson process with average of 4 requests per hour.*
    a. Compute the probability that exactly ten requests are received during a particular 2-hour period.

P(X=10) = dpois(10, 2*4) = pois(2, 4)
= 0.09926153

    b. If the operators of the towing service take a 30-min break for lunch, what is the probability that they do not miss any calls for assistance?

P(x=0) = dpois(0, 4 / 2) = dpois(0, 4 / 2)

    c. How many calls would you expect during their break?

You would expect 2 calls for a half hour period because the average is 4 calls per 1 hour period.

1. A club has 50 members, 10 belonging to the ruling clique and 40 second-class members. Six members are randomly selected for free movie tickets. What is the probability that 3 or more belong to the ruling clique?

P(x>=3) = 1 – P(x<=2) = 1 – phyper(3, 10, 40, 6)
= 0.01095546

---

4. Huck and Jim are waiting for a raft. The number of rafts floating by over intervals of time is a Poisson process with a rate of $\lambda = 0.4$ rafts per day. They agree in advance to let the first raft go and take the second one that comes along. What is the probability that they will have to wait more than a week? Hint: If they have to wait more than a week, what does that say about the number of rafts in a period of 7 days?

P(x=0, lambda*7) = ppois(0, .4 * 7) = 0.06081006

6. For the following scenarios determine the type of probability distribution:
Binomial, Hypergeometric, Poisson or None of these

a. What is the probability of seven or more heads in 10 tosses of a fair coin?

Binomial

b. A certain Starbucks has on average 5 customers per minute. What is the probability that 12 customers arrive in Starbucks in a 30-minute time interval.

Poisson

c. Given a lot with 21 good units and 4 defectives, what is the probability that a sample of five will yield no more than one defective?

Hypergeometric

d. Four companies are bidding for each of the three contacts, with specified success probabilities. What is the probability that a single company will receive all the orders?

None

e. Suppose you first randomly sample one card from a deck of 52. Then, without putting the card back in the deck you sample a second and then (again without replacing cards) a third. Given this sampling procedure, what is the probability that exactly two of the sampled cards will be aces (4 of the 52 cards in the deck are aces).

Hypergeometric

f. The switchboard in a small Denver law office gets an average of 2.5 incoming phone calls during the noon hour on Thursdays. Staffing is reduced accordingly; people are allowed to go out for lunch in rotation. Experience shows that the assigned levels are adequate to handle a high of 5 calls during that hour. What is the chance that 6 calls will be received in the noon hour, some particular Thursday, in which case the firm might miss an important call?

Poisson

g. A company makes electronic components for TV's 95% pass final inspection (and 5% fail and need to be fixed). Suppose 120 components are inspected in one day, what is the probability that 10 fail inspection?

Binomial

7. Suppose that X and Y are independent random variable having the joint probability distribution

$f(x, y)$	x: 2	4
y		
1	0.1	0.15
3	0.2	0.3
5	0.1	0.15

Find:

a) $E(2X - 3Y)$

2 * (2 * (.1+.2+.1) + 4 * (.15+.3+.15)) – 3 * (1 * (.1+.15) + 3 * (.2 + .3) + 5 * (.1 + .15))

= -2.6

b) $E(XY)$

1 * 2 * .1 + 1 * 4 * .15 + 3 * 2 * .2 + 3 * 4 * .3 + 5 * 2 * .1 + 5 * 4 * .15

= 9.6

c) $Cov(X, Y)$

Cov(X,Y) = E[XY] – E[X]*E[Y]

E[X] = 2 * (.1 + .2 + .1) + 4 * (.15 + .3 + .15) = 3.2

E[Y] = 1 * (.1 + .15) + 3 * (.2 + .3) + 5 * (.1 + .15) = 3

E[X] * E[Y] = 9.6

Cov(X, Y) = 0

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

8. The following table is a joint probability table for X = number of dogs a person owns and Y = number of cats a person owns.

	X		
	0	1	2
0	0.20	0.05	0.10
1	0.08	0.15	0.12
2	0.15	0.02	0.13

a. What is the probability $P(X = 0,\ Y = 0)$?

0.20

b. What is the probability that a person owns at least one cat and one dog?

.15 + .12 + .02 + .13 =

c. What is the probability that a person owns one cat, given they own one dog?

.15

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

X

	0	1	2
0	0.20	0.05	0.10
1	0.08	0.15	0.12
2	0.15	0.02	0.13

9.

Use the previous problem to answer the following questions.

a. How many dogs does a person expect to own?

Number dogs person can expect to own = E[X]
E[X] = (E[X=0] + E[X=1] + E[X=2]) / 3
= 0 * (.2+.08+.15) + 1*(.05 + .15 + .02) + 2*(.1 + .12 + .13)
= .92

b. Give the conditional probability distribution of number of dogs, given that they have one cat.

E[X | Y = 1]

0: .08
1: .15
2: .12

c. Is “number of dogs” and “number of cats” independent? Justify your answer.

No. The probabilities for the number of dogs would be the same no matter the number of cats a person owns, and vice versa, if the two were independent.

d. What is the total number of cats and dogs that a person is expected to own, i.e. $E(X + Y)$.

E[X] = .92
E[Y] = 0*(.2+.05+.1) + 1*(.08+.15+.12) + 2*(.15+.02 + .13) = .95

E[X] + E[Y] = 1.87

10. From the previous two exercises, determine the covariance of the number of dogs and cats. Explain what this means.

	X		
	0	1	2
0	0.20	0.05	0.10
1	0.08	0.15	0.12
2	0.15	0.02	0.13

$$Cov(X,Y) = E[XY] – E[X]*E[Y]$$

E[XY] = 0*0*.2 + 0*1*.05 + 0*2*.1 + 1*0*.08 + 1*1*.15 + 1*2*.12 + 2*0*.15 + 2*1*.02 + 2*2*.13

= .95

E[X] = .92
E[Y] = .95

Cov(X,Y) = .95 - (.92*.95) = 0.076

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.