

Thursday, June 10, 2010, 2 - 3:45 pm
Open Book and Notes

- 1.** Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c	
1	2	3	2	1
2	3	2	3,4	1
→ 3	4	1	2	1
4	1	4	/	1

- 2.** Consider the class \mathcal{L}_A of all regular languages that contain only words of odd length, over the fixed two-letter alphabet $A=\{a,b\}$.

- (a) Is \mathcal{L}_A countable?
- (b) Is the class \mathcal{M}_A countable where \mathcal{M}_A consists of all languages over A that are not in \mathcal{L}_A ?
- (c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

- 3.** Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, always using the constructions given in class:

- (a) $(a^2 \cup a^3)^* (a \cup a^2)$ over the alphabet $\{a\}$
- (b) $0^* (01 \cup 10)^* ((01)^* \cup (10)^*)$ over the alphabet $\{0,1\}$

- 4.** Construct a regular expression over the alphabet $\{a,b\}$ for the language accepted by the following automaton:

	a	b	
→ A	•	B	C 0
B	A	/	1
C	/	A,B	0

Points: 1: 12 2: 22 3: 44 4: 22

Exam 1

(79)

	a	b	c			a	b	c
1	2	3	2	1		3	4	1
2	3	2	3,4	1	so	1	2	3
→ 3	4	1	2	1		2	3	2,3,4
4	1	4	/	1		4	1	4

	a	b	c	
→ 3	4	1	2	
4	1	4	/	?
1	2	3	2	
2	3	2	3,4	
1	1	1	1	
3,4	1,4	1,4	2	
1,4	1,2	3,4	2	
1,2	2,3	2,3	2,3,4	
2,4	1,3	2,4	3,4	
2,3	3,4	2,1	2,3,4	
1,3	2,4	1,3	2	
2,3,4	1,3,4	1,2,4	2,3,4	
1,3,4	1,2,4	1,3,4	2	
1,2,4	1,2,3	2,3,4	2,3,4	
1,2,3	2,3,4	1,2,3	2,3,4	

(7)

② a) Once the length of words are ordered from min to max one can make a unique number to each word for it to be countable

b) MA is very complex that can have languages that are not regular which can be a problem and also it can be infinite, so is not countable

c) Is $L_A \cap M_A$ countable?
 By definition a ~~intersection~~ is empty, so $L_A \cap M_A$ is countable

(22)

③ a) $(a \cdot a \cup a \cdot a \cdot a)^*$ $(a \cup a \cdot a)$ over $\{a\}$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 1 & 1 \\ \hline \end{array} \rightarrow 0 \begin{array}{|c|c|} \hline 2 & 0 \\ \hline 1 & 1 \\ \hline \end{array} \rightarrow 0 \begin{array}{|c|} \hline a \\ \hline 3 \\ \hline \end{array} \rightarrow 0 \begin{array}{|c|c|} \hline a & 0 \\ \hline 4 & 1 \\ \hline \end{array} \rightarrow 0 \begin{array}{|c|c|} \hline a & 0 \\ \hline 5 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 1 & 2 \\ \hline 2 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 3 & 0 \\ \hline 3 & 4 \\ \hline 4 & 5 \\ \hline 5 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 1,3 & 0 \\ \hline 1 & 2 \\ \hline 2 & 1 \\ \hline 3 & 4 \\ \hline 4 & 5 \\ \hline 5 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 1,3 & 1 \\ \hline 1 & 2 \\ \hline 2 & 1,3 \\ \hline 3 & 4 \\ \hline 4 & 5 \\ \hline 5 & 1,3 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 6 & 0 \\ \hline 6 & 7 \\ \hline 7 & 1 \\ \hline \end{array} \end{array} \cup \begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 7 & 0 \\ \hline 7 & 8 \\ \hline 8 & 1 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} a \\ \rightarrow 0 \begin{array}{|c|c|} \hline 6,7 & 0 \\ \hline 6 & 7 \\ \hline 7 & 8 \\ \hline 8 & 1 \\ \hline \end{array} \end{array}$$

$(a \cdot a \cup a \cdot a)^\star (a \cup a \cdot a)$

a		a		a	
7	0	1	0	1, 3, 6, 7	0
1	2	2	2	1, 3, 6, 7	2, 4, 8
2	1, 3, 6, 7	3	3	2, 4, 8	1, 3, 6, 7, 5
3	4	4	4	1, 3, 6, 7, 5	2, 4, 8, 1, 3, 6, 7
4	5	5	5	1, 3, 6, 7, 2, 4, 8	2, 4, 8, 1, 3, 6, 7, 5
5	1, 3, 6, 7	6	6	1, 2, 3, 4, 5, 6, 7, 8	2, 1, 3, 6, 7, 4, 5, 8
6	/	/			
7	8	0			
8	/	1			

(22)

a		a	
2	0	1	2' 3' 4' 5' 6
3	1	1	2 3 4 5 6
4	1	A	B
5	1		
6	0		

a	
B	0
B	A

3) b) $0^\star (01 \cup 10)^\star ((01)^\star \cup (10)^\star) \quad \{0, 1\}$

$\rightarrow 0 \begin{smallmatrix} 01 \\ 110 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 210 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 130 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 140 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 510 \end{smallmatrix}$
$1 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$2 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$3 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$4 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$5 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$

$\rightarrow 0 \begin{smallmatrix} 01 \\ 610 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 170 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 18 \end{smallmatrix}$	$\rightarrow 0 \begin{smallmatrix} 01 \\ 910 \end{smallmatrix}$
$6 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$7 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$	$8 \begin{smallmatrix} 11 \\ 11 \end{smallmatrix}$	$9 \begin{smallmatrix} 111 \\ 111 \end{smallmatrix}$

$(01)^\star \begin{smallmatrix} 01 \\ 111 \end{smallmatrix}$	$(01) \rightarrow 0 \begin{smallmatrix} 01 \\ 210 \end{smallmatrix}$	$(10) \rightarrow 0 \begin{smallmatrix} 01 \\ 140 \end{smallmatrix}$	$(01) \rightarrow 0 \begin{smallmatrix} 01 \\ 610 \end{smallmatrix}$	$(10) \rightarrow 0 \begin{smallmatrix} 01 \\ 180 \end{smallmatrix}$

0^* $(01 \cup 10)^*$

$\rightarrow 0 \boxed{1 \ 1 \ 1 \ 1}$

2	1	3	0
3	2	4	1
4	5	1	0
5	2	4	1

$(01)^* \cup (10)^*$

$\rightarrow 0 \boxed{0 \ 1}$

6	1	7	0
7	6	1	1
8	9	1	0
9	8	1	1

$\rightarrow 0 \boxed{0 \ 1}$

2	1	3	0
3	2	6	4,8
4	5	1	0
5	2	6	4,8
6	1	7	0
7	6	1	1
8	9	1	0
9	1	8	1

$0 \boxed{0 \ 1}$

1	2	6	4,8
2	1	7	1
3	1	3	0
4	1,2,6	4,8	1
5	5	1	0
6	1,2,6	4,8	1
7	1	7	0
8	6	1	1
9	9	1	0

(2)

1 $\rightarrow 0 \boxed{0 \ 1}$

1,2,6	4,8	(2)
1	3,7	1
5,9	0	0
1	1	1
1,2,6	4,8	1
1,2,6	4,8	1
1	1	1
0	0	0

$\rightarrow 1 \boxed{0 \ 1}$

3	4
5	6
7	8
5	8
3	4
6	4
7	8

(18)

3 7 | 1 2 4 5 | 6

3	7	14	56	2
3	4	156	4	2
3	7	156	4	2
A	B	C	D	E

$\rightarrow C \boxed{0 \ 1}$

E	A	1
A	C	D
B	B	0
D	DB	1
E	DC	1

$$\textcircled{4} \quad \rightarrow A \begin{array}{c|cc} a & b \\ \hline b & c & 0 \\ c & a & 1 \\ \hline & a, b & 0 \end{array}$$

$$L_A = a \cdot L_B \cup b \cdot L_C \cup \textcircled{E} \times$$

$$L_B = a \cdot L_A \cup \textcircled{E}$$

$$L_C = b \cdot L_A \cup b \cdot L_B$$

$$L_B = b \cdot L_C \cup b \cdot L_B$$

$$L_C = (b^* \cdot b L_B)$$

$$L_C = b^* (b (a \cdot L_A))$$

$$L_A = a (a \cdot L_A) \cup b (b^*) (b (a \cdot L_A)) \cup \textcircled{E}$$

$$L_A = a \cdot a L_A \cup b \cdot b^* b \cdot a \cdot L_A \cup \textcircled{E}$$

$$L_A = \underbrace{(a \cdot a \cup b \cdot b^* \cdot b a)}_L L_A \cup \textcircled{E} \times$$

$$L_A = (a \cdot a \cup b \cdot b^* \cdot b \cdot a)^*$$

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b) My answer is very complex that can be dangerous that are not ready which is the question and also can be a waste of time, so it is not suitable