93

COSC 3340/6309
Examination 2
Tuesday, June 18, 2013, 10 am – 12 noon
Open Book and Notes

15/15 **1**. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B,C}, {a,b}, {S→C|bB, A→aa, B→Sb, C→A }, S).
Note: You must first determine L(G).

8 **2**. Eliminate all ε-productions in the following cfg G:
G = ({S,A,B}, {a,b,c}, {S→aA|BBBa, A→Sc|ε, B→bS|ε}, S).

25 **3**. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(011)^* \cap \overline{1(01^*)^*}\,]^*$$

Note: You must first determine nfas for (101)* and 1(01*)* over {01,2,}, then handle the intersection and complementation, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

15/15 **4**. Construct a Chomsky normal form grammar for L(G) for the following cfg G:
G = ({S,B}, {a,b,c,d}, {S→BBBBaS|B|bcd, B→cSda|S|dcba}, S).
Note: You must first remove all unit productions.

30/30 **5**. Construct a Greibach normal form grammar for L(G) for the following CNF G:
G = ({S,A,B}, {a,b}, {S→AA, A→BaA|a , B→SBA|b}, S).
Note: First derive all the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points: 1: 15 2: 10 3: 30 4: 15 5: 30

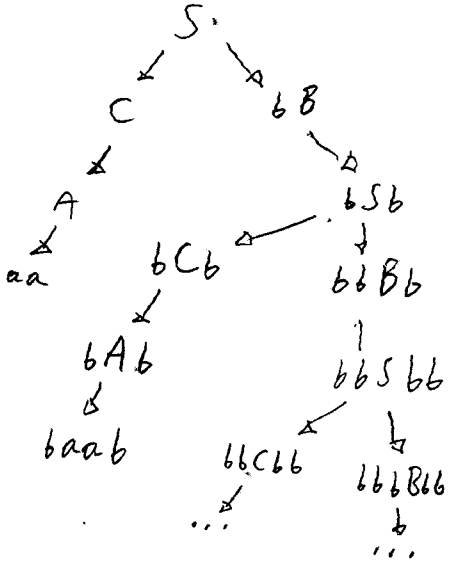
① WE HAVE: $S \to C \mid bB$

$A \to aa$

$B \to Sb$

$C \to A$

SO WE GET →



HENCE, $L(G) = \{b^n aab^n \mid n \geq 0\}$

ASSUMING THAT $L(G)$ IS REGULAR, THEN $\exists$ DFA D WITH n STATES, WHERE: $X = b^n aab^n = Y \cdot Z$ FOR $Y = b^n$, $Z = aab^n$

SINCE $|W| = n$ WE CAN APPLY PUMPING LEMMA:

$W = W_1 W_2 W_3$ WHERE $|W_2| \geq 1$ SO THAT FOR $S \geq 0$ ∄

$$\tau(q_0, W) = \tau(q_0, W_1 (W_2)^S W_3)$$

HOWEVER, IF $S = 0$ THEN $\tau(q_0, W) = \tau(q_0, W_1 W_3)$ BUT

$|W_1 W_3| = n - |W_2| < n$, INDICATING THAT SAID TRANSITION IS NOT PART OF $L(G)$, A CONTRADICTION !!

THEREFORE, $L(G)$ IS NOT REGULAR

② $S \to aA \mid BBBa$
$A \to Sc \mid \varepsilon$
$B \to bS \mid \varepsilon$

$\Rightarrow$ $S \to aSc \mid BBBa \mid a$
$A \to Sc$
$B \to bS \mid \varepsilon$

$\Downarrow$

$S \to Ba \mid aSc \mid bSBBa \mid BbSBa \mid BBbSa \mid BBa \mid a$

$A \to Sc$
$B \to bS$

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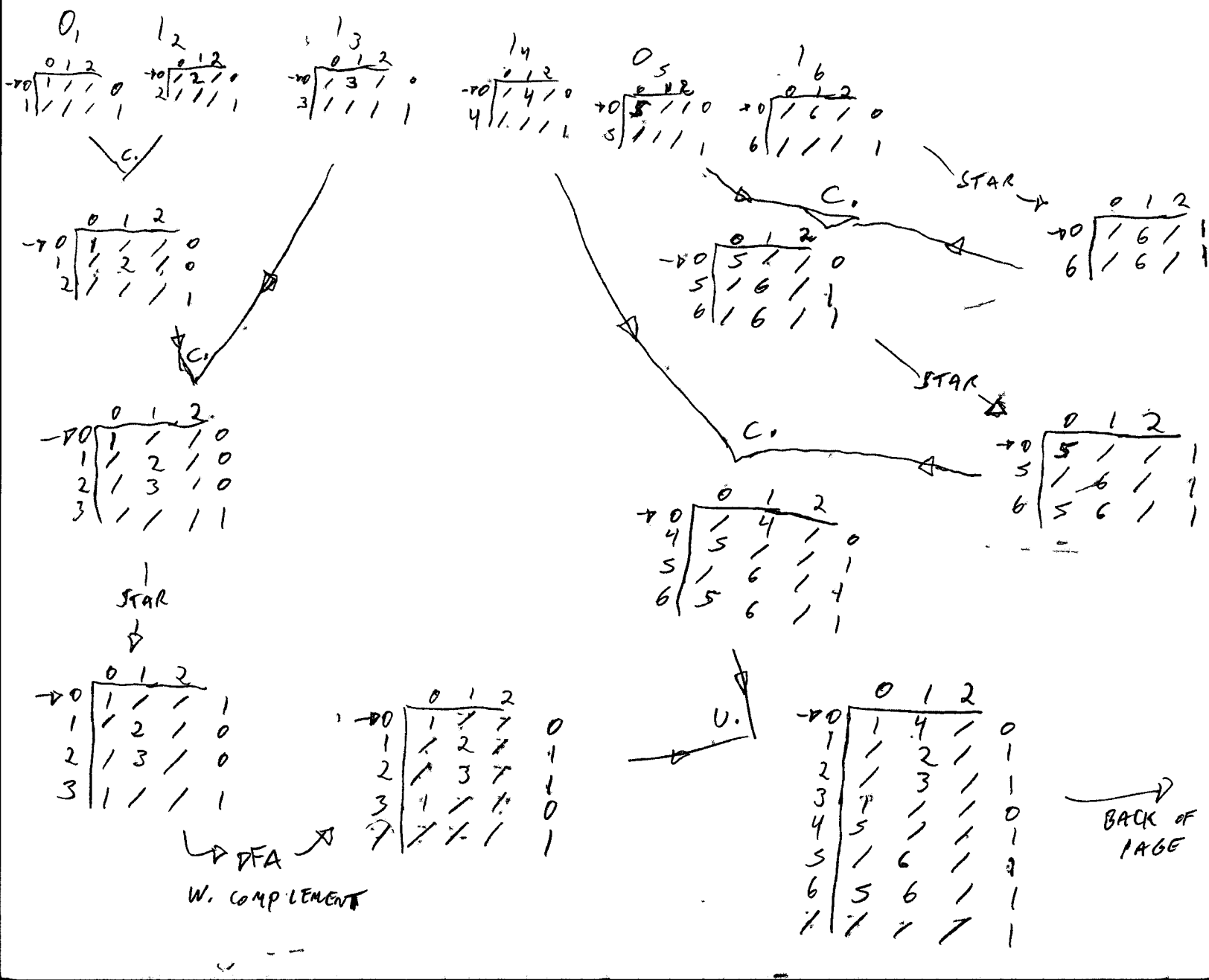
③ $[(011)^* \cap \overline{1(01^*)^*}]^* \Rightarrow [\overline{(0 \cup 1_2 \cup 1_3)^* \cup 1_4(0_5 \cup 1_6)^*}]^*$



	0	1	2	
→ 0	1	4	/	0
1	/	2	/	1
2	/	3	/	1
3	1	/	/	0
4	5	/	/	1
5	4	6	/	1
6	5	6	/	1
/	/	/	/	1

DFA → W. COMPLEMENT →

	0	1	2	
→ 0	1	4	7	1
1	7	2	7	0
2	7	3	7	0
3	1	7	7	1
4	5	7	7	0
5	7	6	7	0
6	5	6	7	0
(7)	7	7	7	0

/ = 7

REDUCTION CHECK

	0	1
	124567	03
	14567	2	03
1	4567	2	03
1	4567	2	03
(A)	(C)	(B)	(D)

	0	1	2	
→ A	C	B	C	0
B	C	D	C	0
C	C	C	C	0
D	A	C	C	1

REDUCED DFA

(4) • $S \to BBBBaS \mid B \mid bcd$ 15/15

• $B \to cSda \mid S \mid dcba$

⇓

• $S \to BBBBaS \mid SBBBaS \mid BSBBaS \mid BBSBaS \mid BBBSaS \mid SSBBaS \mid SBBSaS \mid BBSSaS \mid BSBSaS \mid SSBSaS \mid SBSSaS \mid SSSBaS \mid SSSSaS \mid BSSSaS \mid SBSBaS \mid bcd$

• $B \to cSda \mid cBBBBaS \mid BBBBaS \mid dcba$

⇓

• $S \to \overbrace{BBBB}^{S_1 S_2 S_3 S_4} Xa S \mid SBBBBXaS \mid BSBBXaS \mid BBSBXaS \mid BBBSXaS \mid SSBBXaS \mid SBBSXaS \mid BBSSXaS \mid BSBSXaS \mid SSBSXaS \mid SBSSXaS \mid SSSBXaS \mid SSSSXaS \mid BSSSXaS \mid SBSBXaS \mid XbXcXd$

• $B \to XcSXdXa \mid XcBBBBXaS \mid BBBBXaS \mid XdXcXbXa$

(FOR $Xa \to a$, $Xb \to b$, $Xc \to c$, $Xd \to d$)

⇓

• $S \to BS_1 \mid SS_1 \mid BS_6 \mid BS_7 \mid BS_9 \mid SS_{12} \mid SS_6 \mid BS_{15} \mid BS_{18} \mid SS_{21} \mid SS_{15} \mid SS_{24} \mid SS_{26} \mid BS_{26} \mid SB_{27} \mid X_b S_{28}$

$S_1 \to BS_2$
$S_2 \to BS_3$
$S_3 \to BS_4$
$S_4 \to X_a S$
$S_5 \to SS_6$
$S_6 \to BS_3$
$S_7 \to BS_8$
$S_8 \to SS_3$
$S_9 \to BS_{10}$
$S_{10} \to BS_{11}$
$S_{11} \to SS_4$
$S_{12} \to SS_{13}$
$S_{13} \to BS_{14}$
$S_{14} \to BS_4$
$S_{15} \to BS_{16}$
$S_{16} \to SS_{17}$
$S_{17} \to SS_4$
$S_{18} \to SS_{19}$
$S_{19} \to BS_{20}$
$S_{20} \to SS_4$
$S_{21} \to SS_{22}$
$S_{22} \to BS_{23}$
$S_{23} \to SS_4$
$S_{24} \to SS_{25}$
$S_{25} \to SS_3$
$S_{26} \to SS_{16}$
$S_{27} \to BS_5$
$S_{28} \to X_c X_d$

• $B \to X_c B_1 \mid X_c B_3 \mid BS_1 \mid X_d B_4$

$B_1 \to SB_2$
$B_2 \to XdXa$
$B_3 \to BS_1$
$B_4 \to X_c B_5$
$B_5 \to X_b X_a$

CMF

(5) $S \to AA$

$A \to BaA \mid a$

$B \to SBA \mid b$

$B \to AABA \mid b$

$B \to BaAABA \mid aABA \mid b$

$B \to aABA \mid b \mid aABAB' \mid bB'$

$B' \to aAABA \mid aAABAB'$

*B onto A

$S \to AA$

$A \to aABAaA \mid baA \mid aABAB'aA \mid bB'aA \mid a$

$B \to aABA \mid b \mid aABAB' \mid bB'$

$B' \to aAABA \mid aAABAB'$

*A onto S

$S \to aABAaAA \mid baAA \mid aABAB'aAA \mid bB'aAA \mid aA$

$A \to aABAaA \mid baA \mid aABAB'aA \mid bB'aA \mid a$

$B \to aABA \mid b \mid aABAB' \mid bB'$

$B' \to aAABA \mid aAABAB'$

GNF