COSC 3340/6309 Examination 1

Tuesday, June 11, 2013, 10 - 11:45 am

Open Book and Notes

99

12/12 **1**. Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c	
1	3	3	2	1
→ 2	1	2	1	1
3	4	1	2,4	1
4	2	4	/	1

22/22 **2**. Consider the class $\mathcal{L}_A$ of all regular languages that contain only words of even length, over the fixed two-letter alphabet A={a,b}.

(a) Is $\mathcal{L}_A$ countable?

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

43/44 **3**. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a \cup a^2)^* (a \cup a^4)$ over the alphabet {a}

(b) $((01)^* \cup (10)^*)\, 0^* (01 \cup 10)^*$ over the alphabet {0,1}

22/22 **4**. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

	a	b	
→ A	B	A,C	1
B	A	/	0
C	/	B	0

Points: 1: 12 2: 22 3: 44 4: 22

Ernst Leiss

22

(4)

	a	b	
$\to A$	B	A, C	1
B	A	/	0
C	/	B	0

$L_A = aL_B \cup b(L_A \cup L_C) \cup \varepsilon$

$L_B = aL_A$

$L_C = bL_B$

$* L_B \to L_C$

$\hookrightarrow L_C = baL_A$

$* L_C \to L_A$ & $L_B \to L_A$

$\hookrightarrow L_A = a(aL_A) \cup b[L_A \cup (baL_A)] \cup \varepsilon$

$L_A = aaL_A \cup bL_A \cup bbaL_A \cup \varepsilon$

$L_A = L_A(aa \cup b \cup bba) \cup \varepsilon$

$* \; X = L^*M$

$L_A = (aa \cup b \cup bba)^*$

(1)

	a	b	c	
1	3	3	2	1
→ 2	1	2	1	1
3	4	1	2,4	1
4	2	4	/	1

	a	b	c	
→ 2	1	2	1	1
1	3	3	2	1
3	4	1	2,4	1
4	2	4	/	
2,4	1,2	2,4	1	1
/	/	/	/	
1,2	1,3	2,3	1,2	
1,3	3,4	1,3	2,4	
2,3	1,4	1,2	1,2,4	
3,4	2,4	1,4	2,4	
1,4	2,3	3,4	2	
1,2,4	1,2,3	2,3,4	1,2	
1,2,3	1,3,4	1,2,3	1,2,4	
2,3,4	1,2,4	1,2,4	1,2,4	
1,3,4	2,3,4	1,3,4	2,4	

15 STATES

(2) (A) YES, SINCE L_A HAS ONLY REGULAR LANGUAGES, THERE HAS TO BE AT LEAST ONE DFA THAT ACCEPTS EACH ONE OF ~~THE~~ LANGUAGES. THEREFORE, THEY ARE COUNTABLE.

(2) (B) NO. WE KNOW WE CAN COUNT REGULAR LANGUAGES OVER A; HOWEVER, WE CANNOT COUNT LANGUAGES THAT AREN'T REGULAR, HENCE M_A CAN'T BE COUNTABLE.

(2) (C) OTHER PAGE →

(2) (c) IF L_A HAS ALL REGULAR LANGUAGES AND M_A HAS ALL LANGUAGES THAT ARE NOT IN L_A, THEN WE CAN ASSUME THAT $L_A \cap M_A = \emptyset$

BECAUSE THE CARDINALITY OF THE EMPTY SET IS 0, WE CAN COUNT IT. THEREFORE, $L_A \cap M_A$ IS COUNTABLE

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(3) (A) $(a \cup a^2)^*(a \cup a^4) \Rightarrow (a_1 \cup a_2 a_3)^*(a_4 \cup a_5 a_6 a_7 a_8)$

a_1

	a	
→0	1	0
1	/	1

a_2

	a	
→0	2	0
2	/	1

a_3

	a	
→0	3	0
3	/	1

a_4

	a	
→0	4	0
4	/	1

a_5

	a	
→0	5	0
5	/	1

a_6

	a	
→0	6	0
6	/	1

a_7

	a	
→0	7	0
7	/	1

a_8

	a	
→0	8	0
8	/	1

C.

	a	
→0	2	0
2	3	0
3	/	1

C.

	a	
→0	5	0
5	6	0
6	/	1

U.

	a	
→0	1,2	0
1	/	1
2	3	0
3	/	1

C.

	a	
→0	5	0
5	6	0
6	7	0
7	/	1

STAR

	a	
→0	1,2	1
1	1,2	1
2	3	0
3	1,2	1

C.

	a	
→0	5	0
5	6	0
6	7	0
7	8	0
8	/	1

* CONTINUE ON BACK OF PAGE

	a	
→0	1,2	1
1	1,2	1
2	3	0
3	1,2	1

	a	
→0	4	0
4	/	1

	a	
→0	5	0
5	6	0
6	7	0
7	8	0
8	/	1

U.

	a	
→0	4,5	0
4	/	1
5	6	0
6	7	0
7	8	0
8	/	1

C.

	a	
→0	1,2	0
1	1,2,4,5	0
2	3	0
3	1,2,4,5	0
4	/	1
5	6	0
6	7	0
7	8	0
8	/	1

NFA

		a	
A	→0	1,2	0
B	1,2	1,2,3,4,5	0
C	1,2,3,4,5	1,2,3,4,5,6	1
D	1,2,3,4,5,6	1,2,3,4,5,6,7	1
E	1,2,3,4,5,6,7	1,2,3,4,5,6,7,8	1
F	1,2,3,4,5,6,7,8	1,2,3,4,5,6,7,8	1

DFA

SIMPLIFICATION

	a	
→A	B	0
B	C	0
C	D	1
D	E	1
E	F	1
F	F	1

REDUCTION CHECK

0	1
AB	CDEF
A B	CDEF
A B	CDEF
1 2	3

	a	
→1	2	0
2	3	0
3	3	1

FINAL AUTOMATON

③ Ⓑ $((0_1 1_2)^* \cup (1_3 0_4)^*)\, 0_5^* \,(0_6 1_7 \cup 1_8 0_9)^*$

0_1

	0	1	
→0	1	/	0
1	/	/	1

1_2

	0	1	
→0	/	2	0
2	/	/	1

1_3

	0	1	
→0	/	3	0
3	/	/	1

0_4

	0	1	
→0	4	/	0
4	/	/	1

0_5

	0	1	
→0	5	/	0
5	/	/	1

0_6

	0	1	
→0	6	/	0
6	/	/	1

1_7

	0	1	
→0	/	7	0
7	/	/	1

1_8

	0	1	
→0	/	8	0
8	/	/	1

0_9

	0	1	
→0	9	/	0
9	/	/	1

C.

	0	1	
→0	1	/	0
1	/	2	0
2	/	/	1

C.

	0	1	
→0	/	3	0
3	4	/	0
4	/	/	1

STAR

	0	1	
→0	1	/	1
1	/	2	0
2	1	/	1

STAR

	0	1	
→0	/	3	1
3	4	/	0
4	/	3	1

STAR

	0	1	
→0	5	/	1
5	5	/	1

U.

	0	1	
→0	1	3	1
1	/	2	0
2	1	/	1
3	4	/	0
4	/	3	1

C.

	0	1	
→0	6	/	0
6	/	7	0
7	/	/	1

C.

	0	1	
→0	~~7~~	8	0
8	9	/	0
9	/	/	1

C.

	0	1	
→0	1	3	1
1	/	2	0
2	1,5	/	1
3	4	/	0
4	5	3	1
5	5	/	1

U.

	0	1	
→0	6	8	0
6	/	7	0
7	/	/	1
8	9	/	0
9	/	/	1

* CONTINUE ON BACK PAGE

	0	1	
→ 0	1	3	1
1	/	2	0
2	1,5	/	1
3	4	/	0
4	5	3	1
5	5	/	1

	0	1	
→ 0	6	8	0
6	/	7	0
7	/	/	1
8	9	/	0
9	/	/	1

STAR

	0	1	
→ 0	6	8	1
6	/	7	0
7	6	8	1
8	9	/	0
9	6	8	1

C.

	0	1	
→ 0	1	3	1
1	/	2	0
2	1,5,6	8	1
3	4	/	0
4	5,6	3,8	1
5	5,6	8	1
6	/	7	0
7	6	8	1
8	9	/	0
9	6	8	1

NFA

		0	1	
A	→ 0	1	3	1
B	1	/	2	0
C	3	4	/	0
D	/	/	/	0
E	2	1,5,6	8	1
F	4	5,6	3,8	1
G	1,5,6	5,6	2,7,8	1
H	8	9	/	0
I	5,6	5,6	7,8	1
J	3,8	4,9	/	0
K	2,7,8	1,5,6,9	8	1
L	9	6	8	1
M	6	/	7	0
N	7,8	6,9	8	1
O	4,9	5,6	3,8	1
P	7	6	8	1
Q	1,5,6,9	5,6	2,7,8	1
R	6,9	6	7,8	1

DFA

SIMPLIFICATION

FINAL AUTOMATON

	0	1	
→ A	B	C	1
B	D	E	0
C	F	D	0
D	D	D	0
E	G	H	1
F	I	J	1
G	I	K	1
H	L	D	0
I	I	N	1
J	O	D	0
K	Q	H	1
L	M	H	1
M	D	P	0
N	R	H	1
O	I	J	1
P	M	H	1
Q	I	K	1
R	M	N	1

REDUCTION CHECK

0: BCDHJM | 1: NAEFGIKLOPQR

D | BM | CJH | Q | NEKO | R | F | IG | LAP

D | B | M | C | J | H | Q | N | E | K | O | R | F | I | G | LAP

D | B | M | C | J | H | Q | N | E | K | O | R | F | I | G | L | A

CAN'T BE REDUCED