COSC 3340
Examination 2
Wednesday, March 12, 2008, 1 – 2:30 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: $G = (\{S,A,B\}, \{a,b\}, \{S \to A|bB, A \to a, B \to Sb\}, S)$.
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:

$$G = (\{S,A,B\}, \{a,b,c\}, \{S \to aAb|AB, A \to cS|\varepsilon, B \to Sb|\varepsilon\}, S).$$

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(011)^* \cap \overline{0(01^*)^*}\ ]^*$$

Note: You <u>must</u> first determine nfas for (011)* and 0(10*)*, then handle the intersection, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

$$G = (\{S,B\}, \{a,b,c,d\}, \{S \to aSBb|cd, B \to cSdB|S|cba\}, S).$$

Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

$$G = (\{S,A,B\}, \{a,b\}, \{S \to AA, A \to BS|a, B \to SA|b\}, S).$$

Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points: 1: 15 2: 10 3: 30 4: 15 5: 30

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(1)

1	10
2	10
3	25
4	15
5	30
	90

Ques 2)

$S \rightarrow aAb \mid BA$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow bS \mid \varepsilon$

(i) $B \rightarrow \varepsilon$

$S \rightarrow aAb \mid BA \mid A$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow bS$

(ii) $A \rightarrow \varepsilon$

$S \rightarrow aAb \mid ab \mid BA \mid B \mid A \mid \varepsilon$

$A \rightarrow cS$

$B \rightarrow bS$

(iii) $S \rightarrow \varepsilon$

$S \rightarrow aAb \mid ab \mid BA \mid B \mid A$

$A \rightarrow cS \mid c$ ✓ 10

$B \rightarrow bS \mid b$

Ques 4)

$S \rightarrow aScBb \mid cd$

$B \rightarrow cSdBd \mid S \mid cba$

All symbols are useful and there are no ε productions

We now eliminate unit production $B \rightarrow S$

$S \to aScBb \mid aScSb \mid cd$

$B \to cSdBd \mid cSdSd \mid cba$

We now convert to C.N.F

$S \to X_a S X_c B X_b \mid X_a S X_c S X_b \mid X_c X_d$

$B \to X_c S X_d B X_d \mid X_c S X_d S X_d \mid X_c X_b X_a$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d.$

$S \to X_a S_1 \mid X_a S_4 \mid X_c X_d$

$S_1 \to S S_2$ $\quad$ $S_4 \to S S_5$

$S_2 \to X_c S_3$ $\quad$ $S_5 \to X_c S_6$

$S_3 \to B X_b$ $\quad$ $S_6 \to S X_b$

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$B \to X_c B_1 \mid X_c B_4 \mid X_c B_7$

$B_1 \to S B_2.$ $\quad$ $B_4 \to S B_5$ $\quad$ $B_7 \to X_b X_a$

$B_2 \to X_d B_3$ $\quad$ $B_5 \to X_d B_6$

$B_3 \to B X_d$ $\quad$ $B_6 \to S X_d$

Q5)

$S \to AS$

$A \to BS \mid a$

$B \to SS \mid b$

$i = 1$ We are done

$i = 2$ We are done

$i=3 \quad j=1 \qquad B \to ASS \mid b$

$i=3 \quad j=2 \qquad B \to BSSS \mid aSS \mid b$

✓ $B \to aSS \mid b \mid aSSB' \mid bB'$

$B' \to SSS \mid SSSB'$

Putting production of B in A

✓ $A \to aSSS \mid bS \mid aSSB'S \mid bB'S \mid a$

using A we find production of S

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✓ $S \to aSSSS \mid bSS \mid aSSB'SS \mid bB'SS \mid aS$

substitute S in B'

✓ $B' \to aSSSSSS \mid bSSSS \mid aSSB'SSSS \mid bB'SSSS \mid aSS$
$aSSSSSSB' \mid bSSSSB' \mid aSSB'SSSSB' \mid bB'SSSSB' \mid aSSS$

Ques!) ?

$S \to A \to a \mid b$
$\to bB \to bSb \to bbBb \to b^2Sb^2$

$bSb \to bAb \to bab \quad bbb$

$b^2Sb^2 \to b^2Ab^2 \to b^2ab^2 \quad b^2bb^2$

So the language of Grammar will be

$L \; \{ b^n a b^n : n \geq 0 \}$
$\cup \; b^n b b^n$

To show L is not regular
Assume L is regular

$\Rightarrow \exists$ a DFA $\underset{\sim}{D}$ such that $L(\underset{\sim}{D}) = L$

Let N be number of states of DFA $\underset{\sim}{D}$

Let $u = b^N a b^N$

Now $u \in L$ $\therefore$ $u \in L(D)$

Let $W = b^N$ $v = a b^N$

As $|w| = N$ $\therefore$ By Pumping Lemma $\exists$ w_1, w_2, w_3 such that

$w = w_1 w_2 w_3$ & $|w_2| \geq 1$ and

$\tau(q_0, w) = \tau(q_0, w_1 w_2^s w_3)$ $\forall$ $s \geq 0$ $\in L(D)$

in particular taking $s = 0$ we get

$\tau(q_0, w) = \tau(q_0, w_1 w_3)$

$w = b^N$

$|w_1 w_3| = N - |w_2| < N$ $(\because |w_2| \geq 1)$

$\therefore$ $w_1 w_3 = b^{N - |w_2|} a \cdot b^N \notin L$

Also $\tau(q_0, u) = \tau(q_0, wv)$

$= \tau((q_0, w), v)$

$= \tau((q_0, w_1 w_3), v)$

$\in L(D)$

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but $b^{N-|w_2|} a b^N \notin L$

$\therefore$ we have a contradiction to our assumption that L is regular, $\therefore$ L is not regular

(5)

Ques 3) $[(011^*)^* \cap \overline{0(01^*)^*}]^*$ over $\{0,1,2\}$

Let $A = (011^*)^*$ $B = \overline{0(01^*)^*}$

We have $[A \cap B]^*$

$= [\overline{\overline{A} \cup \overline{B}}]^*$

$= [\overline{\overline{(011^*)^*} \cup \overline{\overline{0(01^*)^*}}}]^*$

$= [\overline{\overline{(011^*)^*} \cup 0(01^*)^*}]^*$

$[\overline{\overline{(011^*)^*} \cup 0(01^*)^*}]^*$

	0	1	2	
→0	1	-	-	0
1	-	-	-	1

	0	1	2	
→0	-	2	-	0
2	-	-	-	1

	0	1	2	
→0	-	3	-	0
3	-	-	-	1

	0	1	2	
→0	4	-	-	0
4	-	-	-	1

	0	1	2	
→0	5	-	-	0
5	-	-	-	1

	0	1	2	
→0	-	6	-	0
6	-	-	-	1

	0	1	2	
→0	1	-	-	0
1	-	2	-	0
2	-	-	-	1

↓*

	0	1	2	
→0	-	3	-	1
3	-	3	-	1

↓*

	0	1	2	
→0	-	6	-	1
6	-	6	-	1

	0	1	2	
→0	5	-	-	0
5	-	6	-	1
6	-	6	-	1

↓*

	0	1	2	
→0	5	-	-	1
5	5	6	-	1
6	5	6	-	1

	0	1	2	
→0	1	-	-	0
1	-	2	-	0
2	-	3	-	1
3	-	3	-	1

↓*

*

	0	1	2	
→0	1	–	–	1
1	–	2	–	0
2	1	3	–	1
3	1	3	–	1

	0	1	2	
→0	4	–	–	0
4	5	–	–	1
5	5	6	–	1
6	5	6	–	1

DFA

		0	1	2	
A	→0	1	–	–	1
B	1	–	2	–	0
C	–	–	–	–	0
D	2	1	3	–	1
E	3	1	3	–	1

Rename

	0	1	2	
→A	B	C	C	1
B	C	D	C	0
C	C	C	C	0
D	B	E	C	1
E	B	E	C	1

Compliment →

	0	1	2	
→A	B	C	C	0
B	C	D	C	1
C	C	C	C	1
D	B	E	C	0
E	B	E	C	0

U

	0	1	2	
→A	B, 4	C	C	0
B	C	D	C	1
C	C	C	C	1
D	B	E	C	0
E	B	E	C	0
4	5	–	–	1
5	5	6	–	1
6	5	6	–	1

↓ Dfa

		0	1	2	
0	→A	B, 4	C	C	0
1	B, 4	C, 5	D	C	1
2	C	C	C	C	1
3	C, 5	C, 5	C, 6	C	1
4	D	B	E	C	0
5	C, 6	C, 5	C, 6	C	1
6	B	C	D	C	1
7	E	B	E	C	0

↓ Rename

	0	1	2	
→0	1	2	2	0
1	3	4	2	1
2	2	2	2	1
3	3	5	2	1
4	6	7	2	0
5	3	5	2	1
6	2	4	2	1
7	6	7	2	0

Compliment →

	0	1	2	
→0	1	2	2	1
1	3	4	2	0
2	2	2	2	0
3	3	5	2	0
4	6	7	2	1
5	3	5	2	0
6	2	4	2	0
7	6	7	2	1

↓ *

	0	1	2	
→0	1	2	2	1
1	3	4	2	0
2	2	2	2	0
3	3	5	2	0
4	1, 6	2, 7	2	1
5	3	5	2	0
6	2	4	2	0
7	1, 6	2, 7	2	1

		0	1	2	
A	→ 0	1	2 2		1
B	1	3	4 2		0
C	2	2	2 2		0
D	3	3	5 2		0
E	4	1,6	2,7	2	1
F	5	3	5	2	0
G	1,6	2,3	4	2	0
H	2,7	12,6	2,7	2	1
I	2,3	23	25	2	0
J	126	23	24	2	0
K	25.	23	25	2	0
L	24	126	27	2	1

Rename →

	0	1	2	
→ A	B	C	C	1
B	D	E	C	0
C	C	C	C	0
D	D	F.	C	0
E	G	H	C	1
F	D	F	C	0
G	I	E	C	0
H	J	H	C	1
I	I	K	C	0
J	I	L	C	0
K	I	K.	C	0
L	J	H	C	1

accepting	rejecting
A E H L	B C D F G I J K
A | EHL	BG | CDFIK | J
A | E | HL.	BG | CDFIK | J
A | E | HL	BG | CDFIK | J
1 2 3	4 5 6.

25

	0	1	2	
→ 1	4	5	5	1
2	4	3	5	1
3	6	3	5	1
4	5	2	5	0
5	5	5	5	0
6.	5	3	5	0