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COSC 3340/6309
Examination 3
Wednesday, June 24, 2015, 10 am – 12 noon
Open Book and Notes

20 **1**. Prove that the following language L is not contextfree:

$L = \{ a^i b^j a^i \mid j > i \geq 1 \}$.

12 **2**. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^{3i} 1^i \mid i \geq 0 \}$ where $L = L(\mathbb{P})$ (acceptance **by final state**).

18 **3**. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, E)$ with $T = \{ id,*,/,(,) \}$,

$N = \{E\}$, and $P = \{ E \to E*E \mid E/E \mid (E) \mid id \}$.

Note: You must use the construction "cfg → pda" given in class. Get G into GNF first!

29 = 29 **4**. Construct a grammar for L(G) for the language N($\mathbb{P}$):

$\mathbb{P} = (\{p,q\}, \{a,b\}, \{Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,b,Z) = \{(p,XZ)\}$ 4 $\delta(q,\varepsilon,Z) = \{(q,\varepsilon)\}$ 1 $\delta(p,b,X) = \{(p,XX)\}$ 4

$\delta(q,b,Z) = \{(p, XZ)\}$ 4 $\delta(q,a,X) = \{(q, \varepsilon)\}$ 1 $\delta(p,a,X) = \{(p,\varepsilon)\}$. 1

20 **5**. Construct a Turing machine for the language in Question 1,

$L = \{ a^i b^j a^i \mid j > i \geq 1 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: 1: 20 2: 12 3: 18 4: 30 5: 20

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Exam 3

Prove

1.) $L = \{a^i b^j a^i \mid j > i \geq 1\}$ is not a CFL

Proof Indirect. Assume L is CFL,
$\Rightarrow \exists\ G = (N, T, P, S)$ in Chomsky Normal Form
s.t. $L = L(G)$

Let $z \in L(G)$, by Pumping Lemma $z = uvwxy$ where
$|vx| \geq 1$, so then $uv^s w x^s y \in L(G)\ \forall s \geq 0$

Consider $z = a^{2^n} b^{2^n} \cdot a^{2^n}$

Case 1: v and x are only in a's on the left side.
if $s = 2$, then there are too many a's on left

Case 2: v, x are for a's on the right side.
if $s = 2$, then right side a's have too many

Case 3: v, x are for only b's.
if $s = 0$, then there are not enough b's

Case 4: v, x are for left a's and b's.
if $s = 2$, then there are too many left a's

Case 5: v, x are for right a's and b's.
if $s = 2$, then there are too many right a's

Case 6: v, x are for both left and right a's.
if $s = 2$, then there are too many a's, not enough

Contradiction: $z \in L(G)$ but $\notin L$
Each case above results in a contradiction

2.) $L = \{0^{3i}1^i \mid i \geq 0\}$ $L = L(P)$ (accepted by final state)

		0	1	ε
q_0	Z_0	$(q_0, ZZZZ_0)$	/	(q_1, ε)
	Z	$(q_0, ZZZZ)$	(q_1, ε)	—
q_1	Z_0	/	/	(q_1, ε)
	Z	/	(q_1, ε)	/

δ'

		0	1	ε
q_0'	Z_0'	/	/	(q_0, Z_0Z_0)
	Z_0	/	/	
	Z	/	/	
q_0	Z_0'	/	/	/
	Z_0	$(q_0, ZZZZ_0)$	/	(q_1, ε)
	Z	$(q_0, ZZZZ)$	(q_1, ε)	/
q_1	Z_0'	/	/	(q_f, ε)
	Z_0	/	/	(q_1, ε)
	Z	/	(q_1, ε)	/
Accepting q_f	Z_0'	/	/	/
	Z_0	/	/	
	Z	/	/	

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Exam 3 cont'd

3.) $L = \mathcal{L}(G)$ where $G = (T, N, P, E)$ with $T = \{id, *, /, (,)\}$
$N = \{E\}$, and $P = \{E \to E*E \mid E/E \mid (E) \mid id\}$

$E \to E*E \mid E/E \mid (E) \mid id$

GNF

remove left recursion

$E \to (E) \mid id \mid (E)E' \mid id\,E'$
$E' \to *E \mid /E \mid *EE' \mid /EE'$

GNF
$E \to (EX_1 \mid id \mid (EX_1E' \mid id\,E'$
$E' \to *E \mid /E \mid *EE' \mid /EE'$

let $X_1 \to)$

		id	*	/	(	)	ε
q	E	$(q, \varepsilon)(q, E')$	–	–	$(q, EX_1)(q, EX_1E')$		–
	E'	–	$(q, E)(q, EE')$	$(q, E)(q, EE')$	–		–
empty	X_1	–	–	–	–	(q, ε)	–

$P_N = (\{q\}, \{id, *, /, (,)\}, \{E, E', X_1\}, \delta, q, E, \emptyset\}$
Top of the stack is the left

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$P = (\{p, q\}, \{a, b\}, \{Z, X\}, \delta, p, Z, \emptyset)$

4.)

		a	b	ε
p	Z	/	(p, XZ)	/
	X	(p, ε)	(p, XX)	/
q	Z	/	(p, XZ)	(q, ε)
	X	(q, ε)	/	/

✓

1. S → [p, Z, p] | [p, Z, q] ✓

2. (p, XZ) ∈ δ(p, b, Z)

$2^2 = 4$ productions

[p, Z,] → b[p, X,][, Z,]

[p, Z, p] → b[p, X, p][p, Z, p] | b[p, X, q][q, Z, p]

[p, Z, q] → b[p, X, p][p, Z, q] | b[p, X, q][q, Z, q]

✓

$2^2 = 4$ productions

(p, XZ) ∈ δ(q, b, Z)

[q, Z,] → b[p, X,][, Z,]

[q, Z, p] → b[p, X, p][p, Z, p] | b[p, X, q][q, Z, p]

[q, Z, q] → b[p, X, p][p, Z, q] | b[p, X, q][q, Z, q]

(p, XX) ∈ δ(q, b, X)

$2^2 = 4$ productions

[p, X,] → b[p, X,][, X,]

[p, X, p] → b[p, X, p][p, X, p] | b[p, X, q][q, X, p] ✓

[p, X, q] → b[p, X, p][p, X, q] | b[p, X, q][q, X, q]

$2^0 = 1$ production	$2^0 = 1$	$2^0 = 1$
(q, ε) ∈ δ(q, ε, Z)	(q, ε) ∈ δ(q, a, X)	(p, ε) ∈ δ(p, a, X)
[q, Z, q] → ε ✓	[q, X, q] → a ✓	[p, X, p] → a ✓

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Exam 3 Cont'd

5.) $L = \{ a^i b^j a^i \mid j > i \geq 1 \}$

ex:

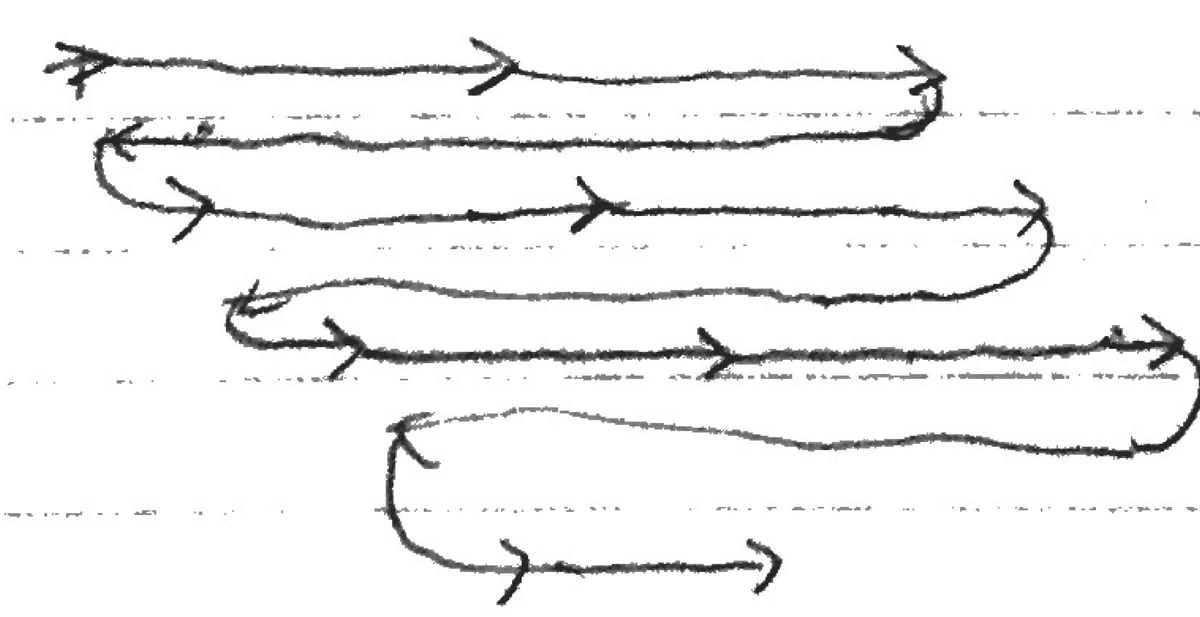
aaa bbbbb $\hat{a}\hat{a}\hat{a}$ $\cancel{b}$

use A, B and $\bar{A}$ as "visited" letters

Let the a's on the right be named $\hat{a}$ and $\cancel{b}$ be blank symbol
# of a's are equal to # of $\hat{a}$'s and are less than # of b's

Configure the Turing machine from the left, find the first a and change it to A, then move right, until we find b, change it to B. Move right until we find $\hat{a}$, change it to $\bar{A}$. Now move left until we find the first A, then stop and move right to find the first a and repeat the above process until B is the next state from A, and then find all b and change them to B.

a a a b b b b $\hat{a}$ $\hat{a}$ $\hat{a}$



	a	b	$\hat{a}$	A	B	$\bar{A}$	$\not b$
q_0	(q_1, A, R)	–	–	–	–	–	–
q_1	(q_1, a, R)	(q_2, B, R)	–	–	–	–	–
q_2	–	(q_2, b, R)	$(q_3, \bar{A}, L)$	–	–	$(q_2, \bar{A}, R)$	–
q_3	(q_3, a, L)	(q_3, b, L)	–	(q_4, A, R)	(q_3, B, L)	$(q_3, \bar{A}, L)$	–
q_4	(q_1, A, R)	–	–	–	(q_5, B, R)		
q_5	–	(q_6, B, R)	–	–	(q_5, B, R)	$(q_5, \bar{A}, R)$	$(q_f, \not b, L)$
accepting state → q_f	–	–	–	–	–	–	–