Math 3339

Homework 6 (Sections 5.1 – 5.4)

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Instructions:

- Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
- Print out this file and complete the problems.
- Use blue or black ink or a dark pencil.
- Write your solutions in the space provided. You must show all work for full credit.
- Submit this assignment at http://www.casa.uh.edu under "Assignments" and choose **HW6.**
- **Total possible points, 15.**

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1. For $0 \le x \le 1$ let $f(x) = kx(1-x)$, where k is a constant. Find the value of k such that f is a density function.

$$\int_0^1 kx(1-x) = 1$$

$$1 = \int_0^1 kx(1-x)\,dx = k\int_0^1 x - x^2\,dx$$

$$= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= k\left[\frac{1}{2} - \frac{1}{3} - 0\right]$$

$$= k\left[\frac{3}{6} - \frac{2}{6}\right] = \frac{k}{6} = 1$$

$$\Rightarrow \boxed{k = 6}$$

2. Find the mean and variance of the distribution in the preceding exercise.

$$E[x] = \int_0^1 x \cdot f(x)\, dx$$
$$= \int_0^1 x \cdot 6x(1-x)\, dx$$
$$= \int_0^1 6x^2(1-x)\, dx$$
$$= 6\int_0^1 x^2 - x^3\, dx$$
$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$
$$= 6\left[\overset{\frac{4}{12}}{\frac{1}{3}} - \overset{\frac{3}{12}}{\frac{1}{4}}\right]$$
$$= 6 \cdot \frac{1}{12} = \boxed{1/2}$$

$$E[V] = E[(x-\mu)^2]$$
$$= \int_0^1 (x - 1/2)^2\, dx$$
$$= \int_0^1 x^2 - x + \frac{1}{4}\, dx$$
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{4}x\right]_0^1$$
$$= \frac{1}{3} - \frac{1}{2} + \frac{1}{4}$$
$$= \frac{8}{24} - \frac{12}{24} + \frac{6}{24} = \frac{2}{24}$$
$$= \boxed{1/12}$$

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

3. Find the cumulative distribution for the previous density function.

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

4. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 \le x \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the probability that over a period of one year, a family runs their vacuum cleaner less than 120 hours.

b) Find the probability that over a period of one year, a family runs their vacuum cleaner between 50 and 100 hours.

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

5. Suppose that a random variable X has a cdf of:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{9}, & 0 \le x < 3 \\ 1, & x \ge 3 \end{cases}$$

a) Determine $P(X \le 1)$.
b) Determine $P(0.75 \le X \le 1.35)$.
c) Determine c for the following probability $P(X \le c) = 0.01$
d) Determine the median of this probability function.
e) Determine the mean, $E(X)$.

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

6. * Suppose the reaction temperature X (in °C) in a certain chemical process has a uniform distribution by $X \sim \text{Unif}(-5, 5)$.

   a. Give the pdf of X.

   b. Compute $P(X < 0)$.

   c. Compute $P(-2.5 < X < 2.5)$

   d. For k satisfying $-5 < k < k+4 < 5$, compute $P(k < X < k+4)$.

* Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

7. Suppose the time spent by a randomly selected student at a campus computer lab has a gamma distribution with mean 20 minutes and variance 80 minutes^2.

   a) What are the values of α and β?

   $E[X] = 20 = \alpha * \beta$

   $V[X] = 80 = \alpha * \beta^2$

   $A = 20 / \beta$

   $80 = (20 / \beta) * \beta^2 = 20 * \beta \Rightarrow \beta = (80 / 20) = 4$

   $20 = \alpha * 4 \Rightarrow \alpha = 5$

   b) What is the probability that a student spends less than 10 minutes at a campus computer lab?

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> pgamma(10, 5, 1/4)
[1] 0.108822
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   c) What is the probability that a student spends at least 1 hours at a campus computer lab?

   $P(X >= 60) = 1 – P(X < 60)$

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> 1 - pgamma(60, 5, 1/4)
[1] 0.0008566412
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∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

8. Let X denote the distance an animal moves from its birth site to the first territorial vacancy it encounters. Suppose that X has an exponential distribution with parameter $\lambda = 0.024$.
   a) What is the probability that the distance is at most 150m?

   P(X <= 150m) = pexp(150, .024) = 0.9726763

   b) What is the expected value of the distance an animal moves from its birth site to the first territorial vacancy it encounters?

   $E[X] = 1 / \lambda = 1 / .024 = 41.66667$

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

For questions 9 & 10 circle your best answer.

9. A continuous random variable may assume
   a. any value in an interval or collection of intervals
   b. only integer values in an interval or collection of intervals
   c. only fractional values in an interval or collection of intervals
   d. only the positive integer values in an interval
   e. none of these

10. A description of the distribution of the values of a random variable and their associated probabilities is called a
    a. probability distribution
    b. random variance
    c. random variable
    d. expected value
    e. none of these

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.