Monday, June 21, 2010, 2 – 3:45 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B,C}, {a,b}, {S→A|Bb, A→C, B→bbS, C→a }, S).
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:

G = ({S,A,B}, {a,b,c}, {S→aA|BBBb, A→cS|ε, B→Sb|ε}, S).

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(010)^* \cap \overline{0(10^*)^*}\ ]^*$$

Note: You <u>must</u> first determine nfas for (010)* and 0(01*)*, then handle the intersection and complementation, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→aSBBB|B|cd, B→cSda|S|cba}, S).

Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

G = ({S,A,B}, {a,b}, {S→AA, A→BSS|a , B→SSA|b}, S).

Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points: 1: 15 2: 10 3: 30 4: 15 5: 30

Production:

$S \to A \mid Bb$

$A \to C$

$B \to bbS$

$C \to a$

① determine $L(G)$

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$S \to A \mid Bb$

$S \to Bb \to bbSb \to bbBbb$

$\to A \to C \to a$ bbS $bbAb$ $bbab$ $bbbbSbb$

$\to bbbbabb \to b^4ab^4$

$L(G) = \{ b^n a b^n \mid n \geq 0 \}$ -5

Assuming $L(G)$ is regular and belongs to dfa, and D will have n states total.

If $x = b^n a b^n = K \cdot \Delta$ $K = b^n$, $\Delta = ab^n$

$\because |K| = n \therefore$ pumping Lemma applies. $K = K_1 K_2 K_3$

such that $|K_2| \geq 1 \therefore \tau(q_0, K) = \tau(q_0, K_1 (K_2)^{\xi} K_3)$ $\xi \geq 0$

when $\xi_1 = 0$

$|K_1 \cdot K_2| = n - |K_2| < n$, so $\tau(q_0, K_1 K_3)$ does Not belong to $L(G)$

$\therefore \tau(q_0, K) \in L(G)$, but $\tau(q_0 K_1 K_3) \notin L(G)$

This is the contradiction which makes the $L(G)$ not regular

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2) $S \to aA \mid BBBb$
$A \to cS \mid \varepsilon$
$B \to Sb \mid \varepsilon$

$A \to \varepsilon$

$S \to aA \mid BBBb \mid a$
$A \to cS$
$B \to Sb$

$B \to \varepsilon$

$S \to aA \mid BBBb \mid a \mid BBb \mid Bb \mid b$
$A \to cS$
$B \to Sb$

$S \to \varepsilon$ -1

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$S \to aA \mid BBBb \mid a \mid BBb \mid Bb \mid b$
$A \to cS \mid c$ ✗
$B \to Sb \mid b$ ✗ -2

$$\left[(010)^* \cap \overline{0(10^*)^*} \right]^* \Rightarrow \left[\overline{(010)^*} \cup 0(10^*)^* \right]^*$$

1 2 3   4 5 6

	0	1	2	
→0	1	–	–	0
1	–	–	–	1

	0	1	2	
→0	–	2	–	0
2	–	–	–	1

	0	1	2	
→0	3	–	–	0
3	–	–	–	1

	0	1	2	
→0	4	–	–	0
4	–	–	–	1

	0	1	2	
→0	–	5	–	0
5	–	–	–	1

	0	1	2	
→0	6	–	–	0
6	–	–	–	1

*

	0	1	2	
→0	6	–	–	1
6	6	–	–	1

	0	1	2	
→0	1	–	–	0
1	–	2	–	0
2	–	–	–	1

	0	1	2	
→0	–	5	–	0
5	6	–	–	1
6	6	–	–	1

*

	0	1	2	
→0	1	–	–	0
1	–	2	–	0
2	3	–	–	0
3	–	–	–	1

	0	1	2	
→0	–	5	–	1
5	6	5	–	1
6	6	5	–	1

*

	0	1	2	
→0	4	–	–	0
4	–	5	–	1
5	6	5	–	1
6	6	5	–	1

	0	1	2	
→0	1	–	–	1
1	–	2	–	0
2	3	–	–	0
3	1	–	–	1

DFA →

		0	1	2	
→A	0	1	–	–	1
B	1	–	2	–	0
C	–	–	–	–	0
D	2	3	–	–	0
E	3	1	–	–	1

rename

	0	1	2	
→A	B	C	C	1
B	C	D	C	0
C	C	C	C	0
D	E	C	C	0
E	B	C	C	1

	0	1	2	
→A	B	C	C	0
B	C	D	C	1
C	C	C	C	1
D	E	C	C	1
E	B	C	C	0

∪

	0	1	2	
→A	B,4	C	C	0
B	C	D	C	1
C	C	C	C	1
D	E	C	C	1
E	B	C	C	0
4	–	5	–	1
5	6	5	–	1
6	6	5	–	1

DFA

	0	1	2	
→A	B,4	C	C	0
B,4	C	D,5	C	1
C	C	C	C	1
D,5	E,6	C,5	C	1
E,6	B,6	C,5	C	1
C,5	C,6	C,5	C	1
B,6	C,6	D,5	C	1
C,6	C,6	C,5	C	1

Rename →

	0	1	2	
1	2	3	3	0
2	3	4	3	1
3	3	3	3	1
4	5	6	3	1
5	7	6	3	1
6	8	6	3	1
7	8	4	3	1
8	8	6	3	1

$\xrightarrow{\overline{\text{compliment}}}$

	0	1	2	
1	2	3	3	1
2	3	4	3	0
3	3	3	3	0
4	5	6	3	0
5	7	6	3	0
6	8	6	3	0
7	8	4	3	0
8	8	6	3	0

$\xrightarrow{*}$

	0	1	2	
1	2	3	3	1
2	3	4	3	0
3	3	3	3	0
4	5	6	3	0
5	7	6	3	0
6	8	6	3	0
7	8	4	3	0
8	8	6	3	0

Reduce $\rightarrow$

1	2 3 4 5 6 7 8

1	2 3 4 5 6 7 8
A	B

	0	1	2	
A	B	B	B	1
B	B	B	B	0

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7)

$S \to aSBBB \mid \underline{B} \mid cd$

$B \to cSda \mid S \mid cba$

$B \to S$

$S \to aSBBB \mid cd \mid aSSBB \mid aSSSB \mid aSSSS \mid aSBSS \mid$

-1 missing / $aSBBS \mid aSSBS$

$B \to cSda \mid cba$

$S \to B$ -S

CNF

$S \to X_a SBBB \mid X_c X_d \mid X_a SSBB \mid X_a SSSB \mid X_a SSSS \mid$

$X_a SBSS \mid X_a SBBS \mid X_a SSBS$

$B \to X_c S X_d X_a \mid X_c X_b X_a$

$X_a \to a$ $\quad X_b \to b$ $\quad X_c \to c$ $\quad X_d \to d$

$S \to X_a S_1 \mid X_c X_d \mid X_a S_4 \mid X_a S_7 \mid X_a S_{10} \mid X_a S_{13} \mid$

$X_a S_{16} \mid X_a S_{19}$

$S_1 \to S S_2$

$S_2 \to B S_3$

$S_3 \to BB$

$S_4 \to S S_5$

$S_5 \to S S_6$

$S_6 \to BB$

$S_7 \to S S_8$

$S_8 \to S S_9$

$S_9 \to SB$

$S_{10} \to S S_{11}$

$S_{11} \to S S_{12}$

$S_{12} \to SS$

$S_{13} \to S S_{14}$

$S_{14} \to B S_{15}$

$S_{15} \to SS$

$S_{16} \to S S_{17}$

$S_{17} \to B S_{18}$

$S_{18} \to BS$

$S_{19} \to S S_{20}$

$S_{20} \to S S_{21}$

$S_{21} \to BS$

$B \to X_c S \mid X_d X_a \mid X_c X_b X_a$

$B \to X_c B_1 \mid X_c B_3$

$B_1 \to S B_2$

$B_2 \to X_d X_a$

$B_c \to X_b X_a$

$X_a \to a \quad X_b \to b \quad X_c \to c \quad X_d \to d$

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$S \to AA$
$A \to BSS \mid a$
$B \to SSA \mid b$

no useless
no ε - production
no unit production

$i=1$ $S \to AA$ no immideate left recurssion

$i=2$, $j=1$ $A \to BSS \mid a$ no immideate left R.

$i=3$ $j=1$ $B \to \underline{A}ASA \mid b$

$j=2$ $B \to \underline{B}SSASA \mid aASA \mid b$

immidiate left R. found

$$B \to B\underbrace{SSASA}_{X} \mid \underbrace{aASA}_{B_1} \mid \underbrace{b}_{B_2}$$

$$B \to aASA \mid b \mid a\cancel{ASA}B' \mid bB'$$

$$B' \to SSASA \mid \cancel{S}SASAB'$$

$S \to AA$
$A \to \underline{B}SS \mid a$
$B \to \underline{aASA} \mid b \mid aASAB' \mid bB'$
$B' \to SSASA \mid SSASAB'$

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$A \to \underline{aASASS} \mid bSS \mid \underline{aASA}B'SS \mid bB'SS \mid a$
$S \to aASASSA \mid bSSA \mid aA\cancel{SA}B'SSA \mid AB'SSA \mid aA$
$B \to aASA \mid b \mid aAS\cancel{AB'} \mid bB'$

$B' \longrightarrow$ back

$B' \rightarrow aASASSASASA \mid bSSASASA \mid aASAB'SSASASA$

$\mid bB'SSASASA \mid aASASA \mid aASASSASASAB' \mid$

$bSSASASAB' \mid aASAB'SSASASAB' \mid$

$bB'SSASASAB' \mid aASASAB'$