COSC 3340
Examination 2
Monday, March 30, 2009, 1 – 2:30 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B}, {a,b}, {S→A|aB, A→b, B→Sa}, S).
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:

G = ({S,A,B}, {a,b,c}, {S→aAb|AB, A→cS|ε, B→Sb|ε}, S).

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(001)^* \cap \overline{0(01^*)^*}\,]^*$$

Note: You <u>must</u> first determine nfas for (001)* and 0(01*)*, then handle the intersection, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→aSBB|cd, B→cSdB|S|cba}, S).
Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

G = ({S,A,B}, {a,b}, {S→AA, A→BS|b , B→SA|a}, S).
Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points:	1: 15	2: 10	3: 30	4: 15	5: 30

1	15
2	10
3	30
4	15
5	30
	100

1 Answer

We have the following productions:

$S \to A \mid aB$

$A \to b$

$B \to Sa$

Determin $L(G)$ from above

$S \to aB \to aSa \to aaBa \to aaSaa \to a^3Ba^2 \to a^3Sa^3 \to \cdots$

$S \to A \to b$

$aSa \to aAa \to aba$

$aaSaa \to a^2Aa^2 \to a^2ba^2$

$a^3Sa^3 \to \cdots \to \cdots$

So, the language of grammer G will be

$L(G) = \{a^n b a^n \mid n \geq 0\}$

Assume $L(G)$ is a regular language, so $\exists$ dfa D which has n states

Consider $x = a^n b a^n = wu$, and $w = a^n$, $u = ba^n$

Since $|w| = n$, we can apply pumping lemma here

$\Rightarrow w = w_1 w_2 w_3$ st. $|w_2| \geq 1$

So, $\tau(q_0, w) = \tau(q_0, w_1 (w_2)^s w_3)$ $s \geq 0$

15

Consider $s = 0$

We have $\tau(q_0, w) = \tau(q_0, w_1 w_3)$

BUT, $|w_1 w_3| = n - |w_2| < n$, so $\tau(q_0, w_1w_3) \notin L(G)$

Since $\tau(q_0, w) \in L(G)$, but $\tau(q_0, w_1 w_3) \notin L(G)$

Hence, we have a contradiction here

Thus, $L(G)$ is not regular.

2 Answer

$S \to aAb \mid AB$
$A \to cS \mid \varepsilon$
$B \to Sb \mid \varepsilon$

① Eliminate $A \to \varepsilon$

$S \to aAb \mid ab \mid AB \mid B$
$A \to cS$
$B \to Sb \mid \varepsilon$

② Eliminate $B \to \varepsilon$

$S \to aAb \mid ab \mid AB \mid A \mid B \mid \varepsilon$
$A \to cS$
$B \to Sb$

③ Eliminate $S \to \varepsilon$

$S \to aAb \mid ab \mid AB \mid A \mid B$.
$A \to cS \mid c$
$B \to Sb \mid b$

10

3 Answer

We have $[(001)^* \cap \overline{0(01^*)^*}]^*$

According to Morgan's Law, we could have:

$$[\overline{\overline{(001)^*} \cup 0(01^*)^*}]^*$$

	0	1	2	
→0	1	–	–	0
1	2	–	–	0
2	–	3	–	0
3	–	–	–	1

↓*

	0	1	2	
→0	1	–	–	1
1	2	–	–	0
2	–	3	–	0
3	1	–	–	1

↓DFA

	0	1	2	
→0	1	–	–	1
1	2	–	–	0
–	–	–	–	0
2	–	3	–	0
3	1	–	–	1

Rename →

	0	1	2	
→A	B	C	C	1
B	D	C	C	0
C	C	C	C	0
D	C	E	C	0
E	B	C	C	1

flip →

	0	1	2	
→A	B	C	C	0
B	D	C	C	1
C	C	C	C	1
D	C	E	C	1
E	B	C	C	0

	0	1	2	
→0	4	–	–	0
4	–	–	–	1

	0	1	2	
→0	5	–	–	0
5	–	–	–	1

	0	1	2	
→0	–	6	–	1
6	–	6	–	1

	0	1	2	
→0	5	–	–	0
5	–	6	–	1
6	–	6	–	1

	0	1	2	
→0	4	–	–	0
4	5	–	–	1
5	5	6	–	1
6	5	6	–	1

↓*

	0	1	2	
→0	5	–	–	1
5	5	6	–	1
6	5	6	–	1

$\cup$

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THX!

3 continued

$\xrightarrow{}$ U

	0	1	2	
→A	B,4	C	C	0
B	D	C	C	1
C	C	C	C	1
D	C	E	C	1
E	B	C	C	0
4	5	–	–	1
5	5	6	–	1
6	5	6	–	1

$\xrightarrow{}$ DFA

	0	1	2	
→A	B4	C	C	0
B4	D5	C	C	1
C	C	C	C	1
D5	C5	E6	C	1
C5	C5	C6	C	1
E6	B5	C6	C	1
C6	C5	C6	C	1
B5	D5	C6	C	1

$\xrightarrow{\text{Rename}}$

	0	1	2	
→1	2	3	3	0
2	4	3	3	1
3	3	3	3	1
4	5	6	3	1
5	5	7	3	1
6	8	7	3	1
7	5	7	3	1
8	4	7	3	1

$\xrightarrow{}$ flip

	0	1	2	
→1	2	3	3	1
2	4	3	3	0
3	3	3	3	0
4	5	6	3	0
5	5	7	3	0
6	8	7	3	0
7	5	7	3	0
8	4	7	3	0

$\xrightarrow{}$ *

	0	1	2	
→1	2	3	3	1
2	4	3	3	0
3	3	3	3	0
4	5	6	3	0
5	5	7	3	0
6	8	7	3	0
7	5	7	3	0
8	4	7	3	0

$\xrightarrow{\text{Reduced}}$

2 3 4 5 6 7 8	1
2 3 4 5 6 7 8	1
B	A

$\xrightarrow{\text{Rename}}$

	0	1	2	
→A	B	B	B	1
B	B	B	B	0

4 Answer

$S \to aSBB \mid cd$
$B \to cSdB \mid S \mid cba$

① Eliminate $B \to S$

$S \to aSBB \mid aSSB \mid aSBS \mid aSSS \mid cd$
$B \to cSdB \mid cSdS \mid cba$

② Construct CNF

<1> $S \to X_aSBB \mid X_aSSB \mid X_aSBS \mid X_aSSS \mid X_cX_d$
$B \to X_cSX_dB \mid X_cSX_dS \mid X_cX_bX_a$
$X_a \to a$, $X_b \to b$, $X_c \to c$ $X_d \to d$

<2> $S \to X_aS_1 \mid X_aS_3 \mid X_aS_5 \mid X_aS_7 \mid X_cX_d$
$S_1 \to SS_2$
$S_2 \to BB$
$S_3 \to SS_4$ 15
$S_4 \to SB$
$S_5 \to SS_6$
$S_6 \to BS$
$S_7 \to SS_8$
$S_8 \to SS$
$B \to X_cB_1 \mid X_cB_3 \mid X_cB_5$
$B_1 \to SB_2$
$B_2 \to X_dB$
$B_3 \to SB_4$ → $X_c \to c$
$B_4 \to X_dS$ $X_d \to d$
$B_5 \to X_bX_a$
$X_a \to a$
$X_b \to b$

5 Answer

$S \rightarrow AA$
$A \rightarrow BS \mid b$
$B \rightarrow SA \mid a$

$i=1$ It's ok
$i=2$ It's OK
$i=3$
$j=1$ Replace $S \rightarrow AA$, we have:
$B \rightarrow AAA \mid a$

$j=2$ Replace $A \rightarrow BS \mid b$, we have:
$B \rightarrow BSAA \mid bAA \mid a$ ✓

Now, we have no left recursion, so we have
$B \rightarrow bAA \mid a \mid bAAB' \mid aB'$
$B' \rightarrow SAA \mid SAAB'$ ✓

Now, putting production of B into A, we have
$A \rightarrow bAAS \mid aS \mid bAAB'S \mid aB'S \mid b$
Using A, we find production of S, ✓
$S \rightarrow bAASA \mid aSA \mid bAAB'SA \mid aB'SA \mid bA$ 30

So, we have
$S \rightarrow bAASA \mid aSA \mid bAAB'SA \mid aB'SA \mid bA$ ✓
$A \rightarrow bAAS \mid aS \mid bAAB'S \mid aB'S \mid b$
$B \rightarrow bAA \mid a \mid bAAB' \mid aB'$
$B' \rightarrow SAA \mid SAAB'$