COSC 3340
Examination 3
Wednesday, April 22, 2009, 1 – 2:30 pm
Open Book and Notes

1. Prove that the following language L is not contextfree:

$L = \{ 0^k 1^j 0^i \mid i>j>k\geq 0 \}$.

2. Construct a pda $\mathbb{P}$ for the following language:

$L = \{ 0^i 1^{4i} \mid i\geq 0 \}$ where $L = L_f(\mathbb{P})$ (acceptance **by final state**).

State on which side you write the top of the stack, left: ____| or right |____ .

Hint: Put four markers on the stack for every 0.

3. Construct a pda $\mathbb{P}$ that accepts the following language **by empty stack**:

$L = L(G)$ where $G = (T, N, P, S)$ with $T = \{<,>,[,]\}$,

$N = \{S,A\}$, and $P = \{ S \to <S>A \mid [A]A \ , \ A \to [A]S \mid <S>S \mid \varepsilon \}$.

State on which side you write the top of the stack, left: ____| or right |____ .

Note: You <u>must</u> use the construction "cfg → pda" given in class. Get G into GNF first!

4. Construct a grammar for L(G) for the language $N(\mathbb{P})$:

$\mathbb{P} = (\{p,q\}, \{a,b\}, Z,X\}, \delta, p, Z, \varnothing)$ where the move function δ is given by

$\delta(p,a,Z) = \{(p,XZ)\}$ $\quad$ $\delta(p,\varepsilon,Z) = \{(p,\varepsilon)\}$ $\quad$ $\delta(p,a,X) = \{(p,XX)\}$

$\delta(q,a,Z) = \{(q,\varepsilon)\}$ $\quad$ $\delta(p,b,X) = \{(q,X)\}$ $\quad$ $\delta(q,b,Z) = \{(p,Z)\}$.

Here, the top of the stack is on the left.

Steven's copy

5. Construct a Turing machine for the language in Question 1,

$L = \{ 0^k 1^j 0^i \mid i>j>k\geq 0 \}$.

Describe first in words what you are doing, then formulate the formal Turing machine.

Points: **1: 20**   **2: 12**   **3: 18**   **4: 30**   **5: 20**

GAURAV GOEL

Automata and Computability

(3)

Examination - 3

1	17	
2	12	
3	18	
4	30	
5	15	
	92	

1) $L = \{0^k 1^j 0^i \mid i > j > k \geq 0$

3 2 1

1 2 3

assume L is CFL

then $\exists$ $G(N,T,P,S)$ in CNF such that $L = L(G)$

assuming t = no. of variables, $k = 2t$, $j = 2t+1$, $i = 2t+2$

take a word $z = 0^{2t} 1^{2t+1} 0^{2t+2}$ $|z| > 2t$

by pumping lemma $z = uvwxy$ where $|vx| > 1$

and $uv^{i'}wx^{i'}y \in L(G)$

for all $i' \geq 0$

0	1	0

Case 1

v & x are all 0's, for $i' = 0$, no of trailing? zeroes can be less than no. of 1, $L(G) \notin L$

Case 2

v & x are all 1's for $i' = 2$, , $L(G) \neq L$, as no. of 1 can be equal > to 0. trailing zero's

Case 3 and 4

v ~~and~~ or x are ~~both~~ only one 0 and one 1 do not fix values in this case if $i' > 1$

$L(G) \neq L$ as the sequence of 0 and 1 might change. It is Applicable for both "01" and "10."

basically v or x contains 2 symbols

Case 5

vx contains 0's before 1 then for $i' > 1$ no. of 0's before 1 can be greater than no. of zeroes after 1 $\therefore$ $L(G) \neq L$

Case 6

No. of 0's after 1 $\in$ v and x than $L(G) \neq L$
for $i' = 0$ ous no. of zeroes after 1 can be ~~...~~ less than no. of 0's before 1.

$\therefore$ there is contradiction in this case.

17 and language L is not context free

how about $v \in 0^+, x \in 1^+$ or $v \in 1^+, x \in 0^+$

Q-2

2) First a pda is constructed for empty stack acception and than change it to being accepted by final state.

$P_e = (\{0,1\}, \{q_0, q_1\}, \{z_0, z\}, \delta, q_0, \phi, z_0)$

Top of stack is on left.

δ

		0	1	ϵ
q_0	z_0	$\{(q_0, zzzzz_0)\}$	/	$\{(q_1, \epsilon)\}$
	z	$\{(q_0, zzzzz)\}$	$\{(q_1, \epsilon)\}$	/
q_1	z_0	/	/	$\{(q_1, \epsilon)\}$
	z	/	$\{(q_1, \epsilon)\}$	/

Now for final state acception.

δ' $\quad P_f = (\{0,1\}, \{(q_0', q_0, q_1, q_f)\}, (Z_0', Z_0, Z), \delta', q_0', q_f, Z_0')$

		0	1	ϵ
q_0'	Z_0'	/	/	$\{(q_0, Z_0 Z_0')\}$
	Z_0	/	/	/
	Z	/	/	/
q_0	Z_0'	/	/	/
	Z_0	$\{(q_0, ZZZZZ_0)\}$	/	$\{(q_1, \epsilon)\}$
	Z	$\{(q_0, ZZZZZ)\}$	$\{(q_1, \epsilon)\}$	/
q_1	Z_0'	/	/	$\{(q_f, \epsilon)\}$
	Z_0	/	/	$\{(q_1, \epsilon)\}$
	Z	/	$\{(q_1, \epsilon)\}$	/
q_f	Z_0'	/	/	/
	Z_0	/	/	/
	Z	/	/	/

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Final state

$=$

3) $S \rightarrow \langle S \rangle A \mid [A] A$

$A \rightarrow [A] S \mid \langle S \rangle S \mid \epsilon$

<u>Eliminating ϵ</u>

$S \rightarrow \langle S \rangle A \mid \langle S \rangle \mid [A]A \mid [A] \mid [\,]A \mid [\,]$

$A \rightarrow [A] S \mid [\,]S \mid \langle S \rangle S$

<u>To GNF</u>

$S \rightarrow \langle S X_{>} A \mid \langle S X_{>} \mid [A X_{]} A \mid [A X_{]} \mid [X_{]} A \mid [X_{]}$

$A \rightarrow [A X_{]} S \mid [X_{]} S \mid \langle S X_{>} S$

$X_{>} \rightarrow >$

$X_{]} \rightarrow]$

$P_N = (\{<, >, [,]\}, \{q_0\}, \{S, A, X_{>}, X_{]}\}, \delta, q_0, \phi, S)$

δ

	$<$	$>$	$]$	$[$	ϵ
S	$\{(q_0, S X_{>} A), (q_0, S X_{>})\}$	/	/	$\{(q_0, A X_{]} A), (q_0, A X_{]}), (q_0, X_{]} A), (q_0, X_{]})\}$	/
A	$\{(q_0, S X_{>} S)\}$	/	/		
$X_{>}$	/	$\{(q_0, \epsilon)\}$	/	$\{(q_0, A X_{]} S), (q_0, X_{]} S)\}$	/
$X_{]}$	/	/	$\{(q_0, \epsilon)\}$	/	/
				/	/

4).

		a	b	ϵ
p	z	$\{(p, XZ)\}$	/	$\{(p, \epsilon)\}$
	x	$\{(p, xx)\}$	$\{(q, x)\}$	/
q	z	$\{(q, \epsilon)\}$	$\{(p, z)\}$	/
	x	/	/	/

$$S \longrightarrow [p, z, p] \mid [p, z, q] \quad \text{— ①}$$

for

$(p, XZ) \in \delta(p, a, z)$

$$[p, z, \substack{p\\q}] \longrightarrow a[p, X, \substack{p\\q}]\,[\substack{p\\q}, z, \substack{p\\q}]$$

② $$[p, z, p] \longrightarrow a[p, X, p]\,[p, z, p] \mid a[p, X, q]\,[q, z, p]$$

③ $$[p, z, q] \longrightarrow a[p, X, p]\,[p, z, q] \mid a[p, X, q]\,[q, z, q]$$

for

$(p, xx) \in \delta(p, a, x)$

④ $$[p, x, \substack{p\\q}] \longrightarrow a[p, X, \substack{p\\q}]\,[\substack{p\\q}, X, \substack{p\\q}]$$

⑤ $$[p\,X\,p] \longrightarrow a[p, X\,p]\,[p, x, p] \mid a[p, X, q]\,[q, X, p]$$

$$[p\,X\,q] \longrightarrow a[p, x, p]\,[p, X, q] \mid a[p, x, q]\,[q, X, q]$$

δ	$<$	$>$	$]$	$[$	ϵ
S	$\{(q_0, S X_> A), (q_0\, S X_>)\}$	/	/	$\{(q_0, A X_] A), (q_0, A X_]), (q_0, X_] A), (q_0, X_])\}$	/
A	$\{(q_0, S X_> S)\}$	/	/	$\{(q_0, A X_] S), (q_0, X_] S)\}$	/
$X_>$	/	$\{(q_0, \epsilon)\}$	/	/	/
$X_]$	/	/	$\{(q_0, \epsilon)\}$	/	/

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Top of Stack is on Left

(5) Turing Machine

$\{0^k 1^j 2^i \mid i > j > k \geq 0\}$

accepted words are :- ~~[illegible], [illegible], [illegible]~~ 100, 011000, 001110000

First we start from left and if a 0 is found we mark it 0'. Than we skip all zeroes and if we found 1 we mark it 1' and skip all 1's and keep on moving right.

Now when we found 2 mark it 2'' and move backward ~~towards~~ Left until we find one 0' and when we find it we again moving right and repeat whole process until we have marked all 0's.

Than we repeat the same process with 1's and 2's until all 1's are marked and when they all are marked we make sure that atleast one (or more) 2's are left. Than we mark it until we reach $\not b$.

In case k=0

If at start no zeroes are found we proceed as if all zeroes have been marked.

for

$(p,\epsilon) \in \delta(p,\epsilon,z)$

⑥ $[p,z,p] \rightarrow \epsilon$

for

$(q,x) \in \delta(p,b,x)$

$[p,x,\genfrac{}{}{0pt}{}{p}{q}] \longrightarrow b[q,x,\genfrac{}{}{0pt}{}{p}{q}]$

⑦ $[p,x,p] \longrightarrow b[q,x,p]$

⑧ $[p,x,q] \longrightarrow b[q,x,q]$

for

$(q,\epsilon) \in \delta(q,a,z)$

⑨ $[q,z,q] \rightarrow a$

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for

$(p,z) \in \delta(q,b,z)$

$[q,z,\genfrac{}{}{0pt}{}{p}{q}] \longrightarrow b[p,z,\genfrac{}{}{0pt}{}{p}{q}]$

⑩ $[q,z,p] \longrightarrow b[p,z,p]$

⑪ $[q,z,q] \longrightarrow b[p,z,q]$

The L(G) is obtained with production $\Rightarrow$ [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]

~~00 111 0000~~

how do you differentiate the two?

for k≠0

	0	1	$\emptyset$	0'	1'	$\emptyset''$	$\not b$
q_0	$(q_1, 0'R)$	$(q_6, 1'R)$	/	/	/	/	/
q_1	$(q_1, 0R)$	$(q_2, 1'R)$	/	/	$(q_1, 1'R)$	/	/
q_2	/	$(q_2, 1R)$	$(q_3, \emptyset''L)$	/	/	$(q_2, \emptyset''R)$	/
q_3	$(q_3, 0L)$	$(q_3, 1L)$	/	$(q_4, 0'R)$	$(q_3, 1'L)$	$(q_3, \emptyset''L)$	/
q_4	$(q_1, 0'R)$	/	/	/	$(q_5, 1'R)$	/	/
q_5	/	$(q_6, 1'R)$	/	/	$(q_5, 1'R)$	/	/
q_6		$(q_6, 1R)$	$(q_7, \emptyset''L)$	/	/	$(q_6, \emptyset''R)$	/
q_7	/	$(q_7, 1L)$	/	/	$(q_8, 1'R)$	$(q_7, \emptyset''L)$	/
q_8	/	$(q_6, 1'R)$	/	/	/	$(q_9, \emptyset''R)$	/
q_9	/	/	$(q_{10}, \emptyset''R)$	/	/	$(q_9, \emptyset''R)$	/
q_{10}	/	/	$(q_{10}, \emptyset''R)$	/	/	/	$(q_f, \not b R)$
q_f	Accepting state.						

Turing Machine for language – $0^k 1^j \emptyset^i$

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0 before 1 → 0'

0 after 1 → 0''