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COSC 3340/6309
Examination 2
Friday, June 22, 2012, 4 – 6 pm
Open Book and Notes

15/15 **1**. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B,C}, {a,b}, {S→A|bB, A→C, B→Sb, C→a }, S).
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:
10 G = ({S,A,B}, {a,b,c}, {S→aA|BBBa, A→cS|ε, B→Sb|ε}, S).

30/30 **3**. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(101)^* \cap \overline{1(01^*)^*}\ ]^*$$

Note: You <u>must</u> first determine nfas for (101)* and 1(01*)* over {01,2,}, then handle the intersection and complementation, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:
10 G = ({S,B}, {a,b,c,d}, {S→BBBBaS|B|bcd, B→cSda|S|cba}, S).
Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:
28 G = ({S,A,B}, {a,b}, {S→AA, A→BaA|a , B→SSA|b}, S).
Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points:	1: 15	2: 10	3: 30	4: 15	5: 30

(1) $S \to A \mid bB \quad A \to C \quad B \to Sb \quad C \to a$

$S \to bB \to bSb \to bbBb \to bbSbb \to \ldots$

$S \to A \to C \to a$, $i=0$

$bSb \to bAb \to bCb \to bab$, $i=1$

$bbSbb \to bbAbb \to bbCbb \to bbabb$, $i=2$

$L(G) = \{ b^i a b^i \mid i \geq 0 \}$

To prove L is not regular, assume L is regular

$\exists$ DFA $\mathcal{D}$ accepting L

Let N be the number of states in $\mathcal{D}$

Consider the case $X = b^i a b^i$ with $|X| = N$

$X = w \cdot v$, $w = b^N$ and $v = ab^N$, $|W| = N$

Pumping-Lemma:

$w = w_1 \cdot w_2 \cdot w_3$ s.t. $|w_2| \geq 1$ and

$I(q_0, w) = I(q_0, w_1 (w_2)^s \cdot w_3) \quad \forall s \geq 0$

where $w \cdot v \in L \quad w_1 (w_2)^s w_3 \in L \quad \forall s \geq 0$

If $s = 0$, $w_1 (w_2)^0 w_3 = w_1 w_3$

$|w_1 (w_2)^0 w_3| = N - |w_2| < N, \quad |w_2| \geq 1$

$\Rightarrow b^{N-|w_2|} a b^N \notin L$

Also, $I(q_0, v) = I(q_0, wv)$

$= I((q_0, w), v)$

$= I((q_0, w_1 w_3), v)$

$\in L(\mathcal{D})$, which is accepting the automata.

But $b^{N-|w_2|} a b^N \notin L$

There is a contradiction $\Rightarrow$ L(G) is not regular

(2) $S \to aA \mid BBBa$

$A \to cS \mid \varepsilon$

$B \to Sb \mid \varepsilon$

Eliminate ε-productions:

When $B \to \varepsilon$: $S \to aA \mid BBBa \mid BBa \mid Ba \mid a$

$A \to cS \mid \varepsilon$

$B \to Sb$

When $A \to \varepsilon$: $S \to aA \mid a \mid BBBa \mid BBa \mid Ba \mid \cancel{a}$

$A \to cS$

$B \to Sb$

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Therefore

$$\boxed{\begin{array}{l} S \to aA \mid a \mid BBBa \mid BBa \mid Ba \\ A \to cS \\ B \to Sb \end{array}}$$

(3) $\left[(101)^* \cap \overline{1(01^*)^*} \right]^* \equiv \left[\overline{\overset{1\,2\,3}{\overline{(101)^*}} \cup \overset{4\,5\,6}{1(01^*)^*}} \right]^*$ over $\{0, 1, 2\}$

(101)

	0	1	2	
→0	–	1	–	0
1	2	–	–	0
2	–	3	–	0
3	–	–	–	1

1_4

	0	1	2	
→0	–	4	–	0
4	–	–	–	1

0_5

	0	1	2	
→0	5	–	–	0
5	–	–	–	1

1_6

	0	1	2	
→0	–	6	–	0
6	–	–	–	1

$* \downarrow$

	0	1	2	
→0	–	6	–	1
6	–	6	–	1

$* \downarrow$

$(101)^*$

	0	1	2	
→0	–	1	–	1
1	2	–	–	0
2	–	3	–	0
3	–	1	–	1

$\cdot$

(01^*)

	0	1	2	
→0	5	–	–	0
5	–	6	–	1
6	–	6	–	1

$(01^*)^* \downarrow *$

	0	1	2	
→0	5	–	–	1
5	5	6	–	1
6	5	6	–	1

Let 7 be null

$\overline{(101)^*}$ dfa $\downarrow$

	0	1	2		
→0	7	1	7	1	0
7	7	7	7	0	1
1	2	7	7	0	1
2	7	3	7	0	1
3	7	1	7	1	0

$\cdot$

$1(01^*)^*$

	0	1	2	
→0	–	4	–	0
4	5	–	–	1
5	5	6	–	1
6	5	6	–	1

rename $\downarrow$

	0	1	2	
→A	B	C	B	0
B	B	B	B	1
C	D	B	B	1
D	B	E	B	1
E	B	C	B	0

$\overline{(101)^*} \cup 1(01^*)^*$ $\downarrow \cup$

	0	1	2	
→0	7	1,4	7	0
1	2	7	7	1
2	7	3	7	1
3	7	1	7	0
4	5	–	–	1
5	5	6	–	1
6	5	6	—	1
7	7	7	7	1

$\xrightarrow{\text{dfa}}$

	0	1	2		
→0	7	1,4	7	0	1
7	7	7	7	1	0
1,4	2,5	7	7	1	0
2,5	5,7	3,6	7	1	0
5,7	5,7	6,7	7	1	0
3,6	5,7	1,6	7	1	0
6,7	5,7	6,7	7	1	0
1,6	2,5	6,7	7	1	0

$\xrightarrow{*}$

	0	1	2	
→0	7	1,4	7	1
7	7	7	7	0
1,4	2,5	7	7	0
2,5	5,7	3,6	7	0
5,7	5,7	6,7	7	0
3,6	5,7	1,6	7	0
6,7	5,7	6,7	7	0
1,6	2,5	6,7	7	0

rename

	0	1	2	
→A	B	C	B	1
B	B	B	B	0
C	D	B	B	0
D	E	F	B	0
E	E	G	B	0
F	E	H	B	0
G	E	G	B	0
H	D	G	B	0

A

A	BCDEFGH
A	BCDEFGH
1	2

→

	0	1	2	
→1	2	2	2	1
2	2	2	2	0

EG

(4) $S \to BBBBaS \mid B \mid bcd$ There does not exist an ε-production

$B \to cSda \mid S \mid cba$ nor useless symbol in this case

Eliminate Unit production:

$S \to B$ $S \to BBBBaS \mid BBBBaB \mid bcd$

$B \to cSda \mid cBda \mid cba \mid S \mid \cancel{B}$

21 missing productions? -5

$B \to S$

$S \to BBBBaS \mid SBBBaS \mid BSBBaS \mid BBSBaS \mid BBBSaS \mid SSBBaS \mid$
$SBSBaS \mid SBBSaS \mid SSSBaS \mid SSBSaS \mid SSSSaS \mid bcd$

$B \to cSda \mid cBda \mid cba$

Convert to CNF

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$S \to BBBBX_aS \mid SBBBX_aS \mid BSBBX_aS \mid BBSBX_aS \mid BBBSX_aS \mid SSBBX_aS \mid$
$SBSBX_aS \mid SBBSX_aS \mid SSSBX_aS \mid SSBSX_aS \mid SSSSX_aS \mid X_bX_cX_d$

$B \to X_cSX_dX_a \mid X_cBX_dX_a \mid X_cX_bX_a$

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CNF

$S \to BS_1 \mid SS_5 \mid BS_9 \mid BS_{13} \mid BS_{17} \mid SS_{21} \mid SS_{25} \mid SS_{29} \mid SS_{33} \mid SS_{37} \mid SS_{41} \mid X_bS_{45}$

$S_1 \to BS_2$	$S_5 \to BS_6$	$S_9 \to SS_{10}$	$S_{13} \to BS_{14}$	$S_{17} \to BS_{18}$	$S_{21} \to SS_{22}$
$S_2 \to BS_3$	$S_6 \to BS_7$	$S_{10} \to BS_{11}$	$S_{14} \to SS_{15}$	$S_{18} \to BS_{19}$	$S_{22} \to BS_{23}$
$S_3 \to BS_4$	$S_7 \to BS_8$	$S_{11} \to BS_{12}$	$S_{15} \to BS_{16}$	$S_{19} \to SS_{20}$	$S_{23} \to BS_{24}$
$S_4 \to X_aS$	$S_8 \to X_aS$	$S_{12} \to X_aS$	$S_{16} \to X_aS$	$S_{20} \to X_aS$	$S_{24} \to X_aS$

$S_{25} \to BS_{26}$	$S_{29} \to BS_{29}$	$S_{33} \to SS_{34}$	$S_{37} \to SS_{38}$	$S_{41} \to SS_{42}$	$S_{45} \to X_cX_d$
$S_{26} \to SS_{27}$	$S_{30} \to BS_{30}$	$S_{34} \to SS_{35}$	$S_{38} \to BS_{39}$	$S_{42} \to SS_{43}$	
$S_{27} \to BS_{28}$	$S_{31} \to SS_{32}$	$S_{35} \to BS_{36}$	$S_{39} \to SS_{40}$	$S_{43} \to SS_{44}$	
$S_{28} \to X_aS$	$S_{32} \to X_aS$	$S_{36} \to X_aS$	$S_{40} \to X_aS$	$S_{44} \to X_aS$	

$B \to X_cB_1 \mid X_cB_3 \mid X_cB_5$

$B_1 \to SB_2$ $B_3 \to BB_4$ $B_5 \to X_bX_a$

$B_2 \to X_dX_a$ $B_4 \to X_dX_a$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d$

It's in CNF so no useless symbols, ϵ, and Unit production

(5) $S \to AA$ 1 ✓ $i=1$ $S \to AA$

$A \to BaA \mid a$ 2 ✓ $i=2$ $j=1$ $A \to BaA \mid a$

$B \to SSA \mid b$ 3 $i=3 \to j=1 =$

$i=B$ $j=1 \Rightarrow B \to AASA \mid b$

$S \to B$

$i=3$ $j=2$ $B \to \underbrace{aASA}_{\beta_1} \mid \underbrace{BaAASA}_{\alpha} \mid \underbrace{b}_{\beta_2}$

$A \to B$

Eliminate immediate left recursion

GNF

$B \to b \mid bB' \mid aASA \mid aASAB'$

$B' \to aAASA \mid aAASAB'$

$A \to baA \mid bB'aA \mid aASAaA \mid aASAB'aA \mid a$

$S \to baAA \mid bB'aAA \mid aASAaAA \mid aASAB'aAA \mid aA$

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