Math 3339

Written Homework 7 (Sections 5.5 & 6.5)

Name: James Richardson PeopleSoft ID: 1555520

Instructions:

- Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
- Print out this file and complete the problems.
- Use blue or black ink or a dark pencil.
- Write your solutions in the space provided. You must show all work for full credit.
- Submit this assignment at http://www.casa.uh.edu under "Assignments" and choose **WH7**
- Total possible points: **15**

1. Let $Z \sim Norm(0, 1)$. Use the normal table and also R's "pnorm" function to find

(a) $Pr(Z \leq 1.45)$
(b) $Pr(Z > -1.28)$
(c) $Pr(-0.674 \leq Z < 1.036)$
(d) $Pr(Z > 0.836)$

```
> pnorm(1.45)
[1] 0.9264707
> 1 - pnorm(-1.28)
[1] 0.8997274
> pnorm(1.036) - pnorm(-.674)
[1] 0.5997433
> 1 - pnorm(.836)
[1] 0.2015775
>
```

2. Use the normal table and also R's "pnorm" function to find

(a) $Pr(X \leq 6.13)$, $X \sim Norm(1, 4)$
(b) $Pr(X > -2.35)$, $X \sim Norm(-1, 2)$
(c) $Pr(-0.872 < X \leq 7.682)$, $X \sim Norm(2.5, 5)$
(d) $Pr(X > 0.698)$, $X \sim Norm(-2, 4)$

```
> pnorm(6.13, 1, 4)
[1] 0.9001663
> 1 - pnorm(-2.35, -1, 2)
[1] 0.7501621
> pnorm(7.682, 2.5, 5)
[1] 0.8499922
> 1- pnorm(.698, -2, 2)
[1] 0.08866848
> 1- pnorm(.698, -2, 4)
[1] 0.2499967
> #a = .9001663, b = .7501621, c= .8499922, d=.2499967
```

3. Use the normal table and also R's "qnorm" function to find

(a) The 90^{th} percentile of $Norm(0, 5)$.
(b) The 15^{th} percentile of $Norm(1, 3)$.
(c) The interquartile range, i.e., the distance from the first to third quartiles of $Norm(\mu, \sigma)$.

```
> qnorm(.9, 0, 5)
[1] 6.407758
> qnorm(.15, 1, 3)
[1] -2.1093
>
```

---

4. Determine the value of the constant c that makes the probability statement correct.

a. P(Z > c) = 0.3859

```
[1] 0.6140919
> c = qnorm(1 - 0.3859)
> c
[1] 0.2900212
> 1 - pnorm(c)
[1] 0.3859
> c
[1] 0.2900212
```

b. ϕ(c) = 0.9838

```
> # ϕ(c) == P(Z<=c)
> qnorm(.9838)
[1] 2.139441
> pnorm(2.139441)
[1] 0.9838
```

c. P(|Z| ≥ c) = 0.05

```
> #The area under the function after c is .05 / 2 so .025
> #this means that our c will appear at the x position
> qnorm(.95 + .025)
[1] 1.959964
> #we could also find our negative c value with
> qnorm(.025)
[1] -1.959964
```

5. *Human body temperatures for healthy individuals have approximately a Normal distribution with mean 98.25°F and standard deviation 0.75°F.
   a. Find the 90th percentile for temperatures of healthy individuals.

      99.21116

   b. Find the 5th percentile for temperatures of healthy individuals.

      97.01636

   c. Determine the first quartile.

      97.74413

```
> qnorm(.9, 98.25, .75)
[1] 99.21116
> qnorm(.05, 98.25, .75)
[1] 97.01636
> qnorm(.25, 98.25, .75)
[1] 97.74413
>
```

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

6. *If adult female heights are Normally distributed, what is the probability that the height of a randomly selected woman is
   a. Within 1.5 SDs of its mean value?

      38.2%

   b. Farther than 2.5 SDs from its mean value?

      .005*2 + .001*2 = 1.2 %

   c. Between 1 and 2 SDs from its mean values?

      .092 * 2 + .044 * 2 = 27.2%

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

7. *The inside diameter of a randomly selected piston ring is a random variable with mean value 12 cm and standard deviation 0.04 cm.
   a. If X is the sample mean diameter for a random sample of n = 16 rings, where is the sampling distribution of X centered, and what is the standard deviation of the X distribution?

E[X’] = 16* E[X] / 16 = 12 cm

sd[X’] = sqrt(16 / 16^2 * var[X]) = sd[X]/sqrt(16)
= .04 / 4 = .01

   b. Answer the questions posted in part (a) for a sample size of n =64 rings.

The mean will still be the same value.
The standard deviation will become
sqrt(64 / 64^2 * Var[X]) = sd[X] = sqrt(64)
= .04 / 8 = .0005

   c. For which of the two random samples, the one of part (a) or the one of part (b), is X more likely to be within 0.01 cm of 12 cm? Explain your reasoning.

Part b where n = 64.
This is because the the standard deviation with a larger sample size is smaller. A smaller standard deviation means it will be closer to the mean.

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

8. *Refer to the previous problem (7). Suppose the distribution of the diameter is normal.

a. Calculate $P(11.99 \leq X \leq 12.01)$ when $n = 16$.

```
> pnorm(12.01, 12, .01) - pnorm(11.99, 12, .01)
[1] 0.6826895
>
```

b. How likely is it that the sample mean diameter exceeds 12.01 when $n = 25$?

```
> 1 - pnorm(12.01, 12, sqrt(.04 / sqrt(25)))
[1] 0.4554896
>
```

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

9. * Suppose only 70% of all drivers in a certain state regularly wear a seat belt. A random sample of 500 drivers is selected. Using the Normal approximation, what is the probability that

a. Between 320 and 370 (inclusive) of the drivers in the sample regularly wear a seat belt?

$\mu = pn = 350, \sigma = \sqrt{(pqn)} = 10.24695$

```
> nu = 350
> sd = 10.24695
> pnorm(370, 350, 10.24695) - pnorm(320, 350, 10.24695)
[1] 0.9728116
>
```

b. Fewer than 325 of those in the sample wear a seatbelt.

```
> pnorm(325, 350, 10.24695)
[1] 0.007348707
>
```

c. Answer parts a and b using pbinomial in R. Do you get the same answers?

```
> pbinom(370, 500, .7) - pbinom(320, 500, .7)
[1] 0.9761352
> pbinom(325, 500, .7)
[1] 0.009057597
```

The numbers aren't exactly the same but they're pretty dang close

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

10. For the following statements, answer True or False.

   a. On a statistics exam, Joe's score was at the 20th percentile and John's score was at the 40th percentile, thus, we can say that John's score was twice Joe's.

False

   b. On the normal curve, the 50th percentile corresponds to the mean.

True

   c. For the standard normal distribution, the mean is 1.

False, for a standard normal the graph is centered at 0!

   d. The sample mean (X) is a random variable.

False, the sample mean is likely to be very close to the mean of the population.
The sample mean is not a numerical value representing a possible outcome of the phenomena

∗ Problems come from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.