COSC 3340/6309 Examination 1

Thursday, June 16, 2011, 4 - 5:45 pm

Open Book and Notes

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1. Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c	
1	2	3	2	1
2	3	2	1,4	1
→ 3	4	1	2	1
4	1	4	/	1

2. Consider the class $\mathcal{L}_A$ of all regular languages that contain only words of even length, over the fixed two-letter alphabet A={a,b}.

(a) Is $\mathcal{L}_A$ countable?

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a^2 \cup a^3)^* (a \cup a^2)$ over the alphabet {a}

(b) $1^* (01 \cup 10)^* ((01)^* \cup (10)^*)$ over the alphabet {0,1}

4. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

	a	b	
→ A	B	C	0
B	A	/	0
C	/	A,B	1

Points:	**1: 12**	**2: 22**	**3: 44**	**4: 22**
	12	10	21+17	22

1.

	a	b	c	
→3	4	1	2	1
4	1	4	∅	1
1	2	3	2	1
2	3	2	1,4	1
∅	∅	∅	∅	1
1,4	1,2	3,4	2	1
1,2	2,3	2,3	1,2,4	1
3,4	1,4	1,4	2	1
2,3	3,4	1,2	1,2,4	1
1,2,4	1,2,3	2,3,4	1,2,4	1
1,2,3	2,3,4	1,2,3	1,2,4	1
2,3,4	1,3,4	1,2,4	1,2,4	1
1,3,4	1,2,4	1,3,4	2	1

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4. $L_A = aL_B \cup bL_C$

$L_B = aL_A$

$L_C = bL_A \cup bL_B \cup \varepsilon$

$L_B \to L_A$ $\quad L_A = aaL_A \cup bL_C$

$L_B \to L_C$ $\quad L_C = bL_A \cup baL_A \cup \varepsilon = (b \cup ba)L_A \cup \varepsilon$

$L_C \to L_A$ $\quad L_A = aaL_A \cup b(b \cup ba)L_A \cup b$

$$\underbrace{L_A}_{X} = \underbrace{[aa \cup b(b \cup ba)]}_{L}\underbrace{L_A}_{X} \cup \underbrace{b}_{M}$$

from theorem we get $L_A = [aa \cup bb \cup bba]^* \cdot b$

$= [a^2 \cup b^2 \cup b^2 \cdot a]^* \cdot b$

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3. a) $(a^2 \cup a^3)^* \cdot (a \cup a^2) \equiv (\overset{1}{a}\cdot\overset{2}{a} \cup \overset{3}{a}\cdot\overset{4}{a}\cdot\overset{5}{a})^* (\overset{6}{a} \cup \overset{7}{a}\cdot\overset{8}{a})$

	$\overset{1}{a}$	
→ 0	1	0
1	∅	1

	$\overset{2}{a}$	
→ 0	2	0
2	∅	1

	$\overset{3}{a}$	
→ 0	3	0
3	∅	1

	$\overset{4}{a}$	
→ 0	4	0
4	∅	1

	$\overset{5}{a}$	
→ 0	5	0
5	∅	1

	$\overset{6}{a}$	
→ 0	6	0
6	∅	1

	$\overset{7}{a}$	
→ 0	7	0
7	∅	1

	$\overset{8}{a}$	
→ 0	8	0
8	∅	1

$a^1 \cdot a^2 =$

	a	
→ 0	1	0
1	2	0
2	/	1

$a^3 a^4 \cdot a^5$

	a	
→ 0	3	0
3	4	0
4	5	0
5	/	1

$a^7 \cdot a^8$

	a	
→ 0	7	0
7	8	0
8	/	1

$\cup$

$a^1 a^2 \cup a^3 a^4 \cdot a^5$

	a	
→ 0	1,3	0
1	2	0
2	/	1
3	4	0
4	5	0
5	/	1

$\xrightarrow{*} (a^1 a^2 \cup a^3 \cdot a^4 \cdot a^5)^*$

	a	
→ 0	1,3	1
1	2	0
2	1,3	1
3	4	0
4	5	0
5	1,3	1

$a^6 \cup a^7 a^8$

	a	
0	6,7	0
→ 6	/	1
7	8	0
8	/	1

$(a^1 a^2 a^3 \cup a^4 \cdot a^5)^* \cdot (a^6 \cup a^7 a^8)$

	a	
→ 0	1, 3,6,7	0
1	2	0
2	1,3,6,7	0
3	4	0
4	5	0
5	1,3,6,7	0
6	/	1
7	8	0
8	/	1

↙ nfa

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subset construction

↙ dfa

		a	
A	→0	1,3,6,7	0
B	1,3,6,7	2,4,8	1
C	2,4,8	1,3,5,6,7	1
D	1,3,5,6,7	1,2,3,4,6,7,8	1
E	1,2,3,4,6,7,8	1,2,3,4,5,6,7,8	1 ✓
F	1-8	1-8	1

	a	
→A	B	0
B	C	1
C	D	1
D	E	1
E	F	1
F	F	1

reduce the dfa

$\boxed{A}$	BCDEF
$\boxed{A}$	BCDEF
1	2

← reduced dfa

	a	
→1	2	0
2	2	1

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3.b) $\overset{1}{1}^*(\overset{2}{0}\overset{3}{1}\cup\overset{4}{1}\overset{5}{0})^*((\overset{6}{0}\overset{7}{1})^*\cup(\overset{8}{1}\overset{9}{0})^*)$

$\overset{1}{1}$

	0	1	
→ 0	/	1	0
1	/	/	1

$\overset{2}{0}$

	0	1	
→ 0	2	/	0
2	/	/	1

$\overset{3}{1}$

	0	1	
→ 0	/	3	0
3	/	/	1

$\overset{4}{1}$

	0	1	
→ 0	/	4	0
4	/	/	1

$\overset{5}{0}$

	0	1	
→ 0	5	/	0
5	/	/	1

$\overset{6}{0}$

	0	1	
→ 0	6	/	0
6	/	/	1

$\overset{7}{1}$

	0	1	
→ 0	/	7	0
7	/	/	1

$\overset{8}{1}$

	0	1	
→ 0	/	8	0
8	/	/	1

$\overset{9}{0}$

	0	1	
→ 0	9	/	0
9	/	/	1

$\overset{1}{1}^*$

	0	1	
→ 0	/	1	1
1	/	1	1

$\overset{2}{0}\cdot\overset{3}{1}$

	0	1	
→ 0	2	/	0
2	/	3	0
3	/	/	1

$\overset{4}{1}\cdot\overset{5}{0}$

	0	1	
→ 0	/	4	0
4	5	/	0
5	/	/	1

$\overset{6}{0}\cdot\overset{7}{1}$

	0	1	
→ 0	6	/	0
6	/	7	0
7	/	/	1

$(\overset{6}{0}\cdot\overset{7}{1})^*$

	0	1	
→ 0	6	/	1
6	/	7	0
7	6	/	1

$\overset{8}{1}\cdot\overset{9}{0}$

	0	1	
→ 0	/	8	0
8	9	/	0
9	/	/	1

$(\overset{8}{1}\cdot\overset{9}{0})^*$

	0	1	
→ 0	/	8	1
8	9	/	0
9	/	8	1

$\overset{2}{0}\overset{3}{1}\cup\overset{4}{1}\overset{5}{0}$

	0	1	
→ 0	2	4	0
2	/	3	0
3	/	/	1
4	5	/	0
5	/	/	1

$(\overset{2}{0}\overset{3}{1}\cup\overset{4}{1}\overset{5}{0})^*$

	0	1	
→ 0	2	4	1
2	/	3	0
3	2	4	1
4	5	/	0
5	2	4	1

6 7 8 9

$(01)^* \cup (10)^*$

	0	1	
→ 0	6	8	1
6	/	7	0
7	6	/	1
8	9	/	0
9	/	8	1

1 2 3 4 5

$1^* \cdot (01 \cup 10)^*$

	0	1	
→ 0	2	1,4	1
1	2	1,4	1
2	/	3	0
3	2	4	1
4	5	/	0
5	2	4	1

nfa ↙

$1^*(01 \cup 10)^* \cdot ((01)^* \cup (10)^*)$

	0	1	
→ 0	2,6	1,4,8	1
1	2,6	1,4,8	1
2	/	3	0
3	2,6	~~1~~,4,8	1
4	5	/	0
5	2,6	~~1~~,4,8	1
6	/	7	0
7	6	/	1
8	9	/	0
9	/	8	1

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		0	1		
A	→ 0	2,4	1,4,8	1	← dfa
B	2,4	5	3	0	
C	1,4,8	2,4,5,9	1,4,8	1	
D	3	2,4	1,4,8	1	
E	5	2,4	1,4,8	1	
F	2,4,5,9	2,4,5	1,3,4,8	1	
G	1,3,4,8	2,4,5,9	1,4,8	1	
H	2,4,5	2,4,5	1,3,4,8	1	

	0	1	
→ A	B	C	1
B	E	D	0
C	F	C	1
D	B	C	1
E	B	C	1
F	H	G	1
G	F	C	1
H	H	G	1

reduced dfa ↙

ACDEFGH | B

A CFGH D E | B

A | CFGH | D | E | B

1 | 2 | 3 | 4 | 5

	0	1	
→ 1	5	2	1
2	2	2	1
3	5	2	1
4	5	2	1
5	4	3	0

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2a). Assume that $\mathcal{L}_A$ is countable and $\mathcal{L}_A=\{L \subseteq A^* \mid$ even length$\}$ where A^* represents all the words over the aphabet $A=\{a,b\}$.

$\mathcal{L}_A=\{L_1, L_2, \ldots\}$ with each L_i containning even length words.
$A^*=\{w_1, w_2, \ldots\}$.

if we take the pairs such as the restriction is satisfied we can get

	w_1	w_2 ---
L_1	0	1
L_2	0	1
L_3	0	0
L_4	1	1
⋮		

0,1 denoting if the word belongs to L_i.

Let L: $w_i \notin L$ if and only if $w_i \in L_i$
But L does not occur in the set of all languages $\mathcal{L}_A$.
The contradiction arises through the countability assumption.

All in all, the restriction to words of even length does not affect the fact that

$\mathcal{L}_A$ is uncountable infinite.

0

2b)
We have that $M_A = \mathcal{L}_A^c = \{L \subseteq A^* | \text{even length}\}^c$

By the same arguements we can show the M_A is uncountable infinite, since the complementary operator does not make a difference in this particular case

M_A = {all regular languages} − {all regular languages | words of even length} 1

which can easily be proved that gives the same response to the question asked.

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2c) M_A consists of all languages over A that are not in $\mathcal{L}_A$, thus $M_A \cap \mathcal{L}_A = \emptyset$ and as a result we get a uncountable since we have only the empty set. is Countable
~~We~~ cannot find a function that goes from the elements of $M_A \cap \mathcal{L}_A$ to the set of the naturals since it is empty!

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