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COSC 3340/6309

Examination 2

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Open Book and Notes

1. Prove that the language $L(G)$ is not regular where G is the following context-free grammar: $G = (\{S, A, B, C\}, \{a, b\}, \{S \rightarrow A|Bb, A \rightarrow C, B \rightarrow aS, C \rightarrow a\}, S)$.
 Note: You must first determine $L(G)$.

2. Eliminate all ϵ -productions in the following cfg G :

$$G = (\{S, A, B\}, \{a, b, c\}, \{S \rightarrow Aa|aBBB, A \rightarrow cS|\epsilon, B \rightarrow Sb|\epsilon\}, S).$$

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet $\{0,1,2\}$ (not $\{0,1\}^*$):

$$[(011)^* \cap \overline{0(11^*)^*}]^*$$

Note: You must first determine nfas for $(010)^*$ and $0(01^*)^*$ over $\{01,2\}$, then handle the intersection and complementation, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for $L(G)$ for the following cfg G :

$$G = (\{S, B\}, \{a, b, c, d\}, \{S \rightarrow aBBB|B|cd, B \rightarrow cSda|S|cba\}, S).$$

Note: You must first remove all unit productions.

5. Construct a Greibach normal form grammar for $L(G)$ for the following CNF G :

$$G = (\{S, A, B\}, \{a, b\}, \{S \rightarrow AA, A \rightarrow BAA|a, B \rightarrow SSS|b\}, S).$$

Note: First derive all the productions for S , A , and B . You may only indicate how the final result looks for whatever primed variables you obtain.

Points: 1: 15 2: 10 3: 30 4: 15 5: 30

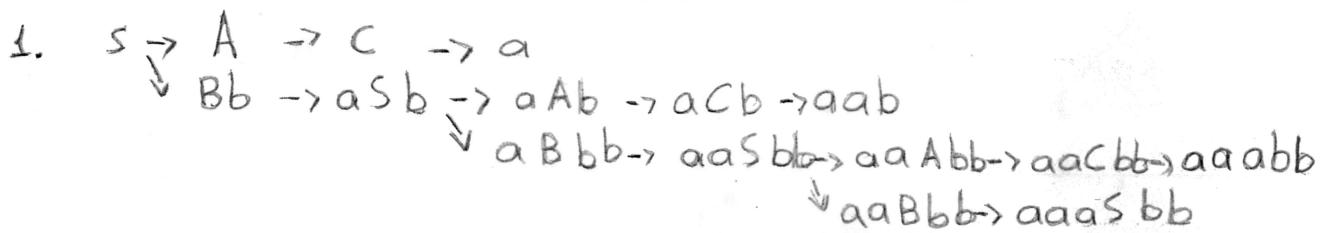
15

10

26

14

36



$$L(G) = \{a^{i+1}b^i \mid i \geq 0\}$$

Claim L is not regular

Assume L is regular

\exists DFA D accepting L

Let N be the number of states in D

Consider $x = a^i a b^i$ with $|x| = N$

$$x = w \cdot v, \quad w = a^N \text{ and } v = ab^N, \quad |w| = N$$

$$w = w_1 \cdot w_2 \cdot w_3 \text{ s.t. } |w_2| \geq 1 \text{ and } \begin{array}{l} \text{if regular \sim pump} \\ I(q_0, w) = I(q_0, w_1(w_2)^s w_3) \quad \forall s \geq 0 \quad \text{given the "right" long} \\ w \cdot v \in L \quad w_1(w_2)^s w_3 \in L \quad \forall s \geq 0 \end{array}$$

if $s = 0$ then:

$$w_1(w_2)^0 w_3 = w_1 w_3$$

$$|w_1(w_2)^0 w_3| = N - |w_2| < N, \quad |w_2| \geq 1$$

$a^{N-|w_2|} a b^N \notin L$ but it is accepted by the automata

we found a contradiction $\Rightarrow L(G)$ is not regular.

$$(A \cap \bar{B})^* = (\bar{A} \cup B)^*$$

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$$3. [(011)^* \cap \overline{0(11^*)^*}]^* = \overline{[(011)^* \cup 0(11^*)^*]}^* \text{ over } \{0, 1, 2\}$$

dfa

	0	1	2	
$\rightarrow 0$	1	1	1	0
1	1	2	1	0
2	1	3	1	0
3	1	1	1	1

$(011)^*$

	0	1	2	
$\rightarrow 0$	1	1	1	1
1	1	2	1	0
2	1	3	1	0
3	1	1	1	1

dfa

	0	1	2	
$\rightarrow 0$	1	4	4	1
1	4	2	4	0
4	4	4	4	0
2	4	3	9	0
3	1	4	4	1

complementary

0
1
1
1
0

	0	1	2	
1	$\rightarrow 0$	1	7	1
7	$\rightarrow 0$	7	1	1
				1^*
				$(11^*)^*$

	0	1	2	
1	$\rightarrow 0$	1	6	1
6	$\rightarrow 0$	6	1	1
				$* \rightarrow$
				$\rightarrow 0$
6	$\rightarrow 0$	6	7	1
7	$\rightarrow 0$	7	1	1

	0	1	2	
0	$\rightarrow 0$	5	1	1
5	$\rightarrow 0$	5	1	1
				$0(11^*)^* \rightarrow 0$
				$\rightarrow 0$
5	$\rightarrow 0$	5	1	1
6	$\rightarrow 0$	6	7	1
7	$\rightarrow 0$	7	1	1

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		0 1 2
3.	$\rightarrow 0$	1,5 4 4 0
	1	4 2 4 1
	2	4 3 4 1
	3	1 4 4 0
	4	4 4 4 1
	5	1 6 1 1
	6	1 6,7 1 1
	7	1 6,7 1 1

did not
star
nfa

dfa		0 1 2	complement of dfa		0 1 2
	$\rightarrow 0$	1,5 4 4 0	1	$\rightarrow A$	B C C 1
we assume	1,5	4 2 4 1	0	B	C D C 0
$\emptyset \neq 4$	4	4 4 4 1	0	C	C C C 0
Exit: corrected to 5	2,5	4 3,6,7 4 1	0	D	C E C 0
	3,6,7	\emptyset 4,6,7 4 1	0	E	C F C 0
	4,6,7	4 4,6,7 4 1	0	F	C F C 0

A	BCDEF
A	BCDEF
1	2

	0 1 2
$\rightarrow 1$	2 2 2 1
2	2 2 2 0

4. $S \rightarrow aBBBS | \boxed{B} | cd$
 $B \rightarrow cSda | S | cba$

We removed unit production
 (There were not ϵ productions
 or useless symbols)

$S \rightarrow aBBBS | aBBBB | cd$
 $B \rightarrow cSda | cBda | \boxed{S} | \boxed{B} | cba$

$S \rightarrow aBBBS | aSBB | aBSBS | aBBSS | aSSBS | aSBSS | aSSSS | \boxed{A} | \boxed{A} | cd$
 $B \rightarrow cSda | cBda | \underline{cSda} | cba$

Incorrect by Grader

$S \rightarrow XaBBBS | XaSBB | XaBSBS | XaBBSS | XaSSBS | XaSBSS | XaSSSS | XcX$
 $B \rightarrow XcSXdXa | XcBXdXa | XcSXdXa | XcXbXa$

$S \rightarrow XaS_1 | XaS_4 | XaS_7 | XaS_{10} | XaS_{13} | XaS_{16} | XaS_{19} | XcXd$
 $B \rightarrow XcB_1 | XcB_3 | XcB_5 | XcB_7$

$S_1 \rightarrow BS_2$	$S_4 \rightarrow SS_5$	$S_7 \rightarrow BS_8$	$S_{10} \rightarrow BS_{11}$	$S_3 \rightarrow SS_{14}$	$S_{16} \rightarrow S S_{17}$
$S_2 \rightarrow BS_3$	$S_5 \rightarrow BS_6$	$S_8 \rightarrow S S_9$	$S_{11} \rightarrow BS_{12}$	$S_{14} \rightarrow SS_{15}$	$S_{17} \rightarrow BS_{18}$
$S_3 \rightarrow BS$	$S_6 \rightarrow BS$	$S_9 \rightarrow BS$	$S_{12} \rightarrow SS$	$S_{15} \rightarrow BS$	$S_{18} \rightarrow SS$

$S_{19} \rightarrow SS_{20}$

$S_{20} \rightarrow SS$

$B_1 \rightarrow SB_2$	$B_3 \rightarrow BB_4$	$B_5 \rightarrow SB_6$	$B_7 \rightarrow XbXa$
$B_2 \rightarrow XdXa$	$B_4 \rightarrow XdXa$	$B_6 \rightarrow XdXa$	

$Xa \rightarrow a$
 $Xb \rightarrow b$
 $Xc \rightarrow c$

2 42 productions

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5. Since we have a CNF it is "free" of useless symbols ϵ and unit productions.

$$\begin{array}{l} S \rightarrow AA \\ A \rightarrow BAA \\ B \rightarrow SSS \end{array} \mid a \quad \mid b$$

$i=3 \checkmark \quad i=2 \checkmark \quad i=3$ plug $S \rightsquigarrow B$ $B \rightarrow AASS \mid b$
 $j=\emptyset \quad j=1 \quad j=1$
 no immediate \Rightarrow
 left recursions $i=3$ plug $A \rightsquigarrow B$ $B \rightarrow aASS \mid BAASS \mid b$
 $j=2$
 eliminate immediate left recursion

$$B \rightarrow aASS \mid b \mid aASSB' \mid bB' \\ B' \rightarrow (\beta)AASS \mid (\beta)AASSB'$$

Back substitution (un-primed)

✓ $B \rightarrow A$ $A \rightarrow a|aASSAA|bAA|aASSB'AA|bB'AA$
 $A \rightarrow S$ $S \rightarrow aA|aASSAAA|bAAA|aASSB'AAA|bB'AAA$
 primed $B' \rightarrow aASSAASS|bAASS|aASSB'AASS|bB'AASS$
 $B \rightarrow B'$ $aASSAASSB'|bAASSB'|aASSB'AASSB'|bB'AASSB'$

Finally, we make any terminals in positions 2,3,... variables

$$A \rightarrow X_a | X_a A S S A A | X_b A A | X_a A S S B' A A | X_b B' A A$$

$$B \rightarrow X_a A S S | X_b | X_a A S S B' | X_b B'$$

$S \rightarrow X_a A_1 X_a A S S A A A A | X_b A A A A | X_a S S B' A A A A | X_b B' A A A A$

$$B' \rightarrow X_g A S S A A S S \mid X_b A A S S \mid X_g A S S B' A A S S \mid X_b B' A A S S$$

$\times_a A \leq S A \leq S B' \mid \times_b A \leq S B' \mid \times_a A \leq S B' A \leq S B' \mid \times_b B' A \leq S B'$

$$x_a \rightarrow a$$

$$x_b \rightarrow b$$

$$X_C \rightarrow C$$

(25) productions