

Math 3339

Homework 8 (Sections 7.1 – 7.3)

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Instructions:

- Homework will NOT be accepted through email or in person. Homework must be submitted through CourseWare BEFORE the deadline.
 - Print out this file and complete the problems.
 - Use blue or black ink or a dark pencil.
 - Write your solutions in the space provided. You must show all work for full credit.
 - Submit this assignment at <http://www.casa.uh.edu> under "Assignments" and choose **Homework8**
 - Total points: **15 points**
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1. *Determine the t critical value that will capture the desired t curve area in each of the following cases;

- Central area = 0.95; df = 10
- Central area = 0.95; df = 20
- Central area = 0.99; df = 30
- Central area = 0.97; df = 20
- Upper-tail = 0.01; df = 25
- Lower-tail = 0.025; df = 5

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> qt(.95, 10)
[1] 1.812461
> qt(.95, 20)
[1] 1.724718
> qt(.99, 30)
[1] 2.457262
> qt(.97, 20)
[1] 1.993713
> qt(1 - .01, 25)
[1] 2.485107
> qt(.025, 5)
[1] -2.570582
> |
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2. *Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and the alcohol content of each bottle is determined. Let μ denote the average alcohol content for the population of all bottles of the brand under study. Suppose that the resulting 95% confidence interval is (7.8, 9.4).

- a. Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.

Narrower. The smaller the range of possible values for our assumption, the smaller our confidence will be.

- b. Consider the following statement: There is a 95% chance that μ is between 7.8 and 9.4. Is this statement correct? Why or why not?

This is true. The confidence interval tells us what values our results should lie between, and the associated percentage is the probability that the value will be within that range for a given sample

- c. Consider the following statement: We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content that is between 7.8 and 9.4. Is this statement correct? Why or why not?

This is technically correct. Our resulting value has a 95% chance of being within the given interval.

- d. Consider the following statement: If the process of selecting a sample of size 50 and then computing the corresponding 95% interval is repeated 100 times, 95 of the resulting intervals will include μ . Is this statement correct? Why or why not?

Again not quite. 95 of the resulting values should lie within the confidence interval.

* Problems came from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

1. A sample of size 36 from a normally distributed population variable with population standard deviation 20 had a sample mean of 88. Find a 90% confidence interval for the population mean.

$$Z = qnorm(.9 + (1-.9)/2), sd = 20, x = 88, n = 36$$

$$X (+/-) Z*(sd/sqrt(n))$$

$$(82.51715, 93.48285)$$

2. A sample of size 90 from a population variable had a sample mean of 4.74 and a sample standard deviation of 0.71. Find a 95% confidence interval for the population mean.

$$t = qt(.95 + (1-.95)/2, 89); n = 90; x = 4.74; s = 0.71$$

$$X (+/-) t*(s/sqrt(n))$$

$$(4.591293, 4.888707)$$

5. We wish to estimate the population mean of a variable that has standard deviation 70.5. We want to estimate it with an error no greater than 5 units with probability 0.99. How big a sample should we take from the population? What happens if the standard deviation and the margin of error are both doubled?

* Problems came from Devore, Jay and Berk, Kenneth, *Modern Mathematical Statistics with Applications*, Thomson Brooks/Cole, 2007.

5.

- a. Among 20 golden hamster litters recorded, there was a sample mean of $\bar{x} = 7.72$ baby hamsters, with a sample standard deviation of $S = 2.5$ hamsters per liter. Create a 90% confidence interval for the mean number of baby hamsters per liter.

$$\bar{X} = 7.72, S = 2.5, n = 20$$

$$\bar{X}(\pm) z^*(sd/\sqrt{n})$$

$$Z^* = \text{qnorm}(.9 + (1-.9)/2) = 1.644854$$

$$7.72 (\pm) Z^*(2.5/\sqrt{20}) =$$

$$(6.800499, 8.639501)$$

- b. Assume that helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with population standard deviation, $\sigma = 0.75$. Compute a 98% confidence interval for the true average porosity of a certain seam for 16 specimens with a sample average porosity of 4.56.

$$N = 16, \bar{x} = 4.56$$

$$Z^* = \text{qnorm}(.98 + (1 - .98)/2)$$

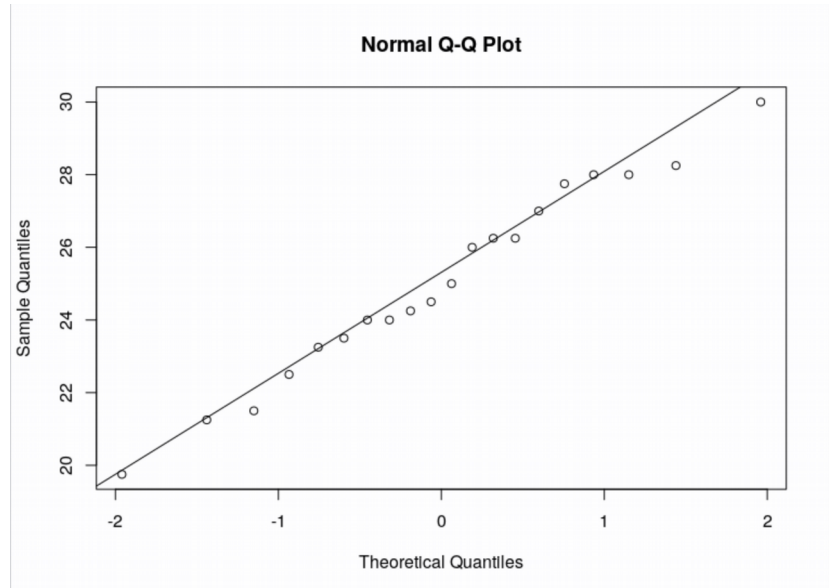
$$(4.56 - Z^*(.75/4), 4.56 + Z^*(.75/4))$$

$$(4.12381, 4.99619)$$

6. *Here is a sample of ACT scores (average of the Math, English, Social Science, and Natural Science scores) for students taking college freshman calculus:

24.00 28.00 27.75 27.00 24.25 23.50 26.25
24.00 25.00 30.00 23.25 26.25 21.50 26.00
28.00 24.50 22.50 28.25 21.25 19.75

- a. Using an appropriate graph, see if it is plausible that the observations were selected from a normal distribution.



- b. Calculate a 95% confidence interval for the population mean.

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> avg = mean(vals)
> standard_dev = sd(vals)
> n = 20
> z = qnorm(.95 + (1 - .95)/2)
> c(avg-z*(standard_dev/sqrt(n)), avg+z*(standard_dev/sqrt(n)))
[1] 23.87123 26.22877
> |
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(23.87, 26.22877)

- c. The university ACT average for entire freshmen that year was about 21. Are the calculus students better than the average as measured by the ACT?

Yes, even the low bound of the confidence interval is above the average measured by the ACT

For questions 7 – 10 circle the best answer.

7. You have measured the systolic blood pressure of a random sample of 25 employees of a company located near you. A 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138). Which of the following statements gives a valid interpretation of the confidence level?
 - a. 95% of the sample of employees have a systolic blood pressure between 122 and 138.
 - b. 95% of the population of employees have a systolic blood pressure between 122 and 138.
 - c. If the procedure were repeated many times, 95% of the resulting confidence intervals would contain
 - d. the population mean systolic blood pressure.
 - e. The probability that the population mean blood pressure is between 122 and 138 is .95.
8. If the 98% confidence limits for the population mean are 73 and 80, which of the following could be the 95% confidence limits?
 - a. 73 and 81
 - b. 72 and 79
 - c. 72 and 81
 - d. 74 and 79
 - e. none of these
9. To assess the precision of a laboratory scale, we measure a block known to have a mass of 1 gram. We measure the block n times and record the mean \bar{x} of the measurements. Suppose the scale readings are normally distributed with unknown mean μ and standard deviation $\sigma = 0.001$ g. How large should n be so that a 95% confidence interval for μ has a margin of error of ± 0.0001 ?
 - a. 20
 - b. 385
 - c. 10,000
 - d. 66,358
 - e. 384
10. As the sample size decreases, the confidence interval for the population mean will
 - a. decrease
 - b. increase
 - c. stay the same
 - d. decrease and then increase