

Thursday, June 14, 2012, 4 - 5:45 pm

Open Book and Notes

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1. Construct a dfa for the following nfa, using the subset construction given in class:

11/12

		a	b	c
→ 1		2	3	2
2		3	2	1
3		4	1	2,4
4		1	4	/

2. Consider the class \mathcal{L}_A of all regular languages that contain only words of odd length, over the fixed two-letter alphabet $A = \{a, b\}$.

(a) Is \mathcal{L}_A countable?

22/22 (b) Is the class \mathcal{M}_A countable where \mathcal{M}_A consists of all languages over A that are not in \mathcal{L}_A ?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, always using the constructions given in class:

(a) $(a \cup a^3)^* (a \cup a^3)$ over the alphabet $\{a\}$

(b) $(01 \cup 10)^* 1^* ((01)^* \cup (10)^*)$ over the alphabet $\{0, 1\}$

4. Construct a regular expression over the alphabet $\{a, b\}$ for the language accepted by the following automaton:

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		a	b
→ A		B	A, C
B		A	/
C		/	B

Points:


1: 12

2: 22


3: 44

4: 22

1	}	44
2		
3		
4		

1) 1)  →

	a	b	b	c	
2	3	2	2	1	1
3	4		1	2,4	1
1	2		3	2	1
4	1		4	/	1
/	/	/	/	/	0
2,4	1,3	2,4		1	1
1,3	2,4	1,3		2,4	1

-1 

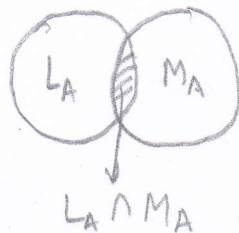
Updated
Answer:

	a	b	c	
→ 2	3	2	1	1
3	4	1	2,4	1
1	2	3	2	1
4	1	4	/	1
2,4	1,3	2,4	1	1
/	/	/	/	0
1,3	2,4	1,3	2,4	1

2) (a) Given from the question, L_A is regular. So, we know there exist a DFA that accepts L_A and that is finite automaton. Therefore, it's countable. ✓

(b) It is given that L_A is regular and countable. Stated by the question, M_A is not L_A . It is uncertain whether or not M_A is regular. By this case, we cannot account for the language defined by M_A . Thus, M_A is not countable. ✓

(c) In an intersection operation, there exist some element found in both sets of L_A and M_A .



M_A was declared that it consists of all languages over A that are not in L_A . With $L_A \cap M_A$, there is no element that is in both L_A and M_A . However, this means that $L_A \cap M_A$ will yield an empty set. Empty Set is consider finite with cardinality of zero. If $L_A \cap M_A$ is finite, it is therefore countable. ✓

$$3 \quad (a_1 \cup a_2 \cdot a_3 \cdot a_4)^* (a_5 \cup a_6 \cdot a_7 \cdot a_8)$$

$$3) (a) (a \cup a^3)^* (a \cup a^3) \text{ over } \{a\}$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 1 & 0 \\ 1 & / & 1 \end{array} \quad \begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 2 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 4 & / & 1 \end{array}$$

$$(a \cup a^3) \downarrow \cup \downarrow$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 1, 2 & 0 \\ 1 & / & 1 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 4 & / & 1 \end{array}$$

$$(a \cup a^3)^* \downarrow *$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 1, 2 & 1 \\ 1 & 1, 2 & 1 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 4 & 1, 2 & 1 \end{array}$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 5 & 0 \\ 5 & / & 1 \end{array} \quad \begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 6 & 0 \\ 6 & 7 & 0 \\ 7 & 8 & 0 \\ 8 & / & 1 \end{array}$$

$$(a \cup a^3) \downarrow \cup \downarrow$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 5, 6 & 0 \\ 5 & / & 1 \\ 6 & 7 & 0 \\ 7 & 8 & 0 \\ 8 & / & 1 \end{array}$$

$$(a \cup a^3)^* (a \cup a^3) \downarrow a$$

$$\begin{array}{c|cc} a & & \\ \hline \rightarrow 0 & 1, 2, 5, 6 & 0 \\ 1 & 1, 2, 5, 6 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 0 \\ 4 & 1, 2, 5, 6 & 0 \\ 5 & / & 1 \\ 6 & 7 & 0 \\ 7 & 8 & 0 \\ 8 & / & 1 \end{array}$$

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Contn.
3a)

dfa			a		
→ 0	1,2,5,6	0	→ 0	1,2,5,6	0
1	1,2,5,6	0	B	1,2,5,6	1,2,3,5,6,7
2	3	0	C	1,2,3,5,6,7	1-8
3	4	0	D	1-8	1-8
4	1,2,5,6	0			
5	/	1			
6	7	0			
7	8	0			
8	/	1			

a		
→ A	B	0
B	C	1
C	D	1
D	D	1

reduce

A	B	C	D
A	B	C	D
1	2		

a		
→ 1	2	0
2	2	1

3) (b) $(0, 1_2 \cup 1_0)^* 1_5 ((0, 1_6)^* \cup (1_0)^*)$ over $\{0, 1\}$

$$0, 1_2 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 1 & 0 \\ 1 & / & 2 & 0 \\ 2 & / & / & 1 \end{array}$$

$$1_3 0_4 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & / & 3 & 0 \\ 3 & 4 & / & 0 \\ 4 & / & / & 1 \end{array}$$

$$1_5 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & / & 5 & 0 \\ 5 & / & / & 1 \end{array}$$

$$0_6 1_7 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 6 & / & 0 \\ 6 & / & 7 & 0 \\ 7 & / & / & 1 \end{array}$$

$$1_8 0_9 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & / & 8 & 0 \\ 8 & 9 & / & 0 \\ 9 & / & / & 1 \end{array}$$

$$1_5^* \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & / & 5 & 1 \\ 5 & / & 5 & 1 \end{array}$$

$$(0_6 1_7)^* \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 6 & / & 1 \\ 6 & / & 7 & 0 \\ 7 & 6 & / & 1 \end{array}$$

$$(0, 1_2 \cup 1_3 0_4) \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 1 & 3 & 0 \\ 1 & / & 2 & 0 \\ 2 & / & / & 1 \\ 3 & 4 & / & 0 \\ 4 & / & / & 1 \end{array}$$

$$(1_8 0_9)^* \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & / & 8 & 1 \\ 8 & 9 & / & 0 \\ 9 & / & 8 & 1 \end{array}$$

$$(0_6 1_7)^* (1_8 0_9)^* \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 6 & 8 & 1 \\ 6 & / & 7 & 0 \\ 7 & 6 & / & 1 \\ 8 & 9 & / & 0 \\ 9 & / & 8 & 1 \end{array}$$

$$(0, 1_2 \cup 1_3 0_4)^* \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 1 & 3 & 1 \\ 1 & / & 2 & 0 \\ 2 & / & 3 & 1 \\ 3 & 4 & / & 0 \\ 4 & 1 & 3 & 1 \end{array}$$

$$(0, 1_2 \cup 1_3 0_4)^* 1_5 \begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 1 & 3, 5 & 1 \\ 1 & / & 2 & 0 \\ 2 & 1 & 3, 5 & 1 \\ 3 & 4 & / & 0 \\ 4 & 1 & 3, 5 & 1 \\ 5 & / & 5 & 1 \end{array}$$

$(0, 1_2 \cup 1_3 0_4)^* 1_5^* ((0_6 1_7)^* \cup (1_8 0_9)^*)$

$$\begin{array}{c|cc} 0 & 1 & \\ \hline \rightarrow 0 & 1, 6 & 3, 5, 8 & 1 \\ 1 & / & 2 & 0 \\ 2 & 1, 6 & 3, 5, 8 & 1 \\ 3 & 4 & / & 0 \\ 4 & 1, 6 & 3, 5, 8 & 1 \\ 5 & 6 & 5, 8 & 1 \\ 6 & / & 7 & 0 \\ 7 & 6 & / & 1 \\ 8 & 9 & / & 0 \\ 9 & / & 8 & 1 \end{array}$$

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Conte...

3b)

	0	1	
→ 0	1,6	3,5,8	1
1	/	2	0
2	1,6	3,5,8	1
3	4	/	0
4	1,6	3,5,8	1
5	6	5,8	1
6	/	7	0
7	6	/	1
8	9	/	0
9	/	8	1

dPa

A	→ 0
B	1,6
C	3,5,8
D	/
E	2,7
F	4,6,9
G	5,8
H	3,5,7,8
I	6,9
J	7,8

	0	1
A	1,6	3,5,8
B	/	2,7
C	4,6,9	5,8
D	/	/
E	1,6	3,5,8
F	1,6	3,5,7,8
G	6,9	5,8
H	4,6,9	5,8
I	/	7,8
J	6,9	/

1
0
1
0
1
1
1
1
1
1

reduce

A	B	C	D	E	F	G	H	I	J
A	C	E	F	G	H	I	J	B	D
A	E	F	I	C	G	H	J	B	D
A	E	F	I	C	G	H	J	B	D
A	E	F	I	C	G	H	J	B	D
A	E	F	I	C	G	H	J	B	D
1	2	3	4	5	6	7			

A E F I

C G H

	0	1	
→ 1	6	3	1
2	7	5	1
3	1	4	1
4	2	4	1
5	2	7	1
6	7	1	0
7	7	7	0

	0	1
A	B	C
B	D	E
C	F	G
D	D	D
E	B	C
F	B	H
G	I	G
H	F	G
I	D	J
J	I	D

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$$4) \rightarrow A \begin{array}{c|cc} & a & b \\ \hline B & B & A, C \\ A & A & / \\ C & / & B \end{array} \begin{array}{c} 0 \\ 0 \\ 1 \end{array}$$

$$A = aB \cup bA \cup bC$$

$$B = aA$$

$$C = bB \cup \epsilon$$

Insert B into A and C =

$$A = aaA \cup bA \cup bC = (aa \cup b)A \cup bC$$

$$C = baA \cup \epsilon$$

Insert C into A:

$$A = (aa \cup b)A \cup bbaA \cup b$$

$$A = \underbrace{(aa \cup b \cup bba)}_L A \cup \underbrace{b}_m$$

Apply Lemma:

$$A = (aa \cup b \cup bba)^*(b)$$