[redacted]

87

COSC 3340/6309
Examination 2
Friday, June 24, 2011, 4 – 6 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B,C}, {a,b}, {S→A|Bb, A→C, B→aS, C→a }, S).
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:
G = ({S,A,B}, {a,b,c}, {S→Aa|aBBB, A→cS|ε, B→Sb|ε}, S).

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(011)^* \cap \overline{0(11^*)^*}\ ]^*$$

Note: You <u>must</u> first determine nfas for (010)* and 0(01*)* over {01,2,}, then handle the intersection and complementation, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:
G = ({S,B}, {a,b,c,d}, {S→aBBBS|B|cd, B→cSda|S|cba}, S).
Note: You <u>must</u> first remove all unit productions.

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:
G = ({S,A,B}, {a,b}, {S→AA, A→BAA|a , B→SSS|b}, S).
Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points:	**1: 15**	**2: 10**	**3: 30**	**4: 15**	**5: 30**
	15	10	21	13	28

[redacted]

(1) $G = (\{S, A, B, C\}, \{a, b\}, \{S \to A \mid Bb, A \to C, B \to aS, C \to a\}, S)$

$S \to A \mid Bb$

$A \to C$

$B \to aS$

$C \to a$

$L(G) = \{ b^n a S b^n ; n \geq 0 \}$

We determine $L(G)$ is regular

$\exists$ dfa $\underset{\sim}{D}$ st $L(G) = L(D)$

dfa has N states if sets of final states of $w \in L(G) \to$

$z(q_0, w) \in F$

$w = a^n s b^{2n} \; \exists \; w = xyz \quad y \notin \varepsilon, \; |xy| \leq n$

$x = \epsilon, \; y = a^n, \; z = a^n s a^{2n}$

$xy^k z \in L(G) \; \forall K \geq 0$

let $K = 0$

$xy^k z = \varepsilon . \varepsilon . a^n s a^{2n}$

$xy^k z \notin L(G) \Rightarrow z(q_0, xy^k z) \notin F$

lemma is contradiction Initial assumption is incorrect

$\Rightarrow L(G)$ is not regular.

$S \to Bb \to baS \to baSb \to bbaSbb \to$

$b^2 a S b^2 \to b^{n+1} a S b^{n+1}$

[redacted]

② cfg G:

$G = (\{S, A, B\}, \{a, b, c\}, \{S \rightarrow Aa \mid aBBB, A \rightarrow cS \mid \varepsilon, B \rightarrow Sb \mid \varepsilon\}, S).$

$S \rightarrow Aa \mid aBBB$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow Sb \mid \varepsilon$

$B \rightarrow \varepsilon$

$S \rightarrow Aa \mid aBBB \mid aBB \mid aB \mid a$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow bS$

$A \rightarrow \varepsilon$

$S \rightarrow Aa \mid aBBB \mid aBB \mid aB \mid a \mid A$

$A \rightarrow cS$

$B \rightarrow Sb$

[redacted]

(3) $\left[(011)^* \cap \overline{0(11^*)^*}\right]^*$ $\alpha \cap B \Rightarrow \overline{\overline{\alpha} \cup \overline{B}}$ demorgan

$\left[\overline{\overline{(\underset{1}{0}\underset{2}{1}\underset{3}{1})^*} \cup \underset{4}{0}(\underset{5}{1}\underset{6}{1}^*)^*}\right]^*$

	0	1	2	
→0	1	-	-	0
1	-	-	-	1

	0	1	2	
→0	-	2	-	0
2	-	-	-	1

	0	1	2	
→0	-	3	-	0
3	-	-	-	1

·

	0	1	2	
→0	1	/	/	0
1	/	2	/	0
2	/	/	/	1

↓*

	0	1	2	
→0	/	3	/	1
3	/	3	/	1

·

	0	1	2	
→0	1	/	/	0
1	/	2	/	0
2	/	3	/	1
3	/	3	/	1

↓*

NFA

	0	1	2	
→0	1	/	/	1
1	/	2	/	0
2	1	3	/	1
3	1	3	/	1

DFA

	0	1	2	
0	1	/	/	1
1	/	2	/	0
	/	/	/	0
2	1	3	/	1
3	1	3	/	1

↓ Renaming

	0	1	2	
→A	B	C	C	1
B	C	D	C	0
C	C	C	C	0
D	B	E	C	1
E	B	E	C	1

↓ Fliping it/compliment

	0	1	2	
→A	B	C	C	0
B	C	D	C	1
C	C	C	C	1
D	B	E	C	0
E	B	E	C	0

	0	1	2	
0	4	/	/	0
4	/	/	/	1

	0	1	2	
0	/	5	/	0
5	/	/	/	1

	0	1	2	
0	/	6	/	0
6	/	/	/	1

↓*

	0	1	2	
0	/	6	/	1
6	/	6	/	1

·

	0	1	2	
0	/	5	/	0
5	/	6	/	1
6	/	6	/	1

↓*

	0	1	2	
0	/	5	/	1
5	/	6	/	1
6	/	6	/	1

·

	0	1	2	
→0	4	/	/	0
4	/	5	/	1
5	4	6	/	1
6	4	6	/	1

U

	0	1	2	
A	B,4	C	C	0
B	C	D	C	1
C	C	C	C	1
D	B	E	C	0
E	B	E	C	0
4	/	5	/	1
5	4	6	/	1
6	4	6	/	1

↓ Dfa

		0	1	2	
0	A	B,4	C	C	0
1	B,4	C	D,5	C	1
2	C	C	C	C	1
3	D,5	B,4	E,6	C	1
4	E,6	B,4	E,6	C	1
5	B	C	D	C	1
6	D	B	E	C	0
7	E	B	E	C	0

21

→

③ →

	0	1	2	
0	1	2	2	0
1	2	3	2	1
2	2	2	2	1
3	1	4	2	1
4	1	4	2	1
5	2	6	2	1
6	5	7	2	0
7	5	7	2	0

$\xrightarrow{\text{compliment}}$

	0	1	2	
0	1	2	2	1
1	2	3	2	0
2	2	2	2	0
3	1	4	2	0
4	1	4	2	0
5	2	6	2	0
6	5	7	2	1
7	5	7	2	1

↓ *

	0	1	2	
0	1	2	2	1
1	2	3	2	0
2	2	2	2	0
3	1	4	2	0
4	1	4	2	0
5	2	6	2	0
6	1,5	2,7	2	1
7	1,5	2,7	2	1

ABC DEFGHIJKLMN

accepting	Rejecting
AGHJKMN	BCDEFIL
A | GJH | KMN	BCD | E | F | I | L
1 2 3	4 5 6 7 8

↓

		0	1	2	
A	0	1	2	2	1
B	1	2	3	2	0
C	2	2	2	2	0
D	3	1	4	2	0
E	4	1	4	2	0
F	5	2	6	2	0
G	6	1,5	2,7	2	1
H	7	1,5	2,7	2	1
I	1,5	2	3,6	2	0
J	2,7	1,2,5	2,7	2	1
K	3,6	1,5	2,4,7	2	1
L	1,2,5	2	2,3,6	2	0
M	2,4,7	1,2,5	2,4,7	2	1
N	2,3,6	1,2,5	2,4,7	2	1

←

	0	1	2	
A	B	C	C	1
B	C	D	C	0
C	C	C	C	0
D	A	E	C	0
E	~~A~~ F	E	C	0
F	C	G	C	0
G	J	J	C	1
H	J	J	C	1
I	C	K	C	0
J	L	J	C	1
K	J	M	C	1
L	C	N	C	0
M	L	M	C	1
N	L	M	C	1

4.

$G = (\{S, B\}, \{a, b, c, d\}, \{S \to aBBBS \mid B \mid cd,$
$B \to cSda \mid S \mid cba\}, S)$.

[redacted]

$S \to aBBBS \mid B \mid cd$
$B \to cSda \mid S \mid cba$

B's $\to$ S
$S \to aSBBB \mid aSSBB \mid aSBSB \mid aSBBS \mid aSSSB \mid aSBSS \mid aSSBS \mid$
$aSSSS \mid \cancel{S} \mid cd$ /- -
$B \to cSda \mid cba$

replace S's $\to$ B's
$aSBBB \mid aBBBB \mid aSSBB \mid aBSBB \mid aSBBB \mid \cancel{aBBBB} \mid aSBSB \mid$
$aBBSB \mid aBBBS \mid aSSSB \mid aBSSB \mid aSBSS \mid aBBSS \mid aSSBS \mid$
$aBSBS \mid aSSSS \mid aBSSS \mid aSBSS \mid aSSBS \mid aSSSB \mid aBBSS$
$aSBBS \mid aSBBB \mid aBBBB \mid cd$.

$B \to cSda \mid cba \mid cBda$ $\quad B \to X_c S X_d X_a \mid X_c X_b X_a$

$X_a \to a$
$X_b \to b$
$X_c \to c$
$X_d \to d$

$B \to X_c B_1 \mid X_c B_2$
$B_1 \to S B_2$
$B_2 \to X_d X_a$
$B_3 \to X_b X_a$

$S \to X_a S_1 \mid X_a S_3 \mid X_a S_4 \mid X_a S_5 \mid X_a S_6 \mid X_a S_7 \mid X_a S_8 \mid X_a S_9 \mid X_a S_{10} \mid X_a S_{11}$
$X_a S_{12} \mid X_a S_{13} \mid X_a S_{14} \mid X_a S_{15} \mid X_a S_{16} \mid X_a S_{17} \mid X_a S_{18} \mid X_a S_{19} \mid X_a S_{20} \mid X_a S_{22}$
$X_a S_{23} \mid X_a S_{24}$.

cX

$B \to X_c B_1 \mid X_c X_b X_a$
$B_1 \to S B_2$
$B_2 \to X_d X_a$
$B_3 \to X_b X_a$

[redacted]

(5)

$G = (\{S, A, B\}, \{a, b\}, \{S \to AA, A \to BAA \mid a, B \to SSS \mid b\}, S)$.

A_1 $S \to AA$ $A_1 = S$ OK

A_2 $A_2 \to B_3AA \mid a$ $A_2 = A$ OK

A_3 $B_3 \to S_1SS \mid b$ $A_3 = B$ violation

$B \to AASS \mid b$

$B \to \underbrace{BAAASS}_{\alpha_1} \mid \underbrace{aASS}_{\beta_1} \mid \underbrace{b}_{\beta_2}$

remove immediate left recursion

$B \to b \mid bB' \mid aASS \mid aASSB'$

$B' \to AAASS \mid AAASSB'$

substitute B, B' in A

$S \to AA$

$A \to bAA \mid bB'AA \mid aASSAA \mid aASSB'AA \mid a$

substitute S in B'

$B' \to bAAAASASA \mid bB'AAAASASA \mid aASASSASASA \mid$
$aASAB'SSASASA \mid aASASA \mid bSSASASAB' \mid$
$bB'SSASASAB' \mid aASASSASASAB' \mid$
$aAASAAB'SSASASAB' \mid aASASAB'$