COSC 3340/6309 Examination 1

Wednesday, June 10, 2009, 2 - 3:45 pm

Open Book and Notes

1. Construct a dfa for the following nfa, using the subset construction given in class:

	a	b	c	
1	2	2	3	1
2	3	2	3,4	1
→ 3	4	1	2	1
4	1	4	/	1

2. Consider the class $\mathcal{L}_A$ of all regular languages that contain only words of even length, over the fixed two-letter alphabet A={a,b}.

(a) Is $\mathcal{L}_A$ countable?

(b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a^2 \cup a^3)^* (a^3 \cup a^2)$ over the alphabet {a}

(b) $(01 \cup 10)^* ((01)^* \cup (10)^*)0^*$ over the alphabet {0,1}

4. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

	a	b	
→ A	B	C	1
B	A	/	0
C	/	B,C	0

Points: 1: 12 2: 22 3: 44 4: 22

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Exam 1.

1)

	a	b	c	
→3	4	1	2	1
1	2	2	3	1
2	3	2	3,4	1
4	1	4	/	1

1	11
2	21
3	44
4	22
	98

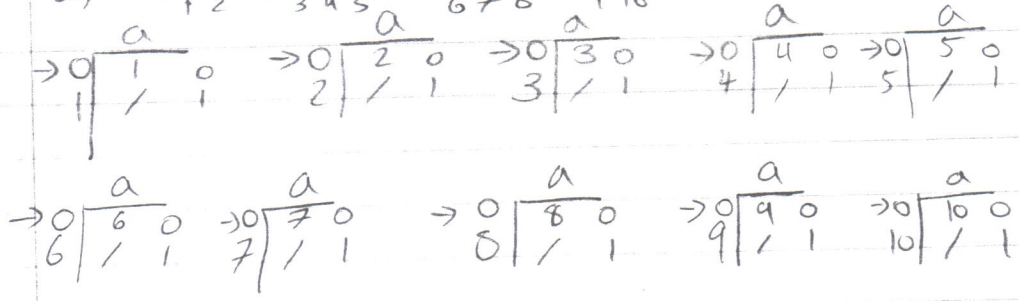
	a	b	c	
→ 3	4	1	2	1
4	1	4	/	1
1	2	2	3	1
2	3	2	3,4	1
/	/	/	/	0
3,4	1,4	1,4	2	1
1,4	1,2	2,4	3	1
1,2	2,3	2	3,4	1
2,4	1,3	2,4	3,4	1
2,3	3,4	1,2	2,3,4	1
1,3	2,4	1,2	2,3	1
2,3,4	1,3,4	1,2,4	2,3,4	1
1,3,4	1,2,4	1,2,4	2,3	1
1,2,4	1,2,3	1,2	2,3,4	1
1,2,3	2,3,4	1,2	2,3,4	1

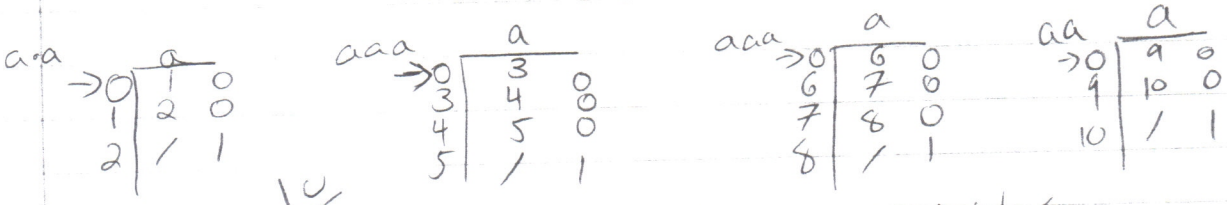
2) (a) L_A is countable because you can sort the words by length, and assign a unique number to each word. ✓

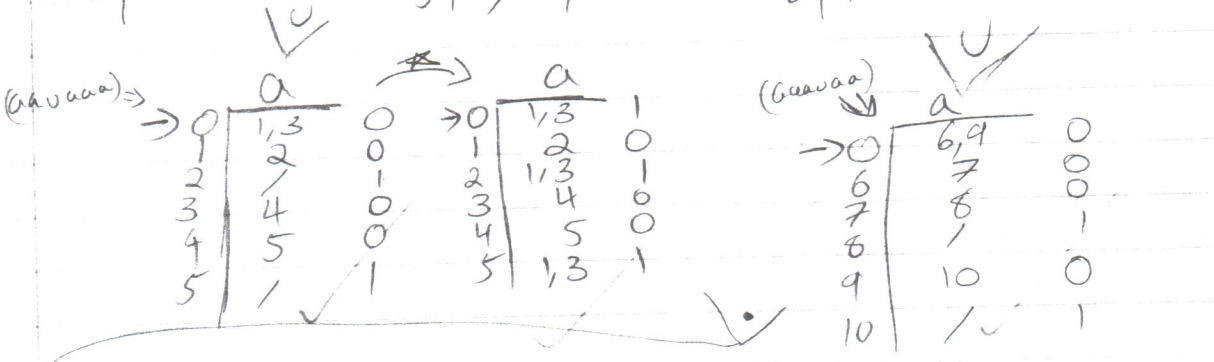
(b) M_A is not countable because it contains languages that are NOT Regular and some of those may have infinite descriptions. ✓

(c) $L_A \cap M_A$ is countable since the intersection is empty. ✓

3) a) $(\underset{1\,2}{aa} \cup \underset{3\,4\,5}{aaa})^* (\underset{6\,7\,8}{aaa} \cup \underset{9\,10}{aa})$







→ on next Page.

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(aa ∪ aaa)* (aaa ∪ aa)

	a	
→ 0	1, 3, 6, 9	0
1	2	0
2	1, 3, 6, 9	0
3	4	0
4	5	0
5	1, 3, 6, 9	0
6	7	0
7	8	0
8	/	1
9	10	0
10	/	1

Dfa ⇒

	a	
→ 0	1, 3, 6, 9	0
1	2, 4, 7, 10	0
2	1, 3, 6, 9, 5, 8	1
3	1, 3, 6, 9, 5, 8	2, 4, 7, 10, 1, 3, 6, 9	1
4	1, 3, 6, 9, 2, 4, 7, 10	2, 4, 7, 10, 1, 3, 6, 9, 5, 8	1
5	1, 2, 3, 4, 5, 6, 7, 8, 9, 10	2, 3, 6, 9, 4, 5, 7, 8, 10	1

⇓

	a	
→ 1	2	0
2	3	0
3	4	1
4	5	1
5	6	1
6	6	1

	12	3456
1	2	3456
1	2	3456
A	B	C

Q2

	a	
→ A	B	0
B	C	0
C	C	1

← reduced

3b) $(01 \cup 10)^*((01)^* \cup (10)^*)0^*$ $\Sigma = \{0, 1\}$

(01 ∪ 10)*

01	0	1
→ 0	1	2
1	/	/
2	/	1

10	0	1
→ 0	/	3
3	4	/
4	/	1

01	0	1
→ 0	5	/
5	/	6
6	/	1

10	0	1
→ 0	/	7
7	8	/
8	/	1

0*	0	1
→ 0	9	/
9	9	/

∪

	0	1
→ 0	1	3
1	/	2
2	/	/
3	4	/
4	/	/

	0	1*
→ 0	5	/
5	/	6
6	5	/

	0	1*
→ 0	/	7
7	8	/
8	/	7

cont. on back →

$(01 \cup 10)^*$

	0	1	
→0	1	3	1
1	/	2	0
2	1	3	1
3	4	/	0
4	1	3	1

$(00)^* \cup (10)^*$

	0	1	
→0	5	7	1
5	/	6	0
6	5	/	1
7	8	/	0
8	/	7	1

0^*

	0	1	
→0	9	/	1
9	9	/	1

	0	1	
→0	1,5	3,7	1
1	/	2	0
2	1,5	3,7	1
3	4	/	0
4	1,5	3,7	1
5	/	6	0
6	5	/	1
7	8	/	0
8	/	7	1

$\cdot 0^*$

	0	1	
→0	1,5,9	3,7	1
1	/	2	0
2	1,5,9	3,7	1
3	4	/	0
4	1,5,9	3,7	1
5	/	6	0
6	5,9	/	1
7	8	/	0
8	9	7	1
9	9	/	1

dfa

		0	1	
1	→0	1,5,9	3,7	1
2	1,5,9	9	2,6	1
3	3,7	4,8	/	0
4	9	9	/	1
5	2,6	1,5,9	3,7	1
6	4,8	1,5,9	3,7	1
7	/	/	/	0

	0	1	
→1	2	3	1
2	4	5	1
3	6	7	0
4	4	7	1
5	2	3	1
6	2	3	1
7	7	7	0

	37	12456			
3	7	1456	2		
→3	7	156	4	2	
3	7	156	4	2	
A	B	C	D	E	

		0	1	
→C		E	A	1
A		C	B	0
B		B	B	0
D		D	B	1
E		D	C	1

Q^2

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4) $\Sigma = \{a, b\}$

	a	b	
→A	B	C	1
B	A	/	0
C	/	B, C	0

$L_A = aL_B \cup bL_C \cup \varepsilon$ ✓
$L_B = aL_A$
$L_C = bL_B \cup bL_C$

$L_G = bL_G \cup bL_B$
$X \quad L \; X \quad M$

$L_C = (b^*)(bL_B)$
⇓
$L_C = (b^*)(b(aL_A))$ ✓

22

$L_A = a(aL_A) \cup b(b^*)(b(aL_A)) \cup \varepsilon$
$L_A = aaL_A \cup bb^*baL_A \cup \varepsilon$
$L_A = (aa \cup bb^*ba)L_A \cup \varepsilon$
$X \quad L \quad X \quad M$

$L_A = (aa \cup bb^*ba)^*\varepsilon$ ✓