COSC 3340/6309 Examination 1

Thursday, June 14, 2012, 4 - 5:45 pm

Open Book and Notes

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1. Construct a dfa for the following nfa, using the subset construction given in class:

11/12

	a	b	c	
1	2	3	2	1
→ 2	3	2	1	1
3	4	1	2,4	1
4	1	4	/	1

2. Consider the class $\mathcal{L}_A$ of all regular languages that contain only words of odd length, over the fixed two-letter alphabet A={a,b}.

(a) Is $\mathcal{L}_A$ countable?

22/22 (b) Is the class $\mathcal{M}_A$ countable where $\mathcal{M}_A$ consists of all languages over A that are not in $\mathcal{L}_A$?

(c) Is the class $\mathcal{L}_A \cap \mathcal{M}_A$ countable?

For each question, you must give a **precise argument substantiating your answer**.

3. Construct an nfa for each of the following regular expressions, then find the corresponding dfa, and then reduce this dfa, **always using the constructions given in class**:

(a) $(a \cup a^3)^* (a \cup a^3)$ over the alphabet {a}

(b) $(01 \cup 10)^* 1^* ((01)^* \cup (10)^*)$ over the alphabet {0,1}

4. Construct a regular expression over the alphabet {a,b} for the language accepted by the following automaton:

22

	a	b	
→ A	B	A,C	0
B	A	/	0
C	/	B	1

Points: **1: 12** **2: 22** **3: 44** **4: 22**

1	
2	
3	44
4	

1)

	a	b	c	
→ 2	3	2	1	1
3	4	1	2,4	1
1	2	3	2	1
4	1	4	/	1
/	/	/	/	0
2,4	1,3	2,4	1	1
1,3	2,4	1,3	2,4	1

-1

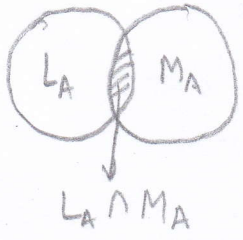
Updated Answer:

	a	b	c	
→ 2	3	2	1	1
3	4	1	2,4	1
1	2	3	2	1
4	1	4	/	1
2,4	1,3	2,4	1	1
/	/	/	/	0
1,3	2,4	1,3	2,4	1

2) (a) Given from the question, L_A is regular. So, we know there exist a DFA that accepts L_A and that is finite automaton. Therefore, it's countable ✓

(b) It is given that L_A is regular and countable. Stated by the question, M_A is not L_A. It is uncertain whether or not M_A is regular. By this case, we cannot account for the language defined by M_A. Thus, M_A is not countable. ✓

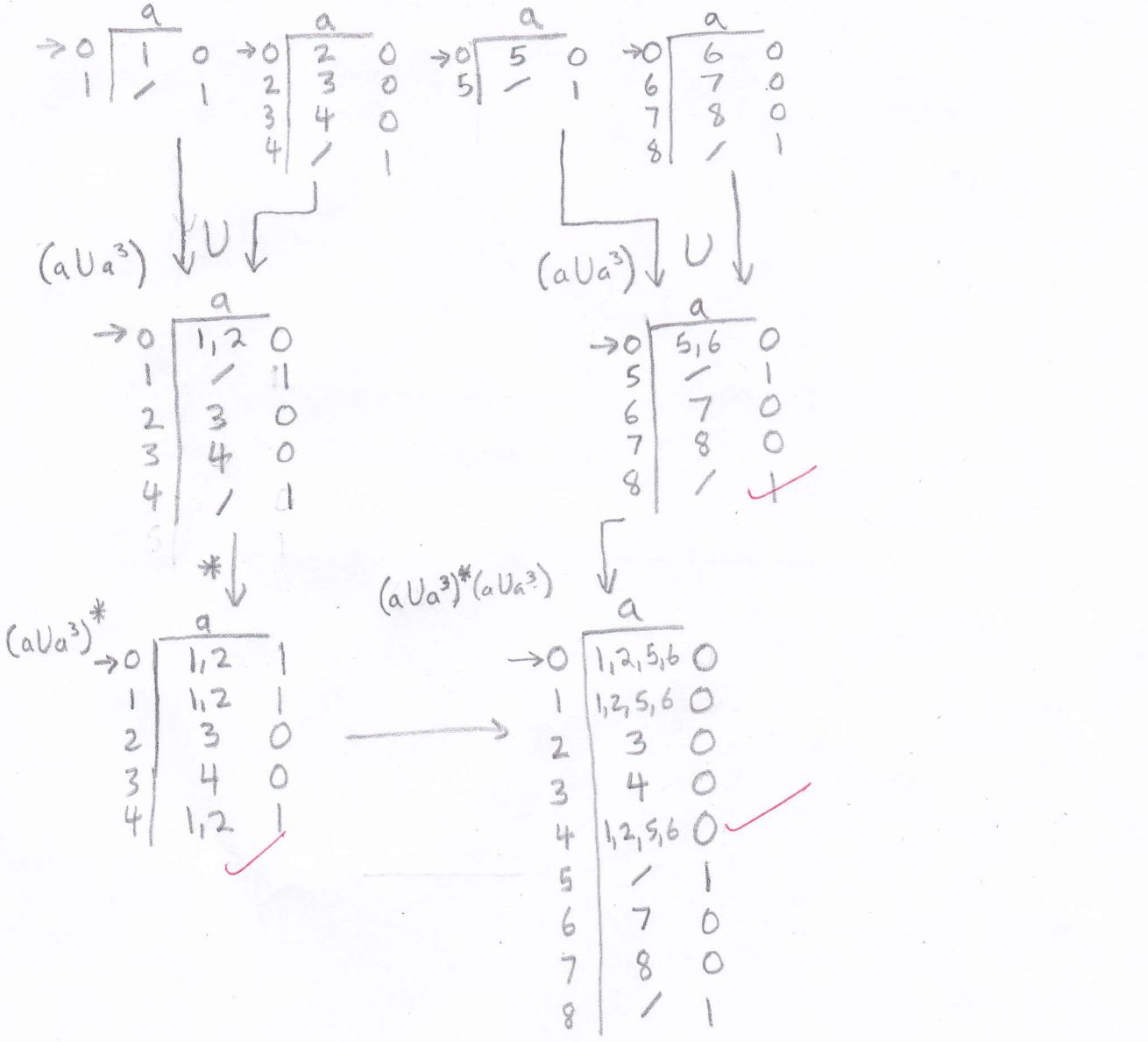
(c) In an intersection operation, there exist some element found in both sets of L_A and M_A.



M_A was declared that it consists of all languages over A that are not in L_A. With $L_A \cap M_A$, there is no element that is in both L_A and M_A. However, this means that $L_A \cap M_A$ will yield an empty set. Empty Set is consider finite with cardinality of zero. If $L_A \cap M_A$ is finite, it is therefore countable. ✓

$(a_1 \cup a_2 \cdot a_3 \cdot a_4)^* (a_5 \cup a_6 \cdot a_7 \cdot a_8)$

3) (a) $(a \cup a^3)^* (a \cup a^3)$ over $\{a\}$



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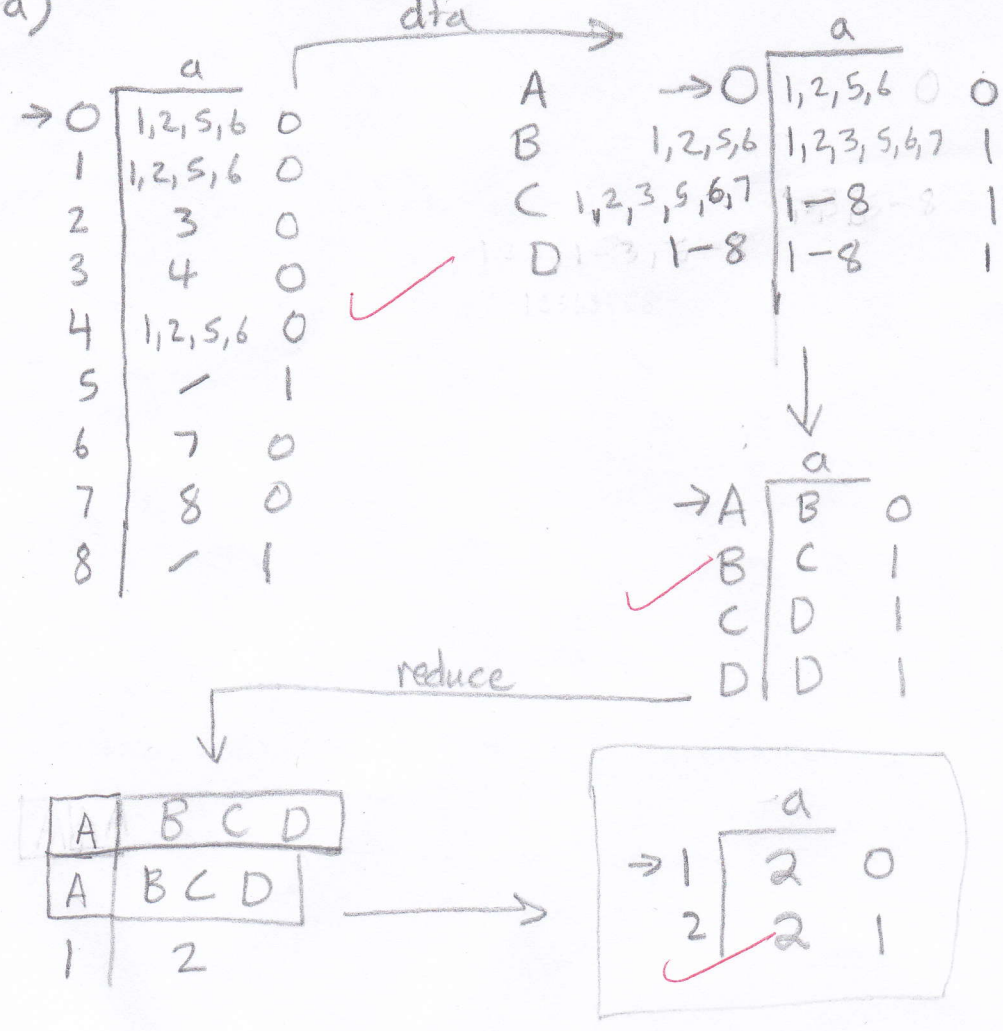
3a)


dfa
a
→0 1,2,5,6 0
1 1,2,5,6 0
2 3 0
3 4 0
4 1,2,5,6 0
5 — 1
6 7 0
7 8 0
8 — 1
A →0 1,2,5,6 0
B 1,2,5,6 1,2,3,5,6,7 1
C 1,2,3,5,6,7 1-8 1
D 1-8 1-8 1
→A B 0
B C 1
C D 1
D D 1
reduce
A B C D
A B C D
1 2
→1 2 0
2 2 1

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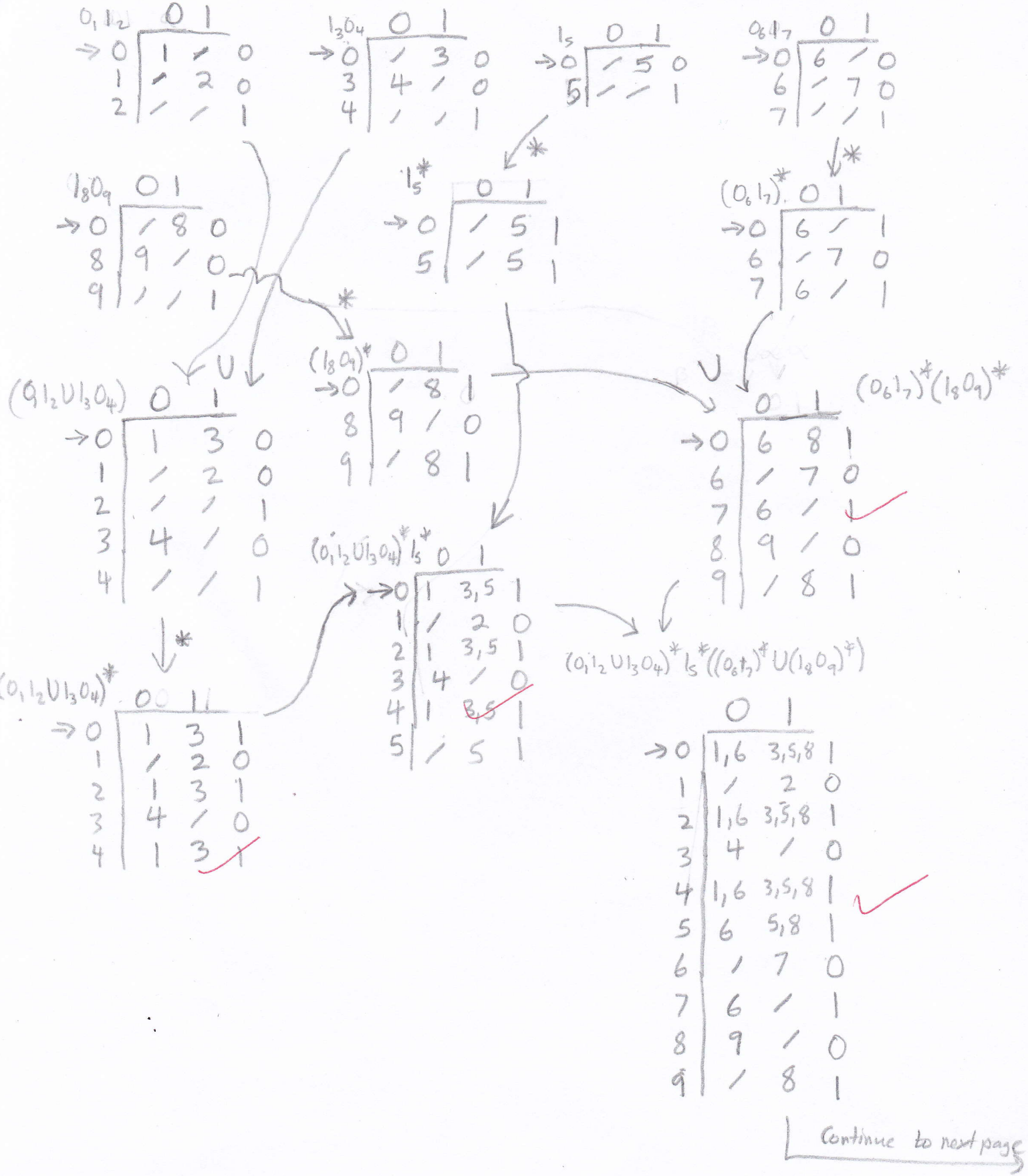
3) (b) $(0_1 1_2 \cup 1_3 0_4)^* \, 1_5^* \, ((0_6 1_7)^* \cup (1_8 0_9)^*)$ over $\{0,1\}$



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3b)

	0	1	
→0	1,6	3,5,8	1
1	/	2	0
2	1,6	3,5,8	1
3	4	/	0
4	1,6	3,5,8	1
5	6	5,8	1
6	/	7	0
7	6	/	1
8	9	/	0
9	/	8	1

dPa →

		0	1	
A	→0	1,6	3,5,8	1
B	1,6	/	2,7	0
C	3,5,8	4,6,9	5,8	1
D	/	/	/	0
E	2,7	1,6	3,5,8	1
F	4,6,9	1,6	3,5,7,8	1
G	5,8	6,9	5,8	1
H	3,5,7,8	4,6,9	5,8	1
I	6,9	/	7,8	1
J	7,8	6,9	/	1

reduce

ABCDEFGHIJ						
ACEFGHIJ					BD	
AEFI		CGH		J	B	D
AEF	I	CGH		J	B	D
AEF	I	CH	G	J	B	D
AEF	I	CH	G	J	B	D
1	2	3	4	5	6	7

		0	1	
	→A	B	C	1
	B	D	E	0
	C	F	G	1
	D	D	D	0
	E	B	C	1
	F	B	H	1
	G	I	G	1
	H	F	G	1
	I	D	J	1
	J	I	D	1

		0	1	
	→1	6	3	1
	2	7	5	1
	3	1	4	1
	4	2	4	1
	5	2	7	1
	6	7	1	0
	7	7	7	0

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4)

	a	b	
→A	B	A,C	0
B	A	/	0
C	/	B	1

$A = aB \cup bA \cup bC$

$B = aA$

$C = bB \cup \varepsilon$

Insert B into A and C:

$A = aaA \cup bA \cup bC = (aa \cup b)A \cup bC$

$C = baA \cup \varepsilon$

Insert C into A:

$A = (aa \cup b)A \cup bbaA \cup b$

$A = \underbrace{(aa \cup b \cup bba)}_{L}\underbrace{A}_{x} \cup \underbrace{b}_{M}$

Appy Lemma:

$$\boxed{A = (aa \cup b \cup bba)^*(b)}$$