

**COSC 3340**  
**Examination 2**  
**Monday, March 30, 2009, 1 – 2:30 pm**  
**Open Book and Notes**

**1.** Prove that the language  $L(G)$  is not regular where  $G$  is the following context-free grammar:  $G = (\{S, A, B\}, \{a, b\}, \{S \rightarrow A|aB, A \rightarrow b, B \rightarrow Sa\}, S)$ .

Note: You must first determine  $L(G)$ .

**2.** Eliminate all  $\epsilon$ -productions in the following cfg  $G$ :

$G = (\{S, A, B\}, \{a, b, c\}, \{S \rightarrow aAb|AB, A \rightarrow cS|\epsilon, B \rightarrow Sb|\epsilon\}, S)$ .

**3.** Construct a **reduced** dfa for the following extended regular expression over the alphabet  $\{0, 1, 2\}$  (not  $\{0, 1\}!$ ):

$$[(001)^* \cap \overline{0(01^*)^*}]^*$$

Note: You must first determine nfas for  $(001)^*$  and  $0(01^*)^*$ , then handle the intersection, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

**4.** Construct a Chomsky normal form grammar for  $L(G)$  for the following cfg  $G$ :

$G = (\{S, B\}, \{\tilde{a}, b, c, d\}, \{S \rightarrow aSBB|cd, B \rightarrow cSdB|S\tilde{c}ba\}, S)$ .

Note: You must first remove all unit productions.

**5.** Construct a Greibach normal form grammar for  $L(G)$  for the following CNF  $G$ :

$G = (\{S, A, B\}, \{a, b\}, \{S \rightarrow AA, A \rightarrow BS|b, B \rightarrow SA|a\}, S)$ .

Note: First derive all the productions for  $S$ ,  $A$ , and  $B$ . You may only indicate how the final result looks for whatever primed variables you obtain.

Points:      1: 15              2: 10              3: 30              4: 15              5: 30

1 Answer

We have the following productions:

$$S \rightarrow A | aB$$

$$A \rightarrow b$$

$$B \rightarrow Sa$$

Determine  $L(G)$  from above

$$\begin{array}{ccccccc}
 S \rightarrow aB \rightarrow aSa \rightarrow aaBa \rightarrow aaSaa \rightarrow a^3Ba^2 \rightarrow a^3Sa^3 \rightarrow \dots \\
 \downarrow A & \downarrow aAa & \downarrow a^2Aa^2 & \downarrow \dots \\
 \downarrow b & \downarrow aba & \downarrow a^2ba^2 & \downarrow \dots
 \end{array}$$

So, the language of grammar  $G$  will be

$$L(G) = \{a^n b a^n \mid n \geq 0\}$$

Assume  $L(G)$  is a regular language, so  $\exists$  dfa  $D$  which has  $n$  statesConsider  $x = a^n b a^n = wu$ , and  $w = a^n$ ,  $u = b a^n$ Since  $|w| = n$ , we can apply pumping lemma here

$$\Rightarrow w = w_1 w_2 w_3 \text{ st. } |w_2| \geq 1$$

$$\text{So, } \tau(q_0, w) = \tau(q_0, w_1 (w_2)^s w_3) \quad s \geq 0$$

Consider  $s = 0$ 

$$\text{We have } \tau(q_0, w) = \tau(q_0, w_1 w_3)$$

BUT,  $|w_1 w_3| = n - |w_2| < n$ , so  $\tau(q_0, w_1 w_3) \notin L(G)$ Since  $\tau(q_0, w) \in L(G)$ , but  $\tau(q_0, w_1 w_3) \notin L(G)$ 

Hence, we have a contradiction here

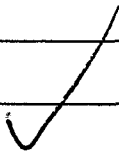
Thus,  $L(G)$  is not regular.

2 Answer

$$S \rightarrow aAb \mid AB$$

$$A \rightarrow cS \mid \epsilon$$

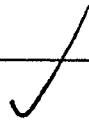
$$B \rightarrow Sb \mid \epsilon$$

① Eliminate  $A \rightarrow \epsilon$ 

$$S \rightarrow aAb \mid ab \mid AB \mid B$$

$$A \rightarrow cS$$

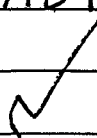
$$B \rightarrow Sb \mid \epsilon$$

② Eliminate  $B \rightarrow \epsilon$ 

$$S \rightarrow aAb \mid ab \mid AB \mid A \mid B \mid \epsilon$$

$$A \rightarrow cS$$

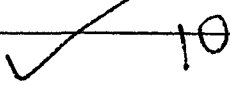
$$B \rightarrow Sb$$

③ Eliminate  $S \rightarrow \epsilon$ 

$$S \rightarrow aAb \mid ab \mid AB \mid A \mid B$$

$$A \rightarrow cS \mid c$$

$$B \rightarrow Sb \mid b$$



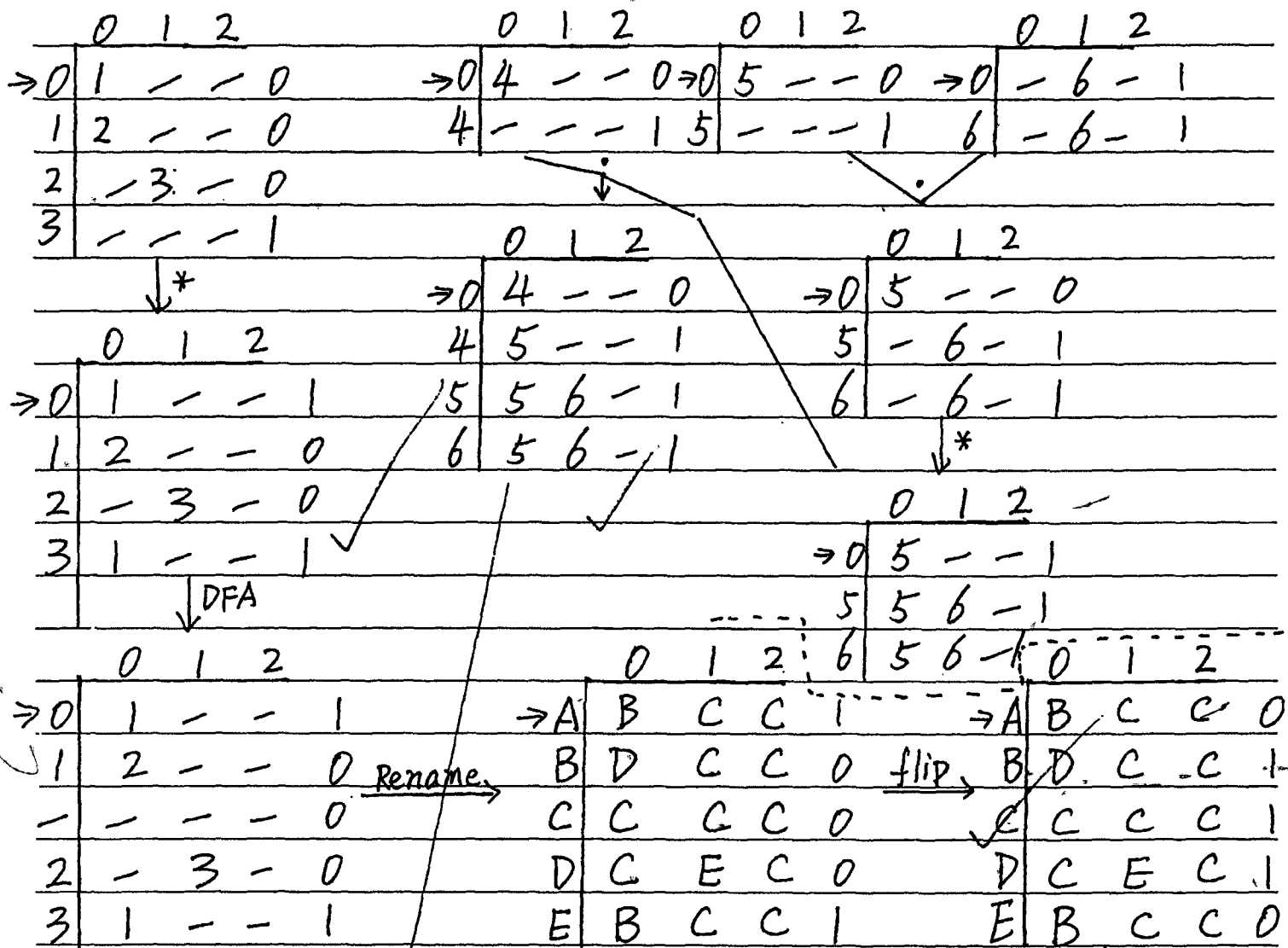
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## 3 Answer

We have  $[(001)^* \cap \overline{0(01)^*}]^*$

According to Morgan's Law, we could have:

$$[(001)^* \cup 0(01)^*]^*$$



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THX!

3 continued

					0	1	2							
L →	A	B	4	C	C	0	DFA	→ A	B	4	C	C	0	
	B	D	C	C	1	B		4	D	5	C	C	1	
	C	C	C	C	1	C		C	C	C	1			
	D	C	E	C	1	D		5	C	5	E	6	C	1
	E	B	C	C	0	C		5	C	5	C	6	C	1
	4	5	-	-	1	E		6	B	5	C	6	C	1
	5	5	6	-	1	6		C	5	C	6	C	1	
6	5	6	-	1	B	5	D	5	C	6	C	1		

Rename →					0	1	2						0	1	2						
→ 1	2	3	3	0	flip →					1	2	3	3	1							
2	4	3	3	1						2	4	3	3	0							
3	3	3	3	1						3	3	3	3	0							
4	5	6	3	1						4	5	6	3	0							
5	5	7	3	1						5	5	7	3	0							
6	8	7	3	1						6	8	7	3	0							
7	5	7	3	1						7	5	7	3	0							
8	4	7	3	1						8	4	7	3	0							
* →					0	1	2	Reduced					2	3	4	5	6	7	8	1	
→ 1	2	3	3	1						2					3	4	5	6	7	8	1
2	4	3	3	0											B					A	
3	3	3	3	0						0					1	2					
4	5	6	3	0	Rename →					A	B	B	B	1	0						
5	5	7	3	0						B	B	B	B	0							
6	8	7	3	0						✓											
7	5	7	3	0																	
8	4	7	3	0																	

## 4 Answer

$$S \rightarrow aSBB \mid cd$$

$$B \rightarrow cSdB \mid S \mid cba$$

① Eliminate  $B \rightarrow S$ 

$$S \rightarrow aSBB \mid aSSB \mid aSBS \mid aSSS \mid cd$$

$$B \rightarrow cSdB \mid cSdS \mid cba$$

② Construct CNF

$$\langle 1 \rangle \quad S \rightarrow X_a S B B \mid X_a S S B \mid X_a S B S \mid X_a S S S \mid X_c X_d$$

$$B \rightarrow X_c (S X_d B) \mid X_c S X_d S \mid X_c X_b X_a$$

$$X_a \rightarrow a, X_b \rightarrow b, X_c \rightarrow c, X_d \rightarrow d$$

$$\langle 2 \rangle \quad S \rightarrow X_a S_1 \mid X_a S_3 \mid X_a S_5 \mid X_a S_7 \mid X_c X_d$$

$$S_1 \rightarrow S S_2$$

$$S_2 \rightarrow B B$$

$$S_3 \rightarrow S S_4$$

$$S_4 \rightarrow S B$$

$$S_5 \rightarrow S S_6$$

$$S_6 \rightarrow B S$$

$$S_7 \rightarrow S S_8$$

$$S_8 \rightarrow S S$$

$$B \rightarrow X_c B_1 \mid X_c B_3 \mid X_c B_5$$

$$B_1 \rightarrow S B_2$$

$$B_2 \rightarrow X_d B$$

$$B_3 \rightarrow S B_4$$

$$B_4 \rightarrow X_d S$$

$$B_5 \rightarrow X_b X_a$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

$$\rightarrow \begin{array}{l} X_c \rightarrow c \\ X_d \rightarrow d \end{array}$$

5 Answer

$$S \rightarrow AA$$

$$A \rightarrow BS \mid b$$

$$B \rightarrow SA \mid a$$

$$i=1 \quad \text{It's OK}$$

$$i=2 \quad \text{It's OK}$$

$$i=3$$

$$j=1 \quad \text{Replace } S \rightarrow AA, \text{ we have:}$$

$$B \rightarrow AAA \mid a$$

$$j=2 \quad \text{Replace } A \rightarrow BS \mid b, \text{ we have:}$$

$$B \rightarrow BSAA \mid bAA \mid a \quad \checkmark$$

Now, we have no left recursion, so we have

$$B \rightarrow bAA \mid a \mid bAAB' \mid aB'$$

$$B' \rightarrow SAA \mid SAAB' \quad \checkmark$$

Now, putting production of B into A, we have

$$A \rightarrow bAAS \mid aS \mid bAAB'S \mid aB'S \mid b$$

Using A, we find production of S,  $\checkmark$

$$S \rightarrow bAASA \mid aSA \mid bAAB'SA \mid aB'SA \mid bA$$

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So, we have

$$S \rightarrow bAASA \mid aSA \mid bAAB'SA \mid aB'SA \mid bA \quad \checkmark$$

$$A \rightarrow bAAS \mid aS \mid bAAB'S \mid aB'S \mid b \quad \checkmark$$

$$B \rightarrow bAA \mid a \mid bAAB' \mid aB'$$

$$B' \rightarrow SAA \mid SAAB'$$