COSC 3340
Examination 2
Wednesday, March 12, 2008, 1 – 2:30 pm
Open Book and Notes

1. Prove that the language L(G) is not regular where G is the following context-free grammar: G = ({S,A,B}, {a,b}, {S→A|bB, A→a, B→Sb}, S).
Note: You <u>must</u> first determine L(G).

2. Eliminate all ε-productions in the following cfg G:

G = ({S,A,B}, {a,b,c}, {S→aAb|AB, A→cS|ε, B→Sb|ε}, S).

3. Construct a **reduced** dfa for the following extended regular expression over the alphabet {0,1,2} (not {0,1}!):

$$[(011)^* \cap \overline{0(01^*)^*}\ ]^* \qquad [\alpha \cap \bar{\beta}]^* \qquad \overline{[\bar{\alpha} \cup \beta]}^*$$

Note: You <u>must</u> first determine <u>nfas for (011)</u>* <u>and 0(10*)*</u>, then handle the intersection, and then deal with the star. Finally reduce the resulting dfa. Consider de Morgan's laws!

4. Construct a Chomsky normal form grammar for L(G) for the following cfg G:

G = ({S,B}, {a,b,c,d}, {S→aSBb|cd, B→cSdB|S|cba}, S).
Note: You <u>must</u> first remove all unit productions.

$$\overline{(\bar{\alpha} \cup B)}^*$$

5. Construct a Greibach normal form grammar for L(G) for the following CNF G:

G = ({S,A,B}, {a,b}, {S→AA, A→BS|a , B→SA|b}, S).
Note: First derive <u>all</u> the productions for S, A, and B. You may only indicate how the final result looks for whatever primed variables you obtain.

Points: **1: 15** **2: 10** **3: 30** **4: 15** **5: 30**

Chomsky → Griebach

Ak ①

$$\left[(0|1)^* \cap 0(01^*)^*\right]^*$$

$$\left[(011)^* \cup 0(01^*)^*\right]^*$$

1	15
2	10
3	5 +10=15
4	10
5	30
	70 =70=80

② -

$S \rightarrow aAb \mid AB$

$A \rightarrow cS \mid \varepsilon$

$B \rightarrow Sb \mid \varepsilon$

$A \rightarrow \varepsilon$

$S \rightarrow aAb \mid ab \mid AB \mid B$

$A \rightarrow cS$

$B \rightarrow Sb \mid \varepsilon$

$B \rightarrow \varepsilon$

$S \rightarrow aAb \mid ab \mid AB \mid A \mid B \mid \varepsilon$

$A \rightarrow cS$

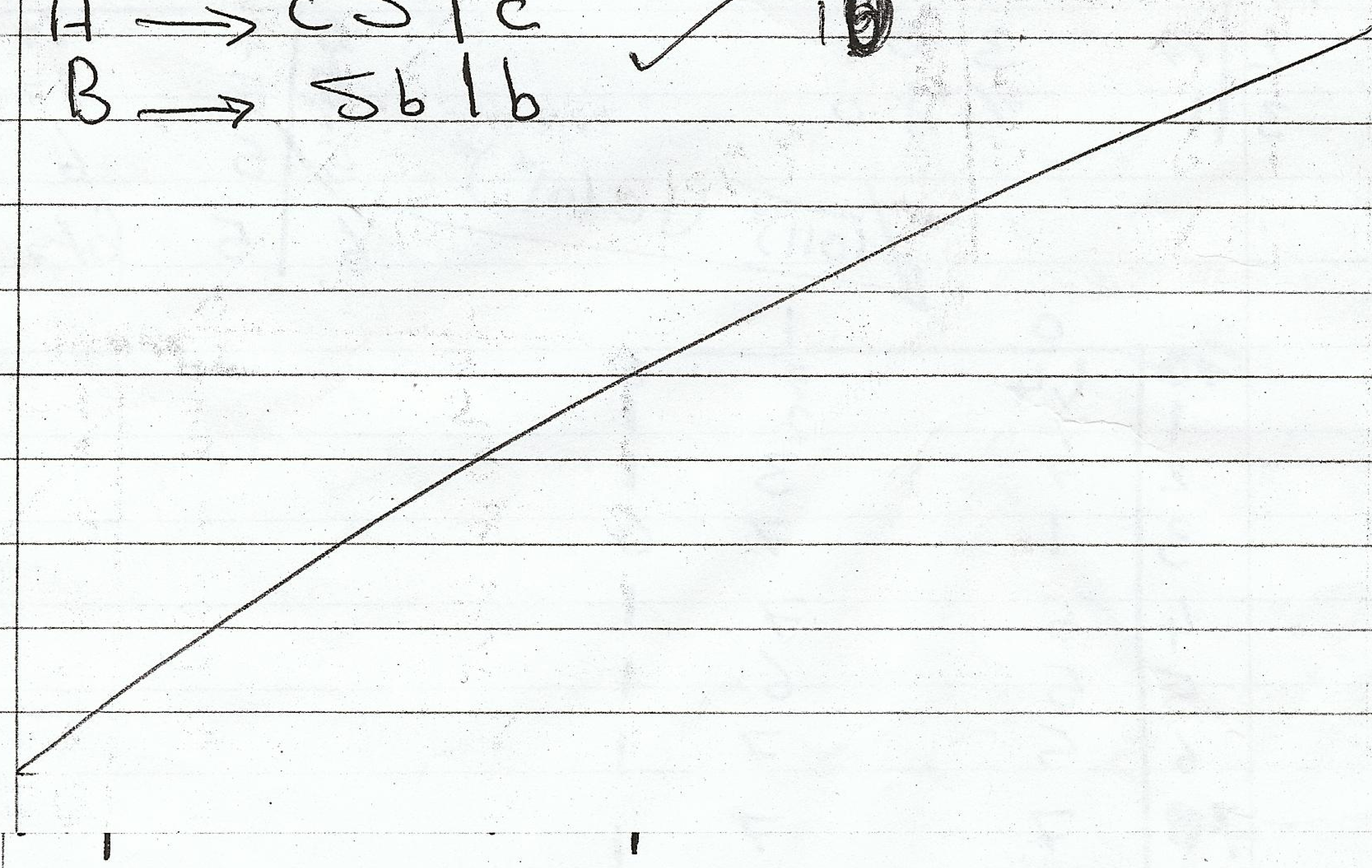
$B \rightarrow Sb$

$S \rightarrow S$

$S \rightarrow aAb \mid ab \mid AB \mid A \mid B$

$A \rightarrow cS \mid c$ ✓ 10

$B \rightarrow Sb \mid b$



②

③ $[(011)^* \cap \overline{0(01^*)^*}]^*$

$[\overline{(011)^* \cup 0(01^*)^*}]^*$ over $\{0,1,2\}$

	0	1	
→0	1	/	0
1	/	/	1

	0	1	
→0	/	2	0
2	/	/	1

	0	1	
→0	/	3	0
3	/	/	1

	0	1	
→0	4	/	0
4	/	/	1

	0	1	
→0	5	/	0
5	/	/	1

	0	1	
→0	/	6	0
6	/	/	1

01

	0	1	
→0	1	/	0
1	/	2	0
2	/	/	1

011

	0	1	
→0	1	/	0
1	/	2	0
2	/	3	0
3	/	/	1

$(011)^*$

	0	1	
→0	1	/	1
1	/	2	0
2	/	3	0
3	1	/	1

dfa $\overline{(011)^*}$

	0		
→0	1	/7	~~1~~ 0
1	~~1~~	2	~~0~~ 1
7 /	/7	7/	~~0~~ 1
2	/7	3	~~0~~ 1
3	1	7/	~~1~~ 0

rename / with 7

$\downarrow *$

	0	1	
→0	/	6	1
6	/	6	1

01^*

	0	1	
→0	5	/	0
5	/	6	1
6	/	6	1

$(01^*)^*$

	0	1	
→0	5	/	1
5	5	6	1
6	5	6/	1

$0(01^*)^*$

	0	1	
→0	4	/7	0
4	5	/7	1
5	5	6	1
6	5	6/7	1

$\overline{(011)^* \cup 0(01^*)^*}$

	0	1	
→0	1,4	7	0
1	7	2	1
2	7	3	1
3	1	7	0
4	5	7	1
5	5	6	1
6	5	7	1
7	7	7	1

Alex Metry (3)

obtaining the Dfa

		0	1	0	1
A	→0	1,4	7		
B	1,4	5,7	2,7	x	0
C	7	7	7	x	0
D	5,7	5,7	6,7	x	0
E	2,7	7	3,7	x	0
F	6,7	5,7	7	x	0
G	3,7	1,7	7	x	0
H	1,7	7	2,7	x	0

the Dfa is

	0	1	
→A	B	C	1
B	D	E	0
C	C	C	0
D	D	F	0
E	C	G	0
F	D	C	0
G	H	C	0
H	C	E	0

Q−F	F
B C D E F G H	A
$\underbrace{\text{B C D E F G H}}_{X}$	
X	

the reduced dfa is

	0	1	
→A	X	X	1
X	X	X	0

Alex Metry ④

④ $S \to aSBb \mid cd$

$B \to cSdB \mid S \mid cba$

remove $B \to S$

$S \to aSBb \mid aSSb \mid cd$

$B \to aSdB \mid aSdS \mid cba$

$S \to X_a S B X_b \mid X_a S S X_b \mid X_c X_d$

$B \to X_a S X_d B \mid X_a S X_d S \mid X_c X_b X_a$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d$

$S \to X_a S_1 \mid X_a S_3 \mid X_c X_d$

$S_1 \to S S_2$

$S_2 \to B X_b$

$S_3 \to S S_4$

$S_4 \to S X_b$

$B \to X_a B_1 \mid X_a B_3 \mid X_c B_5$

$B_1 \to S B_2$

$B_2 \to X_d B_1$

$B_3 \to S B_4$

$B_4 \to X_d S$

$B_5 \to X_b X_a$

$X_a \to a$

$X_b \to b$

$X_c \to c$

$X_d \to d$

10

(5) $S \to AA$
$A \to BS | a$
$B \to SA | b$

$i=1$ $S \to AA$
$i=2$ $A \to BS | a$
$i=3$
$j=1$ $B \to AAA | b$
$j=2$ $B \to \underset{\alpha_1}{BSAA} | \underset{\beta_1}{aAA} | \underset{\beta_2}{b}$

eliminate left recursion

$B \to b | \cancel{bB'} | aAA | bB' | aAAB'$
$B' \to SAA | SAAB'$

$\overbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxx}}^{BS}$

$A \to bS | aAAS | bB'S | aAAB'S | a$
$S \to bSA | aAASA | bB'SA | aAAB'SA | aA$
$B' \to bSAAA | aAASAAA | bB'SAAA | aAAB'SAAA | aAAA$
$| bSAAAB' | aAASAAAB' | bB'SAAAB' | aAAB'SAAAB' | aAAAB'$

GNF

30

$S \to bSA | aAASA | bB'SA | aAAB'SA | aA$
$A \to bS | aAAS | bB'S | aAAB'S | a$
$B \to b | aAA | bB' | aAAB'$
$B' \to bSAAA | aAASAAA | bB'SAAA | aAAB'SAAA$
$| aAAA | bSAAAB' | aAASAAAB' | bB'SAAAB' |$
$aAAB'SAAAB' | aAAAB'$

Alex Metry ⑥

① $S \to A \mid bB$
$A \to a$
$B \to Sb$

~~$S \overset{i}{\Rightarrow} b^i B$~~

$S \Rightarrow bB \Rightarrow bSb \overset{i}{\Rightarrow} b^i S b^i \overset{S \to A}{\Rightarrow} b^i A b^i \Rightarrow b^i a b^i$

$L(G) = \{ b^i a b^i \mid i \geq 0 \}$ no. of b before a = no. of b after a

proof:-

assume $L(G)$ is regular $\Rightarrow \exists$ dfa $\underset{\sim}{D}$

s.t. $L(\underset{\sim}{D}) = \tilde{L}(G)$

Let N be the Number of states of $\underset{\sim}{D}$

Consider $u = b^N a b^N$ $u \in L(G)$

$\exists\, w_1, w_2, w_3$: $w = w_1 w_2 w_3$ and $|w_2| \geq 1$
and

$\tau(q_0, w_1 (w_2)^s w_3) = \tau(q_0, w)\ \forall s \geq 0$

Consider $s = 0$: $|w_1 . (w_2)^0 . w_3| = N - |w_2| < N$

$\therefore \tau(\tau(q_0, w), ab^N) \in F$

$\therefore \tau(\tau(q_0, w_1 . w_3), ab^N) \in F$

but $b^{N-|w_2|} a b^N \notin L$

why?

$\therefore$ There exists a contradiction 15

$\therefore$ The assumption must be wrong

$\therefore$ $L(G)$ is not regular