Contributions to Arc Routing

English Summary

Sanne Wøhlk

May 23, 2005

In this dissertation, mainly two problems are considered, both of which are routing problems. The first part of the dissertation explores the Capacitated Arc Routing Problem (CARP), which was first suggested by Golden and Wond 1981 [1]. This is the problem of servicing a set og edges in a graph, using a fleet of capacity-constrained vehicles. The problem occurs in practice in situations where streets need to be serviced, for example in refuse collection, street sweeping, snow removal, or mail delivery. In the second part of the dissertation, a version of the CARP is considered, where each demand edge has an associated time window within which the service of that edge must begin. This problem is called the Capacitated Arc Routing Problem with Time Windows (CARP-TW), and has only been studied in the acadamic literature in few papers. The third part of the dissertation contains material that is related to the first two parts, but which does not fit into those parts. This includes an online varsion of the CARP and a setup with several Depots and fixed vehicle costs.

A natural starting point when studying an Operations Research problem, is to construct a mathematical model for the problem. Various such models have previously been presented for the CARP. To our knowledge, no mathematical model for the undirected CARP-TW has been presented before. In the dissertation, we discuss the difficulties of constructing such a model for the CARP-TW, and present two models for the problem, one based directly on arc routing and the other based on a transformation of the problem into node routing.

A Lower Bounds for an optimization problems is a value, LB, for which it can be proved that any feasible solution to the problem must have objective value at least as large as this number. Lower Bounds are used for evaluation of the quality of feasible solutions, and for the use in methods, such as Branch and Bound, for finding an optimal feasible solution. Several Lower Bounds have been presented for the CARP. We present a new Lower Bound, the Multiple Cuts Node Duplication Lower Bound (MCNDLB), for the CARP, and prove that this bound outperforms the existing Lower Bounds. Furthermore, we extend the MCNDLB for the CARP to be valid for the CARP-TW and show how the bound can be tightened based on the time windows.

A way of obtaining a feasible solution to a problem is by using a Problem-Specific Heuristic, i.e. an algorithm designed specifically for that problem. Several problem-specific heuristics have been proposed for the CARP in the academic literature. We show how one of the existing heuristics, the Path-Scanning Heuristic, can be improved, and propose two new problem-specific heuristics, the Double Outher Scan Heuristic and the Node Duplicated Heuristic. The latter of these perform relatively well with a performance of about 15 percent above lower bound. For the CARP-TW, we show how two of the problem-specific heuristics for the classical CARP can be tuned to work for the CARP-TW. Furthermore, we present a new problem-specific heuristic, the Preferable Neighbor Heuristics, for the CARP-TW. For the test set, this algorithm finds an optimal solution for 17 instances out of 24.

During the past decade, Meta Heuristics have been very popular when solving optimization problems. Meta heuristics are based on a neighborhood structure, which defines a way of moving from one feasible solution to another. Unlike problem-specific heuristics, Meta Heuristics can be used for several different problems depending on the specification of the cost structure and the neighborhood structure. Various Meta Heuristic strategies have been applied to the classical CARP. We consider a Meta Heuristic strategy where we combine Simulated Annealing and Dynamic Programming, and show how this strategy applies to the classical CARP. Furthermore, we use the same idea to obtain an algorithm for the CARP-TW, and finally, we show how the same idea can be used for several other routing and scheduling problems.

Approximation Algorithms are a special kind of algorithms, that give a feasible solution to a problem like heuritics, but unlike heuristics, approximation algorithms have a performance guarantee on the solution constructed. That is, the cost of a solution returned by an approximation algorithm is at most some constant times the cost of an optimal solution. Approximation algorithms have been studied for the General Capacitated Routing Problem, which has the CARP as a special case. After describing this work in the content of the CARP, we present a new approximation algorithm for the CARP, and show that this algorithm outperforms the other one. Besides being an approximation algorithm, this new algorithm can also be considered a problem-specific heuristic for the CARP, and is endeed competitive to other heuristics for that problem, with a performance of about 15 percent above lower bound.

Optimal Solutions for optimization problems are desirable for several reasons. First of all, the optimal solution is the one true solution of the problem and is therefore interesting in itself. Second, from the company point of view, the optimal solution is the absolutely best possible, and therefore desirable. Third, knowing the optimal solution gives a way of measuring the performanse of heuristics and lower bounding procedures, and a clear indication of which should be improved. Optimal solutions are computationally very hard to obtain, and can only be found for relatively small instances of the problem at hand.

We present a strategy for obtaining optimal solutions for the CARP-TW, which is based on a primal-dual subproblem simplex for solving the LP-relaxation, and uses a two-phase algorithm for solving the ILP.

In the third part of the dissertation, we consider a Computer Science oriented problem, which we refer to as Online Arc Routing. The idea in this setup is that the demand for service of edges is not known in advance, and every time a request for a demand edge is revealed, a decision has to be made about which vehicle is to service this edge, and in which direction. Next, we consider a problem referred to as the Restricted Arc Routing Problem. In this problem, we fix the order in which the demand edges are serviced, and decisions must be made about the routing, and about when to return to the Depot to recharge capacity. The solution to this problem can be considered as the optimal offline solution to the online problem, and is also the basis of the Meta Heuristic procedure presented earlier.

Finally, we consider an extention of the classical CARP with several Depots instead of just one. Furthermore, in this model, vehicles are allowed to have different capacities, and to induce a fixed cost if they are used. This problem is referred to as the Multiple Depot Capacitated Arc Routing Problem (CARP-MD), and has been studied in the literature to some extent. We consider the problem from a theoretical point of view, where a mathematical model is presented along with a lower-bounding algorithm, and heuristic solution procedures are suggested.

References

[1] Bruce L. Golden and Richard T. Wong. Capacitated Arc Routing Problems. *Networks*, 11:305–315, 1981.