Econometrics Assignment e
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(a) $y = \mathbf{X}\beta + \epsilon$
Full model (unrestricted): p_1 regressors
Restricted model: $p_0 < p_1$ regressors
$AIC = \log s^2 + \frac{2k}{n}$

If smallest model is preferred

$$\log s_0^2 + \frac{2p_0}{n} < \log s_1^2 + \frac{2p_1}{n}$$

$$\log s_0^2 - \log s_1^2 < \frac{2}{n}(p_1 - p_0)$$

$$\log \frac{s_0^2}{s_1^2} < \frac{2}{n}(p_1 - p_0)$$

$$\frac{s_0^2}{s_1^2} < e^{\frac{2}{n}(p_1 - p_0)}$$

(b) Using $e^x \approx 1 + x$ for small values of x

For very large n, $\frac{2}{n}$ becomes very small

$$\frac{s_0^2}{s_1^2} < e^{\frac{2}{n}(p_1 - p_0)}$$

$$\frac{s_0^2}{s_1^2} < 1 + \frac{2}{n}(p_1 - p_0)$$

$$\frac{s_0^2}{s_1^2} - 1 < \frac{2}{n}(p_1 - p_0)$$

$$\frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{n}(p_1 - p_0)$$

(c) At large n, $s_i \approx e_i' e_i$

$$\frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{n}(p_1 - p_0)$$

$$\frac{e_R' e_R - e_U' e_U}{e_U' e_U} < \frac{2}{n}(p_1 - p_0)$$

(d) At large n, using $\frac{e_R' e_R - e_U' e_U}{e_U' e_U} < \frac{2}{n}(p_1 - p_0)$

$$F = \frac{e_R' e_R - e_U' e_U/(p_1 - p_0)}{e_U' e_U/(n - p_1)}$$

$$F = \frac{n - p_1}{p_1 - p_0}\frac{2}{n}(p_1 - p_0)$$

$$F = \frac{2(n - p_1)}{n}$$

Since n is large, $n - p_1 \approx n$

$$F \approx \frac{2n}{n}$$

$$F \approx 2$$