Math Logic: Assignment 2

Oct 9, 2021

Attention: To get full credits, you *must provide explanations to your answers*! You will get at most $1/3$ of the points if you only present the final results.

1. (3pt) Let R be real numbers between the interval $(-1, 0]$ (i.e., real numbers less than or equal to 0 and greater than -1). Show that R is uncountable by using the diagonal argument.

2. (3pt) Let $\mathbb{P}$ be the set of prime numbers. Write down an algorithm for determining membership in $\mathbb{P}$.

3. (3pt) Write down an algorithm for enumerating prime numbers.

4. (4pt) Let $f : \mathbb{N} \to \mathbb{N}$ be a strictly increasing, effectively computable function. Show that the range of f is effectively decidable. (Hint: Given an algorithm for computing f, how to construct another algorithm for deciding the membership in the range of f?)

5. (3pt) Show if $A(A \subseteq \mathbb{N})$ and $\mathbb{N} \setminus A$ are both effectively enumeratble, then A is effectively decidable.

6. (4pt) Suppose that R is an effectively enumerable subset of $\mathbb{N}$, and let

$$P = \{n \in \mathbb{N} \mid \forall x < n, x \in R\}.$$

Is P effectively enumerable? Explain why.