Math Logic Assignment #4 💯

tags: `Math Logic` 作业

> — raoxiangyun 520030910366
>
> — Tue, Nov 8, 2022 8:36 AM

1.

(1) Solution:

First let's make a truth table:

A	B	C	$(A \leftrightarrow B) \leftrightarrow C$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

From the truth value table we can construct the DNF of $(A \leftrightarrow B) \leftrightarrow C$:
$(A \leftrightarrow B) \leftrightarrow C = (A \wedge B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$

(2) Solution:

The process is:
$(A \wedge B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge C)$
$\models=\mid (A \vee (A \wedge \neg B \wedge \neg C) \wedge (B \vee (A \wedge \neg B \wedge \neg C) \wedge (C \vee (A \wedge \neg B \wedge \neg C))$
$\vee(\neg A \vee (\neg A \wedge \neg B \wedge C)) \wedge (B \vee (\neg A \wedge \neg B \wedge C)) \wedge (\neg C \vee (\neg A \wedge \neg B \wedge C))$

$\models=\mid (((A \vee A) \wedge (A \vee \neg B) \wedge (A \vee \neg C)) \wedge ((A \vee B) \wedge (B \vee \neg B) \wedge (B \vee \neg C))$
$\wedge((A \vee C) \wedge (\neg B \vee C) \wedge (C \vee \neg C))) \vee (((\neg A \vee \neg A) \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C))$
$\wedge((\neg A \vee B) \wedge (B \vee \neg B) \wedge (B \vee C)) \wedge ((\neg A \vee \neg C) \wedge (\neg B \vee \neg C) \wedge (\neg C \vee C)))$

$\models\!=\!\mid ((A \wedge (A \vee \neg B) \wedge (A \vee \neg C)) \wedge ((A \vee B) \wedge (B \vee \neg C))$
$\wedge((A \vee C) \wedge (\neg B \vee C))) \vee ((\neg A \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C))$
$\wedge((\neg A \vee B) \wedge (B \vee C)) \wedge ((\neg A \vee \neg C) \wedge (\neg B \vee \neg C)))$ **-----simplify some clauses**

$\models\!=\!\mid (A \wedge (A \vee \neg B) \wedge (A \vee \neg C) \wedge (A \vee B) \wedge (B \vee \neg C)$
$\wedge(A \vee C) \wedge (\neg B \vee C)) \vee (\neg A \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C)$
$\wedge(\neg A \vee B) \wedge (B \vee C) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee \neg C))$ **-----delete some parentheses**

- *Denote* $(\neg A \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C) \wedge (\neg A \vee B) \wedge (B \vee C) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee \neg C))$ *as* D

$\models\!=\!\mid (A \wedge (A \vee \neg B) \wedge (A \vee \neg C) \wedge (A \vee B) \wedge (B \vee \neg C) \wedge (A \vee C) \wedge (\neg B \vee C)) \vee D$

$\models\!=\!\mid$
$(A \vee D) \wedge (((A \vee \neg B) \vee D)) \wedge ((A \vee \neg C) \vee D) \wedge ((A \vee B) \vee D) \wedge ((B \vee \neg C) \vee D)$
$\wedge((A \vee C) \vee D) \wedge ((\neg B \vee C) \vee D)$

- *Now I will simplify each clause of the current result above.*

$A \vee D$
$\models\!=\!\mid$
$A \vee (\neg A \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C) \wedge (\neg A \vee B) \wedge (B \vee C) \wedge (\neg A \vee \neg C) \wedge (\neg B \vee \neg C)$

$\models\!=\!\mid (A \vee \neg A) \wedge (A \vee \neg A \vee \neg B) \wedge (A \vee \neg A \vee C) \wedge (A \vee \neg A \vee B)$
$\wedge(A \vee B \vee C) \wedge (A \vee \neg A \vee \neg C) \wedge (A \vee \neg B \vee \neg C))$

$\models\!=\!\mid T \wedge T \wedge T \wedge T(A \vee B \vee C) \wedge T \wedge (A \vee \neg B \vee \neg C)$**-----simplify some clauses**

$\models\!=\!\mid (A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C)$

$(A \vee \neg B) \vee D$
$\models\!=\!\mid (A \vee \neg B) \vee (\neg A \wedge (\neg A \vee \neg B) \wedge (\neg A \vee C) \wedge (\neg A \vee B) \wedge (B \vee C) \wedge (\neg A \vee \neg C)$
$\wedge(\neg B \vee \neg C)$

$\models\!=\!\mid ((A \vee \neg B) \vee \neg A) \wedge ((A \vee \neg B) \vee \neg A \vee \neg B) \wedge ((A \vee \neg B) \vee \neg A \vee C)$
$\wedge((A \vee \neg B) \vee \neg A \vee B) \wedge ((A \vee \neg B) \vee B \vee C) \wedge ((A \vee \neg B) \vee \neg A \vee \neg C)$
$\wedge((A \vee \neg B) \vee \neg B \vee \neg C))$

$\models\!=\!\mid T \wedge T \wedge T \wedge T \wedge T \wedge T \wedge T$**-----simplify some clauses**

$\models\!=\!\mid T$

- *Now we can simplify* $((A \vee \neg C) \vee D), ((A \vee B) \vee D), ((B \vee \neg C) \vee D), ((A \vee C) \vee D),((\neg B \vee C) \vee D)$ *like above:*

$((A \vee \neg C) \vee D) \models\!=\!\mid (A \vee \neg B \vee \neg C)$

$((A \vee B) \vee D) \models=| (A \vee B \vee C)$

$((B \vee \neg C) \vee D) \models=| (\neg A \vee B \vee \neg C)$

$((A \vee C) \vee D) \models=| (A \vee B \vee C)$

$((\neg B \vee C) \vee D) \models=| (\neg A \vee \neg B \vee C)$

- **Therefore, the final CNF is:**
  $(A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (\neg A \vee \neg B \vee C) \wedge (\neg A \vee B \vee \neg C)$

2. Solution:

$\Sigma \vee \neg\alpha$ is unsatisfiable.
$\Leftrightarrow \Sigma \vee \neg\alpha$ is not finite-satisfiable.
$\Leftrightarrow \exists$ finite set $\Delta \subset \Sigma$ that is unsatisfiable.

1. If $\neg\alpha \in \Delta$, $\Delta = \Delta^{'} \vee \{\neg\alpha\}$ is unsatisfiable $\Leftrightarrow \Delta^{'} \models \alpha$ no matter whether $\Delta^{'}$ is satisfiable because when $\Delta^{'}$ is satisfiable, it doesn't contain $\{\neg\alpha\}$, then it must contain $\{\alpha\}$; when $\Delta^{'}$ is unsatisfiable, $\Delta^{'} \models \alpha$ is always true because no truth assignment $s.t.$ $\Delta^{'}$.
2. If $\neg\alpha \notin \Delta$, then $\{\alpha\} \in \Delta$,$\Delta \models \alpha$.

Therefore, there exists some finite set Δ such that $\Delta \subseteq \Sigma$ and $\Delta \models \alpha$.

3. Solution:

First from ppt-4 page-71, we can know that the set of semantic consequences of Σ is effectively enumerable. Then for any wff α, we can do the listing like this:
Let $\beta_1, \beta_2, \ldots\ldots, \beta_n, \ldots\ldots$ be an effective enumeration of Σ;
Let $\Delta_n = \{\beta_1, \beta_2, \ldots\ldots, \beta_n\}$;
Now let's make a interesting table:

	Δ_1	Δ_1	......	Δ_n	......
α	$\Delta_1 \models \alpha$	$\Delta_2 \models \alpha$	......	$\Delta_n \models \alpha$	......
$\neg\alpha$	$\Delta_1 \models \neg\alpha$	$\Delta_2 \models \neg\alpha$	......	$\Delta_n \models \neg\alpha$	......

We can check from Δ_1 to check whether $\alpha, \neg\alpha$ can be tautologically implied by the current subset of Σ. Now let's construct an Algorithm:

```
On input alpha:
    for Delta in Delta_enumeration_list:
        if Delta \models alpha:
            outputs yes;
        elif Delta \models \neg alpha:
            outputs no;
        continue
```

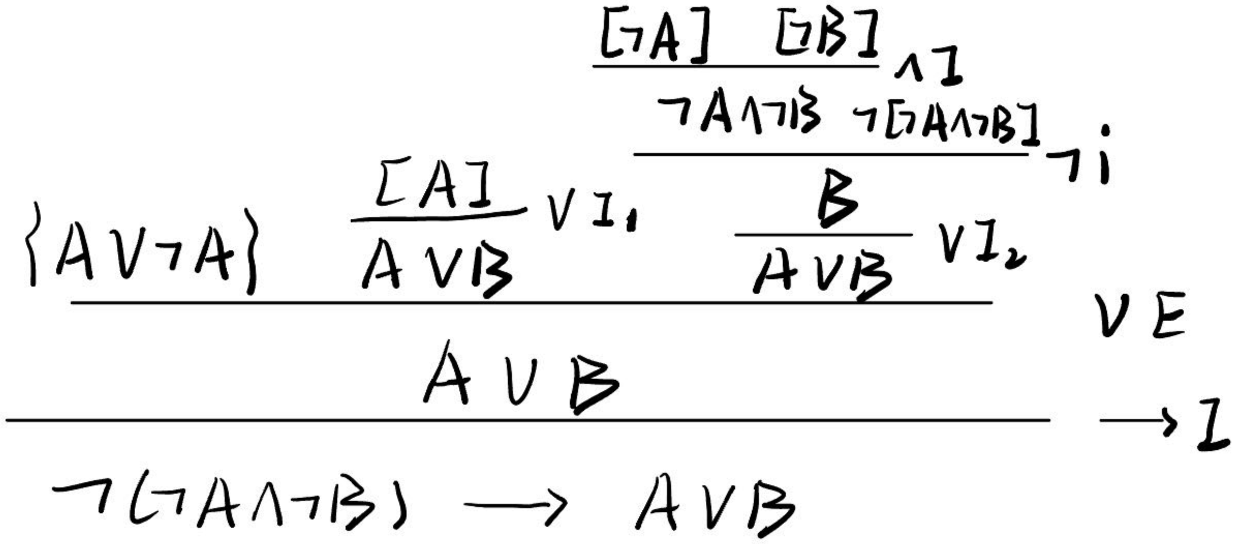
Because for evert wff α, either $\Sigma \models \alpha$ or $\Sigma \models \neg\alpha$, and from problem 2 we know $\exists \Delta \subsetneq \Sigma$, $\Delta \models \alpha$ or $\Delta \models \neg\alpha$. We can find this finite set Δ in our enumeration list because Σ is effectively enumerable.Therefore, the set of semantic consequences of Σ is effectively decidable.

4.

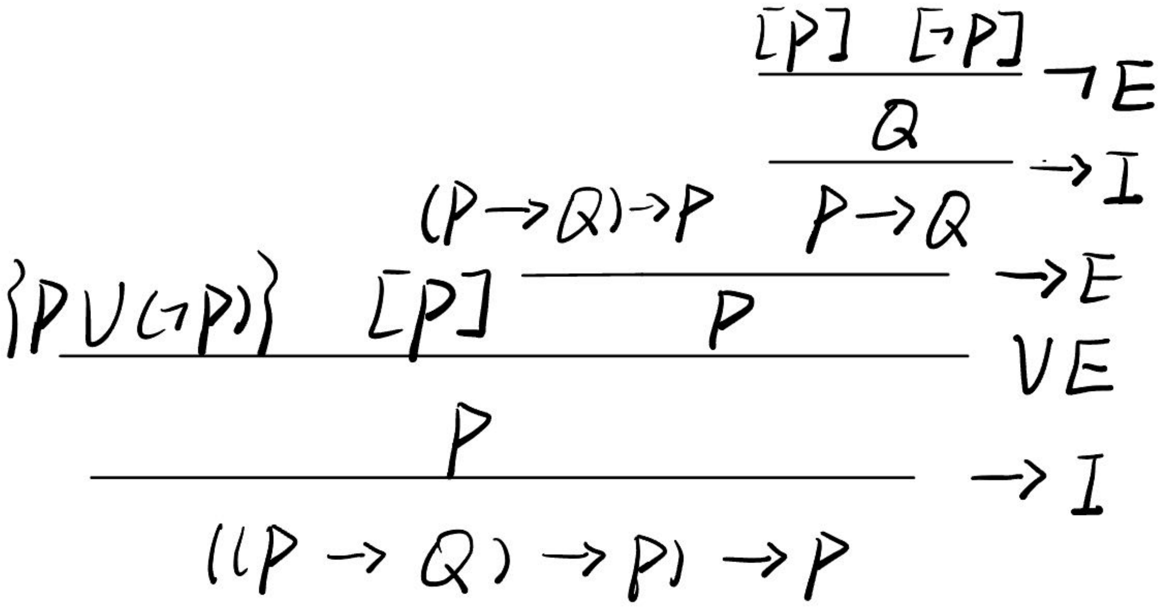
(1). Solution:

$$\dfrac{\dfrac{\dfrac{[A\wedge(B\vee C)]}{B\vee C}\wedge E_2 \qquad \dfrac{\dfrac{\dfrac{[A\wedge(B\vee C)]}{A}\wedge E_1 \quad [B]}{A\wedge B}\wedge\text{-}I}{(A\wedge B)\vee(A\wedge C)}\vee E_1 \qquad \dfrac{\dfrac{\dfrac{[A\wedge(B\vee C)]}{A}\wedge E_1 \quad [C]}{A\wedge C}\wedge I}{(A\wedge B)\vee(A\wedge C)}\vee E_2}{(A\wedge B)\vee(A\wedge C)}\vee e}{A\wedge(B\vee C)\longrightarrow(A\wedge B)\vee(A\wedge C)}\to i$$

(2). Solution:



(3). Solution:



5.

(1). Solution:

First we can know from the picture that $\Sigma;\beta \vdash \sigma \wedge \neg\sigma$, $\Sigma \vdash \neg\beta$. From lower height partial proof tree we can know that $\Sigma;\beta \models \sigma \wedge \neg\sigma$. That's to say, if a truth assignment $s.t.$ $\Sigma;\beta$, it also $s.t$ $\sigma \wedge \neg\sigma$. But no truth assignment $s.t.$ $\sigma \wedge \neg\sigma$. Therefore, $\Sigma \vee \beta$ is unsatisfiable. And if a truth assignment v $s.t.$ Σ, it will not $s.t.$ β ($v(\beta) = False$), and $\bar{v}(\neg\beta) = \bar{v}(\alpha) = True$, that is, $\Sigma \models \alpha$.

(2). Solution:

From the partial proof tree we know that $\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_n\} \vdash \beta$ and $\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\} \vdash \neg\beta$, and from lower height case, we know $\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_n\} \models \beta$ and $\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\} \models \neg\beta$. That is if a truth assignment $s.t\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_n\}$, it $s.t.\ \beta$;if a truth assignment $s.t\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\}$, it $s.t.\ \neg\beta$. But $\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_n\}$, $\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\} \subset \Sigma$, so we know that if a truth assignmet $v\ s.t.\ \Sigma$, it will $s.t.$ $\{\gamma_1, \gamma_2, \ldots\ldots, \gamma_n\}$,$\{\sigma_1, \sigma_2, \ldots\ldots, \sigma_m\}$, and it will $s.t.\ \beta, \neg\beta$.($\bar{v}(\beta) = \bar{v}(\neg\beta) = True$), it is impossible. So there is no truth assignment $s.t.\ \Sigma$. Therefore, $\Sigma \models \alpha$.