Mathematical Logic Assignment 1

— Raoxiangyun 520030910366

— Tue, Sep 20, 2022 11:26 AM

Problem 1

(a). According to the definition of binary relation, the domain is

$$\{x \mid \exists y, < x, y > \in R\}$$

And the range is

$$\{y \mid \exists x, x, y \in R\}$$

.
Therefore, the domain of R is:

$$\{1, 2, 3\}$$

And the range of R is:

$$\{1.1, 3.2, 2.0, 1.1\}$$

(b). R is not a function from B to R.
According to the definition of function, for any element a from the domain, there exists a unique element b from the range that <a,b>R if R is a function. We can observe that there exist 2 elements from the range that corresponds to 2 from the domain. Therefore R is not a function.

Problem 2

(a).Proof:
$f:\mathbb{N}\longmapsto A$ is a surjective
$\Rightarrow \forall y\in A, \exists x\in\mathbb{N}, f(x)=y.$
$\because$ f is a function, every element in $\mathbb{N}$ has a unique corresponding element $\in A$.
$\therefore |A|\leq|\mathbb{N}|$

1. if A is **finite**, A is obviously countable.
2. if A is **infinite**:
   Now we can construct a listing of A:

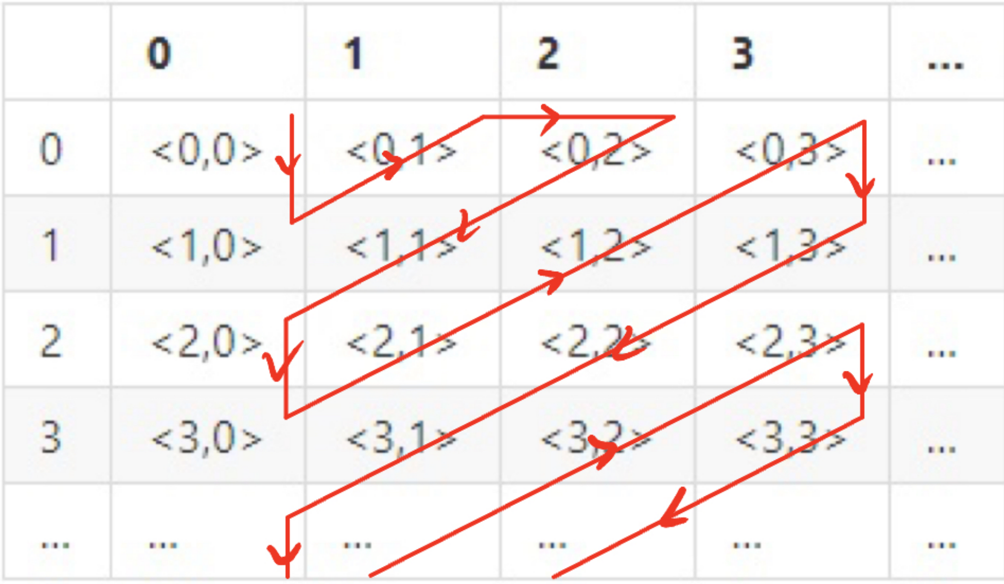
   $$a_0, a_1, a_2, \cdots a_n, \cdots$$

   And $a_n=f(n)$. **This listing with possible repetitions of A proves that A is countable**.

(b).Proof:$''$
is a surjective
$f:A\longmapsto\mathbb{N}$
$\Rightarrow \forall y\in\mathbb{N}, \exists x\in A, f(x)=y.$
Suppose A is finite, which means there exists a bijection between A and $\{0,1,\cdots,n\}, n=|A|$. In other words, there exists a bijection between A and a subset B of $\mathbb{N}$ with a size of $|A|$. But we can always find a element b from the supplymentary set of B of $\mathbb{N}$. And according to the definition of surjection there is also a element a from A that $<a,b>\in f$. But a has been used to correspond to a item c from B. That means $<a,b>,<a,c>\in f$ while f is a funcion, **contradictory!**
Therefore, A is infinite.

Problem 3

From the definition of order pair we know that $< a,b > \neq < b,a >$ if $a \neq b$. If we regard the left as numerator, the right as denominator, we can list them in the way we list $\mathbb{Q}$.
We can constuct a listing of $\mathbb{N} \times \mathbb{N}$ in the following way:



	0	1	2	3	...
0	<0,0>	<0,1>	<0,2>	<0,3>	...
1	<1,0>	<1,1>	<1,2>	<1,3>	...
2	<2,0>	<2,1>	<2,2>	<2,3>	...
3	<3,0>	<3,1>	<3,2>	<3,3>	...
...	...	...	...	...	...

Each pair of natural numbers will be encode into a item in $\mathbb{N}$ according to the order of being reached by the red line.