

# Math Logic: Assignment 6

Dec 13, 2022

**Attention:** To get full credits, you *must provide explanations to your answers!* You will get at most 1/3 of the points if you only present the final results.

1. (6pt) Given a 2-ary predicate symbol  $Q$ , show that

$$\models_{\mathfrak{A}} \forall v_2 Q v_1 v_2 \llbracket c^{\mathfrak{A}} \rrbracket \iff \models_{\mathfrak{A}} \forall v_3 Q c v_3.$$

2. (18pt) For each of the following relations, give a wff that defines it in the structure  $\mathfrak{A} = (\mathbb{N}, +, \times)$  (Assume this language has  $\dot{+}$ ,  $\dot{\times}$  and  $\dot{=}$  with standard interpretations in  $\mathfrak{A}$ ).

- (6pt)  $\{0, 1\}$ ;
- (6pt)  $\{2\}$ ;
- (6pt)  $\{n \in \mathbb{N} \mid n \text{ is an even number}\}$  (Hint: you may reuse the previous defining formula  $\varphi$  and use  $\varphi(x)$  to denote the renaming of free occurrences of  $v_1$  in  $\varphi$  to  $x$ .)

3. (6pt) Let  $\mathbb{L}$  be a first-order language with  $\dot{=}$ , a constant  $\dot{0}$ , a 2-ary function symbol  $\dot{+}$ . Given the following two structures for  $\mathbb{L}$ :

- $\mathfrak{N}_1$  where  $|\mathfrak{N}_1| = \mathbb{N}$ ,  $\dot{0}^{\mathfrak{N}_1} = 0$  and  $\dot{+}^{\mathfrak{N}_1} = +$ ;
- $\mathfrak{N}_2$  where  $|\mathfrak{N}_2| = \mathbb{N}$ ,  $\dot{0}^{\mathfrak{N}_2} = 1$  and  $\dot{+}^{\mathfrak{N}_2} = \times$ .

Show that there is a homomorphism from  $\mathfrak{N}_1$  to  $\mathfrak{N}_2$ . You need to prove that there is a function that satisfies the properties of homomorphisms. (Hint: consider the simplest case where the function is constant.)