Math Logic Assignment #6 💯

tags: `Math Logic` 作业

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1. Solution:

$\models_{\mathfrak{A}} \forall v_2 Q v_1 v_2[|c^{\mathfrak{A}}|]$
$\Leftrightarrow$for all a $\in |\mathfrak{A}|, \models_{\mathfrak{A}} Q v_1 v_2[|c^{\mathfrak{A}}, a|]$
$\Leftrightarrow$for all a $\in |\mathfrak{A}|$, Qca.
$\Leftrightarrow$for all a $\in |\mathfrak{A}|$,$(c, a) \in Q^{\mathfrak{A}}$
$\Leftrightarrow$for all a $\in |\mathfrak{A}|$,$\models_{\mathfrak{A}} Q c v_3[|a|]$
$\Leftrightarrow \models_{\mathfrak{A}} \forall v_3 Q c v_3$

2.

(1) Solution:

$$\forall v_2 (v_2 \dot{\times} v_1 \doteq v_1 \vee v_2 \dot{\times} v_1 \doteq v_2)$$

(2) Solution:

$$\forall v_2 (v_2 \dot{\times} v_1 \doteq v_2 \dot{+} v_2)$$

(3) Solution:

Mark $\psi(x)$ as the wff in subproblem 2 that replace v_1 with x. Then the answer will be:
$\exists v_2 v_3 (\psi(v_2) \wedge \psi(v_3)) v_2 \dot{\times} v_3 \doteq v_1$

3. Solution:

I think there is a function:$h(n) = 2^n$ satisfies all the needs.
For constant 0: $h(0^{\mathfrak{A}}) = h(0) = 2^0 = 1 = 0^{\mathfrak{B}}$
For functions label $\dot{+}$: for any
$n_1, n_2 \in |\mathfrak{A}|, h(n_1 +^{\mathfrak{A}} n_2) = h(n_1 + n_2) = 2^{n_1+n_2} = 2^{n_1} \times 2^{n_2} = h(n_1) +^{\mathfrak{B}} h(n_2)$