Mathematical Logic: Assignment 1

Sep 19, 2022

Attention: To get full credits, you *must provide explanations to your answers*! You will get at most $1/3$ of the points if you only provide the final results without any explanation.

1. (4pt) Let $R = \{\langle 1, 1.1\rangle, \langle 2, 3.2\rangle, \langle 3, 2.0\rangle, \langle 2, 1.1\rangle\}$ be a binary relation between $\mathbb{N}$ and $\mathbb{R}$.
   (a) (2pt) What are the domain and range of R?
   (b) (2pt) Let $B = \{1, 2, 3\}$, is R a function from B to $\mathbb{R}$? Explain why.
2. (6pt) Prove the following propositions:
   (a) (3pt) If $f : \mathbb{N} \to A$ is surjective (i.e. f maps $\mathbb{N}$ onto A) then A is countable.
   (b) (3pt) If $f : A \to \mathbb{N}$ is surjective then A is infinite.
3. (5pt) $\mathbb{N} \times \mathbb{N}$ is the set of order pairs of $\mathbb{N}$, prove $\mathbb{N} \times \mathbb{N}$ is enumerable. (Hint: You may use the fact that a non-repetitive listing is enumerable.)