MF 796: Computational Methods of Mathematical Finance

Professor Chris Kelliher   Fall 2018

Problem set # 1
Due: Wednesday, September 12th, by 6:30pm.

1. **Historical Analysis of Sector ETFs:**

Ticker	Description
SPY	S&P Index
XLB	Materials
XLE	Energy
XLF	Financials
XLI	Industrials
XLK	Technology
XLP	Consumer Staples
XLU	Utilities
XLV	Healthcare
XLY	Consumer Discretionary

   (a) Download historical price data from January 1st 2010 for the following ETFs on ya-hoo finance or another site of your choice. Clean/check the data for splits and other anomalies.

   (b) Calculate the annualized return and standard deviation of each ETF

   (c) Calculate the covariance matrix of daily and monthly returns. Comment on the differences in correlations at different frequencies.

   (d) Calculate a rolling 90-day correlation of each sector ETF with the S&P index. Do the correlations appear to be stable over time? What seems to cause them to vary the most?

   (e) Consider a single factor CAPM model where returns are composed of the market return and an idiosyncratic component:

$$r_{i,t} = \beta r_{\text{mkt},t} + \epsilon_{i,t} \tag{1}$$

   For each sector ETF, compute it's β to the market using the above model. This will require computing a linear regression on stock returns. Compute the β for the entire historical period and also compute rolling 90-day β's. Are the β's that you calculated consistent over the entire period? How do they compare to the rolling correlations?

(f) Compute the auto-correlation of each ETF by regressing each ETFs current days return against its previous days return:

$$r_{i,t} = \alpha r_{i,t-1} + \epsilon_{i,t} \tag{2}$$

Is there any evidence of auto-correlation in this ETF universe? Present the α's for each ETF and comment on your results.

2. **Exotic Option Pricing via Simulation**

Consider a one-year **fixed strike lookback option** which enables the buyer to choose the point of exercise for the option at its expiry. Recall that the dynamics of the Black-Scholes model can be written as:

$$dS_t = rS_t \, dt + \sigma S_t \, dW_t \tag{3}$$

And that each Brownian motion increment dW_t is normally distributed with mean zero and variance t. Assume that $r = 0$, $S_0 = 100$ and $\sigma = 0.25$.

(a) Generate a series of normally distributed random numbers and use these to generate simulated paths for the underlying asset. What is the mean and variance of the terminal value of these paths? Does it appear to be consistent with the underlying dynamics?

(b) Calculate the payoff of a European put option with strike 100 along all simulated paths. Make a histogram of the payoffs for the European option. What is the mean and standard deviation of the payoffs?

(c) Calculate a simulation approximation to the price of a European put option by taking the average discounted payoff across all paths.

(d) Compare the price of the European put option obtained via simulation to the price you obtain using the Black-Scholes formula. Comment on any difference between the two prices.

(e) Calculate the payoff of a fixed strike lookback put option with stike 100 along all simulated path. (HINT: The option holder should exercise at the minimum price along each simulated path). Calculate the simulation price for the lookback option by averaging the discounted payoffs.

(f) Calculate the premium that the buyer is charged for the extra optionality embedded in the lookback. When would this premium be highest? Lowest? Can it ever be negative?

(g) Try a few different values of σ and comment on what happens to the price of the European, the Lookback option and the spread/premium between the two.

NOTE: All code for completing these exercises should be completed either in Python or C++ and should be written generically. You may end up using this code on future assignments so I encourage you to code thoughtfully.