6th Homework Report

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1、 Covariance Matrix Decomposition:

(b)

Covariance matrix

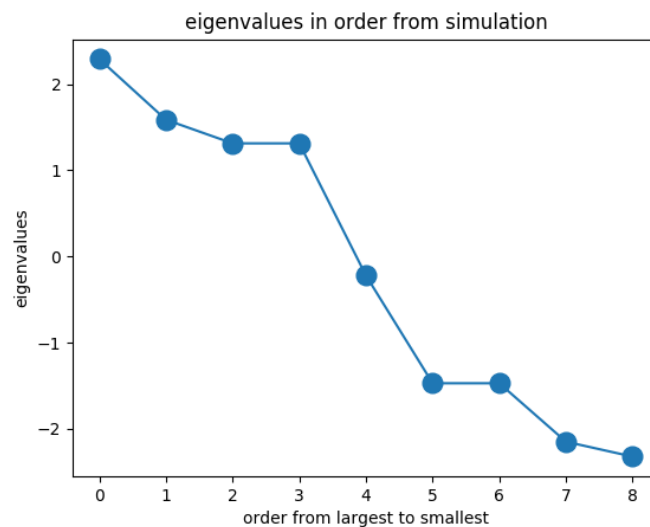|	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
XLB	0.00015	0.000135	0.000124	0.000117	0.000098	0.000058	0.000049	0.000081	0.0001
XLE	0.000135	0.000187	0.000122	0.000113	0.000095	0.000056	0.000051	0.000079	0.000096
XLF	0.000124	0.000122	0.000192	0.000118	0.0001	0.00006	0.000049	0.000086	0.000105
XLI	0.000117	0.000113	0.000118	0.000119	0.000093	0.000057	0.000047	0.000078	0.000096
XLK	0.000098	0.000095	0.0001	0.000093	0.000108	0.000051	0.000041	0.000073	0.000091
XLP	0.000058	0.000056	0.00006	0.000057	0.000051	0.000055	0.000043	0.000049	0.000054
XLU	0.000049	0.000051	0.000049	0.000047	0.000041	0.000043	0.000078	0.000039	0.000043
XLV	0.000081	0.000079	0.000086	0.000078	0.000073	0.000049	0.000039	0.000087	0.000074
XLY	0.0001	0.000096	0.000105	0.000096	0.000091	0.000054	0.000043	0.000074	0.000104

(c)

All of the 9 eigenvalues are positive, and they should be. That's because eigenvalues are the elements of the diagonal matrix derived from the diagonalized positive definite covariance matrix.
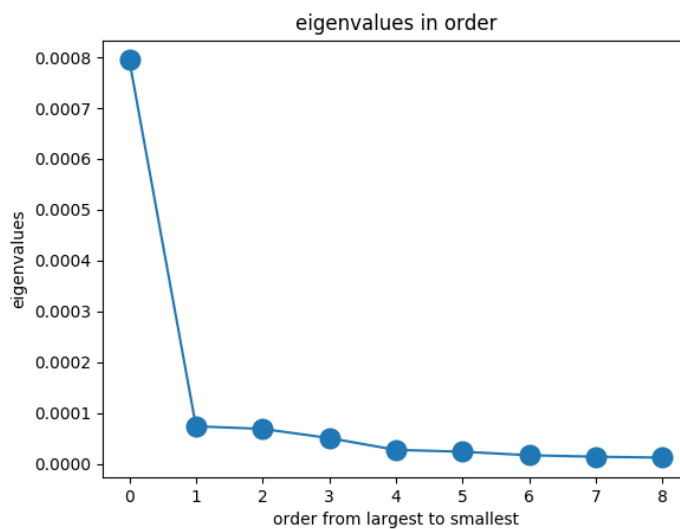
(e)

Some of the eigenvalues from simulated matrix are complex numbers, which is different from covariance matrix. Plot them in order by casting complex values to real discards the imaginary part.

eigenvalues in order from simulation

2、 Portfolio Optimization:

(a)

Annualized returns for ETFs

Ticker	XLB	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY
Return	7.55%	3.60%	13.87%	12.52%	13.79%	11.00%	9.91%	13.61%	15.75%

(b)

weights are:
[-2.32002732 -1.7796722 0.34265559 1.163832   0.17139991 -0.9709274
  0.03888691 0.9229273 3.43092521]

(c)

sigma = [0.005, 0.01, 0.05, 0.01]
weights are:
[-3.23755225 -1.63033231 0.19052644 1.56705611 0.81910781 -1.77845168
  0.34197811 1.03070568 3.69696209]
[-3.03504498 -1.67401422 0.22182382 2.31556479 2.52519044 -2.01740917
  0.81171247 0.44207064 1.41010621]
[1.072513   -4.45410034 -1.29097268 8.10151813 0.41007826 2.13494509
 -0.67853645 -1.35931692 -2.93612808]
[0.87877683 -6.73477301 -3.05646518 14.210227   -11.50056824
 -4.09094955 1.96820368 -7.03778571 16.36333419]

weights of portfolio are not stable to changes in expected returns.

(e)

when δ = 1, eigenvalues of regularized covariance matrix are:
[0.03777738 0.04721946 0.04839646 0.02988708 0.02727644 0.01385994
  0.01976396 0.02199124 0.0262778]
regularized covariance matrix are full rank (9).

(f)

tried δ = np.arange(0.1, 1, 0.1). All 9 eigenvalues are positive.

(g)

delta = 0.1

[-1.3925694 -1.86199512 0.4949962   0.52776655 0.43332337 -0.64987744
 -0.13259689 1.17154675 2.40940598]
[-1.41502036 -1.32059592 0.3429081 -0.29993318 0.20962929 -1.90050464
  0.60545534 1.50188555 3.27617582]
[-1.8005793   0.47395226 2.3115232 -7.62207672 -0.34415133 -0.21357926
  0.08782286 -2.66270374 10.76979202]
[1.7160164   4.95154503 3.22665714 -5.47437593 5.98907293 0.84033649
 -1.3768918 -7.28197901 -1.59038125]

delta = 0.2

[-1.39309034 -1.52421684 0.13636244 0.81646422 0.84012271 -0.70706722
  0.15188619 0.589512   2.09002685]
[-2.10729529 -1.34420139 0.39891167 0.21824894 1.72661771 0.11071852
 -0.60418227 1.32566411 1.27551799]
[-0.45339391 -0.4028535 -1.57332029 4.93642541 1.3891311 -3.4909433
  0.36659701 3.13612253 -2.90776505]
[-5.66745806 -0.75742807 -0.20483305 1.80293958 4.39495936 -3.55805378
  3.50447846 3.05979988 -1.57440432]

delta = 0.3

[-1.09954356 -1.34498912 0.39694013 0.33382353 0.41204074 -0.10957328
 -0.29334427 0.9206609   1.78398494]
[-1.30857267e+00 -1.33935083e+00 1.51629851e-01 5.60791211e-01
  1.02017096e+00 -2.91354608e-01 5.86270485e-04 8.86397671e-01
  1.31970214e+00]
[-4.78079761 0.73060445 0.35438453 0.93220627 4.53613552 1.08300938
 -1.29813434 -0.56665344 0.00924525]
[-1.91986199 0.34085471 -0.94150128 -5.36783208 11.34574171 -1.46052099
 -2.55111285 3.55509607 -2.00086331]

delta = 0.4

[-0.89592021 -1.38290214 0.30224352 0.38720477 0.72947607 0.12670813
 -0.47904713 0.91922539 1.2930116]
[-0.75300572 -1.57166603 0.719667   0.31798162 0.56537595 -0.89246517
  0.07650843 0.99752632 1.5400776]
[-2.5636052 -0.57848539 0.97054626 -1.70031059 -0.02427081 3.69904131
 -0.77419714 0.943189   1.02809255]
[-3.27697222 -1.42267576 5.04350111 -5.41929695 2.42340299 2.16849358

1.39464043 1.92768001 -1.83877317]

delta = 0.5

[-0.68651263 -1.16640827 0.35703474 0.54739941 0.53666032 0.09680393
 -0.14508868 0.36139721 1.09871396]
[-0.42344871 -0.9823476   0.37388953 0.435206   0.22947062 -0.33969559
 -0.34103695 0.9556788   1.09228392]
[-1.28727083 -2.23540963 -0.00526154 0.64392154 2.30499576 -3.00724749
  0.60928512 1.92421431 2.05277277]
[-2.6948599   1.62453927 0.89538824 6.67571115 0.19333214 -1.99645531
  2.06111309 0.60165912 -6.3604278]

delta = 0.6

[-0.57703525 -1.036071   0.26089254 0.15206147 0.69194779 -0.42494522
  0.04660982 0.50544496 1.3810949]
[-0.494579   -1.14202538 0.10321084 -0.1091232   0.55337406 0.25423428
 -0.25647107 0.84778601 1.24359345]
[-0.45181217 0.05815054 -0.71753255 0.47561501 -1.18522506 -0.67615254
  0.50688989 2.81403187 0.176035]
[-1.60667133 -1.77174586 1.08553018 -1.27758327 -2.04851516 5.47800307
  4.02174503 -4.5884077   1.70764505]

delta = 0.7

[-0.60460158 -1.05233631 0.3548399   0.29716113 0.57723624 0.18822518
 -0.2743115   0.57464482 0.93914212]
[-0.87485623 -0.89710473 0.31023082 0.47672456 0.2094646   0.52791079
 -0.60334964 0.5749975   1.27598233]
[0.00916636 -0.77422748 -0.12752857 0.07824579 1.49859156 -3.70415451
  0.6279288   2.02566274 1.36631531]
[-0.80820235 1.29223765 -0.64933197 0.21459182 1.12761723 0.34708398
 -3.94180996 1.87122078 1.54659283]

delta = 0.8

[-0.61966914 -0.95765235 0.18544123 0.27125311 0.60366737 -0.06648238
 -0.01370822 0.76227188 0.83487851]
[-0.95652797 -0.96907724 0.26855329 0.64431657 0.40563659 0.35875546
 -0.36313924 0.76969887 0.84178367]
[-0.58528829 -0.80359602 0.71245514 1.3179233 -0.55427226 0.62871399
  0.94232364 -0.76730463 0.10904513]
[-1.76672199 -1.69722299 -0.40355532 0.06342213 4.55980252 3.42751165
 -1.99039217 0.27670721 -1.46955105]

delta = 0.9

[-0.56552388 -0.71644   0.24529796 0.30208231 0.33279152 0.32166608
 -0.45220766 0.51101878 1.02131489]
[-0.49373198 -1.05453938 0.38671737 0.0512077   0.57634311 -0.15040508
  0.00544872 0.5693263   1.10963324]
[-1.0582705   0.00630626 -0.48433679 0.42398254 1.25762126 -1.53393637
  0.24363208 1.11526899 1.02973253]
[1.32510261 -0.34309491 1.48135458 -1.85564033 1.190589   -2.6860985
  3.28926135 -1.33262663 -0.06884716]

the weights becomes more stable as delta increases.