Fuzzy Inferencing and Fuzzy Control

Outline

- **Fuzzy Reasoning**
  - Case 1: Single Rule with Single Antecedent
  - Case 2: Single Rule with Multiple Antecedents
  - Case 3: Multiple Rules with Multiple Antecedents
- **Fuzzy Inference System (FIS)**
  - Mamdani Fuzzy Model
  - Sugeno Fuzzy Model
  - Tsukomoto Fuzzy Model
- **Case Studies**

Fuzzy Reasoning

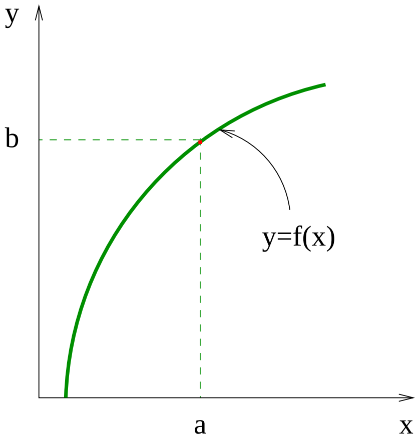
Definition: Fuzzy Reasoning

Fuzzy reasoning, also known as approximate reasoning (AR), is an inference procedure that derives conclusions from a set of if-then rules.

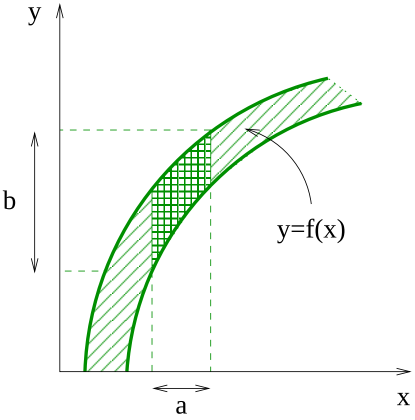
Definition: Composition Rule of Inference

- Let us assume that F is a fuzzy relation on $X \times Y$.
- Let A be a fuzzy set in X.
- To obtain the resulting fuzzy set B, we first construct a cylindrical extension of A, $C(A)$.
- The inference of $C(A)$ and F leads to the antecedent of the projection of $C(A) \cap F$.

Derivation of $y = b$ from $x = a$ and $y = f(x)$ in crisp logic setting:



a and b are points
$y = f(x)$ is a curve



a and b are intervals
$y = f(x)$ is an interval-valued function

$$A \xrightarrow{\text{CE}} C(A) \xrightarrow{\cap F} \overset{\text{antecedent}}{C(A)\cap F} \xrightarrow{P} \boxed{P\left(C(A)\cap F\right)} = B$$

This leads to the fuzzy set B.

$\underset{\substack{\uparrow \\ R_1}}{C(A)} \underset{\substack{\uparrow \\ \circ}}{\cap} \underset{\substack{\uparrow \\ R_2}}{F}(x, y)$ known as the composition operator

Definition: Fuzzy Reasoning

Fuzzy reasoning is basically the extension of the well known composition of elements and functions.

$$\underbrace{\mu_C(A)}_{C(A)} \cap F(x, y) = \min\left(\mu_{C(A)}(x), \mu_F(x, y)\right)$$

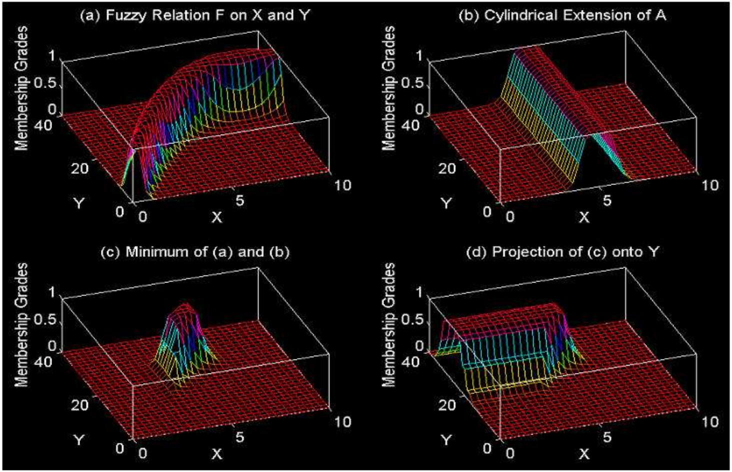
- Projecting $\mu_C(A) \cap F(x,y)$ provides

$$\begin{aligned} B &\equiv \mu_B(y) \\ &= \max_x \left[\min\left(\mu_{C(A)}(x), \mu_F(x,y)\right)\right] \\ &= \bigvee_x \left[\mu_{C(A)}(x) \wedge \mu_F(x,y)\right] \end{aligned}$$

- This is basically the max-min composition of two relations of A (unary relation) and F (binary relation)

$$B = A \circ F \qquad\qquad \text{Compositional rule of inference}$$

Graphical Illustration



a is a fuzzy set and $y = f(x)$ is a fuzzy relation

Modus Ponens (MP)

- The rule of inference in conventional logic is *modus ponens*.
- MP leads to inference of truth of a proposition B from the truth of A and the implementation $A \rightarrow B$.

Example

- Rule: If tomato is red, then tomato is ripe ($A \rightarrow B$).
- Fact: Tomato is red (A).
- Conclusion: Tomato is ripe (B).

Modus Ponens (MP)

Crisp Case

Premise 1: fact	x is A
Premise 2: rule	If x is A, then y is B
Conclusion: consequence	y is B

Fuzzy Case: Fuzzy Reasoning

Premise 1: fact	x is A'
Premise 2: rule	If x is A, then y is B
Conclusion: consequence	y is B'

- A' could be close to A
- B' could be close to B
- A, B, A', and B' are fuzzy sets

Fuzzy Inferencing

- Let A, A', and B be fuzzy sets over X, X, and Y, respectively.
- Assume that the fuzzy implication $A \to B$ be expressed as fuzzy relation $R_{X \times Y}$.
- Then the fuzzy set B induced by (x is A') for the relation "x is A then y is B" is given by

$$\mu_{B'}(y) = \max_{x} [\min (\mu_{A'}(x), \mu_R(x, y))]$$

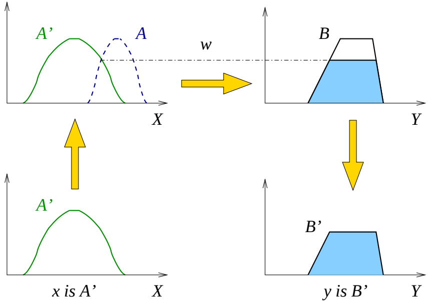
$$= \bigvee_{x} [\mu_{A'}(x) \wedge \mu_R(x, y)]$$

$$B' = A' \circ R = A' \circ (A \to B)$$

Case 1: Single Rule with Single Antecedent

$$\mu_{B'}(y) = \underbrace{\bigvee_x [\mu_{A'}(x) \wedge \mu_A(x)]}_{\text{degree of validity}=w} \wedge \mu_B(y) = w \wedge \mu_B(y)$$

Rule: if x is A then y is B
Fact: x is A'
Conclusion: y is B'



Case 2: Single Rule with Multiple Antecedents

If x is A and y is B, then z is C

Premise 1: fact	x is A' and y is B'
Premise 2: rule	If x is A and y is B, then z is C
Conclusion: consequence	z is C'

$$R : A \times B \to C \quad \Longrightarrow \quad R : A \times B \times C$$

$$R = \int_{X \times Y \times Z} \frac{\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)}{(x, y, z)}$$

Result: $C' : (A' \times B') \circ (A \times B \to C)$

$$\mu_{C'}(z) = \bigvee_{x,y} \underbrace{[\mu_{A'}(x) \wedge \mu_{B'}(y)]}_{(A' \times B')} \wedge \underbrace{[\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)]}_{(A \times B \to C)}$$

$$= \bigvee_{x,y} [\mu_{A'}(x) \wedge \mu_{B'}(y) \wedge \mu_A(x) \wedge \mu_B(y)] \wedge \mu_C(z)$$

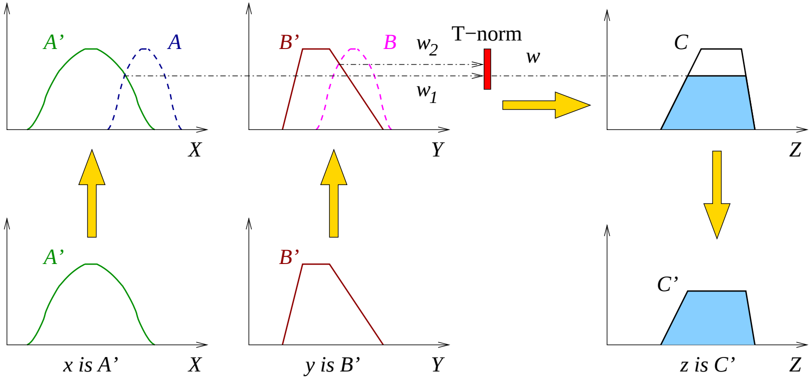
$$= \underbrace{\bigvee_{x,y} [\mu_{A'}(x) \wedge \mu_A(x)]}_{\text{degree of validity of } x \ (w_1)} \wedge \underbrace{\bigvee_{x,y} [\mu_{B'}(y) \wedge \mu_B(y)]}_{\text{degree of validity of } y \ (w_2)} \wedge \mu_C(z)$$

$$= \underbrace{(w_1 \wedge w_2)}_{\text{firing strength}} \wedge \mu_C(z)$$

General Case

In the case of n antecedents: $\mu_{C'}(z) = (w_1 \wedge w_2 \wedge \cdots \wedge w_n) \wedge \mu_C(z)$

Rule: If x is A and y is B, then z is C
Fact: x is A' and y is B'
Conclusion: z is C'



Case 3: Multiple Rules with Multiple Antecedents

Premise 1:	x is A' and y is B'
Premise 2:	If x is A_1 and y is B_1, then z is C_1, **else**
	If x is A_2 and y is B_2, then z is C_2
Consequence	z is C'

Let $R_1 : A_1 \times B_1 \to C_1$ and $R_2 : A_2 \times B_2 \to C_2$.
Since the max-min operator is distributive over Union, we get,

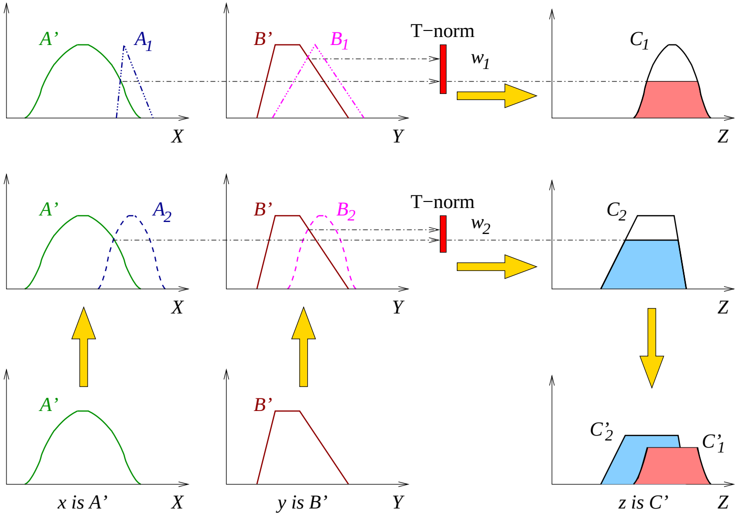
$$C' = \underbrace{(A' \times B')}_{\text{antecedents}} \circ \underbrace{(R_1 \cup R_2)}_{\text{rules}} = \underbrace{[(A' \times B') \circ R_1]}_{\substack{\text{rule 1 with}\\ \text{multiple}\\ \text{antecedents}}} \cup \underbrace{[(A' \times B') \circ R_2]}_{\substack{\text{rule 2 with}\\ \text{multiple}\\ \text{antecedents}}}$$

$$= C_1' \cup C_2', \text{ where } C_i' \text{ is the inferred consequent of rule } i$$

As shown previously, for $C_i' = (A' \times B') \circ R_i$,

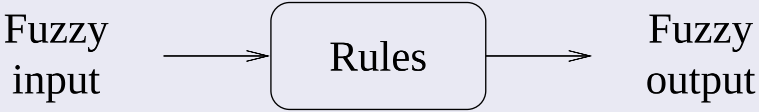
$$\mu_{C_i'}(z) = \bigvee_x [\mu_{A'}(x) \wedge \mu_{A_i}(x)] \wedge \bigvee_y [\mu_{B'}(y) \wedge \mu_{B_i}(y)] \wedge \mu_{C_i}(z)$$

Graphical Illustration



A Typical Fuzzy System

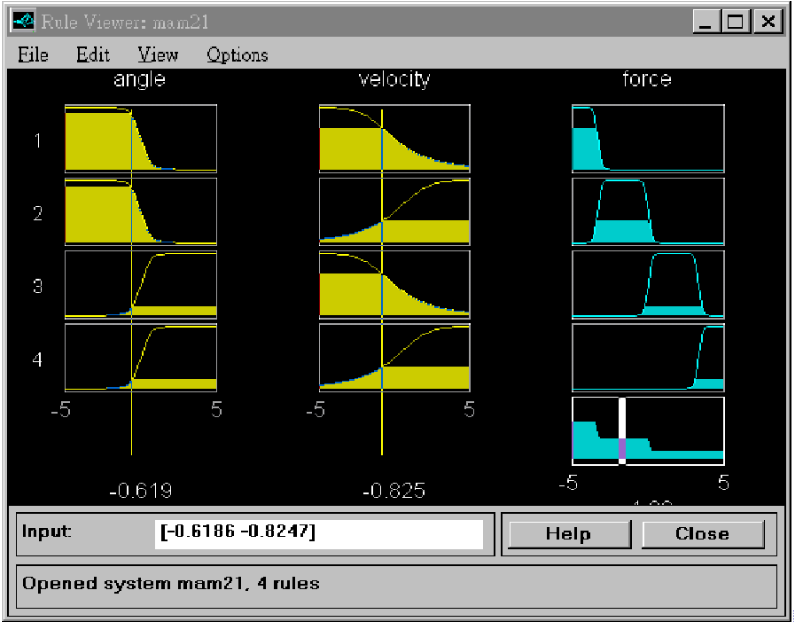
Fuzzy Reasoning



Required Steps for Extended Modus Ponens (Fuzzy Reasoning)

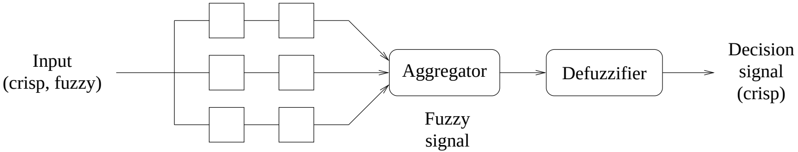
1. Obtain degree of compatibility
   - Compare the known facts with the antecedents of the fuzzy rules $\rightarrow$ degree of compatibility
2. Find the firing strength which combines the degree of compatibility using fuzzy “and” or “or”
   - This indicates the degree at which the antecedent part of the rule is satisfied
3. Qualified consequent
   - Apply the firing strength to the consequent membership function of a rule to generate a qualified consequent membership function
4. Overall output MF aggregates all qualified consequent MF’s to obtain the overall MF

Fuzzy Reasoning: A Graphical Example



Fuzzy Inference System (FIS)

- The basic structure of any FIS is made of:
  1. Rule base
  2. Database of rules
  3. Reasoning mechanism (fuzzification, defuzzification)



Fuzzification

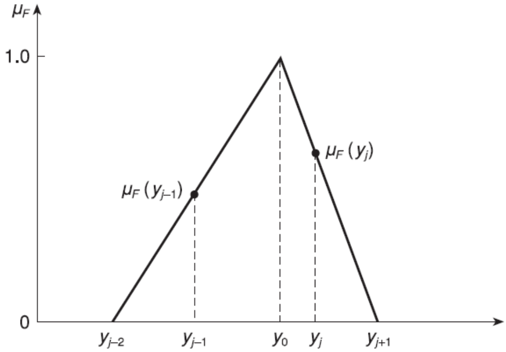
- Fuzzification refers to the representation of a crisp value by a membership function.
- It is needed prior to applying the composition (CRI), when the data (measurements) are crisp values, as common in control applications.
- It may be argued that the process of fuzzification amounts to giving up the accuracy of crisp data.
- This is not so in general. The reason is, a measured piece of data may not be known to be 100% accurate.

Different Fuzzification Methods

- Discrete case of fuzzification
- Continuous case of fuzzification
  - Singleton method
  - Triangular function method
  - Gaussian function method

Discrete Case of Fuzzification

- In the case of discrete membership functions, the crisp quantity y0 may not correspond to one of the discrete points of the membership function of the fuzzy variable Y (or fuzzy state Y_j).
- Suppose that the crisp data value y_0 falls between the discrete values y_{j-1} and y_j of the membership function.



Discrete Case of Fuzzification (cont.)

- Assigning a membership grade of 1 for y_0, the membership grades of the fuzzified quantity F at y_{j-1} and y_j are determined through linear interpolation as $\frac{y_{j-1}-y_{j-2}}{y_0-y_{j-2}}$, $\frac{y_{j+1}-y_j}{y_{j+1}-y_0}$, respectively.
- Accordingly, the discrete membership function of the fuzzified quantity F is given by:

$$F = \{\frac{\frac{y_{j-1}-y_{j-2}}{y_0-y_{j-2}}}{y_{j-1}}, \frac{\frac{y_{j+1}-y_j}{y_{j+1}-y_0}}{y_j}\}$$

- The approach can be extended to include more than two discrete points, thereby providing a wider membership function (greater fuzziness).

Fuzzification: Singleton Method

- Consider a crisp measurement y_0 of a fuzzy variable Y.
- It is known that the measurement y_0 is perfectly accurate.
- y_0 may be represented by a fuzzy quantity F with the singleton membership function

$$\mu_F(y) = \delta(y - y_0) = \begin{cases} 1 & \text{when } y = y_0 \\ 0 & \text{elsewhere} \end{cases}$$

- Since the measured data are not perfectly accurate, a more appropriate method of using fuzzy singleton to fuzzify a crisp value is given now.

Fuzzification: Singleton Method (cont.)

- Suppose that a crisp measurement y_0 is made of a fuzzy variable Y. Let Y can take n fuzzy states $Y_1, Y_2, \cdots, Y_n$.
- Since $Y = Y_1 \ OR \ Y_2 \ OR \ \cdots \ OR \ Y_n$, the membership function of Y is given as the union of the membership functions of the individual fuzzy states:

$$\mu_Y(y) = max_{j=1}^{n} \mu_{Y_j}(y)$$

- The membership function of the fuzzified quantity F is given according to the extended singleton method by a set of fuzzy singletons. For state j:

$$\mu_F(y) = \mu_{Y_j}(y)\delta(y - y_0)$$

Fuzzification: Triangular Function Method

- A triangular membership function may be used to represent the fuzzified quantity for each fuzzy state.
- A triangular membership function (continuous case) may be expressed as

$$\mu_A(y) = \begin{cases} 1 - \frac{|y-y_0|}{s} & \text{for } |y - y_0| \leq s \\ 0 & \text{elsewhere} \end{cases}$$

  - where y_0 shows the peak point and s denotes the base length (support set).

Fuzzification: Triangular Function Method (cont.)

- Again $Y = Y_1 \ OR \ Y_2 \ OR \ \cdots \ OR \ Y_n$.
- The membership function of the fuzzified quantity F is such that, for state j,

$$\mu_F(y) = \mu_{Y_j}(y_0)\mu_{A_j}(y),$$

  - where

$$\mu_{A_j}(y) = \begin{cases} 1 - \frac{|y-y_0|}{s_j} & \text{for } |y - y_0| \leq s_j \\ 0 & \text{elsewhere} \end{cases}$$

    - s_j : base length for state j.

Fuzzification: Gaussian Function Method

- A Gaussian membership function may be used to represent the fuzzified quantity for each fuzzy state.
- A Gaussian membership function (continuous case) may be expressed as

$$\mu_A(y) = exp(\frac{y - y_0}{s})^2$$

- The smaller the s the sharper (or less fuzzy) the membership function.

Fuzzification: Gaussian Function Method (cont.)

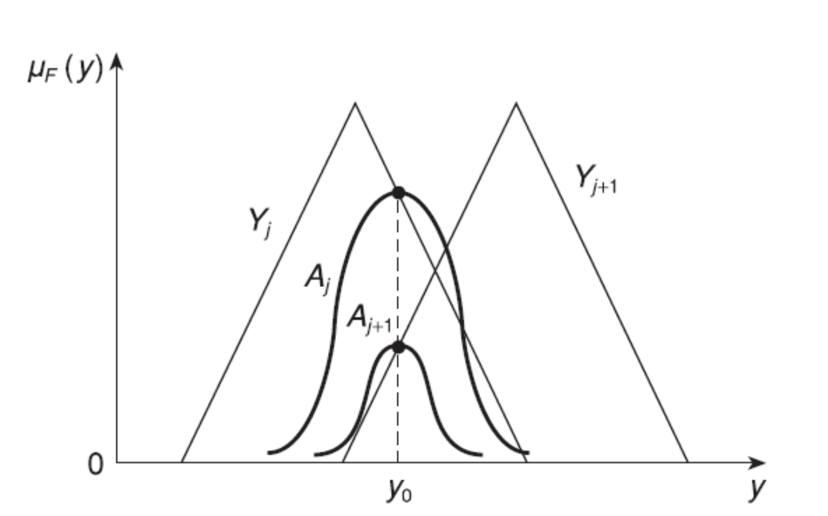
- Again $Y = Y_1 \; OR \; Y_2 \; OR \; \cdots \; OR \; Y_n$.
- The membership function of the fuzzified quantity F is such that, for state j,

$$\mu_F(y) = \mu_{Y_j}(y_0)\mu_{A_j}(y),$$

  - where

$$\mu_{A_j}(y) = exp(\frac{y - y_0}{s_j})^2$$

Graphical Representation



Defuzzification

- Usually, the decision (control action) of a fuzzy logic controller is a fuzzy value and is represented by a membership function.
- Because low-level control actions are typically crisp, the control inference must be defuzzified for physical purposes such as actuation.
- Methods of defuzzification:
    - Centroid method
    - Mean of maxima method
    - Threshold methods

Centroid Method (Center of Gravity)

- Suppose that the membership function of a control inference is $\mu_C(c)$, and its support set is given by : $S = c|\mu_C(c) > 0 \cdot$
- The centroid method of defuzzification is expressed as:

Continuous Case

$$\hat{c} = \frac{\int_{c \in S} c\mu_C(c)dc}{\int_{c \in S} \mu_C(c)dc}$$

Discrete Case

$$\hat{c} = \frac{\sum_{c_i \in S} c_i\mu_C(c_i)}{\sum_{c_i \in S} \mu_C(c_i)}$$

Mean of Maxima Method

- If the membership function of the control inference is unimodal, the control value at the peak membership grade is chosen as the defuzzified control action.

$$\hat{c} = c_{max} \text{ such that } \mu_C(c_{max}) = \mu_C(c)$$

- If the control membership function is multi-modal, the mean of the control values at these peak points, weighted by the corresponding membership grades, is used as the defuzzified value.

$$c_i \text{ such that } \mu_C(c_i)\Delta = \mu_i = max_{c \in S}\mu_C(c), \quad i = 1, 2, \cdots, P$$

  - Then

$$\hat{c} = \frac{\sum_{i=1}^{P} \mu_i c_i}{\sum_{i=1}^{P} \mu_i}, \quad p : \text{total number of modes (peaks)}$$

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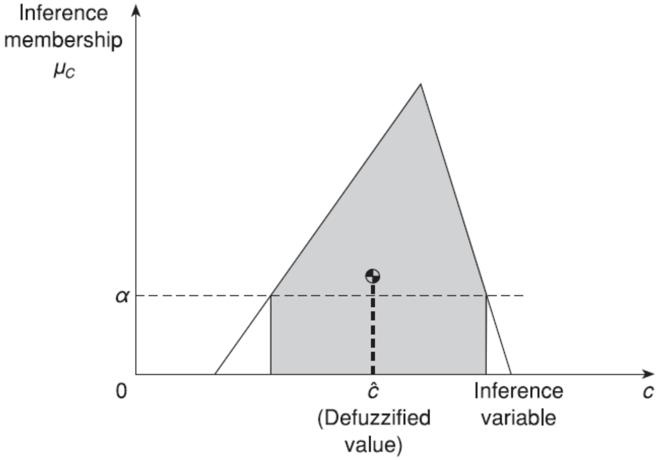
  - Then

$$\hat{c} = \frac{\sum_{i=1}^{P} \mu_i c_i}{\sum_{i=1}^{P} \mu_i}, \quad p : \text{total number of modes (peaks)}$$

Threshold Methods

- Sometimes, it may be desirable to leave out the boundaries of the control inference membership function.
- Only the main core of the control inference is used, not excessively diluting or desensitizing the defuzzified value.
- The corresponding procedures of defuzzification are known as threshold methods.
- The formulae remain the same as given before. However, we use an α-cut of the control inference set not the entire support set.

Threshold Methods: Graphical Illustration

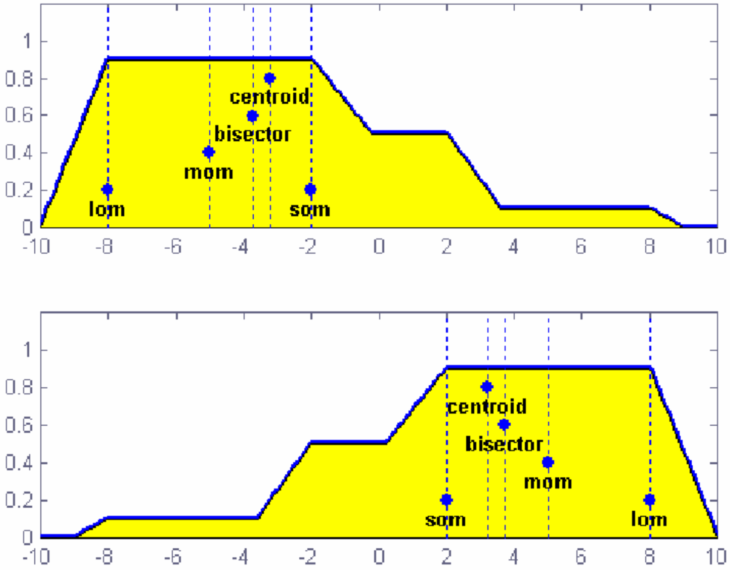


Defuzzification: Other Methods

Defuzzification

Bisector of Area	$\Rightarrow$	c_{BOA} such that, $\int_{\alpha}^{c_{\text{BOA}}} \mu_C(c)\,dc = \int_{c_{\text{BOA}}}^{\beta} \mu_C(c)\,dc$, with $\alpha = \min\{c\}$ and $\beta = \max\{c\}$
Smallest of Maximum	$\Rightarrow$	min of max μ_C
Largest of Maximum	$\Rightarrow$	max of max μ_C

Defuzzification Methods: Graphical Example



Different Inferencing Systems

- Different inferencing procedures have been used in the literature.
- The main difference among them is the aggregation and the defuzzification.

Fuzzy Inference Systems

1. Mamdani fuzzy model
2. Sugeno fuzzy model
3. Tsukomoto fuzzy model

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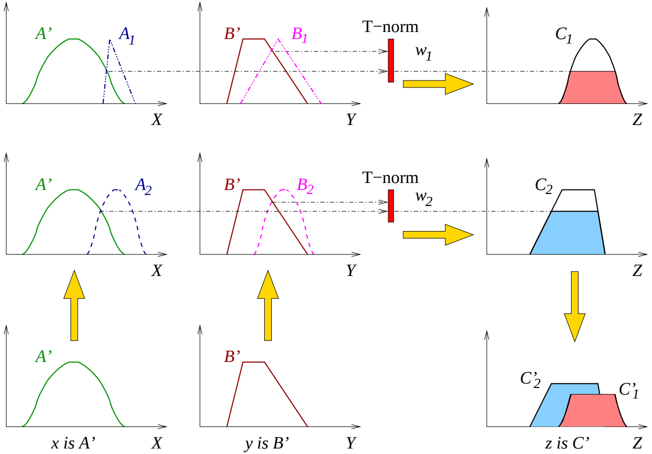
Mamdani Fuzzy Model

- (max-min operator) for aggregation part.



Example: Mamdani Fuzzy Model

Example

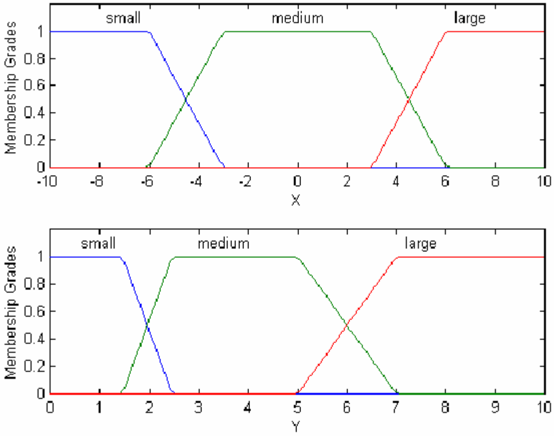
- Let $x \in [-10, 10]$ and $y \in [0, 10]$ be two linguistic variables representing the antecedent and consequent of a fuzzy inference system, respectively.
- Rules

  If x is small then y is small

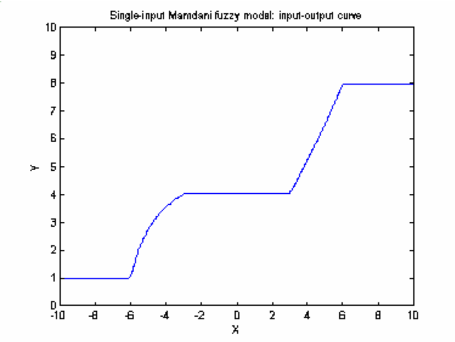
  If x is medium then y is medium

  If x is large then y is large
- Obtain the output of this system using the Mamdani fuzzy inferencing model. Use the center of area as defuzzification operator.

Membership Functions for Input and Output Variables



Input-Output Curve



- The advantage of this curve is that we can directly calculate the value of output y for a given value of x.
- Hence, in practice the system will be faster.

Sugeno Fuzzy Model

- The consequent in the Sugeno fuzzy model is a function of the antecedent.

$$\textit{If } x \textit{ is } A_1 \textit{ and } y \textit{ is } B_1 \textit{ then } z = f_1(x, y)$$

$$\textit{If } x \textit{ is } A_2 \textit{ and } y \textit{ is } B_2 \textit{ then } z = f_2(x, y)$$

  where f_i, $i = 1, 2, \ldots$, is a crisp polynomial function in its arguments.

Sugeno Fuzzy Model

Sugeno Zero Order Model: $f_i(x, y) = \text{constant}$

Sugeno First Order Model: $f_i(x, y) = a_i x + b_i y + c_i$

$\Rightarrow$ The overall output (aggregation) is obtained through a weighted average.

Example 1: Sugeno Fuzzy Model

If x is small then $y = 0.1x + 6.4$

If x is medium then $y = 0.5x + 4$

If x is large then $y = x - 2$

Sugeno Fuzzy Model

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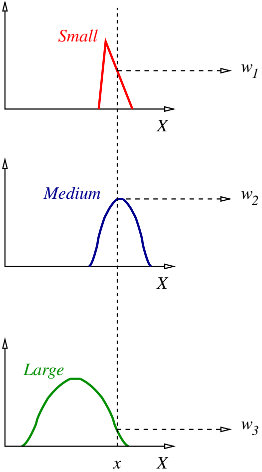
Example 1: Graphical Solution



$$y_1 = 0.1x + 6.4$$

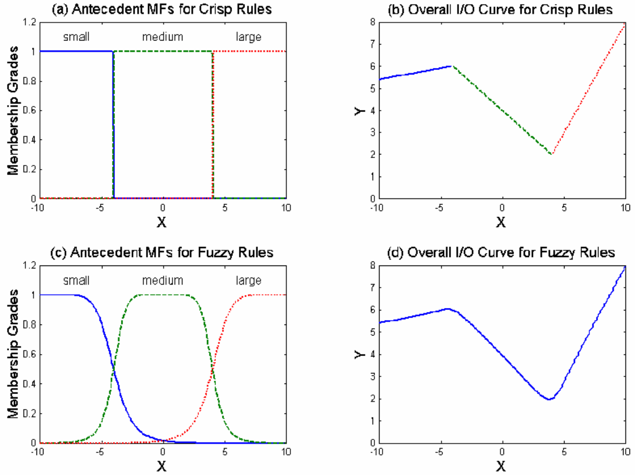
$$y_2 = 0.5x + 4$$

$$y_3 = x - 2$$

$$y = \frac{w_1 y_1 + w_2 y_2 + w_3 y_3}{w_1 + w_2 + w_3}$$

Example 1 (cont.)

Crisp Rules vs Fuzzy Rules



Example 2: Sugeno Fuzzy Model

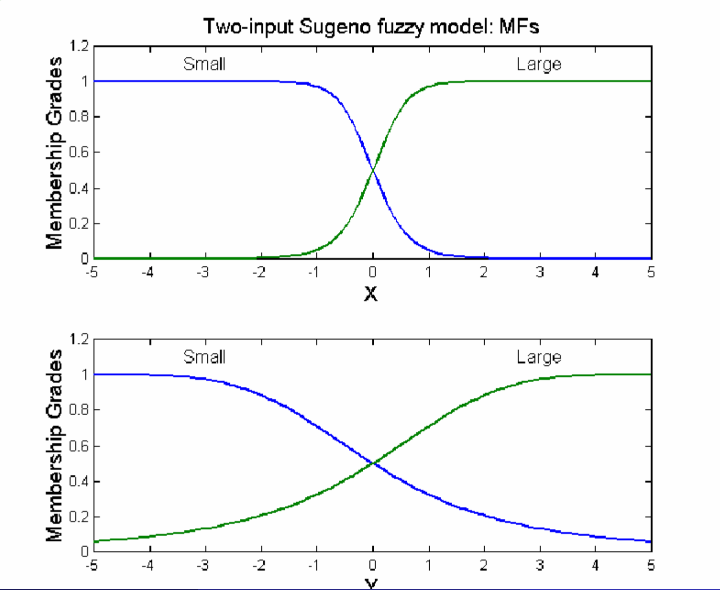
Rules: Two Inputs - One Output

If x is small and y is small then $z = -x + y + 1$
If x is small and y is large then $z = -y + 3$
If x is large and y is small then $z = -x + 3$
If x is large and y is large then $z = x + y + 2$

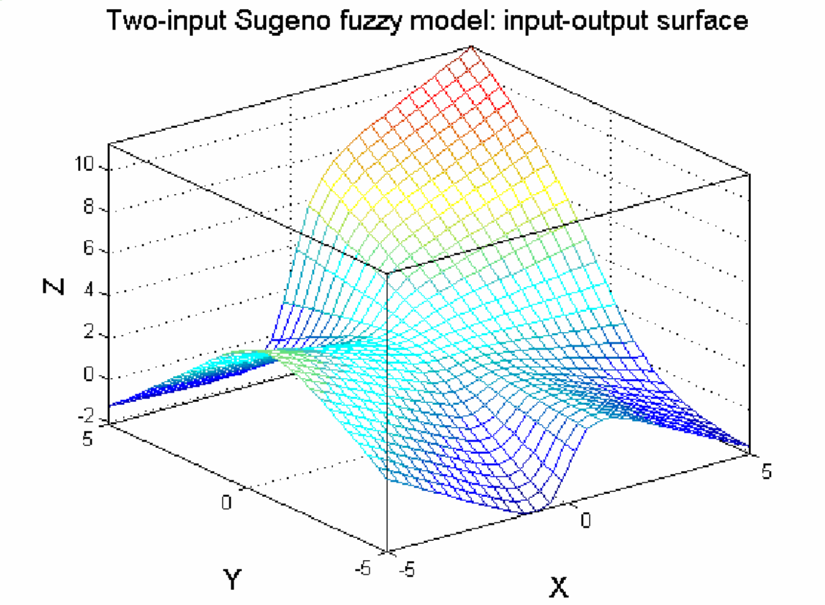
Aggregation

$$z = \frac{\sum_{i=1}^{\#\text{ of rules}} w_i z_i}{\sum_{i=1}^{\#\text{ of rules}} w_i} \quad \Rightarrow \quad z = \frac{w_1 z_1 + w_2 z_2 + w_3 z_3 + w_4 z_4}{w_1 + w_2 + w_3 + w_4}$$

Example 2: Membership Functions for Input Variables

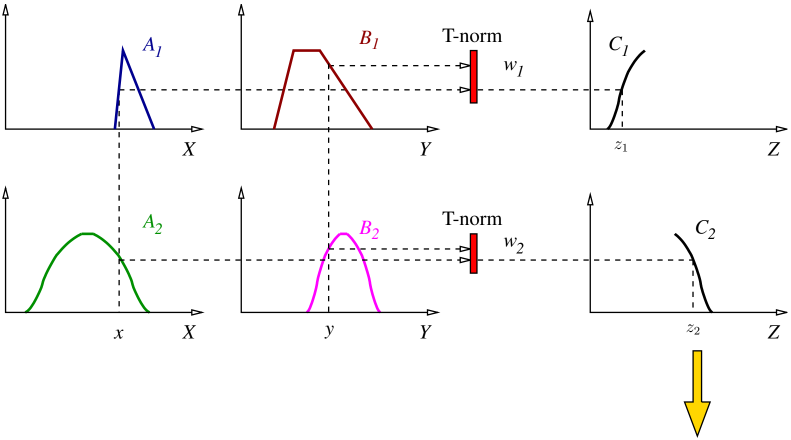


Example 2: Input-Output Surface



Tsukomoto Fuzzy Model

The output membership function is a monotonic mapping (f^{-1} exists).



$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

Example: Tsukomoto Inference Model

Rules

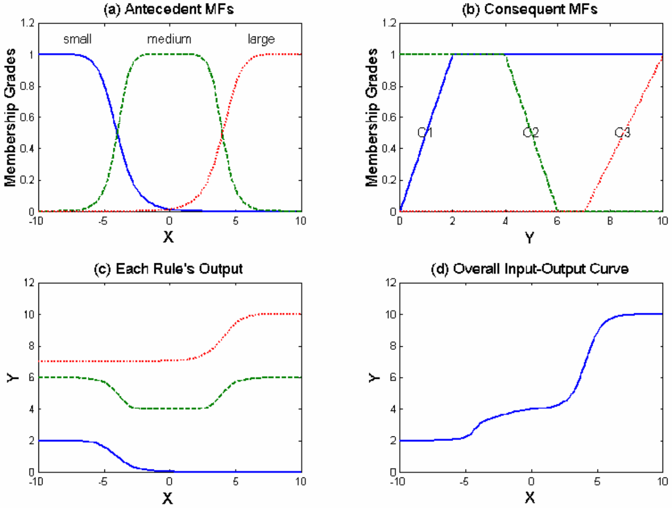
If x *is small then* y *is* C_1

If x *is medium then* y *is* C_2

If x *is large then* y *is* C_3

where C_1, C_2, and C_3, are the consequent membership function variables.

Example: Input and Output Variables and Input-Output Curve



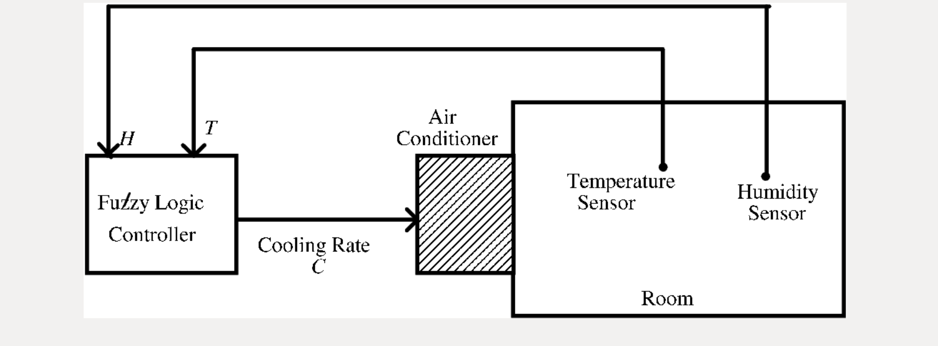
Case Study 1: Room Comfort Control System

Room Comfort Control System

Problem Statement

- The temperature (T) and humidity (H) are the process variables that are measured.
- These sensor signals are provided to the fuzzy logic controller.
- The fuzzy logic controller determines the cooling rate (C) that should be generated by the air conditioning unit.
- The objective is to maintain a particular comfort level inside the room.

Structure of Room Comfort Control System



Input Variables

- Temperature level (T): There are two fuzzy states (HG, LW), which denote high and low, respectively, with the corresponding membership functions.

- Humidity level (H): There exist two other fuzzy states (HG, LW) with associated membership functions.

- The membership functions of T are quite different from those of H, even though the same nomenclature is used.

Room Comfort Control System

Nomenclature Used for the Fuzzy States

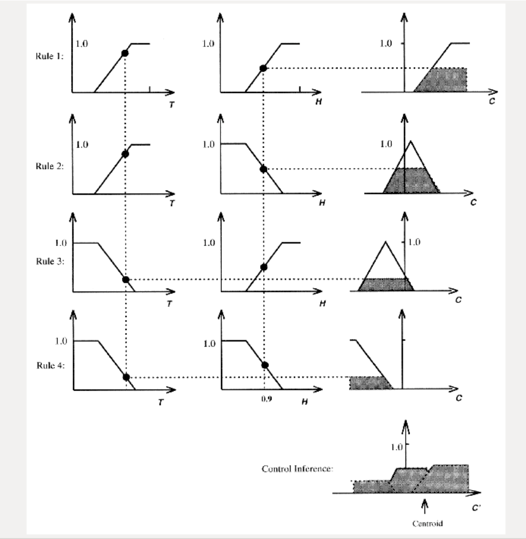
Temperature (T)	Humidity (H)	Change in Cooling Rate (C)
HG = High	HG = High	PH = Positive High
LW = Low	LW = Low	PL = Positive Low
		NH = Negative High
		NL = Negative Low

Room Comfort Control System : Knowledge Base

Rule Base

Rule 1:		If	T	is	HG	and	H	is	HG	then	C	is	PH
Rule 2:	else	if	T	is	HG	and	H	is	LW	then	C	is	PL
Rule 3:	else	if	T	is	LW	and	H	is	HG	then	C	is	NL
Rule 4:	else	if	T	is	LW	and	H	is	LW	then	C	is	NH
	and	if											

Mamdani Inferencing Model



Fuzzy Reasoning

- Application of the compositional rule of inference is done here by using individual rule-based composition.
- For example, suppose that the room temperature is 30^oC and the relative humidity is 0.9.
- Lines are drawn at these points to determine the corresponding membership grades for the fuzzy states in the four rules.

Fuzzy Reasoning (cont.)

- In each rule the lower value of the two grades of process response variables is used to clip (or modulate) the corresponding membership function of C (a *min* operation).
- The resulting ”clipped” membership functions of C for all four rules are superimposed (a *max* operation) to obtain the control inference C' as shown.
- This result is a fuzzy set, and it must be defuzzified to obtain a crisp control action $\hat{c}$ for changing the cooling rate.
- The centroid method may be used, as explained in the next section.

Case Study 2

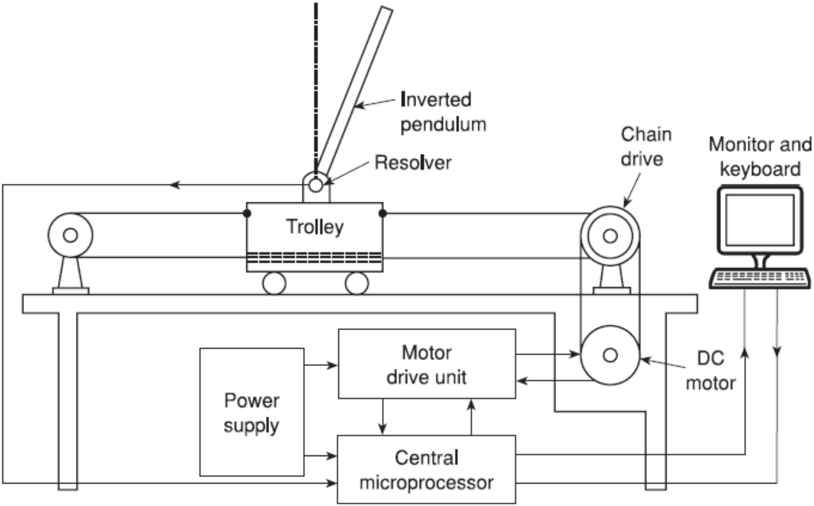
Computer-Controlled Inverted Pendulum

Computer-Controlled Inverted Pendulum

Problem Statement

- The process measurements are the angular position, about the vertical (ANG) and the angular velocity (VEL) of the pendulum.
- The fuzzy logic controller determines the control action (CNT), i.e. the current of the motor driving the positioning trolley.
- The objective of the fuzzy logic control is to keep the inverted pendulum upright.

Computer-controlled Inverted Pendulum



Input Variables

Angular Position (ANG)

Angular position takes two fuzzy states:

- Positive large (PL)
- Negative large (NL)

Angular Velocity (VEL)

Angular velocity takes two fuzzy states:

- Positive large (PL)
- Negative large (NL)

Output Variable

Control Action (CNT)

Control action takes three fuzzy states:

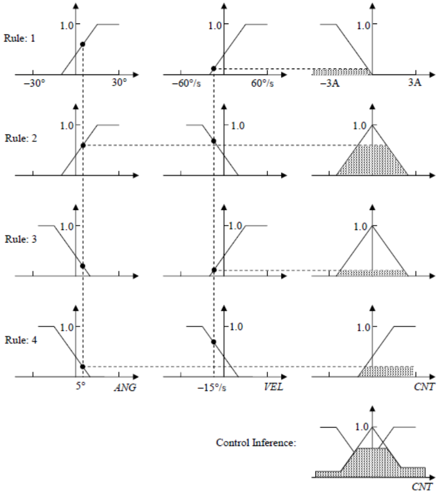
- Positive large (PL)
- No Change (NC)
- Negative large (NL)

Rule Base

Governing Rules

1. If ANG is PL and VEL is PL then CNT is NL
2. else if ANG is PL and VEL is NL then CNT is NC
3. else if ANG is NL and VEL is PL then CNT is NC
4. else if ANG is NL and VEL is NL then CNT is LP

Mamdani Fuzzy Inference System



Fuzzy Reasoning

- Application of the compositional rule of inference is done here by using individual rule-based composition.
- For example, suppose that the angular position and velocity are 5° and $-15^\circ/S$, respectively;
- Lines are drawn at these points to determine the corresponding membership grades for the fuzzy states in each of the four rules.

Fuzzy Reasoning (cont.)

- For each rule the lower value of the membership grades of process response variables is used to clip (or modulate) the corresponding membership function of CNT (*min* operation).
- The resulting clipped membership functions of CNT for all four rules are superimposed (*max* operation) to obtain the control inference.
- The inference result is a fuzzy set, and it must be defuzzified to obtain a crisp control action.
- The centroid method may be used for defuzzification.

Case Study 3

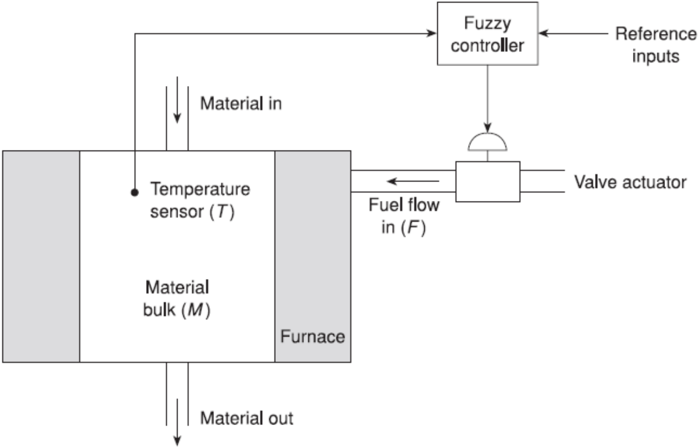
Metallurgical Heat Treatment Process

Metallurgical Heat Treatment Process

Problem Statement

- A metallurgical process consists of heat treatment of a bulk of material for a specified duration of time at a suitable temperature.
- The heater is controlled by its fuel supply rate, which is in turned controlled by a fuzzy controller.
- Four fuzzy quantities are defined, meterial temperature (T), material mass (M), process time (P), and fuel rate (F).

Metallurgical Heat Treatment Process



I/O Variables

Temperature of the Material (T)

Temperature takes two fuzzy states (LW,HG), denoting low and high, respectively.

Mass of the Material (T)

Mass takes two fuzzy states (SM,LG), denoting small and large, respectively.

I/O Variables (cont.)

Process Termination Time (P)

Process termination time takes two fuzzy states (FR,NR), denoting far and near, respectively.

Fuel Supply Rate (F)

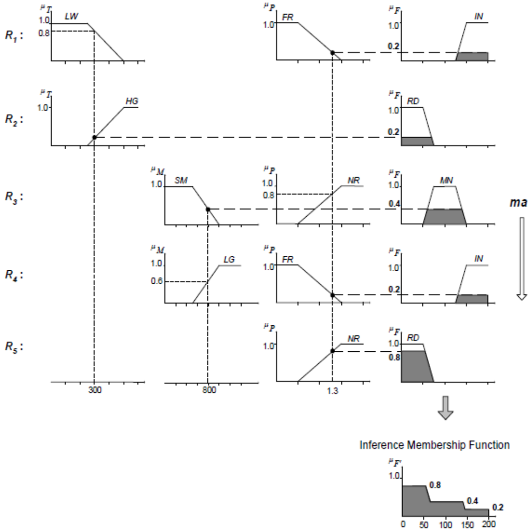
Fuel supply rate takes three fuzzy states (RD,MN,IN), denoting reduce, maintain, and increase, respectively.

Rule Base

Governing Rules

1. If T is LW and P is FR then F is IN
2. or if T is HG then F is RD
3. or if M is SM and P is NR then F is MN
4. or if M is LG and P is FR then F is IN
5. or if P is NR then F is RD

Mamdani Fuzzy Inference System



Fuzzy Reasoning

- Application of the compositional rule of inference is done here by using individual rule-based composition.
- For example, suppose that the material temperature and mass are $300\,^{\circ}C$ and 800 kg, respectively, with process operation time of 1.3 hr.
- Lines are drawn at these points to determine the corresponding membership grades for the fuzzy states in each of the five rules.

Fuzzy Reasoning (cont.)

- For each rule the lower value of the membership grades of process response variables is used to clip (or modulate) the corresponding membership function of F (*min* operation).
- The resulting clipped membership functions of F for all five rules are superimposed (*max* operation) to obtain the control inference.
- The inference result is a fuzzy set, and it must be defuzzified to obtain a crisp control action.
- The defuzzification can be performed using centroid method.