Solutions

Problem # 1

- (a) Linguistic representation:
  x_1 is strongly related to y_1 and weakly related to y_2, y_3 and y_4.
  x_2 is strongly related to y_2 and weakly related to y_1, y_3 and y_4.
  x_3 is strongly related to y_3 and weakly related to y_1, y_2 and y_4.
  x_4 is strongly related to y_4 and weakly related to y_1, y_2 and y_3.
  x_5 is moderately related to y_2 and weakly related to y_1, y_3 and y_4.

- Suppose, the fuzzy diagnostic relationship is a union
  $$R_d = \frac{1.0}{(x_1,y_1)} + \frac{0.1}{(x_1,y_2)} + \frac{0.05}{(x_1,y_3)} + \frac{0.0}{(x_1,y_4)} +$$
  $$\frac{0.0}{(x_2,y_1)} + \frac{0.9}{(x_2,y_2)} + \frac{0.0}{(x_2,y_3)} + \frac{0.0}{(x_2,y_4)} +$$
  $$\frac{0.05}{(x_3,y_1)} + \frac{0.2}{(x_3,y_2)} + \frac{0.95}{(x_3,y_3)} + \frac{0.0}{(x_3,y_4)} +$$
  $$\frac{0.1}{(x_4,y_1)} + \frac{0.0}{(x_4,y_2)} + \frac{0.01}{(x_4,y_3)} + \frac{1.0}{(x_4,y_4)} +$$
  $$\frac{0.0}{(x_5,y_1)} + \frac{0.9}{(x_5,y_2)} + \frac{0.0}{(x_5,y_3)} + \frac{0.0}{(x_5,y_4)}.$$
  It can be expressed in the following forms respectively,

- (b) A directed graph Fig. 1.

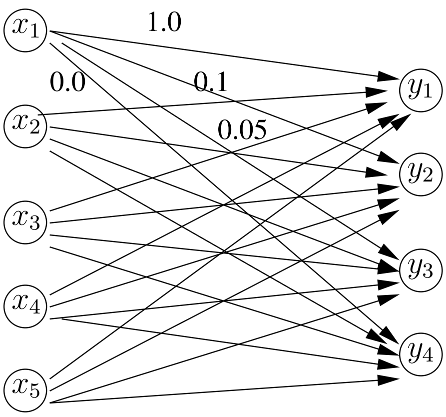

Figure 1: A directed graph

- (c) Tabular form:

	y_1	y_2	y_3	y_4
x_1	1.0	0.1	0.05	0.0
x_2	0.0	0.9	0.0	0.0
x_3	0.05	0.2	0.95	0.0
x_4	0.1	0.0	0.01	1.0
x_5	0.0	0.9	0.0	0.0

- (d) As a matrix:

$$R = \begin{bmatrix} 1.0 & 0.1 & 0.05 & 0.0 \\ 0.0 & 0.9 & 0.0 & 0.0 \\ 0.05 & 0.2 & 0.95 & 0.0 \\ 0.1 & 0.0 & 0.01 & 1.0 \\ 0.0 & 0.9 & 0.0 & 0.0 \end{bmatrix}$$

- (e) In ordered pairs:

$$\begin{aligned} R_d = {} & \tfrac{1.0}{(x_1,y_1)} + \tfrac{0.1}{(x_1,y_2)} + \tfrac{0.05}{(x_1,y_3)} + \tfrac{0.0}{(x_1,y_4)} + \\ & \tfrac{0.0}{(x_2,y_1)} + \tfrac{0.9}{(x_2,y_2)} + \tfrac{0.0}{(x_2,y_3)} + \tfrac{0.0}{(x_2,y_4)} + \\ & \tfrac{0.05}{(x_3,y_1)} + \tfrac{0.2}{(x_3,y_2)} + \tfrac{0.95}{(x_3,y_3)} + \tfrac{0.0}{(x_3,y_4)} + \\ & \tfrac{0.1}{(x_4,y_1)} + \tfrac{0.0}{(x_4,y_2)} + \tfrac{0.01}{(x_4,y_3)} + \tfrac{1.0}{(x_4,y_4)} + \\ & \tfrac{0.0}{(x_5,y_1)} + \tfrac{0.9}{(x_5,y_2)} + \tfrac{0.0}{(x_5,y_3)} + \tfrac{0.0}{(x_5,y_4)}. \end{aligned}$$

Problem # 2

$$\begin{array}{cccc} & R = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} & \begin{array}{c} \text{First projection} \\ 1.0 \\ 1.0 \\ 1.0 \\ 1.0 \end{array} & \\ \text{Second projection} \rightarrow & 1.0 \;\; 1.0 \;\; 1.0 \;\; 1.0 & [1.0] & \leftarrow \text{total projection} \end{array}$$

In other words:
First projection: $R^1 = \frac{1.0}{x_1} + \frac{1.0}{x_2} + \frac{1.0}{x_3} + \frac{1.0}{x_4}$
Second projection: $R^2 = \frac{1.0}{y_1} + \frac{1.0}{y_2} + \frac{1.0}{y_3} + \frac{1.0}{y_4}$
Total projection:

$$R^T = \mathrm{CE}(R^2) = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

Problem # 3

$$R_1 = \begin{bmatrix} 1.0 & 0.3 & 0.9 & 0.0 \\ 0.3 & 1.0 & 0.8 & 1.0 \\ 0.9 & 0.8 & 1.0 & 0.8 \\ 0.0 & 1.0 & 0.8 & 1.0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.0 & 0.5 \\ 0.3 & 0.1 & 0.0 \\ 0.2 & 0.3 & 0.1 \end{bmatrix}$$

- (a) max-product:

$$R1 \bullet R2 = R_{m-p} \quad \Rightarrow \quad R_1 \bullet R2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.3 & 0.5 \\ 0.9 & 0.9 & 0.81 \\ 1.0 & 0.3 & 0.5 \end{bmatrix}$$

Example calculations for first row:

$$(1.0 * 1.0) \vee (0.3 * 1.0) \vee (0.9 * 0.3) \vee (0.0 * 0.2) = (1.0) \vee (0.3) \vee (0.27) \vee (0.0) = 1.0$$

$$(1.0 * 1.0) \vee (0.3 * 0.0) \vee (0.9 * 0.1) \vee (0.0 * 0.3) = (1.0) \vee (0.0) \vee (0.09) \vee (0.0) = 1.0$$

$$(1.0 * 0.9) \vee (0.3 * 0.5) \vee (0.9 * 0.0) \vee (0.0 * 0.1) = (0.9) \vee (0.15) \vee (0.0) \vee (0.0) = 0.9$$

- (b) max-average:

$$R1 < + > R2 = R_{m-a} \quad \Rightarrow \quad R_1 \quad R2 = \begin{bmatrix} 1.0 & 1.0 & 0.95 \\ 1.0 & 0.65 & 0.75 \\ 0.95 & 0.95 & 0.9 \\ 1.0 & 0.65 & 0.75 \end{bmatrix}$$

Example calculations for first row:

$$\frac{1}{2}[(1.0+1.0)\vee(0.3+1.0)\vee(0.9+0.3)\vee(0.0+0.2) = \frac{1}{2}[(2.0) \vee (1.3) \vee (1.2) \vee (0.2)] = \frac{1}{2}(2.0) = 1.0$$

$$\frac{1}{2}[(1.0+1.0)\vee(0.3+0.0)\vee(0.9+0.1)\vee(0.0+0.3) = \frac{1}{2}[(2.0) \vee (0.3) \vee (1.0) \vee (0.3)] = \frac{1}{2}(2.0) = 1.0$$

$$\frac{1}{2}[(1.0+0.9)\vee(0.3+0.5)\vee(0.9+0.0)\vee(0.0+0.1) = \frac{1}{2}[(1.9) \vee (0.8) \vee (0.9) \vee (0.1)] = \frac{1}{2}(1.9) = 0.95$$

- (c) max-min:

$$R1 \quad R2 = R_{max-min} \quad \Rightarrow \quad R_1 \quad R2 = \begin{bmatrix} 1.0 & 1.0 & 0.9 \\ 1.0 & 0.3 & 0.5 \\ 0.9 & 0.9 & 0.9 \\ 1.0 & 0.3 & 0.5 \end{bmatrix}$$

Example calculations for first row:

$$(1.0 \wedge 1.0) \vee (0.3 \wedge 1.0) \vee (0.9 \wedge 0.3) \vee (0.0 \wedge 0.2) = (1.0) \vee (0.3) \vee (0.3) \vee (0.0) = 1.0$$

$$(1.0 \wedge 1.0) \vee (0.3 \wedge 0.0) \vee (0.9 \wedge 0.1) \vee (0.0 \wedge 0.3) = (1.0) \vee (0.0) \vee (0.1) \vee (0.0) = 1.0$$

$$(1.0 \wedge 0.9) \vee (0.3 \wedge 0.5) \vee (0.9 \wedge 0.0) \vee (0.0 \wedge 0.1) = (0.9) \vee (0.3) \vee (0.0) \vee (0.0) = 0.9$$

Problem # 4

max-min:

$$R1 \quad R2 = R_{max-min} \quad \Rightarrow \quad R_1 \quad R2 = \begin{bmatrix} 0.4 & 0.7 & 0.3 & 0.7 \\ 0.3 & 1.0 & 0.5 & 0.8 \\ 0.8 & 0.3 & 0.7 & 1.0 \end{bmatrix}$$

Example calculations for first row:

$$(0.1\wedge0.9)\vee(0.2\wedge0.2)\vee(0.0\wedge0.8)\vee(1.0\wedge0.4)\vee(0.7\wedge0.0) = (0.1)\vee(0.2)\vee(0.0)\vee(0.4)\vee(0.0) = 0.4$$

$$(0.1\wedge0.0)\vee(0.2\wedge1.0)\vee(0.0\wedge0.0)\vee(1.0\wedge0.2)\vee(0.7\wedge1.0) = (0.0)\vee(0.2)\vee(0.0)\vee(0.2)\vee(0.7) = 0.7$$

$$(0.1\wedge0.3)\vee(0.2\wedge0.8)\vee(0.0\wedge0.7)\vee(1.0\wedge0.3)\vee(0.7\wedge0.0) = (0.1)\vee(0.2)\vee(0.0)\vee(0.3)\vee(0.0) = 0.3$$

$$(0.1\wedge1.0)\vee(0.2\wedge0.0)\vee(0.0\wedge1.0)\vee(1.0\wedge0.0)\vee(0.7\wedge0.8) = (0.1)\vee(0.0)\vee(0.0)\vee(0.0)\vee(0.7) = 0.7$$