ECE 657
INTELLIGENT SYSTEMS DESIGN
COMPUTATIONAL INTELLIGENCE

ASSIGNMENT #1

Problem 1. Suppose that the state of "fast speed" of a machine is denoted by the fuzzy set F with membership function $\mu_F(v)$. Then the state of "very fast speed", where the *linguistic hedge* "very" has been incorporated, may be represented by $\mu_F(v-v_o)$ with $v_o > 0$. Also, the state "presumably fast speed", where the linguistic hedge "presumably" has been incorporated, may be represented by $\mu_F^2(v)$.

(a) Discuss the appropriateness of the use of these membership functions to represent the respective linguistic hedges.

(b) In particular, if

$$F = \{\frac{0.1}{10}, \frac{0.3}{20}, \frac{0.6}{30}, \frac{0.8}{40}, \frac{1.0}{50}, \frac{0.7}{60}, \frac{0.5}{70}, \frac{0.3}{80}, \frac{0.1}{90}\}$$

in the discrete universe $V = \{0, 10, 20, \ldots, 190, 200\}$ rev/s and $v_o = 50$ rev/s, determine the membership functions of "very fast speed" and "presumably fast speed".

(c) Suppose that power p is given by the relation (crisp)

$$p = v^2$$

For the given fast speed (fuzzy) F, determine the corresponding membership function for power. Compare/contrast this with "presumably fast speed".

Problem 2. Sketch the membership function $\mu_A(x) = e^{-\lambda(x-a)^n}$ for $\lambda = 2$, $n = 2$, and $a = 3$ for the support set $S = [0,6]$. On this sketch separately show the shaded areas that represent the following fuzziness measures:

(a) $M_1 = \int_S f(x)dx$ where
$$f(x) = \mu_A(x) \quad \text{for} \quad \mu_A(x) \le 0.5$$
$$= 1 - \mu_A(x) \quad \text{for} \quad \mu_A(x) > 0.5$$

(b) $M_2 = \int_S \left|\mu_A(x) - \mu_{A_{1/2}}(x)\right| dx$

where $\mu_{A_{1/2}}$ is the $\alpha-$cut of $\mu_A(x)$ for $\alpha = 1/2$

(c) $M_3 = \int_S \left|\mu_A(x) - \mu_{\overline{A}}(x)\right| dx$

where $\overline{A}$ is the complement of the fuzzy set A. Evaluate the values of M_1, M_2 and M_3 for the given membership function.

i. Establish relationships between M_1, M_2 and M_3.

ii. Indicate how these measures can be used to represent the degree of fuzziness of a membership function.

iii. Compare your results with the case $\lambda = 1$, $a = 3$, and $n = 2$ for the same support set, by showing the corresponding fuzziness measures on a sketch of the new membership function.

Problem 3. The grade of inclusion of fuzzy set A in fuzzy set B is given by:

$$g(x) = 1 \quad \text{for } \mu_A(x) \le \mu_B(x)$$
$$= \mu_B(x) \quad \text{for } \mu_A(x) > \mu_B(x)$$

Show that $g(x) = sup\ \{c \in [0,1] \mid \mu_A(x) T c \le \mu_B(x)\}$

where the *t-norm, T* may be interpreted as the *min* operation.

Problem 4. Show that $max[0, x + y - 1]$ is a t-norm. Also, determine the corresponding t-conorm (i.e., s-norm). *Hint:* Show that the non-decreasing, commutative, and associative properties and the boundary conditions are satisfied.

Problem 5.

(a) Consider the membership function $\mu_A(x) = e^{-\lambda|x-a|^n}$, for a fuzzy set A.
Interpret the meaning of the parameters a, λ and n. In particular, discuss how (1) fuzziness and (2) a fuzzy adjective or fuzzy modifier such as "very" or "somewhat" of a fuzzy state may be represented using these parameters.

(b) Using the general membership function expression used in part (a), give an analytical representation for temperature inside a living room, that has the three fuzzy states "cold, comfortable, and hot". You must give appropriate numerical values for the parameters of the analytical expression.