# Fuzzy Logic in the Real World

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# Overview

- Motivation
- What is Fuzzy Logic?
- Fuzzy Logic Systems
- Example Applications
- Uncertainty and Fuzziness
- The Future of Fuzzy Systems

# Crisp Sets and Logic

- A well defined, unordered collection of items which are identifiable and distinct
- Six nations rugby teams = $\{$England, Scotland, France, Italy, Ireland, Wales$\}$

# Crisp Sets and Logic

## Definition

A crisp set $A$ over the universe for discourse $X$ is subset of the domain $X$ based on some condition(s):

$$A = \{x | x \text{ meets some condition(s)}\}$$

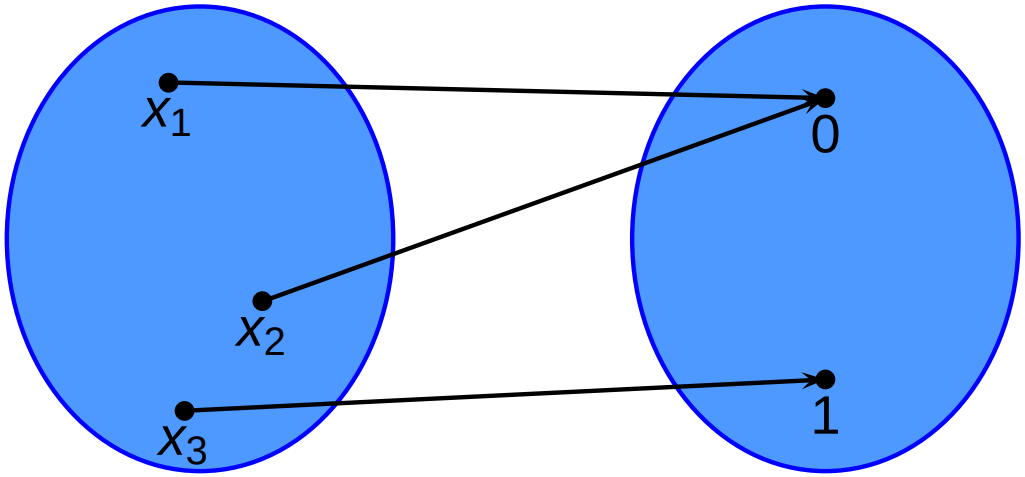
A membership function $\mu_A$ is used to map elements of $X$ to their respective membership in $A$ of zero or one:

$$\mu_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

# Crisp Sets and Logic

So a crisp set is a relation from some domain to binary values:

$$A : X \times \{0, 1\}$$



# The Trouble with Crisp Sets

## The Sorites Paradox

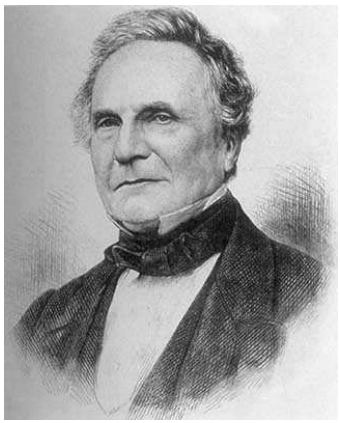
- Premise 1: Consider 100,000 grains of sand to be a heap
- Premise 2: A heap of sand minus one grain is still a heap of sand

- But at some point it **must** stop being a heap

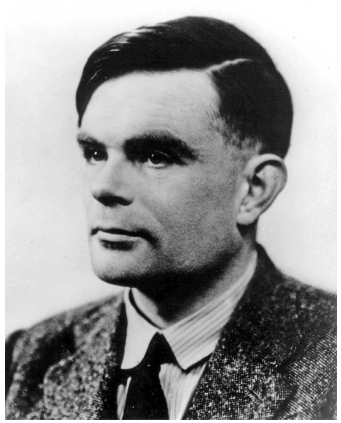
# The Trouble with Crisp Sets

The Sorites Paradox - Bertrand Russell's view

- Person *x* is *tall* if their height is 170cm or more
- Tall = $\{\text{person} \mid \text{height(person)} \geq 170\}$



Charles' height is 169cm



Alan's height 170cm

Jon's height is 185cm

# The Trouble with Crisp Sets

Perhaps we need:

- A softer model
- Degrees of set membership
- Some conceptual vagueness

# Fuzzy Sets

Fuzzy sets - proposed by Lotfi Zadeh in 1965

- Set membership is graduated
- Degrees of belonging measured as real numbers between zero and one
- Boundaries of the set are soft, not crisp

# Fuzzy Sets

## Definition

A fuzzy set $A$ over the universe for discourse $X$ is a set of ordered pairs mapping domain elements their respective degrees of belonging measured as a real number between zero and one:

$$A = \{(x_1, 0.4), (x_2, 0.3), (x_3, 1), (x_4, 0.6)\}$$

Or using Zadeh's notation:

$$A = \{0.4/x_1 + 0.3/x_2 + 1/x_3 + 0.6/x_4\}$$

A fuzzy set $A$ is usually expressed in terms of its membership function $\mu_A$ which maps domain elements ($x$) their respective degrees of of belonging in the interval $[0, 1]$:

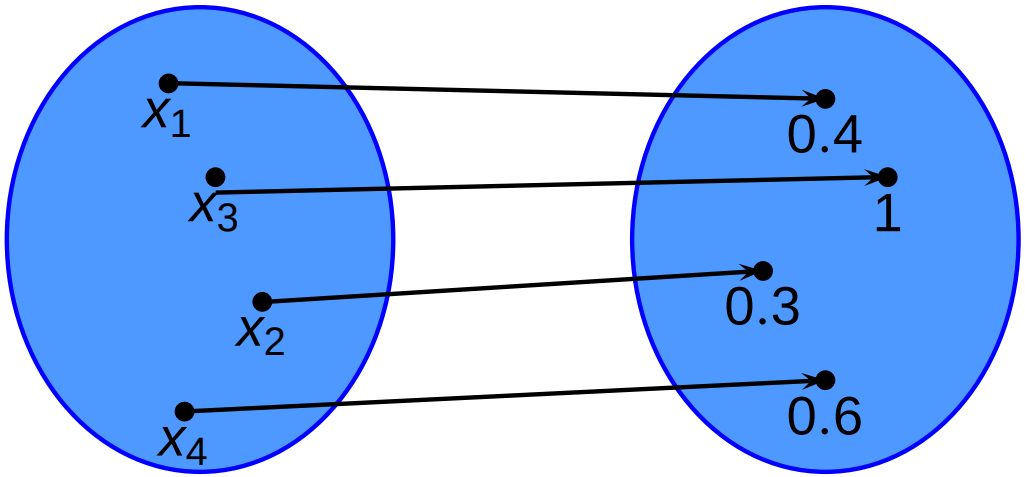
$$A = \{(x, \mu_A(x)) | x \in X\}$$

# Fuzzy Sets

## A Graphical Comparison with Crisp Sets

So a fuzzy set is a relation from some domain to real numbers:

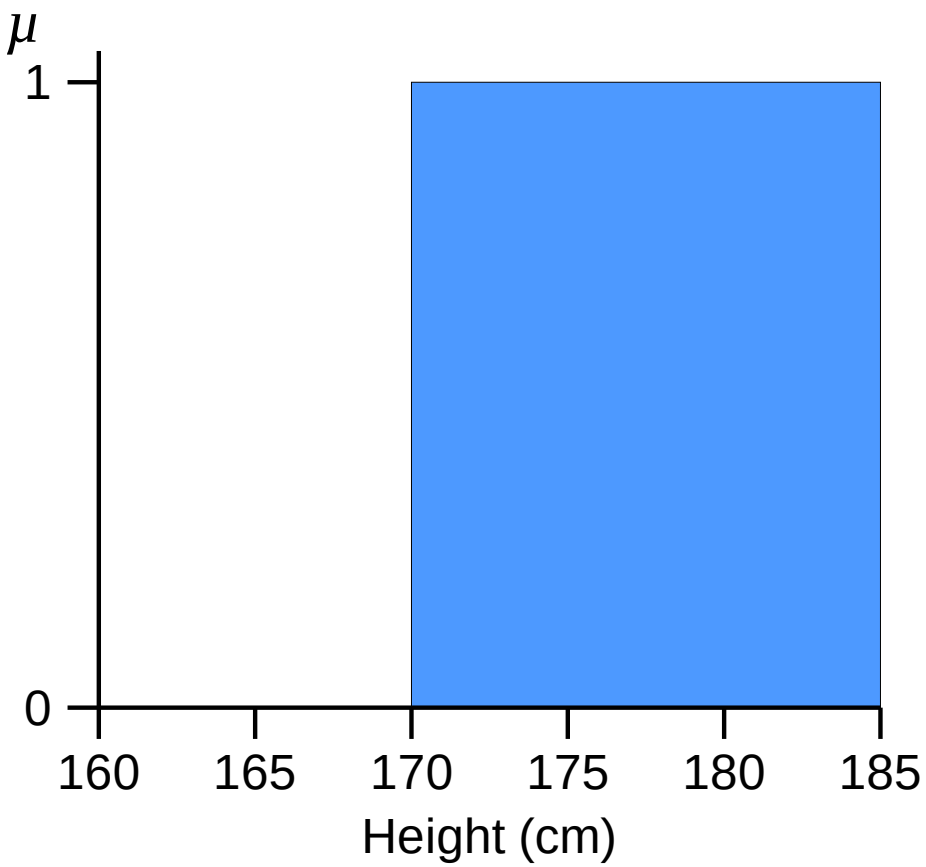
$$A : X \times \{0, 1\}$$



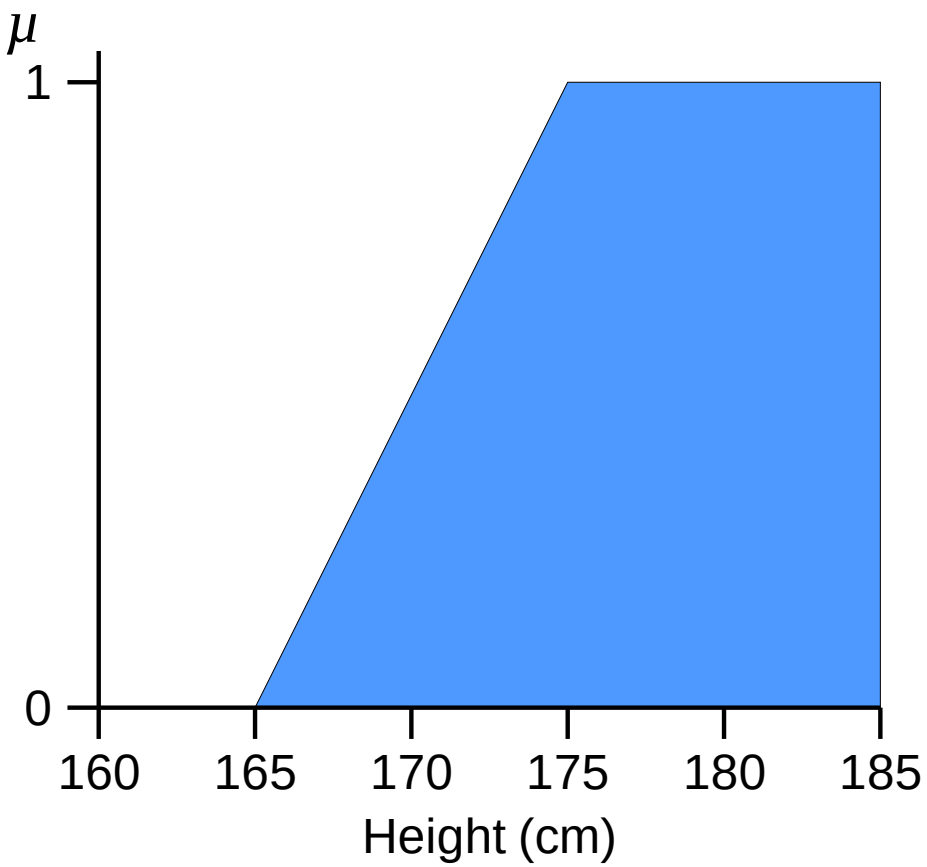
# Fuzzy Sets

## A Graphical Comparison with Crisp Sets

The Membership Function of the *Crisp* Set Tall




The Membership Function of the *Fuzzy* Set Tall

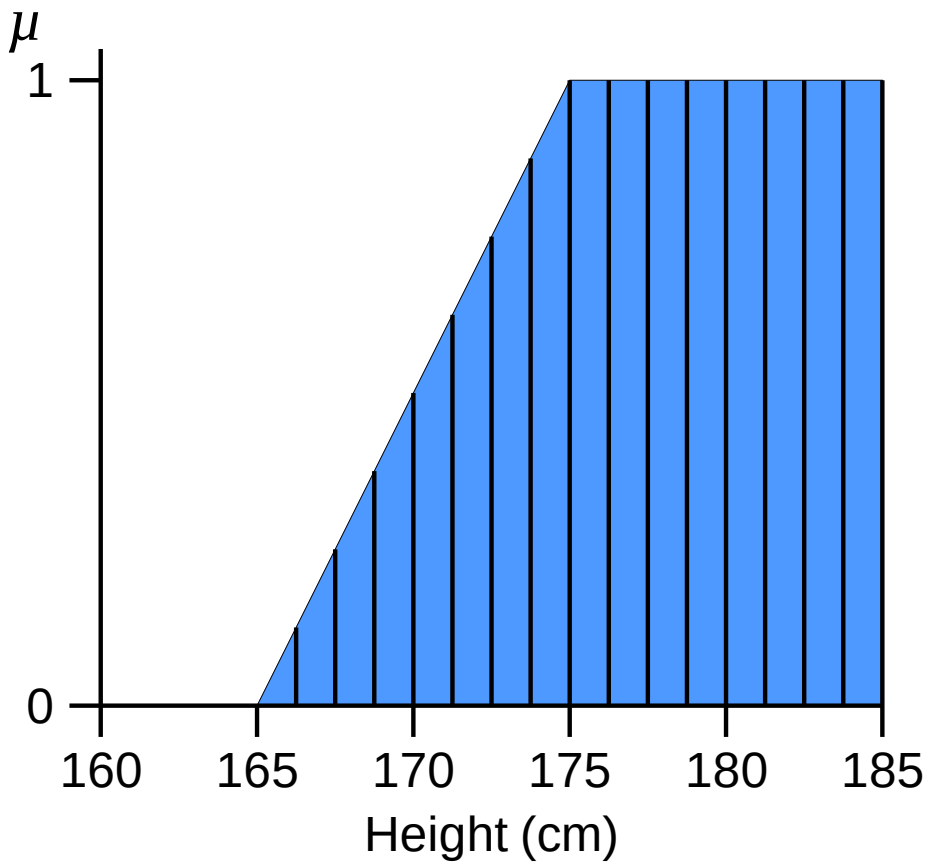


# Fuzzy Sets

## Implementation Reality

- Computers don't like continuous functions
- Instead use discrete approximations
- A number of ordered pairs mapping $x$ values to the $\mu$

The Membership Function of the *Fuzzy* Set Tall



# Fuzzy Sets and Probability

## A Cautionary Tale

- Quite different meanings
- Example - bottles of liquid:

Fuzzy Bottle

0.7 Drinkable

Probabilistic Bottle

0.7 Drinkable

# Fuzzy Logic Operators

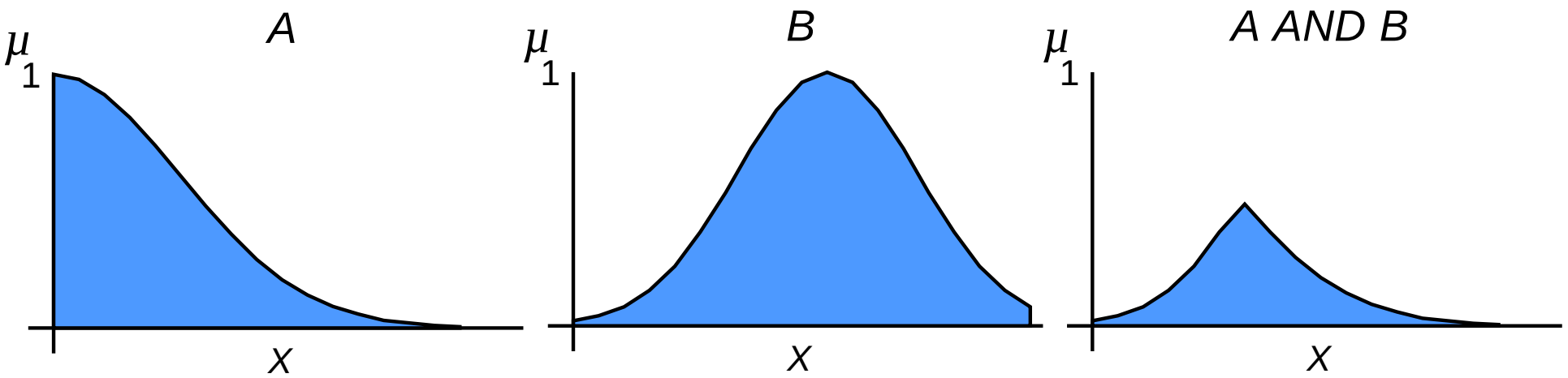
- Logical operations of fuzzy sets are well defined
- Together these form fuzzy logic:
  - AND
  - OR
  - NOT
  - IMPLIES
- Crucial for rule based fuzzy logic systems

# Fuzzy Logic Operators

## Logical AND

- Defined for each point in the membership function
- Extends Boolean AND
- Any t-norm but usually minimum:

$$\mu_{A \text{ AND } B}(x) = \mu_A(x) \wedge \mu_B(x)$$

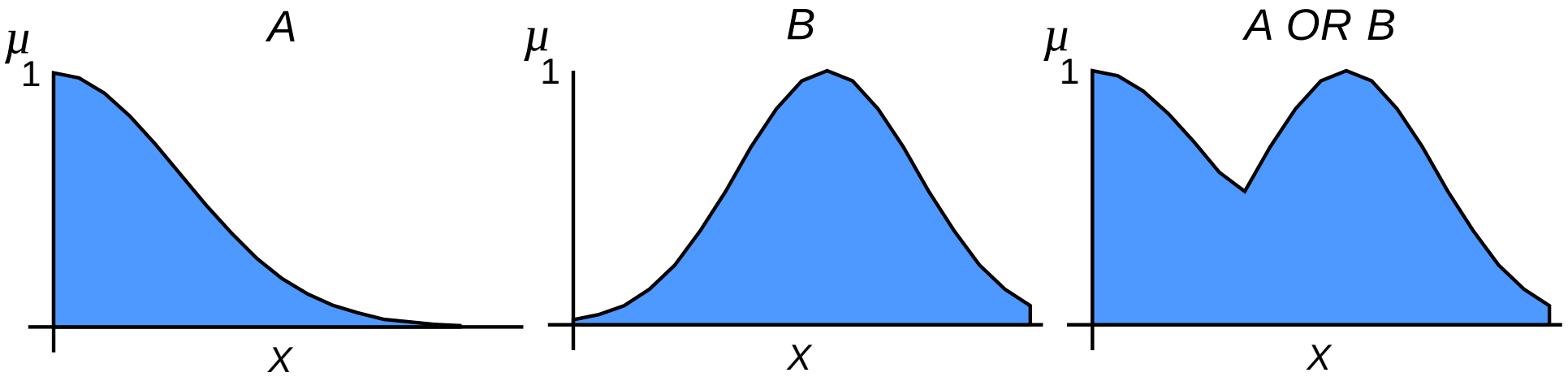


# Fuzzy Logic Operators

## Logical OR

- Defined for each point in the membership function
- Extends Boolean OR
- Any t-norm but usually maximum:

$$\mu_{A \text{ AND } B}(x) = \mu_A(x) \vee \mu_B(x)$$

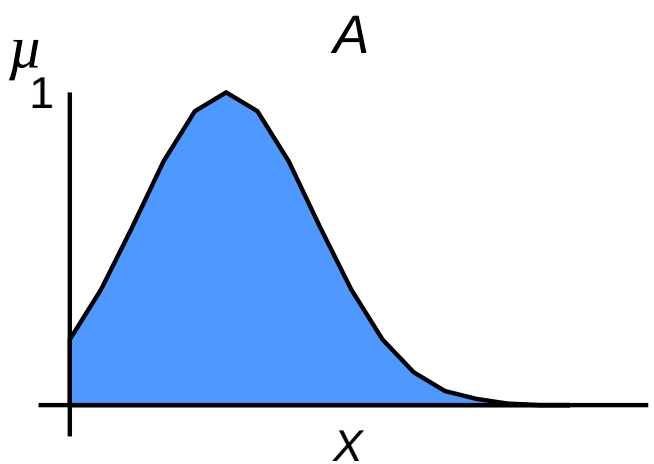


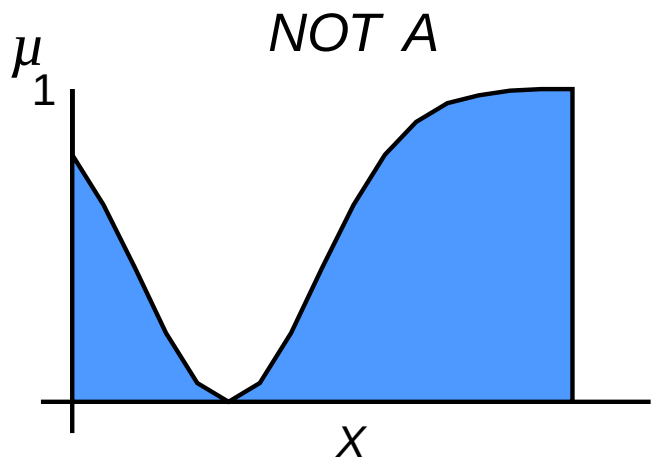
# Fuzzy Logic Operators

## Logical NOT

- Defined for each point in the membership function
- Extends Boolean NOT:

$$\neg \mu_A(x) = 1 - \mu_A(x)$$



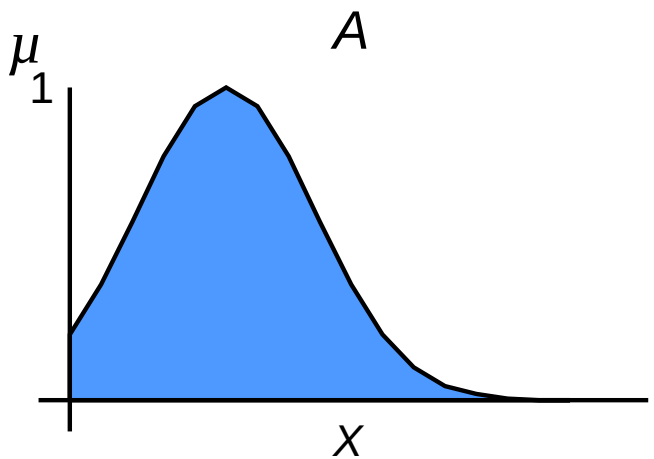



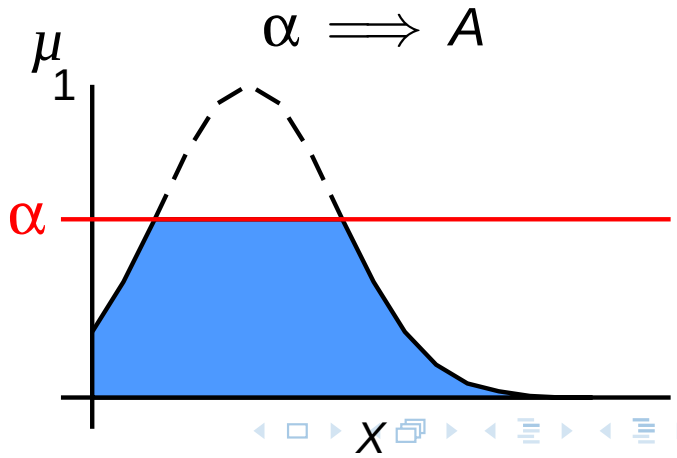
# Fuzzy Logic Operators

Logical IMPLIES

- Defined for each point in the membership function
- A variety of operators
- Most commonly used is generalised modus ponens:
  - Modus ponens: If $X$ THEN $Y$. X, therefore Y
  - Generalised modus ponens: If $X$ THEN $Y$. X to degree 0.6, therefore Y to degree 0.6
- Any t-norm but usually minimum:

$$\mu_{\alpha \Longrightarrow A}(x) = \alpha \vee \mu_A(x)$$

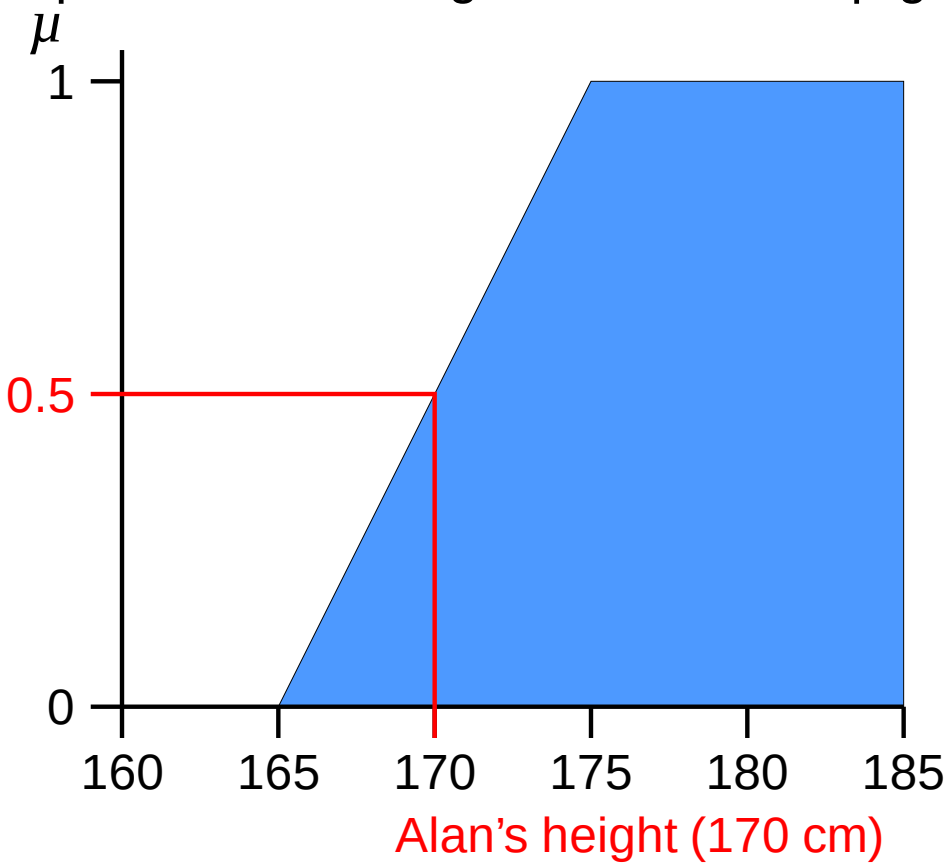


# Fuzzy Logic Operators

## Fuzzification

- The process of finding the membership grade of an input:



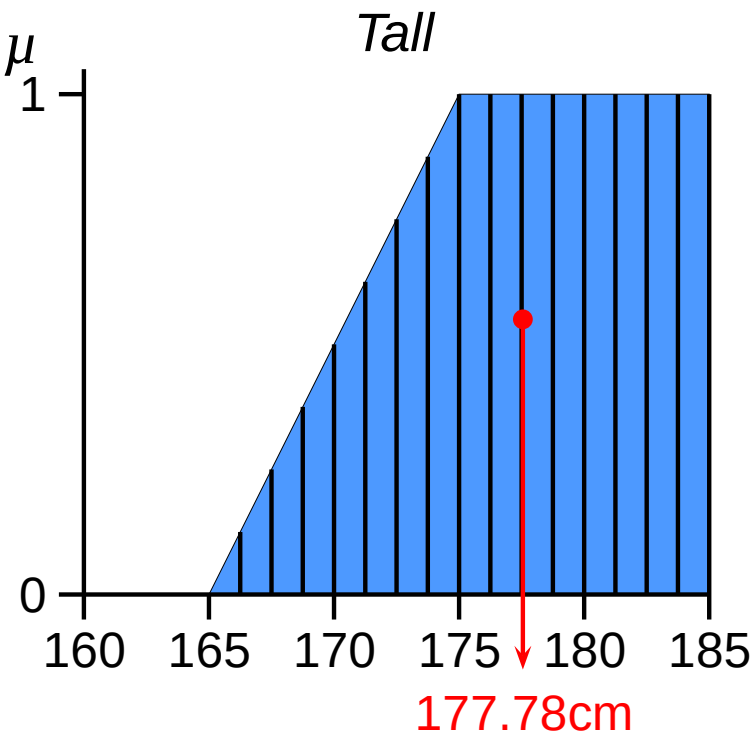

$$\mu_{Tall}(170) = 0.5$$

# Fuzzy Logic Operators

## Defuzzification

- The process of reducing a fuzzy set to a single crisp value
- Centre of area is most commonly used:

$$C_A = \frac{\sum \mu_A(x)x}{\sum \mu_A(x)}$$




$$C_{Tall} = \frac{0.13 \times 166.25 + \ldots + 1 \times 185}{0.13 + \ldots + 1} = 177.78$$

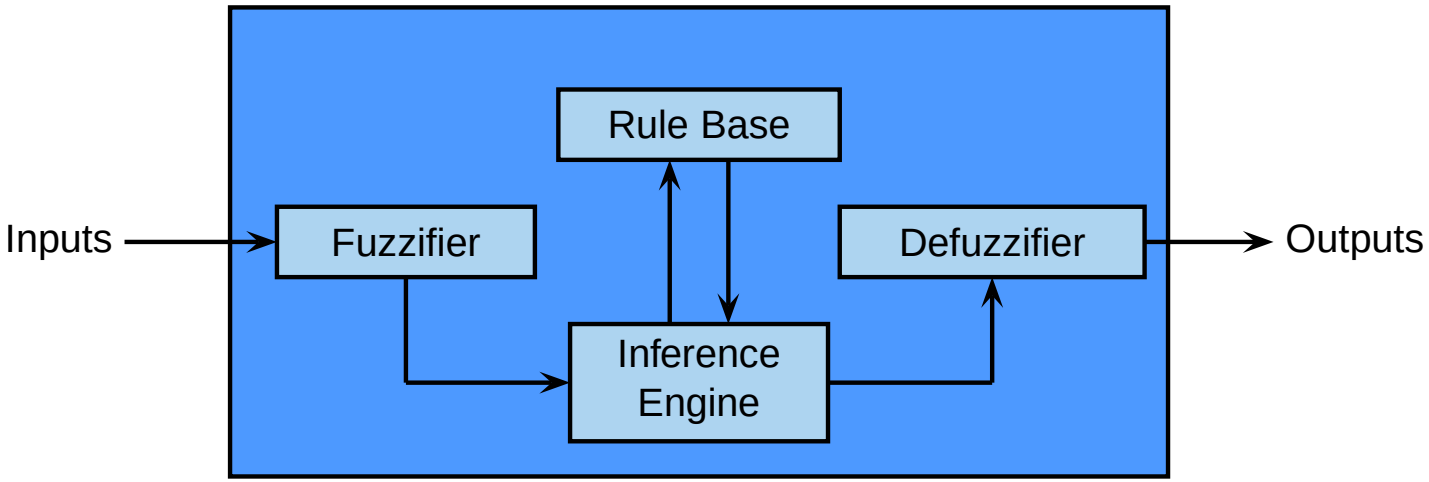
# Fuzzy Logic Systems

## Fitting it all Together

- Typically rule-based:

  IF *age* is **Young** AND *wealth* is **Rich** THEN *disposition* is **Very Happy**

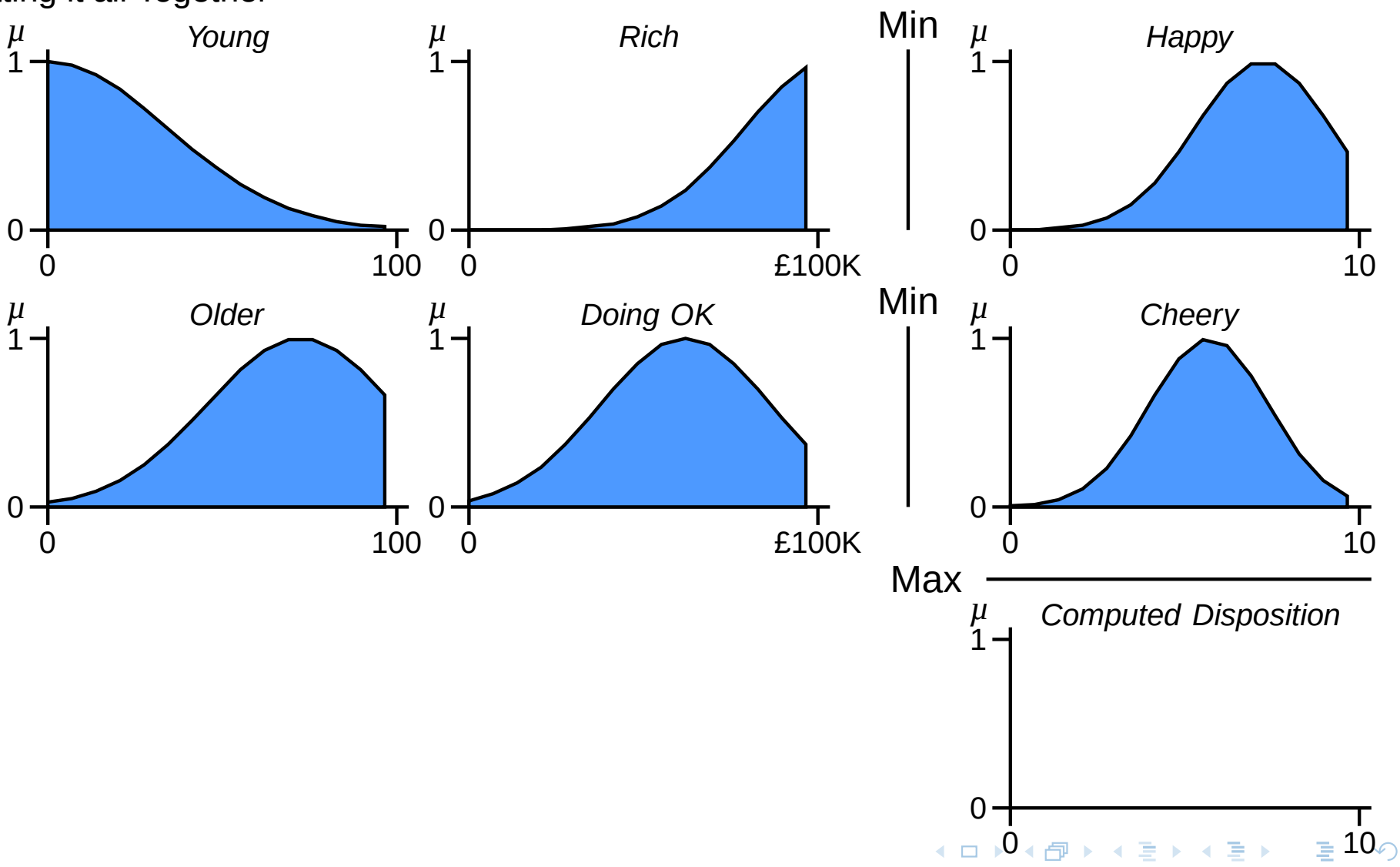
- Combine fuzzy sets with logical operators
- Crisp inputs, often crisp outputs:




Fuzzy Logic System

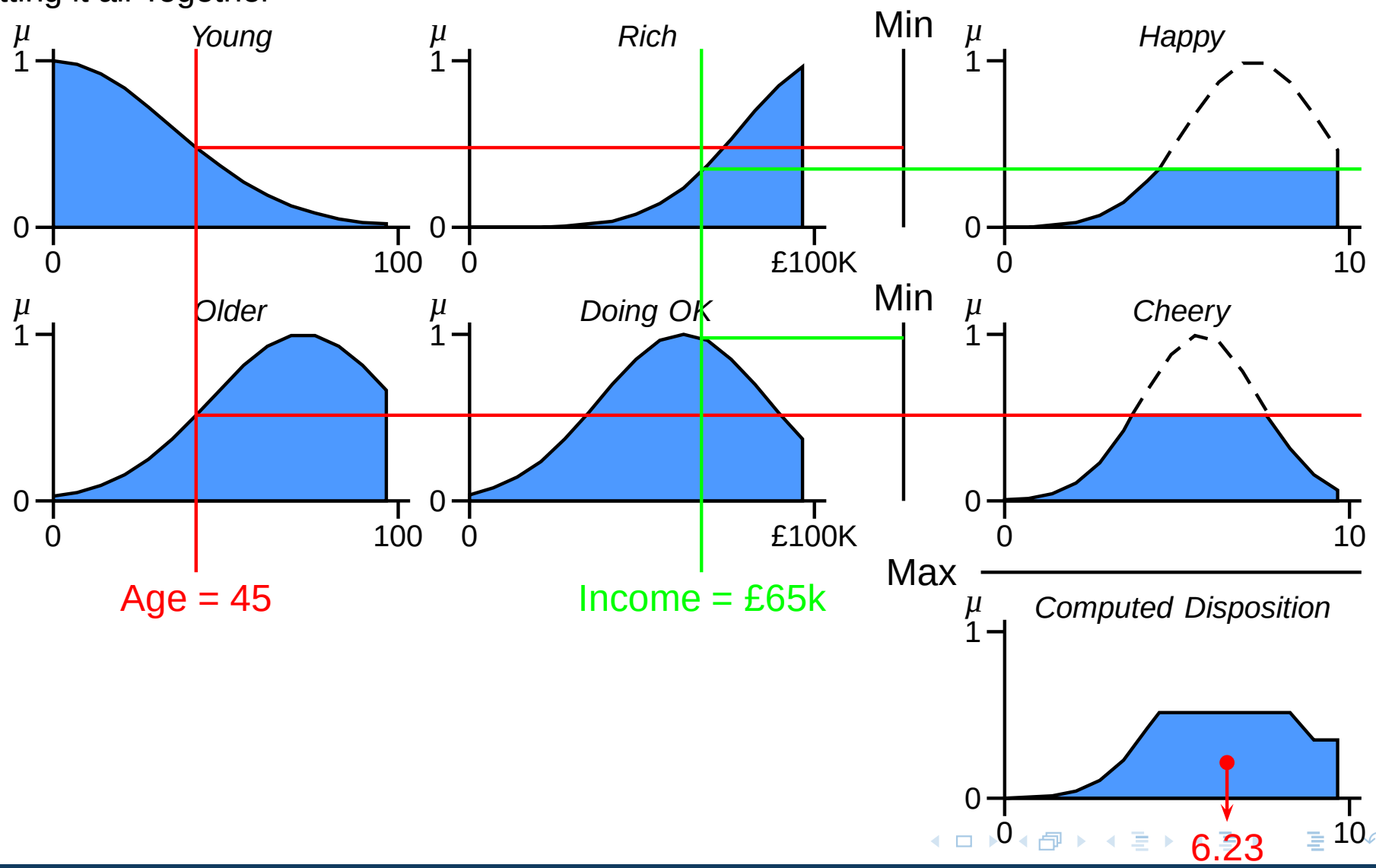
# Fuzzy Logic Systems

## Fitting it all Together



# Fuzzy Logic Systems

## Fitting it all Together



# Fuzzy Logic Systems

What I haven't told you

Many other approaches:

- Logical operator choices
- Neuro-fuzzy systems
- Defuzzification operator choices
- Adaptive systems

# Application Areas

Applied to a wide range of problems including:

- Industrial control
- Human decision making
- Image processing

# Industrial Control

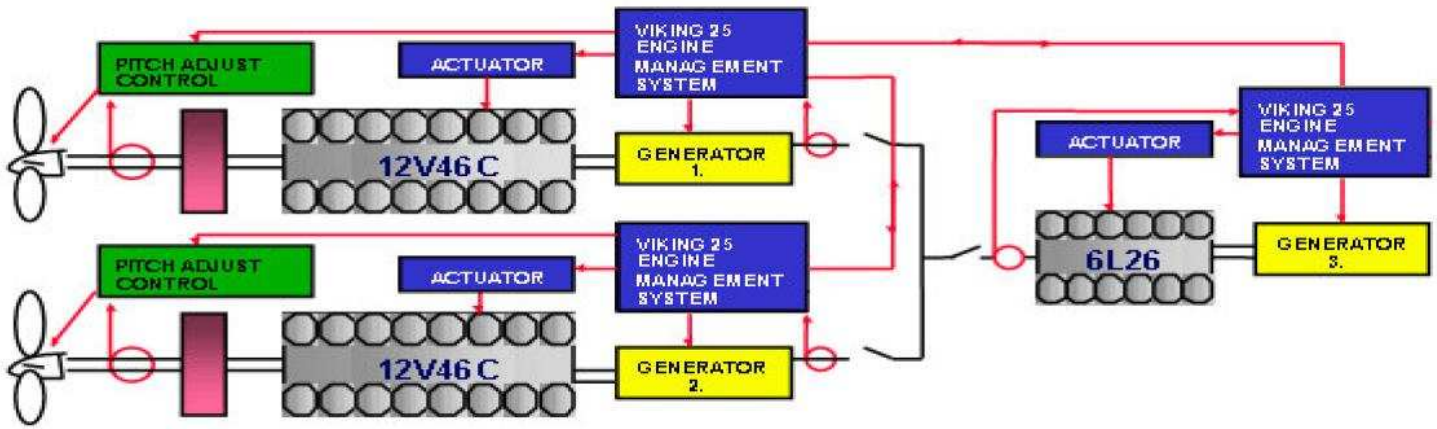
Control of marine diesel engines

- MAN 9000kW Cathedral Engines
- Low overshoot tolerance
- Highly dynamic and uncertain environments
- Require robust and accurate control

# Industrial Control

Control of marine diesel engines

- Typically three engines
- Two drive props and generators
- One solely for power generation

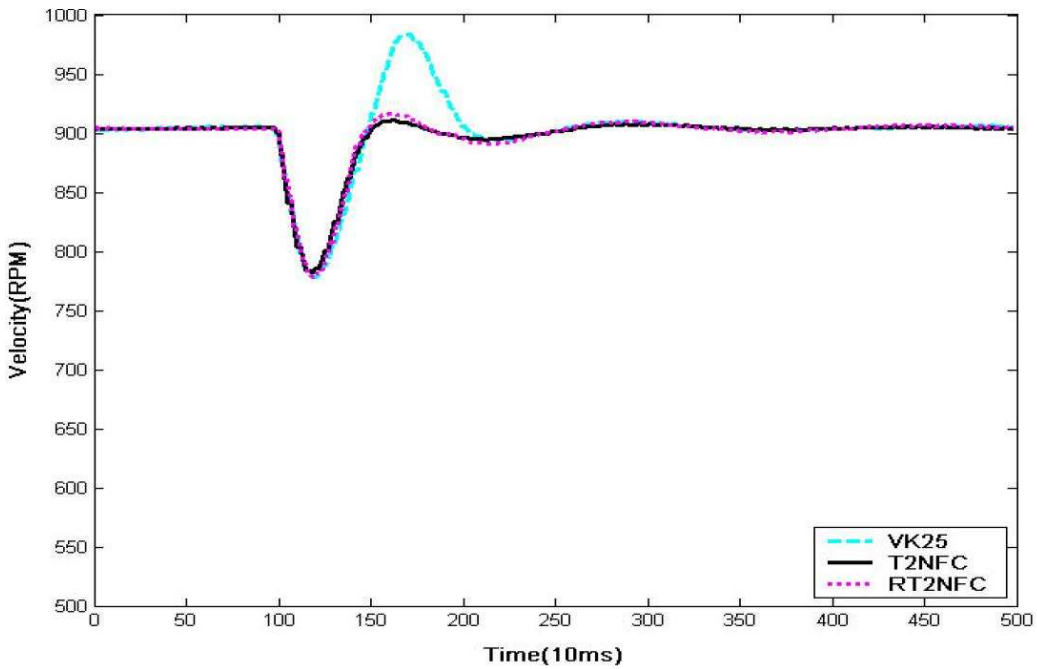



From Lynch, C. *et al*, Using Uncertainty Bounds in the Design of an Embedded Real-Time Type-2 Neuro-Fuzzy Speed Controller for Marine Diesel Engines, FUZZ-IEEE 2006.

# Industrial Control

Control of marine diesel engines

- VK25 is the current control system
- T2NFC and RT2NFC are both fuzzy
- Type-2 fuzzy sets
- Different defuzzification techniques

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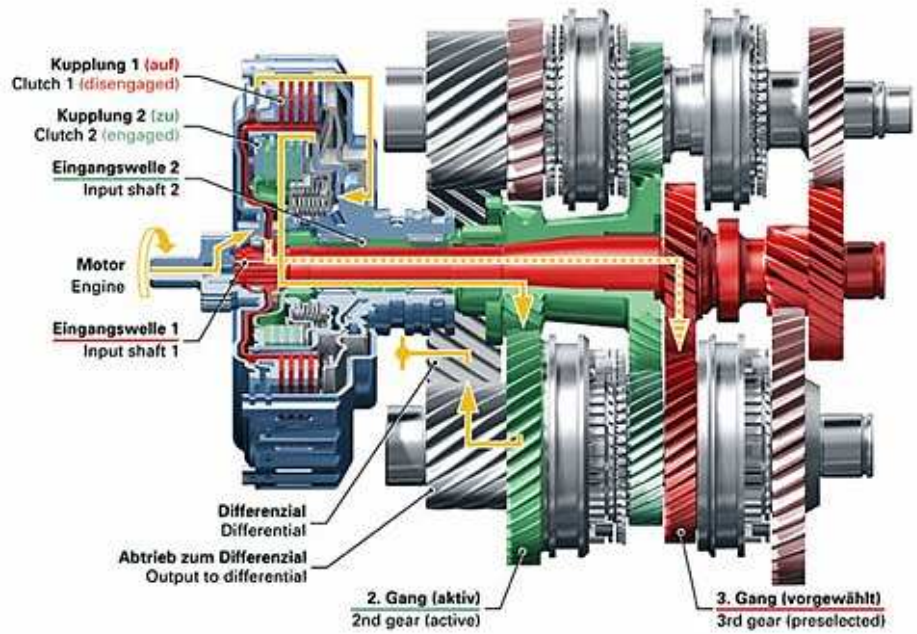
# Human Decision Making

Volkswagen Direct-Shift Gearbox

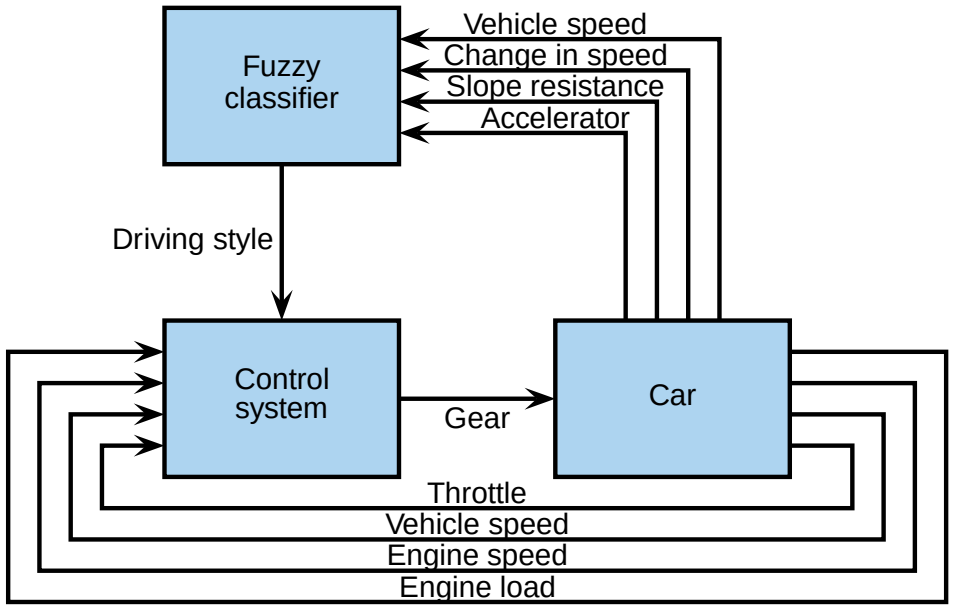
- Automatic gear selection behaviour
- Gear choice can be inferred from sensor readings
- Need to account for human factor



# Human Decision Making

Volkswagen Direct-Shift Gearbox

- Two fuzzy systems are used:
  - Infer driving style
  - Select gear
- Gear selection based on:
  - Sensor data
  - Fuzzy judgement of current driving style



# Human Decision Making

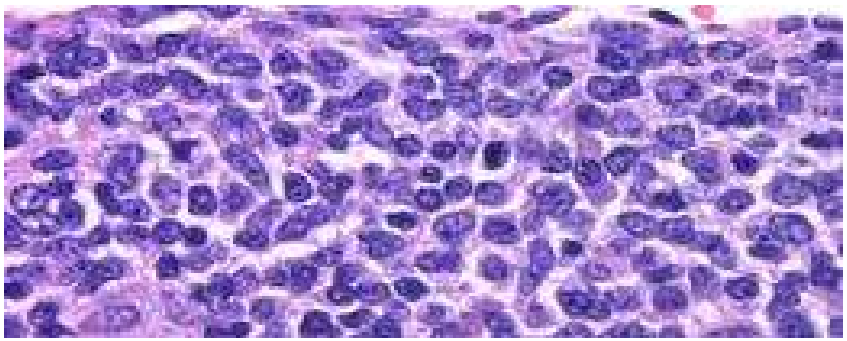
## Volkswagen Direct-Shift Gearbox

- Adaptive fuzzy systems
- Gradually adjusts the fuzzy sets
- Tailored to suit your personal driving style

# Image Processing

## Segmentation of Histopathology Images

- Identify regions of the image as:
  - Nuclei
  - Lumen
  - Cytoplasm
- Classify tissue



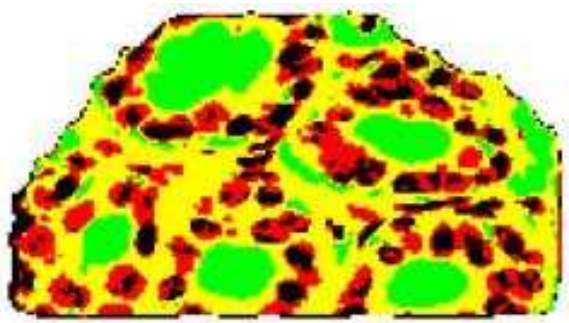
# Image Processing

## Segmentation of Histopathology Images

1. Set the number of classes $n$ (3)
2. Initialise a fuzzy description of each
3. Find the set of fuzzy descriptions of $n$ with the lowest overlap

# Image Processing

## Segmentation of Histopathology Images

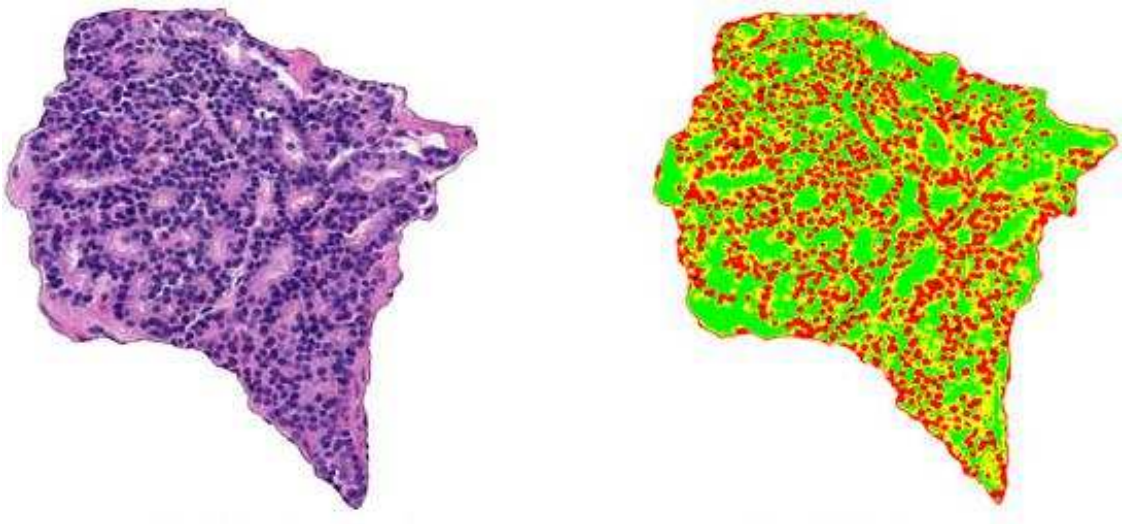


Nuclei in red and black, lumen in green and cytoplasm in yellow

From Adel Hafiane *et al*, Lecture Notes in Computer Science. 5259: 903914 (2008)

# Image Processing

## Segmentation of Histopathology Images



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# Application of Fuzzy Methods

- Useful wherever vagueness of uncertainty exists
- Relatively simple paradigm
- Not a panacea - good science is still the key
- Areas not mentioned:
  - White goods - fridges, freezers, washing machines
  - Camera anti-shake - Minolta and Canon
  - Scheduling - Seattle traffic light control system

# Uncertainty and Vagueness

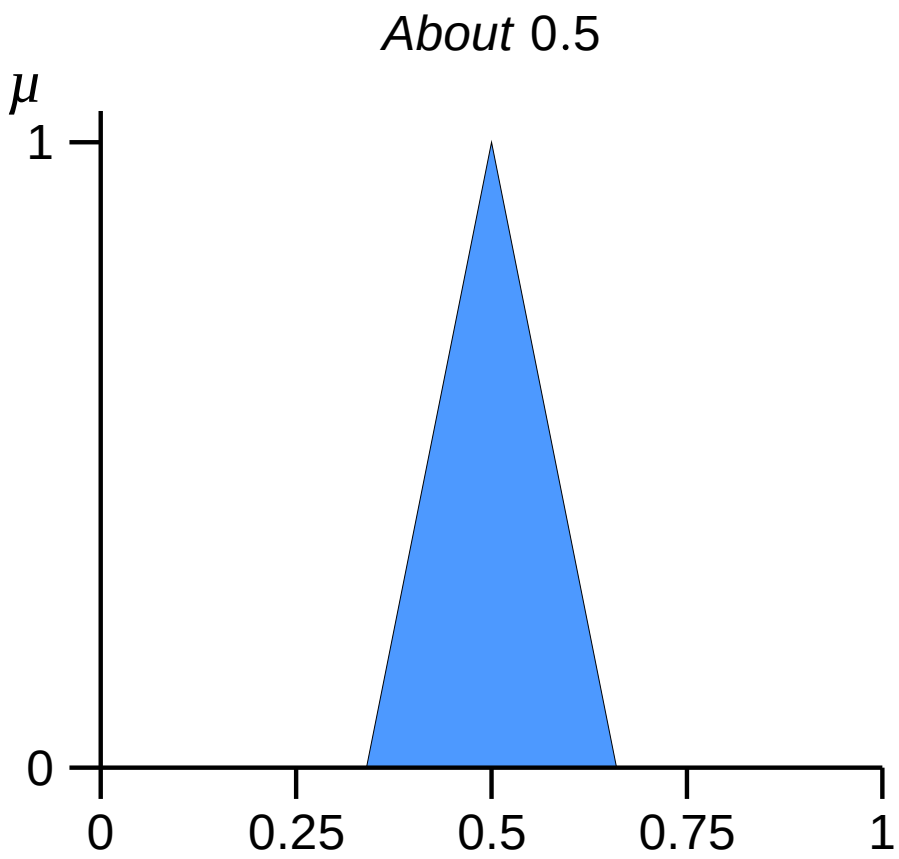
## The Trouble with (Type-1) Fuzzy Sets

Fuzzy sets and systems:

- Vagueness
- Partial truth
- Degrees of set membership

But what about uncertainty?

- Alan is 0.5 *Tall*
- 0.5 is crisp!
- Alan is *about* 0.5 *Tall*



# Uncertainty and Vagueness

## The Trouble with (Type-1) Fuzzy Sets

Type-2 Fuzzy Sets:

- Set membership measured as a fuzzy number
- Alan is *about* 0.5 *Tall*
- Where *about* 0.5 is a fuzzy set (number)
- DMU lead the world in this field
- Example type-2 fuzzy set - run program

# The Future of Fuzzy Systems

A Personal View

- Uncertainly management is key
- Type-2 fuzzy systems have a **big** role to play
- Other extensions will also be important
- Computing with Words has potential
- Worth measured by applications

# Summary

- Fuzzy sets are sets with soft boundaries
- Fuzzy logic performs inference on fuzzy sets
- Applied in a variety of areas
- Future developments are likely to be concerned with uncertainty models


ASV robo