Appendix

A.1. The impact without manipulation

1.1 No platform governance

We directly derive the derivative of the firms' profit functions with respect to price in the absence of platform governance. And we can get the results in Eq. (A.1).

$$\begin{cases} \dfrac{\partial \pi_{ANU}}{\partial p_A} = \dfrac{(1-\lambda)(t-2p_A+p_B+x_R)}{2t} = 0 \\ \dfrac{\partial \pi_{BNU}}{\partial p_B} = \dfrac{(1-\lambda)(t+p_A-2p_B-x_R)}{2t} = 0 \end{cases} \tag{A.1}$$

By calculating these two equations, we can get the equilibrium prices in Eqs. (A.2) and (A.3).

$$p_A = \frac{3t+x_R}{3} \tag{A.2}$$

$$p_B = \frac{3t-x_R}{3} \tag{A.3}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $0 < x_R < 3t$ is to ensure that the equilibrium prices are greater than 0. Substituting the equilibrium prices into Eq. (8), the equilibrium profits of all three players are derived.

1.2 Platform governance

Given that the sequence of the game involves the platform first deciding whether to govern, followed by the firms deciding whether to manipulate. Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eq. (12). And we can get the results in Eq. (A.4).

$$\begin{cases} \dfrac{\partial \pi_{ANU}}{\partial p_A} = \dfrac{(1-\lambda)(t-2p_A+p_B+x_R)}{2t} = 0 \\ \dfrac{\partial \pi_{BNU}}{\partial p_B} = \dfrac{(1-\lambda)(t+p_A-2p_B-x_R)}{2t} = 0 \end{cases} \tag{A.4}$$

Based on the backward induction, we should first take the derivative with respect to the platform's decision variable β. However, since no firms engage in manipulation, the platform cannot impose any penalties (i.e. $\beta = 0$). Then, we can get the equilibrium prices by calculating Eq. (A.4).

$$p_A = \frac{3t+x_R}{3} \tag{A.5}$$

$$p_B = \frac{3t-x_R}{3} \tag{A.6}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $0 < x_R < 3t$ is to ensure that the equilibrium prices are greater than 0. Substituting the equilibrium prices into Eq. (12), the equilibrium profits of all three players are derived.

A.2. The impact of manipulation by the superior firm

2.1 No platform governance

Firm's optimization problem is characterized by the first-order conditions of Eqs. (16-1) and (16-2). And we can get the results in Eq. (A.7).

$$\begin{cases} \dfrac{\partial \pi_{ASU}}{\partial p_A} = \dfrac{(1-\lambda)(t-2p_A+p_B+x_R+e_A)}{2t} = 0 \\ \dfrac{\partial \pi_{ASU}}{\partial e_A} = \dfrac{(1-\lambda)p_A}{2t} - 2e_A\mu_1 = 0 \\ \dfrac{\partial \pi_{BSU}}{\partial p_B} = \dfrac{(1-\lambda)(t+p_A-2p_B-x_R-e_A)}{2t} = 0 \end{cases} \tag{A.7}$$

By calculating these three equations, we can get the equilibrium prices and manipulation effort in Eqs. (A.8) - (A.10).

$$p_A = \frac{4t\mu_1(3t+x_R)}{12t\mu_1 - 1 + \lambda} \tag{A.8}$$

$$p_B = \frac{2t\left(2\mu_1(3t-x_R)-1+\lambda\right)}{12t\mu_1 - 1 + \lambda} \tag{A.9}$$

$$e_A = \frac{(1-\lambda)(3t+x_R)}{12t\mu_1 - 1 + \lambda} \tag{A.10}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < \frac{6t\mu_1 - 1 + \lambda}{2\mu_1}$ are to ensure that the equilibrium prices are greater than 0. Substituting the equilibrium prices into Eqs. (16-1) – (16-3), the equilibrium profits of all three players are derived.

Proposition 1

When the platform does not choose to govern, suppose that the superior firm manipulates alone, firm A will be better off if and only if $\pi_{ASU} > \pi_{ANU}$. The conditions $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < \frac{6t\mu_1 - 1 + \lambda}{2\mu_1}$ are to ensure the scenario with no firm manipulation and the scenario with the manipulation by only the superior firm are comparable.

According to the profits we calculated in Eq. (19-1) and (10-1), we know that π_{ASU} is always higher than π_{ANU}. Therefore, the superior firm A will always benefit from its manipulation. Then, by comparing the profits of the inferior firm B when it manipulates versus when it does not manipulate, we determine firm B's decision. Firm B will choose not to manipulate reviews if and only if $\pi_{BSN} > \pi_{BBN}$. Through calculation, we find that under conditions $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < \frac{6t\mu_1 - 1 + \lambda}{2\mu_1}$, the equation has no solution. That is, the inferior firm will not choose to abstain from manipulation when the superior firm manipulates alone and the platform chooses not to govern.

2.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eq. (21). And we can get the results in Eq. (A.11).

$$\begin{cases} \dfrac{\partial \pi_{ASG}}{\partial p_A} = -\dfrac{(1-\lambda)p_A}{2t} + (1-\lambda)\dfrac{t - p_A + p_B + x_R + \tau e_A}{2t} = 0 \\ \dfrac{\partial \pi_{ASG}}{\partial e_A} = \dfrac{(1-\lambda)\tau p_A}{2t} - 2e_A\mu_1 - \beta(1-\tau) = 0 \\ \dfrac{\partial \pi_{BSG}}{\partial p_B} = -\dfrac{(1-\lambda)p_B}{2t} + (1-\lambda)\dfrac{t + p_A - p_B - x_R - \tau e_A}{2t} = 0 \end{cases} \tag{A.11}$$

Based on these equations in Eq. (A-11), the equilibrium prices and effort are derived as follows in Eqs. (A-12) -(A-14).

$$p_A = \frac{2t\left(6t\mu_1 + 2x_R\mu_1 - (1-\tau)\beta\tau\right)}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.12}$$

$$p_B = \frac{2t\left(6t\mu_1 - 2x_R\mu_1 + \tau\left((1-\tau)\beta - (1-\lambda)\tau\right)\right)}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.13}$$

$$e_A = \frac{(1-\lambda)x_R\tau + 3t\left((1-\lambda)\tau - 2(1-\tau)\beta\right)}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.14}$$

Substitute Eqs. (A.12) - (A.14) into the platform's revenue function (i.e. Eq. (21-3)), then take the derivative with respect to the punishment intensity.

$$\frac{\partial \pi_{PSG}}{\partial \beta} = \frac{(1-\tau)\begin{pmatrix} -x_R(1-\lambda)^2\tau^3 + 36t^2\mu_1\left((1-\lambda)\tau - 4\beta(1-\tau)\right) \\ +t\tau\left(4x_R\mu_1(3-7\lambda) + \tau\begin{pmatrix} 4\beta(3-\lambda)(1-\tau) \\ +\tau\left(-3+2\lambda+\lambda^2\right)\end{pmatrix}\right)\end{pmatrix}}{\left(12t\mu_1 - (1-\lambda)\tau^2\right)^2} \tag{A.15}$$

We can get the punishment intensity by solving the equation.

$$\beta = \frac{4\mu_1\tau\left(9+3x_R-9\lambda-7x_R\lambda\right)-\left(1-\lambda\right)\tau^3\left(3+\left(1-\lambda\right)x_R+\lambda\right)}{4\left(1-\tau\right)\left(36\mu_1-\left(3-\lambda\right)\tau^2\right)} \tag{A.16}$$

Based on the punishment intensity, the equilibrium prices and effort are derived as follows.

$$p_A = \frac{24t\mu_1\left(3t+x_R\right)-\tau^2\left(x_R\left(1-\lambda\right)+t\left(3+\lambda\right)\right)}{72t\mu_1-2\left(3-\lambda\right)\tau^2} \tag{A.17}$$

$$p_B = \frac{24t\mu_1\left(3t-x_R\right)+\tau^2\left(x_R\left(1-\lambda\right)+t\left(9-5\lambda\right)\right)}{72t\mu_1-2\left(3-\lambda\right)\tau^2} \tag{A.18}$$

$$e_A = \frac{\tau\left(x_R\left(3+\lambda\right)+9t\left(1-\lambda\right)\right)}{72t\mu_1-2\left(3-\lambda\right)\tau^2} \tag{A.19}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{(9-5\lambda)\tau^2}{72t}$ and $0 < x_R < \frac{t\left(72t\mu_1-(9-5\lambda)\tau^2\right)}{24t\mu_1-(1-\lambda)\tau^2}$ are to ensure that the equilibrium prices and demands are positive. Substituting the equilibrium prices into Eq. (21), the equilibrium profits are derived.

Proposition 2

When the platform chooses to govern, suppose that the superior firm manipulates alone, firm A will be better off if and only if $\pi_{ASG} > \pi_{ANG}$. The conditions $\mu_1 > \frac{(9-5\lambda)\tau^2}{72t}$ and $0 < x_R < \frac{t\left(72t\mu_1-(9-5\lambda)\tau^2\right)}{24t\mu_1-(1-\lambda)\tau^2}$ are to ensure the scenario with no firm manipulation and the scenario with the manipulation by only the superior firm are comparable.

According to the profits we calculated in Eq. (25-1) and (14-1), we know that there are two cases where $\pi_{ASG} > \pi_{ANG}$ is satisfied.

(1) $\frac{\left(9-5\lambda\right)\tau^2}{72t} < \mu_1 < \frac{2\lambda\tau^2}{9t}$, $\frac{3}{7} < \lambda < 1$, $0 < x_R < \frac{t\left(72t\mu_1-\left(9-5\lambda\right)\tau^2\right)}{24t\mu_1-\left(1-\lambda\right)\tau^2}$;

(2) $\mu_1 > \max\left\{\frac{2\lambda\tau^2}{9t}, \frac{\lambda\tau^2(1-\lambda)}{27t\lambda-9t}\right\}$, $\frac{1}{3} < \lambda < 1$ $\frac{3\left(1-\lambda\right)\left(2\lambda\tau^2-9\mu_1\right)}{9\mu_1\left(1-5\lambda\right)+2\lambda\tau^2\left(1-\lambda\right)} < x_R < \frac{t\left(72t\mu_1-\left(9-5\lambda\right)\tau^2\right)}{24t\mu_1-\left(1-\lambda\right)\tau^2}$.

Proposition 3

Under the manipulation by only the superior firm, both firms will be better off if and only if $\pi_{ASG} > \pi_{ANG}$ and $\pi_{BSG} > \pi_{BBG}$. From Proposition 2, we already know that π_{ASG} is higher than

π_{ANG} in two cases. Moreover, under the conditions $\mu_1 > \frac{(9-5\lambda)\tau^2}{72t}$ and $0 < x_R < \frac{t\left(72t\mu_1-(9-5\lambda)\tau^2\right)}{24t\mu_1-(1-\lambda)\tau^2}$ and solving $\pi_{BSG} > \pi_{BBG}$, we have one case:

(1) $\frac{1}{3} < \lambda < 1$,

$$\mu_1 > \max\left\{\frac{(1-\lambda)\tau^2\lambda}{t(27\lambda-9)}, \frac{(6-\lambda)\tau^2}{18t}+\sqrt{\frac{\tau^4\left(63+6\lambda-5\lambda^2\right)}{6t^2}}\right\},$$

$$\max\left\{\frac{3(1-\lambda)\left(2\lambda\tau^2-9\mu_1\right)}{9\mu_1(1-5\lambda)+2\lambda\tau^2(1-\lambda)}, x_1\right\} < x_R < \min\left\{x_2, \frac{t\left(72t\mu_1-(9-5\lambda)\tau^2\right)}{24t\mu_1-(1-\lambda)\tau^2}\right\}.$$

Proposition 4

When firm A manipulates online reviews alone, the platform will choose to govern if and only if $\pi_{PSG} > \pi_{PSU}$. According to the profits we calculated in Eqs. (25-3) and (19-3), there are three cases.

(1) $0 < \mu_2 < \min\{\mu_{21}, \mu_{22}\}, \tau_1 < \tau < \tau_2$, $\max\left\{\frac{1-\lambda}{6t}, \frac{(1-\lambda)^2}{12t-28t\lambda}, \mu_{11}\right\} < \mu_1 < \mu_{12}$, $x_3 < x_R < x_4$;

(2) $0 < \mu_2 < \mu_{21}$, $\max\left\{\frac{1-\lambda}{6t}, \mu_{11}\right\} < \mu_1 < \mu_{12}$, $\tau_2 < \tau < 1, x_R > x_4$;

(3) $\mu_2 > \max\{\mu_{21}, \mu_{22}\}$, $\max\left\{\frac{1-\lambda}{6t}, \mu_{11}\right\} < \mu_1 < \mu_{12}$, $0 < \tau < \tau_3, x_R < x_3$.

The specific values of notations are all shown in the following.

The values of x are as follows in Eqs. (A.20) - (A.23),

$$x_1 = \frac{2t\mathrm{B}}{\mathrm{A}} - \frac{1}{2\sqrt{2}}\sqrt{\frac{tC\left(36t\mu_1-(3-\lambda)\tau^2\right)^2\left(6t\mu_1-(1-\lambda)\tau^2\right)^2}{\mathrm{A}^2}} \tag{A.20}$$

$$x_2 = \frac{2t\mathrm{B}}{\mathrm{A}} + \frac{1}{2\sqrt{2}}\sqrt{\frac{tC\left(36t\mu_1-(3-\lambda)\tau^2\right)^2\left(6t\mu_1-(1-\lambda)\tau^2\right)^2}{\mathrm{A}^2}} \tag{A.21}$$

$$x_3 = \frac{t(1-\lambda)F}{G} - 2\sqrt{2}\sqrt{\frac{t\left(12t\mu_1-1+\lambda\right)^2\left(12t\mu_1-(3-\lambda)\tau^2\right)E}{D^2}} \tag{A.22}$$

$$x_4 = \frac{t(1-\lambda)F}{G} + 2\sqrt{2}\sqrt{\frac{t\left(12t\mu_1-1+\lambda\right)^2\left(12t\mu_1-(3-\lambda)\tau^2\right)E}{D^2}} \tag{A.23}$$

where

$$\begin{aligned}\mathrm{A} &= 288\mu_1^3t^3(17\lambda-9)+4\mu_1^2t^2\tau^2\left(153+\lambda(257\lambda-426)\right) \\ &-2\mu_1t\tau^4(1-\lambda)\left(21+\lambda(29\lambda-54)\right)+\tau^6(1-\lambda)^2\end{aligned},$$

$$\mathrm{B} = 432\mu_1^3 t^3 (7\lambda - 3) + 6\mu_1^2 t^2 \tau^2 \left(63 + \lambda(155\lambda - 234)\right)$$
$$-\mu_1 t\tau^4 (1-\lambda)\left(27 + \lambda(83\lambda - 138)\right) + 2\tau^6 \lambda (1-\lambda)^2 (\lambda - 2),$$

$$C = 32\mu_1^3 t^3 \left(63 + \lambda(247\lambda - 246)\right) + 4\mu_1^2 t^2 \tau^2 \left(183 + \lambda(463\lambda - 566)\right)$$
$$+2t\mu_1 \tau^4 (1-\lambda)^2 \left(43 + \lambda(67\lambda - 106)\right) - 3\tau^6 (1-\lambda)^5,$$

$$D = (1-\lambda)^4 \tau^2 + 16t^2\mu_1^2 \left(9\tau^2 - 6\lambda(8-\tau^2) + \lambda^2(48+\tau^2)\right) + 8t\mu_1 (1-\lambda)^2 \left(-3\tau^2 + \lambda(4+3\tau^2)\right),$$

$$E = (1-\lambda)^4 \mu_1 \tau^3 + 8t(1-\lambda)^2 \mu_1\mu_2\tau \left(-3\tau^2 + \lambda(4+\tau^2)\right)$$
$$+16t^2\mu_1 \begin{pmatrix} 9\mu_1\mu_2\tau^3 + 6\lambda\mu_1\mu_2\tau(\tau^2 - 8) + 4\lambda^3(\tau^2 - 1) - 2\lambda^4(\tau^2 - 1) \\ +\lambda^2\left(2 - 2\tau^2 + \mu_1\mu_2\tau(\tau^2 + 48)\right) \end{pmatrix},$$

$$F = -(1-\lambda)^2 (3+\lambda)\tau^2 + 8t\mu_1\tau^2 \left(9 + \lambda(\lambda - 18)\right) - 144t^2\mu_1^2 \left(-8\lambda + (3+\lambda)\tau^2\right),$$

$$G = 32t\lambda\tau\mu_1 (1-\lambda)(24t\mu_1 - 1 + \lambda) - \tau^3\left((1-\lambda)^4 - 24t\mu_1(1-\lambda)^3 + 16t^2\mu_1^2(3+\lambda)^2\right).$$

The values of μ_2 are as follows in Eqs. (A.24) and (A.25),

$$\mu_{21} = \frac{t(1-\lambda)^2}{8\tau}\left(\frac{9\tau^2}{36t\mu_1 - (3-\lambda)\tau^2} - \frac{8\lambda}{(12t\mu_1 - 1 + \lambda)^2}\right) \tag{A.24}$$

$$\mu_{22} = \frac{32t^2(1-\lambda)^2 \lambda^2 \mu_1^2 (1-\tau^2)}{G} \tag{A.25}$$

The values of τ are as follows in Eqs. (A.26) and (A.28),

$$\tau_1 = 12\sqrt{2}\sqrt{\frac{t\lambda\mu_1}{\lambda^2 + 9(1-12t\mu_1)^2 + 6\lambda(1+36t\mu_1)}} \tag{A.26}$$

$$\tau_2 = 4\sqrt{2}\sqrt{\frac{t\lambda\mu_1(1-\lambda)(24t\mu_1 - 1 + \lambda)}{(1-\lambda)^4 - 24t\mu_1(1-\lambda)^3 + 16t^2\mu_1^2(3+\lambda)^2}} \tag{A.27}$$

$$\tau_3 = 12\sqrt{2}\sqrt{\frac{t\lambda\mu_1}{\lambda^2 + 9(1-12t\mu_1)^2 + 6\lambda(1+36t\mu_1)}} \tag{A.28}$$

A.3. The impact of manipulation of the inferior firm

3.1 No platform governance

Firm's optimization problem is characterized by the first-order conditions of Eqs. (27-1) and (27-2). And we can get the results in Eq. (A.29).

$$
\begin{cases}
\dfrac{\partial \pi_{AIU}}{\partial p_A} = \dfrac{(1-\lambda)\left(t-2p_A+p_B+x_R-e_B\right)}{2t} = 0 \\
\dfrac{\partial \pi_{BIU}}{\partial p_B} = \dfrac{(1-\lambda)\left(t+p_A-2p_B-x_R+e_B\right)}{2t} = 0 \\
\dfrac{\partial \pi_{BIU}}{\partial e_B} = \dfrac{(1-\lambda)\,p_B}{2t} - 2e_B\mu_1 = 0
\end{cases}
\tag{A.29}
$$

By calculating these three equations, we can get the equilibrium prices and manipulation effort in Eqs. (A.30) - (A.32).

$$
p_A = \frac{2t\left(2\mu_1\left(3t+x_R\right)-1+\lambda\right)}{12t\mu_1-1+\lambda} \tag{A.30}
$$

$$
p_B = \frac{4t\mu_1\left(3t-x_R\right)}{12t\mu_1-1+\lambda} \tag{A.31}
$$

$$
e_B = \frac{(1-\lambda)\left(3t-x_R\right)}{12t\mu_1-1+\lambda} \tag{A.32}
$$

By substituting the equilibrium prices of the both firms into the profit functions of the firms and the platform, we can obtain the profits of all three players in the main text.

Proposition 5

When the platform does not choose to govern, suppose that the inferior firm manipulates alone, firm B will be better off if and only if $\pi_{BIU} > \pi_{BNU}$. According to the profits we calculated in Eq. (30-2) and (10-2), we know that π_{BIU} is always higher than π_{BNU} when $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < 3t$. Therefore, the inferior firm B will always benefit from its manipulation in this case. Then, by comparing the profits of the superior firm A when it manipulates versus when it does not manipulate, firm A make its decision. Firm A will choose not to manipulate reviews if and only if $\pi_{AIU} > \pi_{ABU}$. Through calculation, we find that under conditions $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < 3t$, the equation has no solution. That is, the superior firm will not choose to abstain from manipulation when the inferior firm manipulates alone and the platform chooses not to govern.

3.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eq. (32-1) and (32-2). And we can get the results in Eq. (A.33).

$$\begin{cases} \dfrac{\partial \pi_{AIG}}{\partial p_A} = -\dfrac{(1-\lambda)p_A}{2t} + (1-\lambda)\dfrac{t - p_A + p_B + x_R + \tau e_A}{2t} = 0 \\ \dfrac{\partial \pi_{BIG}}{\partial p_B} = -\dfrac{(1-\lambda)p_B}{2t} + (1-\lambda)\dfrac{t + p_A - p_B - x_R - \tau e_A}{2t} = 0 \\ \dfrac{\partial \pi_{BIG}}{\partial e_B} = \dfrac{(1-\lambda)\tau p_B}{2t} - 2e_B\mu_1 - \beta(1-\tau) = 0 \end{cases} \tag{A.33}$$

Based on these equations in Eq. (A-33), the equilibrium prices and manipulation effort are derived as follows in Eqs. (A-34) - (A-36).

$$p_A = \frac{2t\left(6t\mu_1 + 2x_R\mu_1 + \tau\left(\beta + (-1-\beta+\lambda)\tau\right)\right)}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.34}$$

$$p_B = \frac{2t\left(6t\mu_1 - 2x_R\mu_1 - \beta\tau(1-\tau)\right)}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.35}$$

$$e_B = \frac{3t\left((1-\lambda)\tau - 2(1-\tau)\beta\right) - (1-\lambda)x_R\tau}{12t\mu_1 - (1-\lambda)\tau^2} \tag{A.36}$$

Substitute Eqs. (A-34) - (A-36) into the platform's revenue function (i.e. Eq. (32-3)), then take the derivative with respect to the punishment intensity.

$$\frac{\partial \pi_{PIG}}{\partial \beta} = \frac{(1-\tau)\begin{pmatrix} x_R(1-\lambda)^2\tau^3 + 36t^2\mu_1\left((1-\lambda)\tau - 4\beta(1-\tau)\right) \\ +t\tau\left(4x_R\mu_1(-3+7\lambda) + \tau\begin{pmatrix} 4\beta(3-\lambda)(1-\tau) \\ +\tau\left(-3+2\lambda+\lambda^2\right)\end{pmatrix}\right)\end{pmatrix}}{\left(12t\mu_1 - (1-\lambda)\tau^2\right)^2} \tag{A.37}$$

We can get the punishment intensity:

$$\beta = \frac{4t\mu_1\tau\left(x_R(7\lambda-3) + 9t(1-\lambda)\right) - (1-\lambda)\tau^3\left(t(3+\lambda) - x_R(1-\lambda)\right)}{4t(1-\tau)\left(36t\mu_1 - (3-\lambda)\tau^2\right)} \tag{A.38}$$

Based on the punishment intensity, the equilibrium prices and effort are derived as follows.

$$p_A = \frac{24t\mu_1(3t + x_R) - \tau^2\left(x_R(1-\lambda) + t(9-5\lambda)\right)}{72t\mu_1 + 2\tau^2(-3+\lambda)} \tag{A.39}$$

$$p_B = \frac{24t\mu_1(x_R - 3t) + \tau^2\left(t(3+\lambda) - x_R(1-\lambda)\right)}{72t\mu_1 + 2\tau^2(-3+\lambda)} \tag{A.40}$$

$$e_B = \frac{\tau\left(9t(1-\lambda) - x_R(3+\lambda)\right)}{72t\mu_1 + 2\tau^2(-3+\lambda)} \tag{A.41}$$

Substituting the equilibrium prices into Eq. (32-1) - (32-3), the equilibrium profits of three players are derived.

Proposition 6

When the platform chooses to govern, suppose that the inferior firm manipulates alone, firm A will be better off if and only if $\pi_{BIG} > \pi_{BNG}$. According to Eqs. (36-2) and (14-2), there is one case satisfied.

(1) $\frac{\lambda\tau^2(1-\lambda)}{9t-27t\lambda} < \mu_1 < \frac{(9-5\lambda)\tau^2}{72t}$, $\frac{3}{7} < \lambda < 1$, $\max\left\{0, \frac{t\tau^2(9-5\lambda)-72t^2\mu_1}{24t\mu_1-\tau^2(1-\lambda)}\right\} < x_R < \frac{3t(1-\lambda)(9t\mu_1-2\lambda\tau^2)}{9t\mu_1(1-5\lambda)+2(1-\lambda)\lambda\tau^2}$

Proposition 7

Under the manipulation by only the inferior firm, both firms will be better off if and only if $\pi_{BIG} > \pi_{BNG}$ and $\pi_{AIG} > \pi_{ABG}$. From Proposition 6, we already know that π_{ASG} is higher than π_{ANG} in one case. Moreover, by solving $\pi_{BSG} > \pi_{BBG}$, we have one case:

(1) $\frac{\lambda\tau^2(1-\lambda)}{9t-27t\lambda} < \mu_1 < \frac{(9-5\lambda)\tau^2}{72t}$, $\frac{83+4\sqrt{61}}{219} < \lambda < 1$, $\frac{t\tau^2(9-5\lambda)-72t^2\mu_1}{24t\mu_1-\tau^2(1-\lambda)} < x_R < x_4$ or

$x_5 < x_R < \frac{3t(1-\lambda)(9t\mu_1-2\lambda\tau^2)}{9t\mu_1(1-5\lambda)+2(1-\lambda)\lambda\tau^2}$

Proposition 8

When firm B manipulates online reviews alone, the platform will choose to govern if and only if $\pi_{PIG} > \pi_{PIU}$. According to the profits we calculated in Eqs. (36-3) and (30-3), there are three cases.

(1) $0 < \mu_2 < \mu_{21}$, $\max\{0, \tau_2\} < \tau < \tau_3$, $0 < \mu_1 < \frac{9-6\lambda+5\lambda^2}{108t(1-\lambda)}$, $x_R > x_6$;

(2) $\mu_2 > \mu_{22}$, $\mu_1 > \frac{1-\lambda}{24t}$, $0 < \tau < \tau_2$, $x_7 < x_R < x_6$;

(3) $\mu_2 > \mu_{21}$, $\tau_1 < \tau < \tau_3$, $0 < \mu_1 < \frac{3+\lambda}{36t}$, $x_R < x_7$.

The specific values of notations are all shown in the following.

The values of x are as follows in Eqs. (A.42) - (A.45).

$$x_4 = -\frac{1}{2\sqrt{2}}\sqrt{\frac{tC\left(36t\mu_1-(3-\lambda)\tau^2\right)^2\left(6t\mu_1-(1-\lambda)\tau^2\right)^2}{\mathrm{A}^2}} - \frac{2t\mathrm{B}}{\mathrm{A}} \tag{A.42}$$

$$x_5 = \frac{1}{2\sqrt{2}}\sqrt{\frac{tC\left(36t\mu_1-(3-\lambda)\tau^2\right)^2\left(6t\mu_1-(1-\lambda)\tau^2\right)^2}{\mathrm{A}^2}} - \frac{2t\mathrm{B}}{\mathrm{A}} \tag{A.43}$$

$$x_6 = 2\sqrt{2}\sqrt{\frac{t\left(12t\mu_1 - 1 + \lambda\right)^2\left(36t\mu_1 - \left(3-\lambda\right)\tau^2\right)E}{D^2}} - \frac{t\left(1-\lambda\right)F}{G} \tag{A.44}$$

$$x_7 = -2\sqrt{2}\sqrt{\frac{t\left(12t\mu_1 - 1 + \lambda\right)^2\left(36t\mu_1 - \left(3-\lambda\right)\tau^2\right)E}{D^2}} - \frac{t\left(1-\lambda\right)F}{G} \tag{A.45}$$

A.4. The impact of manipulation of both firms

4.1 No platform governance

Firm's optimization problem is characterized by the first-order conditions of Eqs. (38-1) and (38-2). And we can get the results in Eq. (A.46).

$$\begin{cases} \dfrac{\partial \pi_{ABU}}{\partial p_A} = \dfrac{\left(1-\lambda\right)\left(t - 2p_A + p_B + x_R + e_A - e_B\right)}{2t} = 0 \\ \dfrac{\partial \pi_{ABU}}{\partial e_A} = \dfrac{\left(1-\lambda\right)p_A}{2t} - 2e_A\mu_1 = 0 \\ \dfrac{\partial \pi_{BBU}}{\partial p_B} = \dfrac{\left(1-\lambda\right)\left(t + p_A - 2p_B - x_R - e_A + e_B\right)}{2t} = 0 \\ \dfrac{\partial \pi_{ABU}}{\partial e_B} = \dfrac{\left(1-\lambda\right)p_B}{2t} - 2e_B\mu_1 = 0 \end{cases} \tag{A.46}$$

By calculating these four equations, we can get the equilibrium prices and manipulation efforts in Eqs. (A.47) - (A.50).

$$p_A = \frac{t\left(2\mu_1\left(3t + x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.47}$$

$$p_B = \frac{t\left(2\mu_1\left(3t - x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.48}$$

$$e_A = \frac{\left(1-\lambda\right)\left(2\mu_1\left(3t + x_R\right) - 1 + \lambda\right)}{4\mu_1\left(6t\mu_1 - 1 + \lambda\right)} \tag{A.49}$$

$$e_B = \frac{\left(1-\lambda\right)\left(2\mu_1\left(3t - x_R\right) - 1 + \lambda\right)}{4\mu_1\left(6t\mu_1 - 1 + \lambda\right)} \tag{A.50}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $0 < x_R < \dfrac{6t\mu_1 - 1 + \lambda}{2\mu_1}$ is to ensure that the equilibrium prices are greater than 0.

Substituting the equilibrium prices into Eqs. (38-1) - (38-3), the equilibrium profits of all three players are derived.

Proposition 9

When the platform does not choose to govern, suppose that both firms choose to manipulate, they will be better off if and only if $\pi_{ABU} > \pi_{AIU}$ and $\pi_{BBU} > \pi_{BSU}$. According to the profits we calculated, we can derive that when $\mu_1 > \frac{1-\lambda}{4\sqrt{3t}}$, $\mu_1 \neq \frac{1-\lambda}{6t}$ and $0 < x_R < \frac{6t\mu_1 - 1 + \lambda}{2\mu_1}$ both firms are better off compared to no manipulation.

4.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eqs. (43-1) and (43-2). And we can get the results in Eq. (A.51).

$$\begin{cases} \frac{\partial \pi_{ABG}}{\partial p_A} = -\frac{(1-\lambda)p_A}{2t} + (1-\lambda)\frac{t - p_A + p_B + x_R + \tau e_A}{2t} = 0 \\ \frac{\partial \pi_{ABG}}{\partial e_A} = \frac{(1-\lambda)\tau p_A}{2t} - 2e_A\mu_1 - \beta(1-\tau) = 0 \\ \frac{\partial \pi_{BBG}}{\partial p_B} = -\frac{(1-\lambda)p_B}{2t} + (1-\lambda)\frac{t + p_A - p_B - x_R - \tau e_A}{2t} = 0 \\ \frac{\partial \pi_{BBG}}{\partial e_B} = \frac{(1-\lambda)p_B}{2t} - 2e_B\mu_1 - \beta(1-\tau) = 0 \end{cases} \tag{A.51}$$

Based on these equations in Eq. (A-51), the equilibrium prices and effort are derived as follows in Eqs. (A-52) - (A-55).

$$p_A = t + \frac{2tx_R\mu_1}{6t\mu_1 - (1-\lambda)\tau^2} \tag{A.52}$$

$$p_B = t - \frac{2tx_R\mu_1}{6t\mu_1 - (1-\lambda)\tau^2} \tag{A.53}$$

$$e_A = \frac{1}{4}\left((1-\lambda)\tau\left(\frac{1}{\mu_1} + \frac{2x_R}{6t\mu_1 - (1-\lambda)\tau^2}\right) - \frac{2(1-\tau)\beta}{\mu_1}\right) \tag{A.54}$$

$$e_B = \frac{1}{4}\left((1-\lambda)\tau\left(\frac{1}{\mu_1} - \frac{2x_R}{6t\mu_1 - (1-\lambda)\tau^2}\right) - \frac{2(1-\tau)\beta}{\mu_1}\right) \tag{A.55}$$

Substitute Eqs. (A.52) - (A.55) into the platform's revenue function (i.e. Eq. (43-3)), then take the derivative with respect to the punishment intensity.

$$\frac{\partial \pi_{PSG}}{\partial \beta} = \frac{(1-\tau)\left((1-\lambda)\tau - 4\beta(1-\tau)\right)}{2\mu_1} \tag{A.56}$$

We can get the punishment intensity:

$$\beta = \frac{(1-\lambda)\tau}{4(1-\tau)} \tag{A.57}$$

Based on the punishment intensity, the equilibrium prices and effort are derived as follows.

$$p_A = \frac{t\left(2\mu_1\left(3t + x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.58}$$

$$p_B = \frac{t\left(2\mu_1\left(3t - x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.59}$$

$$e_A = \frac{\left(1-\lambda\right)\left(2\mu_1\left(3t + x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.60}$$

$$e_B = \frac{\left(1-\lambda\right)\left(2\mu_1\left(3t - x_R\right) - 1 + \lambda\right)}{6t\mu_1 - 1 + \lambda} \tag{A.61}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{\left(1-\lambda\right)\tau^2}{6t}$ and $0 < x_R < \frac{6t\mu_1 - \left(1-\lambda\right)\tau^2}{4\mu_1}$ are to ensure that the equilibrium prices and demands are positive. Substituting the equilibrium prices into Eqs. (43-1) - (43-3), the equilibrium profits are derived.

Proposition 10

When the platform chooses to govern, suppose that both firms manipulate alone, firm A will be better off if and only if $\pi_{ABG} > \pi_{AIG}$ and $\pi_{BBG} > \pi_{BSG}$. The conditions $\mu_1 > \frac{\left(1-\lambda\right)\tau^2}{6t}$ and $0 < x_R < \frac{6t\mu_1 - \left(1-\lambda\right)\tau^2}{4\mu_1}$ are to ensure the scenario with no firm manipulation and the scenario with the manipulation by only the superior firm are comparable. Thus, there is one case.

(1) $\frac{\left(1-\lambda\right)\tau^2}{6t} < \mu_1 < \frac{\left(6-\lambda\right)\tau^2}{18t} + \frac{\tau^2}{36}\sqrt{\frac{63+6\lambda-5\lambda^2}{t^2}}$, $0 < x_R < \min\left\{\frac{6t\mu_1 - \left(1-\lambda\right)\tau^2}{4\mu_1}, x_1, x_2\right\}$.

Proposition 11

When both firms choose to manipulate online reviews, the platform will choose to govern if and only if $\pi_{PBG} > \pi_{PBU}$. According to the profits we calculated in Eqs. (47-3) and (41-3), there are three cases.

(1) $0 < \mu_2 < \frac{\left(1-\lambda\right)^2\tau^2}{16\mu_1\left(1-\tau\right)}$, $\frac{\left(1-\lambda\right)\tau^2}{6t} < \mu_1 < \frac{\left(1-\lambda\right)\left(1+\tau^2\right)}{12t}$, $x_R > 0$;

(2) $0 < \mu_2 < \frac{\left(1-\lambda\right)^2\tau^2}{16\mu_1\left(1-\tau\right)}$, $\mu_1 \geq \frac{\left(1-\lambda\right)\left(1+\tau^2\right)}{12t}$, $0 < x_R < x_8$;

(3) $\mu_2 > \frac{\left(1-\lambda\right)^2\tau^2}{16\mu_1\left(1-\tau\right)}$, $\frac{\left(1-\lambda\right)\tau^2}{6t} < \mu_1 < \frac{\left(1-\lambda\right)\left(1+\tau^2\right)}{12t}$, $x_R > x_8$.

The specific value of x_8 is shown in Eq. (A.62).

$$x_8 = \frac{1}{8}\sqrt{\frac{\left(6t\mu_1 - 1 + \lambda\right)^2 \tau\left(\tau\left(1+\lambda\right)^2 - 16\mu_1\mu_2\right)\left(6t\mu_1 - \left(1-\lambda\right)\tau^2\right)^2}{t\lambda\mu_1^3\left(1-\lambda\right)\left(1-\tau^2\right)\left(12t\mu_1 - 1 + \lambda - \left(1-\lambda\right)\tau^2\right)}} \tag{A.62}$$

A.4 Proof of Proposition 12

When there is no governance, we summarize four scenarios in the following Table 2.

Table 2. Four scenarios when there is no governance.

		A	
		No manipulation	manipulation
B	No manipulation	$\left(\pi_{ANU}, \pi_{BNU}\right)$	$\left(\pi_{ASU}, \pi_{BSU}\right)$
	manipulation	$\left(\pi_{AIU}, \pi_{BIU}\right)$	$\left(\pi_{ABU}, \pi_{BBU}\right)$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{1-\lambda}{6t}$ and $x_R < \frac{6t\mu_1 - 1 + \lambda}{2\mu_1}$ are to ensure that the equilibrium prices and demands are positive in different scenarios. In this case, if firm A does not choose to manipulate, firm B is more likely to choose manipulation. This is because π_{BIU} is always higher than π_{BNU}. Similarly, if firm A chooses to manipulate, B is also better off choosing manipulation, because π_{BBU} is greater than π_{BSU}. Then, this paper checks firm A's best response to firm B's manipulation. If firm B does not manipulate online reviews, firm A's best response is to manipulate online reviews because π_{ASU} is always larger than π_{ANU}. If firm B manipulates online reviews, firm A will always choose to manipulate online reviews because π_{ABU} is always higher than π_{AIU}. Therefore, both firms will always choose to manipulate online reviews in equilibrium.

A.5 Proof of Corollary 1

For firms A and B, they will fall into a prisoner's dilemma where both choosing manipulation is a suboptimal choice for them. That is, under the manipulation by both firms, both firms are worse off only if $\pi_{ABU} < \pi_{ANU}$ and $\pi_{BBU} < \pi_{BNU}$. By calculating $\pi_{ABU} < \pi_{ANU}$ and $\pi_{BBU} < \pi_{BNU}$, we have $\mu_1 > \frac{1-\lambda}{6t}$ and $0 < x_R < \frac{3}{4}\left(2\sqrt{2}\sqrt{\frac{t\left(6t\mu_1 - 1 + \lambda\right)^2\left(8t\mu_1 - 1 + \lambda\right)}{\mu_1\left(15t\mu_1 - 2 + 2\lambda\right)^2}} + t\left(-1 + \frac{3t\mu_1}{15t\mu_1 - 2 + 2\lambda}\right)\right)$. There is one case where prisoner's dilemma exists.