Appendix

A.1 The impact without manipulation

1.1 No platform governance

We directly derive the derivative of the firms' profit functions with respect to price in the absence of platform governance. And we can get the results in Eq. (A.1).

$$\begin{cases} \dfrac{\partial \pi_{ANU}}{\partial p_A} = \left(\dfrac{1}{2} + \dfrac{-p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_A \lambda}{2\omega} = 0 \\ \dfrac{\partial \pi_{BNU}}{\partial p_B} = \left(\dfrac{1}{2} - \dfrac{-p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_B \lambda}{2\omega} = 0 \end{cases} \tag{A.1}$$

By calculating these two equations, we can get the equilibrium prices in Eqs. (A.2) and (A.3).

$$p_A = \frac{3\omega + x_R}{3} \tag{A.2}$$

$$p_B = \frac{3\omega - x_R}{3} \tag{A.3}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $0 < x_R < 3t$ is to ensure that the equilibrium prices are greater than 0. Substituting the equilibrium prices into Eq. (8), the equilibrium profits of all three players are derived.

$$\pi_{ANU} = \frac{\lambda\left(3\omega + x_R\right)^2}{18\omega}$$

$$\pi_{BNU} = \frac{\lambda\left(3\omega - x_R\right)^2}{18\omega} \tag{A.4}$$

$$\pi_{PNU} = \frac{\left(1-\lambda\right)\left(x_R^2 + 9\omega^2\right)}{9\omega}$$

1.2 Platform governance

Given that the sequence of the game involves the platform first deciding whether to govern, followed by the firms deciding whether to manipulate. Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eq. (12). And we can get the results in Eq. (A.5).

$$\begin{cases} \dfrac{\partial \pi_{ANG}}{\partial p_A} = -\dfrac{p_A \lambda}{2\omega} + \lambda\left(\dfrac{-p_A + p_B + x_R}{2\omega} + \dfrac{1}{2}\left(1 + kt\tau\right)\right) = 0 \\ \dfrac{\partial \pi_{BNG}}{\partial p_B} = -\dfrac{p_B \lambda}{2\omega} + \lambda\left(-\dfrac{-p_A + p_B + x_R}{2\omega} + \dfrac{1}{2}\left(1 + kt\tau\right)\right) = 0 \end{cases} \tag{A.5}$$

By substituting the current prices into the Eq. (12), we can get the revenue of the platform. Based on the backward induction, we should first take the derivative with respect to the platform's decision variable τ. Then, we can get the equilibrium prices as follows.

$$\tau = \frac{2kt\omega(1-\lambda)-\mu_2}{2k^2t^2\omega(-1+\lambda)} \tag{A.6}$$

$$p_A = \frac{x_R}{3} + \frac{\mu_2}{2kt(1-\lambda)} \tag{A.7}$$

$$p_A = -\frac{x_R}{3} + \frac{\mu_2}{2kt(1-\lambda)} \tag{A.8}$$

Substituting the equilibrium prices into Eq. (12), the equilibrium profits of all three players are derived.

$$\pi_{ANG} = \frac{\lambda\left(3\mu_2 + 2kt(1-\lambda)x_R\right)^2}{72k^2t^2\omega(1-\lambda)^2} \quad \text{(A.9).}$$

$$\pi_{BNG} = \frac{\lambda\left(3\mu_2 - 2kt(1-\lambda)x_R\right)^2}{72k^2t^2\omega(1-\lambda)^2} \quad \text{(A.10).}$$

$$\pi_{PNG} = \frac{4k^2t^2x_R^2(1-\lambda)^2 + 9\mu_2\left(4kt\omega(1-\lambda)-\mu_2\right)}{36k^2t^2\omega(1-\lambda)} \quad \text{(A.11).}$$

A.2. The impact of manipulation by the superior firm

2.1 No platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of the profit functions of both firms in Eq. (17).

$$\begin{cases} \dfrac{\partial \pi_{ASU}}{\partial p_A} = \left(\dfrac{1}{2} + \dfrac{e_A - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_A\lambda}{2\omega} = 0 \\ \dfrac{\partial \pi_{ASU}}{\partial e_A} = \dfrac{p_A\lambda}{2\omega} - 2e_a\mu_1 = 0 \\ \dfrac{\partial \pi_{BSU}}{\partial p_B} = \left(\dfrac{1}{2} - \dfrac{e_A - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_B\lambda}{2\omega} = 0 \end{cases} \tag{A.12}$$

Based on these equations in Eq. (A-12), the equilibrium prices and effort are derived as follows.

$$p_A = \frac{4\omega(x_R + 3\omega)\mu_1}{-\lambda + 12\omega\mu_1} \tag{A.13}$$

$$p_B = -\frac{2\left(\omega\lambda + 2x_R\omega\mu_1 - 6\omega^2\mu_1\right)}{-\lambda + 12\omega\mu_1} \tag{A.14}$$

$$e_A = \frac{(x_R + 3\omega)\lambda}{-\lambda + 12\omega\mu_1} \tag{A.15}$$

Substitute the equilibrium prices and effort into the revenue functions of firms and the platform, then we can obtain their equilibrium revenue.

$$\pi_{ASU} = \frac{\mu_1\lambda\left(8\omega\mu_1 - \lambda\right)\left(3\omega + x_R\right)^2}{\left(12\omega\mu_1 - \lambda\right)^2} \quad \text{(A.16).}$$

$$\pi_{BSU} = \frac{2\omega\lambda\left(\lambda + 2\mu_1\left(x_R - 3\omega\right)\right)^2}{\left(12\omega\mu_1 - \lambda\right)^2} \quad \text{(A.17).}$$

$$\pi_{PSU} = \frac{2\omega\left(1-\lambda\right)\left(\lambda^2 + 4\mu_1\lambda\left(x_R - 3\omega\right) + 8\mu_1^2\left(x_R^2 + 9\omega^2\right)\right)}{\left(12\omega\mu_1 - \lambda\right)^2} \quad \text{(A.18).}$$

2.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of the profit functions of both firms in Eq. (22).

$$\begin{cases} \dfrac{\partial\pi_{ASG}}{\partial p_A} = -\dfrac{p_A}{2\omega} + \lambda\left(\dfrac{-p_A + p_B + x_R + me_A}{2\omega} + \dfrac{1+kt\tau}{2}\right) = 0 \\ \dfrac{\partial\pi_{ASG}}{\partial e_A} = \dfrac{\lambda m p_A}{2\omega} - 2e_A\mu_1 - \tau = 0 \\ \dfrac{\partial\pi_{BSG}}{\partial p_B} = -\dfrac{\lambda p_B}{2\omega} + \lambda\left(\dfrac{p_A - p_B - x_R - me_A}{2\omega} + \dfrac{1+kt\tau}{2}\right) = 0 \end{cases} \quad \text{(A.19)}$$

Based on these equations in Eq. (A.19), the equilibrium prices and effort are derived as follows in Eqs. (A.20) -(A.22).

$$p_A = \frac{2\omega\left(6\omega\mu_1 + 2x_R\mu_1 - m\tau + 6\mu_1 kt\omega\tau\right)}{12\omega\mu_1 - \lambda m^2} \quad \text{(A.20)}$$

$$p_B = \frac{-2\left(m^2\omega\lambda + 2\omega x_R\mu_1 - 6\mu_1\omega^2 - m\omega\tau + km^2 t\lambda\omega\tau - 6\mu_1 kt\omega^2\tau\right)}{12\omega\mu_1 - \lambda m^2} \quad \text{(A.21)}$$

$$e_A = \frac{mx_R\lambda + 3m\lambda\omega + 6\omega\tau - 3kmt\lambda\omega\tau}{12\omega\mu_1 - \lambda m^2} \quad \text{(A.22)}$$

Substitute the equilibrium prices into the platform's revenue function (i.e. Eq. (22-3)), then take the derivative concerning the governance intensity.

$$\frac{\partial \pi_{PSG}}{\partial \tau} = \frac{\begin{array}{l} 144\omega^2 \mu_1^2 \left(2kt\omega(1-\lambda) - \mu_2 + 2k^2t^2\omega(1-\lambda)\tau\right) \\ -16mx_R\omega(1-\lambda)\mu_1 - 4m^3\omega(1-\lambda)\lambda(1+2kt\tau) \\ +m^4\lambda^2\left(4kt\omega(1-\lambda) - \mu_2 + 4k^2t^2\omega(1-\lambda)\tau\right) \\ +8m^2\omega\begin{pmatrix} kt\lambda\mu_1(x_R - 6\omega)(1-\lambda) + 3\lambda\mu_1\mu_2 \\ +(1-\lambda)\tau - 6k^2t^2\omega(1-\lambda)\lambda\mu_1\tau \end{pmatrix} \end{array}}{\left(m^2\lambda - 12\omega\mu_1\right)^2} \tag{A.23}$$

By setting the first derivative of the τ to zero, we can calculate the optimal governance intensity that the platform should choose when only firm A engages in manipulation. We can get the optimal governance intensity:

$$\tau = \frac{\begin{array}{l} \mu_2\left(m^2\lambda - 12\omega\mu_1\right)^2 \\ -4\omega(1-\lambda)\left(m^3\lambda(ktm\lambda - 1) + 2m\left(72kt\omega^2\mu_1^2 - 2x_R + kmt\lambda(x_R - 6\omega)\right)\right) \end{array}}{4\omega(1-\lambda)\left(m^2\left(2 + kmt\lambda(kmt\lambda - 2)\right) - 12\mu_1k^2m^2t^2\omega\lambda + 72k^2\omega^2t^2\mu_1^2\right)} \tag{A.24}$$

Based on the optimal governance intensity, the equilibrium prices and effort are derived as follows.

$$p_A = \frac{\begin{array}{l} 4\omega(1-\lambda)\left(m + 2\mu_1ktx_R\right)\left(m - km^2t\lambda + 6\mu_1kt\omega\right) \\ +\mu_2\left(m^2\lambda - 12\mu_1\omega\right)\left(m - 6\mu_1kt\omega\right) \end{array}}{2(1-\lambda)\left(m^2\left(2 + kmt\lambda(kmt\lambda - 2)\right) - 12\mu_1k^2m^2t^2\omega\lambda + 72k^2\omega^2t^2\mu_1^2\right)} \tag{A.25}$$

$$p_B = \frac{\begin{array}{l} 4\omega(1-\lambda)\left(m + 2\mu_1ktx_R\right)\left(m - 6\mu_1kt\omega\right) \\ -\mu_2\left(m^2\lambda - 12\mu_1\omega\right)\left(6\mu_1kt\omega + m(1 - \omega mkt)\right) \end{array}}{2(1-\lambda)\left(m^2\left(2 + kmt\lambda(kmt\lambda - 2)\right) - 12\mu_1k^2m^2t^2\omega\lambda + 72k^2\omega^2t^2\mu_1^2\right)} \tag{A.26}$$

$$e_A = \frac{\begin{array}{l} 3\mu_2(2 - ktm\lambda)\left(m^2\lambda - 12\omega\mu_1\right) \\ +4(1-\lambda)\begin{pmatrix} -m\left(3\omega ktm\lambda + x_R\left(2 + ktm\lambda(ktm\lambda - 2)\right)\right) \\ +6\omega kt\mu_1(6\omega + ktm\lambda x_R) \end{pmatrix} \end{array}}{4(1-\lambda)\left(m^2\left(2 + kmt\lambda(kmt\lambda - 2)\right) - 12\mu_1k^2m^2t^2\omega\lambda + 72k^2\omega^2t^2\mu_1^2\right)} \tag{A.27}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $\mu_1 > \frac{m^2}{6\omega}$ is to ensure that the equilibrium prices and demands are positive. Substituting the equilibrium prices into Eq. (22), the equilibrium profits are derived.

$$\pi_{ASG} = \frac{\begin{array}{c}\Delta + 384m\omega^2\mu_1^2\begin{pmatrix}2kt\omega x_R(1-\lambda)^2\left(2\mu_1k^2t^2\lambda(x_R+3\omega)-3\right)\\+3\mu_2(1-\lambda)\left(x_R+6\mu_1k^2t^2\lambda\omega^2\right)-9\mu_1kt\lambda\omega\mu_2^2\end{pmatrix}\\+576\omega^3\mu_1^3\begin{pmatrix}4k^2t^2\omega(1-\lambda)^2\left(9\omega+2\mu_1k^2t^2\lambda x_R^2\right)\\+12\mu_2kt\omega(1-\lambda)\left(2\mu_1k^2t^2\lambda x_R\right)+9\mu_2^2\left(1+2\mu_1k^2t^2\lambda\omega\right)\end{pmatrix}\end{array}}{16\omega(1-\lambda)^2\left(m^2\left(2+kmt\lambda(kmt\lambda-2)\right)-12\mu_1k^2m^2t^2\omega\lambda+72k^2\omega^2t^2\mu_1^2\right)^2} \tag{A.28}$$

$$\pi_{BSG} = \frac{\lambda\begin{bmatrix}4\omega(1-\lambda)(m+2\mu_1ktx_R)(m-6\mu_1kt\omega)\\+\mu_2\left(m^2\lambda-12\mu_1\omega\right)\left(m(kmt\lambda-1)-6\mu_1kt\omega\right)\end{bmatrix}^2}{8\omega(1-\lambda)^2\left(m^2\left(2+kmt\lambda(kmt\lambda-2)\right)-12\mu_1k^2m^2t^2\omega\lambda+72k^2\omega^2t^2\mu_1^2\right)^2} \tag{A.29}$$

$$\pi_{PSG} = \frac{\begin{array}{c}16\omega^2(1-\lambda)^2(m+2\mu_1kt\omega)^2+\left(m^2\lambda-12\mu_1\omega\right)^2\mu_2^2\\+8\omega(1-\lambda)\left(m^3\lambda(kmt\lambda-1)+2\mu_1m\left(kmt\lambda(x_R-6\omega)-2x_R\right)+72\mu_1^2kt\omega^2\right)\end{array}}{8\omega(1-\lambda)\left(m^2\left(2+kmt\lambda(kmt\lambda-2)\right)-12\mu_1k^2m^2t^2\omega\lambda+72k^2\omega^2t^2\mu_1^2\right)} \tag{A.30}$$

Proposition 2

When the platform chooses to govern, suppose that the superior firm manipulates alone, firm A will be better off if and only if $\pi_{ASG} > \pi_{ANG}$. The condition $\mu_1 > \frac{m^2}{6\omega}$ is to ensure the scenario with no firm manipulation and the scenario with the manipulation by only the superior firm are comparable.

According to the profits we calculated in Eq. Eqs. (27-1) and (15-1), we know that there are two cases where $\pi_{ASG} > \pi_{ANG}$ is satisfied. It is important to note that, since the value of m ranges between 0 and 1, we simplify the calculations by neglecting the cubic and higher-order terms of m while ensuring accuracy. This approximation has a small impact on the results while significantly reducing computational complexity.

(1) $\frac{m^2}{6\omega} < \mu_1 < \frac{m}{24kt\omega-9k^2mt^2\omega\lambda} - \frac{1}{3}\sqrt{2}\sqrt{\frac{m(-12+5kmt\lambda)}{k^3t^3\omega^2\lambda(8-3kmt\lambda)^2}}$, $x_R > x_1$;

(2) $\mu_1 > \frac{m}{24kt\omega-9k^2mt^2\omega\lambda} - \frac{1}{3}\sqrt{2}\sqrt{\frac{m(-12+5kmt\lambda)}{k^3t^3\omega^2\lambda(8-3kmt\lambda)^2}}$, $\mu_2 < \mu_{21}$, $x_R < x_1$.

Proposition 3

Under the manipulation by only the superior firm, the inferior firm B will choose not to manipulate if and only $\pi_{ASG} > \pi_{ANG}$ and $\pi_{BSG} > \pi_{BBG}$. From Proposition 2, we already know that π_{ASG} is higher than π_{ANG} in two cases. Moreover, under the conditions and solving $\pi_{BSG} > \pi_{BBG}$, we have one case:

(1) $\lambda > \frac{8}{3mtk}$, $\frac{m}{24kt\omega - 9k^2mt^2\omega\lambda} - \frac{1}{3}\sqrt{2}\sqrt{\frac{m(-12+5kmt\lambda)}{k^3t^3\omega^2\lambda(8-3kmt\lambda)^2}} < \mu_1 < \frac{2m}{-24kt\omega + 9k^2mt^2\omega\lambda}$,

$\frac{8kt\omega(-1+\lambda)(m-3kt\omega\mu_1)}{3m(-2+k^2t^2\omega\lambda\mu_1)} < \mu_2 < \mu_{21}$, $x_2 < x_R < \min\{x_1, x_3\}$.

Proposition 4

When firm A manipulates online reviews alone, the platform will choose to govern if and only if $\pi_{PSG} > \pi_{PSU}$. According to the profits we calculated in Eqs. (27-3) and (20-3), there is one case as $\mu_1 < \mu_{11}$, $\mu_{22} < \mu_2 < \mu_{23}$.

The specific values of notations are all shown in the following.

The values of x are as follows in Eqs. (A.31) - (A.33),

$$x_1 = \frac{3\mathrm{B}}{2(1-\lambda)\mathrm{A}} + 3\sqrt{\frac{mB^2 + AC\omega\mu_1\left(-2\mu_2 + kt\left(4\omega(1-\lambda)+m\lambda\mu_2\right)\right)}{m(1-\lambda)^2\mathrm{A}^2}} \quad \text{(A.31)}$$

$$x_2 = \frac{E}{4D} - \frac{1}{4}\sqrt{\frac{m\lambda k^2t^2\omega\mu_1E^2 + \left(m^2\lambda - 6\omega\mu_1\right)^2 DF}{m\lambda k^2t^2\omega\mu_1D^2}} \quad \text{(A.32)}$$

$$x_3 = \frac{E}{4D} + \frac{1}{4}\sqrt{\frac{m\lambda k^2t^2\omega\mu_1E^2 + \left(m^2\lambda - 6\omega\mu_1\right)^2 DF}{m\lambda k^2t^2\omega\mu_1D^2}} \quad \text{(A.33)}$$

where $\mathrm{A} = 24k^3t^3\omega^2\lambda\mu_1^2 - m\left(-1+2k^2t^2\omega\lambda\mu_1 + 9k^4t^4\omega^2\lambda^2\mu_1^2\right)$,

$\mathrm{B} = 2kt\omega^2(1-\lambda)\mu_1\left(6-kt\lambda(7m+12kt\omega\mu_1)\right) + \mu_2\left(-6\omega\mu_1 + m\lambda\left(m+3kt\omega\mu_1\left(3+k^2t^2\omega\lambda\mu_1\right)\right)\right)$,

$C = \left(7m^2\lambda - 18\omega\mu_1\right)\mu_2 - kt\omega\left(10m^2(1-\lambda)\lambda + 36z(1-\lambda)\mu_1 - 3m\lambda\mu_1\mu_2\right)$,

$D = kt\omega(1-\lambda)\mu_1\left(2m - 3kt\omega(3kmt\lambda - 8)\mu_1\right)$,

$E = \left(6\omega\mu_1 - m^2\lambda\right)\left(8kt\omega(1-\lambda)(m-3kt\omega\mu_1) + 3m\left(k^2t^2\omega\lambda\mu_1 - 2\right)\mu_2\right)$,

$$F = \left(kt\left(4\omega(1-\lambda)+m\lambda\mu_2 - 2\mu_2\right)\right)\begin{pmatrix} 2m^2(1-\lambda) - m^2\mu_2 + kt\mu_1\left(5m^2\lambda - 18\omega\mu_1\right)\mu_2 \\ +k^2t^2\omega\mu_1\left(2(1-\lambda)\left(18\omega\mu_1 - 7m^2\lambda\right) + 3m\lambda\mu_1\mu_2\right) \end{pmatrix}.$$

A.3. The impact of manipulation of the inferior firm

3.1 No platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of the profit functions of both firms in Eq. (28).

$$\begin{cases} \dfrac{\partial \pi_{AIU}}{\partial p_A} = \left(\dfrac{1}{2} + \dfrac{-e_B - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_A \lambda}{2\omega} = 0 \\ \dfrac{\partial \pi_{BIU}}{\partial e_B} = \dfrac{p_B \lambda}{2\omega} - 2e_B \mu_1 = 0 \\ \dfrac{\partial \pi_{BIU}}{\partial p_B} = \left(\dfrac{1}{2} - \dfrac{-e_B - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_B \lambda}{2\omega} = 0 \end{cases} \tag{A.34}$$

Based on these equations in Eq. (A.34), the equilibrium prices and effort are derived as follows.

$$p_A = \frac{2\omega\left(\lambda - 2\left(x_R + 3\omega\right)\mu_1\right)}{\lambda - 12\omega\mu_1} \tag{A.35}$$

$$p_B = \frac{4\left(x_R - 3\omega\right)\omega\mu_1}{\lambda - 12\omega\mu_1} \tag{A.36}$$

$$e_B = \frac{\left(x_R - 3\omega\right)\omega\lambda}{\lambda - 12\omega\mu_1} \tag{A.37}$$

Substitute the equilibrium prices and effort into the revenue functions of firms and the platform, then we can obtain their equilibrium revenue.

$$\pi_{AIU} = \frac{2\omega\lambda\left(\lambda - 2\left(x_R + 3\omega\right)\mu_1\right)^2}{\left(\lambda - 12\omega\mu_1\right)^2} \quad \text{(A.38).}$$

$$\pi_{BIU} = \frac{\left(x_R - 3\omega\right)^2 \lambda\mu_1\left(-\lambda + 8\omega\mu_1\right)}{\left(\lambda - 12\omega\mu_1\right)^2} \quad \text{(A.39).}$$

$$\pi_{PIU} = \frac{2\omega(1-\lambda)\left(\lambda^2 - 4\left(x_R + 3\omega\right)\lambda\mu_1 + 8\left(x_R^2 + 9\omega^2\right)\mu_1^2\right)}{\left(\lambda - 12\omega\mu_1\right)^2} \quad \text{(A.40).}$$

3.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of profit functions of both firms in Eq. (33).

$$\begin{cases} \dfrac{\partial \pi_{AIG}}{\partial p_A} = -\dfrac{p_A \lambda}{2\omega} + \lambda\left(\dfrac{-me_B - p_A + p_B + x_R}{2\omega} + \dfrac{1}{2}(1 + kt\tau)\right) = 0 \\ \dfrac{\partial \pi_{BIG}}{\partial p_B} = -\dfrac{p_B \lambda}{2\omega} + \lambda\left(-\dfrac{-me_B - p_A + p_B + x_R}{2\omega} + \dfrac{1}{2}(1 + kt\tau)\right) = 0 \\ \dfrac{\partial \pi_{BIG}}{\partial e_B} = \dfrac{mp_B \lambda}{2\omega} - 2e_B\mu_1 - \tau = 0 \end{cases} \tag{A.41}$$

Based on these equations in Eq. (A.41), the equilibrium prices and effort are derived as follows in Eqs. (A.18) - (A.20).

$$p_A=\frac{2\left(m^2\omega\lambda-2x_R\omega\mu_1-6\omega^2\mu_1-m\omega\tau+km^2t\omega\lambda\tau-6kt\omega^2\mu_1\tau\right)}{m^2\lambda-12\omega\mu_1} \tag{A.42}$$

$$p_B=\frac{2\omega\left(2x_R\mu_1-6\omega\mu_1+m\tau-6kt\omega\mu_1\tau\right)}{m^2\lambda-12\omega\mu_1} \tag{A.43}$$

$$e_B=-\frac{-mx_R\lambda+3m\omega\lambda-6\omega\tau+3kmt\omega\lambda\tau}{m^2\lambda-12\omega\mu_1} \tag{A.44}$$

Substitute Eqs. (A.42) - (A.44) into the platform's revenue function (i.e. Eq. (33-3)), then take the derivative with respect to the governance intensity.

$$\frac{\partial\pi_{PIG}}{\partial\tau}=\frac{\begin{array}{l}8m^2\omega\begin{pmatrix}-kt\left(x_R+6\omega\right)\left(1-\lambda\right)\lambda\mu_1\\+3\lambda\mu_1\mu_2+\left(1-\lambda\right)\tau-6k^2t^2\omega\left(1-\lambda\right)\lambda\mu_1\tau\end{pmatrix}+16mx_R\omega\left(1-\lambda\right)\mu1\\-4m^3\omega\left(1-\lambda\right)\lambda\left(1+2kt\tau\right)+144z^2\mu_1^2\left(2kt\omega\left(1-\lambda\right)-\mu_2+2k^2t^2\omega\left(1-\lambda\right)\tau\right)\\+m^4\lambda^2\left(4kt\omega\left(1-\lambda\right)-\mu_2+4k^2t^2\omega\left(1-\lambda\right)\tau\right)\end{array}}{\left(m^2\lambda-12\omega\mu_1\right)^2} \tag{A.45}$$

We can get the governance intensity:

$$\tau=\frac{\begin{array}{l}4z\left(1-\lambda\right)\left(m^3\lambda\left(1-kmt\lambda\right)+2m\left(-2x_R+kmt\left(x_R+6\omega\right)\lambda\right)\mu_1+72kt\omega^2\mu_1^2\right)\\+\left(m^2\lambda-12\omega\mu_1\right)^2\mu_2\end{array}}{4\omega\left(1-\lambda\right)\left(m^2\left(2+kmt\lambda\left(kmt\lambda-2\right)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)} \tag{A.46}$$

Based on the governance intensity, the equilibrium prices and effort are derived as follows.

$$p_A=\frac{\begin{array}{c}4z\left(1-\lambda\right)\left(m-2ktx_R\mu_1\right)\left(m-6kt\omega\mu_1\right)\\+\left(m^2\lambda-12\omega\mu_1\right)\left(m\left(kmt\lambda-1\right)-6kt\omega\mu_1\right)\mu_2\end{array}}{2\left(1-\lambda\right)\left(m^2\left(2+kmt\lambda\left(kmt\lambda-2\right)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)} \tag{A.47}$$

$$p_B=\frac{\begin{array}{c}4z\left(1-\lambda\right)\left(m-2ktx_R\mu_1\right)\left(m-km^2t\lambda+6kt\omega\mu_1\right)\\+\left(m^2\lambda-12\omega\mu_1\right)\left(m-6kt\omega\mu_1\right)\mu_2\end{array}}{2\left(1-\lambda\right)\left(m^2\left(2+kmt\lambda\left(-2+kmt\lambda\right)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)} \tag{A.48}$$

$$e_B = \frac{\begin{array}{l} 3(kmt\lambda-2)\left(m^2\lambda-12\omega\mu_1\right)\mu_2 \\ +4(1-\lambda)\begin{pmatrix} m\left(x_R\left(2+kmt\lambda(-2+kmt\lambda)\right)-3kmt\omega\lambda\right) \\ +6kt\omega\left(kmtx_R\lambda-6\omega\right)\mu_1 \end{pmatrix} \end{array}}{4(1-\lambda)\left(m^2\left(2+kmt\lambda(kmt\lambda-2)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)} \tag{A.49}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the condition $\mu_1 > \frac{m^2}{6\omega}$ is to ensure that the equilibrium prices and demands are positive. Substituting the equilibrium prices into Eq. (33), the equilibrium profits are derived as follows.

$$\pi_{AIG} = \frac{\lambda\begin{pmatrix} 4\omega(1-\lambda)\left(m-2ktx_R\mu_1\right)\left(m-6kt\omega\mu_1\right) \\ +\left(m^2\lambda-12\omega\mu_1\right)\left(m(-1+kmt\lambda)-6kt\omega\mu_1\right)\mu_2 \end{pmatrix}^2}{8\omega(1-\lambda)^2\left(m^2\left(2+kmt\lambda(-2+kmt\lambda)\right)-12k^2m^2t^2\omega\mu_1\lambda+72k^2t^2\omega^2\mu_1^2\right)^2} \tag{A.50}$$

$$\pi_{BIG} = \frac{\begin{array}{l} \Lambda+384m\omega^2\mu_1^2\begin{pmatrix} 2ktx_R\omega(1-\lambda)^2\left(3+2k^2t^2\left(x_R-3\omega\right)\lambda\mu_1\right) \\ -3(1-\lambda)\left(x_R-6k^2t^2\omega^2\lambda\mu_1\right)\mu_2-9kt\omega\lambda\mu_1\mu_2^2 \end{pmatrix} \\ +576\omega^3\mu_1^3\begin{pmatrix} 4k^2t^2\omega(1-\lambda)^2\left(9\omega+2k^2t^2x_R^2\lambda\mu_1\right)+9\left(1+2k^2t^2\omega\lambda\mu_1\right)\mu_2^2 \\ +12kt\omega(-1+\lambda)\left(3+2k^2t^2x_R\lambda\mu_1\right)\mu_2 \end{pmatrix} \end{array}}{16\omega(1-\lambda)^2\left(m^2\left(2+kmt\lambda(-2+kmt\lambda)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)^2} \tag{A.51}$$

$$\pi_{PIG} = \frac{\begin{array}{l} 16\omega^2(1-\lambda)^2\left(m-2ktx_R\mu_1\right)^2-\left(m^2\lambda-12\omega\mu_1\right)^2\mu_2^2 \\ +8\omega(1-\lambda)\begin{pmatrix} m^3\lambda(-1+kmt\lambda)+72kt\omega^2\mu_1^2 \\ -2m\left(-2x_R+kmt\left(x_R+6\omega\right)\lambda\right)\mu_1 \end{pmatrix}\mu_2 \end{array}}{8\omega(1-\lambda)\left(m^2\left(2+kmt\lambda(-2+kmt\lambda)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)} \tag{A.52}$$

Proposition 6

When the platform chooses to govern, suppose that the inferior firm manipulates alone, firm B will be better off compared to no manipulation if and only if $\pi_{BIG} > \pi_{BNG}$. It is important to note that, since the value of m ranges between 0 and 1, we simplify the calculations by neglecting the cubic and higher-order terms of m while ensuring accuracy. This approximation has a small impact on the results while significantly reducing computational complexity. According to Eqs. (38-2) and (15-2), there are three cases satisfied.

(1) $\mu_1 > \frac{m^2}{6\omega}$, $0 < m < \frac{24k^3t^3\omega^2\lambda\mu_1^2}{-1+2k^2t^2\omega\lambda\mu_1+9k^4t^4\omega^2\lambda^2\mu_1^2}$, $x_R < x_5$ or $x_R > x_6$;

(2) $\frac{24k^3t^3\omega^2\lambda\mu_1^2}{-1+2k^2t^2\omega\lambda\mu_1+9k^4t^4\omega^2\lambda^2\mu_1^2} < m < 1$,

$\mu_2 < \mu_{24}$, $\frac{m^2\lambda\left(10kt\omega(-1+\lambda)+7\mu_2\right)}{3\omega\left(6\mu_2+kt\left(12\omega(-1+\lambda)-m\lambda\mu_2\right)\right)} \le \mu_1 < \frac{2-3kmt\lambda+2k^2m^2t^2\lambda^2}{k^3mt^3\omega\lambda^2}$, $x_R < x_6$;

(3) $\frac{24k^3t^3\omega^2\lambda\mu_1^2}{-1+2k^2t^2\omega\lambda\mu_1+9k^4t^4\omega^2\lambda^2\mu_1^2} < m < 1$, $\mu_1 \ge \frac{2-3kmt\lambda+2k^2m^2t^2\lambda^2}{k^3mt^3\omega\lambda^2}$, $\mu_2 > \mu_{24}$,

$x_R < x_6$.

Proposition 7

Under the manipulation by only the inferior firm, the superior firm A will choose not to manipulate if and only if $\pi_{BIG} > \pi_{BNG}$ and $\pi_{AIG} > \pi_{ABG}$. From Proposition 6, we already know that π_{BIG} is higher than π_{BNG} in three cases. Moreover, by solving $\pi_{AIG} > \pi_{ABG}$, we have two cases:

(1) $\lambda > \frac{8}{3tmk}$, $0 < m < \frac{24k^3t^3\omega^2\lambda\mu_1^2}{-1+2k^2t^2\omega\lambda\mu_1+9k^4t^4\omega^2\lambda^2\mu_1^2}$,

$\mu_1 > \frac{2m}{-24kt\omega+9k^2mt^2\omega\lambda}$, $x_R < \min\{x_5, x_7\}$ or $x_R > \max\{x_6, x_8\}$;

(2) $\lambda > \frac{8}{3tmk}$, $\frac{24k^3t^3\omega^2\lambda\mu_1^2}{-1+2k^2t^2\omega\lambda\mu_1+9k^4t^4\omega^2\lambda^2\mu_1^2} < m < 1$,

$\frac{2-3kmt\lambda+2k^2m^2t^2\lambda^2}{k^3mt^3\omega\lambda^2} < \mu_1 < \frac{2m}{-24kt\omega+9k^2mt^2\omega\lambda}$, $\mu_2 > \mu_{24}$, $x_7 < x_R < \{x_6, x_8\}$.

Proposition 8

When firm B manipulates online reviews alone, the platform will choose to govern if and only if $\pi_{PIG} > \pi_{PIU}$. According to the profits we calculated in Eqs. (38-3) and (32-3), there is one case as $\mu_1 > \frac{m^2}{6\omega}$, $\mu_{25} < \mu_2 < \mu_{26}$..

The specific values of notations are all shown in the following.

The values of x_R are as follows in Eqs. (A.53) - (A.56).

$$x_5 = \frac{3H}{2(1-\lambda)A} - \frac{3}{2}\sqrt{\frac{m\omega\mu_1\left(H^2 - A\left(-2\mu_2 + kt\left(4\omega(1-\lambda)+m\lambda\mu_2\right)\right)C\right)}{m(1-\lambda)^2 A^2}} \tag{A.53}$$

$$x_6=\frac{3H}{2(1-\lambda)A}+\frac{3}{2}\sqrt{\frac{m\omega\mu_1\left(H^2-A\left(-2\mu_2+kt\left(4\omega(1-\lambda)+m\lambda\mu_2\right)\right)C\right)}{m(1-\lambda)^2A^2}} \tag{A.54}$$

$$x_7=\frac{-E}{4D}-\frac{1}{4}\sqrt{\frac{m\lambda k^2t^2\omega\mu_1E^2+\left(m^2\lambda-6\omega\mu_1\right)^2DF}{m\lambda k^2t^2\omega\mu_1D^2}} \tag{A.55}$$

$$x_8=\frac{-E}{4D}+\frac{1}{4}\sqrt{\frac{m\lambda k^2t^2\omega\mu_1E^2+\left(m^2\lambda-6\omega\mu_1\right)^2DF}{m\lambda k^2t^2\omega\mu_1D^2}} \tag{A.56}$$

where $\begin{array}{l}H=-6\mu_2+3k^3t^3\omega\lambda\mu_1\left(288\omega^3\left(-1+\lambda\right)\mu_1^2+m\lambda\mu_2\right)\\-2k^2mt^2\lambda\left(7\omega(1-\lambda)+3m\lambda\mu_2\right)+3kt\left(4\omega(1-\lambda)+3m\lambda\mu_2\right)\end{array}.$

The values of μ_2 are as follows in Eqs. (A.57)- (A.59).

$$\mu_{24}=-\frac{2kt\omega(1-\lambda)\left(-6+kt\lambda\left(7m+432kt\omega^3\mu_1^3\right)\right)}{-6+3kmt\lambda\left(3+kt\lambda\left(-2m+kt\omega\mu_1\right)\right)} \tag{A.57}$$

$$\mu_{25}=\frac{4\omega(1-\lambda)\begin{pmatrix}4mx_R\mu_1+km^4t\lambda^2\left(1-2ktG\right)+72kt\omega^2\mu_1^2\left(1-2ktG\right)\\-\lambda m^3\left(1-4ktG\right)-2m^2\left(2G+kt\lambda\mu_1\left(x_R+6\omega(1-2ktG)\right)\right)\end{pmatrix}}{\left(m^2\lambda-12\omega\mu_1\right)^2} \tag{A.58}$$

$$\mu_{26}=\frac{4\omega(1-\lambda)\begin{pmatrix}4mx_R\mu_1+km^4t\lambda^2\left(1+2ktG\right)+72kt\omega^2\mu_1^2\left(1+2ktG\right)\\-\lambda m^3\left(1+4ktG\right)-2m^2\left(2G+kt\lambda\mu_1\left(x_R+6\omega(1+2ktG)\right)\right)\end{pmatrix}}{\left(m^2\lambda-12\omega\mu_1\right)^2} \tag{A.59}$$

where $I=\sqrt{\dfrac{\left(1-m^2\right)\left(x_R-3\omega\right)\lambda\mu_1\left(-m^2\lambda^2+2\left(1+m^2\right)\left(x_R+3\omega\right)\lambda\mu_1-48x_R\omega\mu_1^2\right)}{\left(\lambda-12\omega\mu_1\right)^2\left(m^2\left(2+kmt\lambda\left(kmt\lambda-2\right)\right)-12k^2m^2t^2\omega\lambda\mu_1+72k^2t^2\omega^2\mu_1^2\right)}}$.

A.3. The impact of manipulation of both firms

4.1 No platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of the profit functions of both firms in Eq. (39).

$$\begin{cases} \dfrac{\partial \pi_{ABU}}{\partial p_A} = \left(\dfrac{1}{2} + \dfrac{e_A - e_B - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_A \lambda}{2\omega} \\ \dfrac{\partial \pi_{ABU}}{\partial e_A} = \dfrac{p_A \lambda}{2\omega} - 2e_A \mu_1 = 0 \\ \dfrac{\partial \pi_{BBU}}{\partial p_B} = \left(\dfrac{1}{2} - \dfrac{e_A - e_B - p_A + p_B + x_R}{2\omega}\right)\lambda - \dfrac{p_B \lambda}{2\omega} = 0 \\ \dfrac{\partial \pi_{BBU}}{\partial e_B} = \dfrac{p_B \lambda}{2\omega} - 2e_B \mu_1 = 0 \end{cases} \tag{A.60}$$

Based on these equations in Eq. (A.60), the equilibrium prices and effort are derived as follows.

$$p_A = \frac{\omega\left(-\lambda + 2\left(x_R + 3\omega\right)\mu_1\right)}{-\lambda + 6\omega\mu_1} \tag{A.61}$$

$$p_B = \frac{\omega\left(\lambda + 2\left(x_R - 3\omega\right)\mu_1\right)}{\lambda - 6\omega\mu_1} \tag{A.62}$$

$$e_A = \frac{\lambda\left(\lambda - 2\left(x_R + 3\omega\right)\mu_1\right)}{4\mu_1\left(\lambda - 6\omega\mu_1\right)} \tag{A.63}$$

$$e_B = \frac{\lambda\left(\lambda + 2\left(x_R - 3\omega\right)\mu_1\right)}{4\mu_1\left(\lambda - 6\omega\mu_1\right)} \tag{A.64}$$

Substitute the equilibrium prices and effort into the revenue functions of firms and the platform, then we can obtain their equilibrium revenue.

$$\pi_{ABU} = \frac{\lambda\left(8\omega\mu_1 - \lambda\right)\left(\lambda - 2\left(x_R + 3\omega\right)\mu_1\right)^2}{16\mu_1\left(\lambda - 6\omega\mu_1\right)^2} \tag{A.65}$$

$$\pi_{BBU} = \frac{\lambda\left(\lambda + 2\left(x_R - 3\omega\right)\mu_1\right)^2\left(8\omega\mu_1 - \lambda\right)}{16\mu_1\left(\lambda - 6\omega\mu_1\right)^2} \quad \text{(A.66).}$$

$$\pi_{PBU} = \frac{\omega\left(1-\lambda\right)\left(\lambda^2 - 12\omega\lambda\mu_1 + 4\left(x_R^2 + 9\omega^2\right)\mu_1^2\right)}{\left(\lambda - 6\omega\mu_1\right)^2} \tag{A.67}$$

4.2 Platform governance

Based on the backward induction, we can first calculate the equilibrium value by derivation of the profit functions of both firms in Eq. (44).

$$\begin{cases}\dfrac{\partial \pi_{ABG}}{\partial p_A}=-\dfrac{p_A\lambda}{2\omega}+\lambda\left(\dfrac{me_A-me_B-p_A+p_B+x_R}{2\omega}+\dfrac{1}{2}\left(1+kt\tau\right)\right)=0\\ \dfrac{\partial \pi_{ABG}}{\partial e_A}=\dfrac{m\lambda p_A}{2\omega}-2e_A\mu_1-\tau=0\\ \dfrac{\partial \pi_{BBG}}{\partial p_B}=-\dfrac{p_B\lambda}{2\omega}+\lambda\left(\dfrac{-me_A+me_B+p_A-p_B-x_R}{2\omega}+\dfrac{1}{2}\left(1+kt\tau\right)\right)=0\\ \dfrac{\partial \pi_{BBG}}{\partial e_B}=\dfrac{m\lambda p_B}{2\omega}-2e_B\mu_1-\tau=0\end{cases} \quad \text{(A.68)}$$

Based on these equations in Eq. (A.68), the equilibrium prices and effort are derived as follows in Eqs. (A.69) - (A.72).

$$p_A=\frac{\omega\left(-m^2\lambda+2x_R\mu_1+6\omega\mu_1-km^2t\lambda\tau+6kt\omega\mu_1\tau\right)}{6\omega\mu_1-m^2\lambda} \quad \text{(A.69)}$$

$$p_B=\frac{\omega\left(-m^2\lambda-2x_R\mu_1+6\omega\mu_1-km^2t\lambda\tau+6kt\omega\mu_1\tau\right)}{6\omega\mu_1-m^2\lambda} \quad \text{(A.70)}$$

$$e_A=\frac{m^3\lambda^2-2m\lambda x_R\mu_1-6m\lambda\omega\mu_1-2m^2\lambda\tau+km^3t\lambda^2\tau+12\omega\mu_1\tau-6kmt\lambda\omega\mu_1\tau}{4\mu_1\left(m^2\lambda-6\omega\mu_1\right)} \quad \text{(A.71)}$$

$$e_B=\frac{m^3\lambda^2+2m\lambda x_R\mu_1-6m\lambda\omega\mu_1-2m^2\lambda\tau+km^3t\lambda^2\tau+12\omega\mu_1\tau-6kmt\lambda\omega\mu_1\tau}{4\mu_1\left(m^2\lambda-6\omega\mu_1\right)} \quad \text{(A.72)}$$

Substitute these equilibrium prices and efforts into the platform's revenue function (i.e. Eq. (45-3)), then take the derivative with respect to the punishment intensity.

$$\frac{\partial \pi_{PBG}}{\partial \tau}=2kt\omega\left(1-\lambda\right)-\mu_2+2k^2t^2\omega\left(1-\lambda\right)\tau \quad \text{(A.73)}$$

We can get the governance intensity:

$$\tau=\frac{2kt\left(1-\lambda\right)-\mu_2}{2k^2t^2\omega\left(-1+\lambda\right)} \quad \text{(A.74)}$$

Based on the governance intensity, the equilibrium prices and efforts are derived as follows.

$$\begin{cases}p_A=-\dfrac{2x_R\omega\mu_1}{m^2\lambda-6\omega\mu_1}+\dfrac{\mu_2}{2kt\left(1-\lambda\right)}\\ p_B=\dfrac{2x_R\omega\mu_1}{m^2\lambda-6\omega\mu_1}+\dfrac{\mu 2}{2kt\left(1-\lambda\right)}\end{cases} \quad \text{(A.75)}$$

$$e_A = \frac{\begin{array}{c} kt\left(m^2\lambda - 6\omega\mu_1\right)\left(4\omega(1-\lambda) + m\lambda\mu_2\right) \\ -4k^2mt^2x_R\omega(1-\lambda)\lambda\mu_1 - 2\left(m^2\lambda - 6\omega\mu_1\right)\mu_2 \end{array}}{8k^2t^2\omega(1-\lambda)\mu_1\left(m^2\lambda - 6\omega\mu_1\right)} \tag{A.76}$$

$$e_B = \frac{\begin{array}{c} kt\left(m^2\lambda - 6\omega\mu_1\right)\left(4\omega(1-\lambda) + m\lambda\mu_2\right) \\ +4k^2mt^2x_R\omega(1-\lambda)\lambda\mu 1 - 2\left(m^2\lambda - 6\omega\mu_1\right)\mu_2 \end{array}}{8k^2t^2\omega(1-\lambda)\mu_1\left(m^2\lambda - 6\omega\mu_1\right)} \tag{A.77}$$

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{m^2\lambda}{6\omega}$ and $0 < x_R < \frac{m^2\lambda\mu_2 - 6\omega\mu_1\mu_2}{-4kt\omega\mu_1 + 4kt\lambda\omega\mu_1}$ are to ensure that the equilibrium prices and demands are positive. Substituting the equilibrium prices into Eq. (44), the equilibrium profits are derived.

Proposition 10

When the platform chooses to govern, suppose that both firms manipulate alone, firm A will be better off if and only if $\pi_{ABG} > \pi_{AIG}$ and $\pi_{BBG} > \pi_{BSG}$. The conditions $\mu_1 > \frac{m^2\lambda}{6\omega}$ and $0 < x_R < \frac{m^2\lambda\mu_2 - 6\omega\mu_1\mu_2}{-4kt\omega\mu_1 + 4kt\lambda\omega\mu_1}$ are to ensure the scenario with no firm manipulation and the scenario with the manipulation by only the superior firm are comparable. It is important to note that, since the value of m ranges between 0 and 1, we simplify the calculations by neglecting the cubic and higher-order terms of m while ensuring accuracy. This approximation has a small impact on the results while significantly reducing computational complexity. There are two cases.

(1) $\lambda > \frac{8}{3mtk}$, $\frac{m^2\lambda}{6\omega} < \mu_1 < \frac{2m}{-24kt\omega + 9k^2mt^2\omega\lambda}$, $0 < x_R < \min\{x_3, x_7\}$ or $x_R > \max\{x_4, x_8\}$;

(2) $\lambda > \frac{8}{3mtk}$, $\mu_1 > \frac{2m}{-24kt\omega + 9k^2mt^2\omega\lambda}$, $0 < \mu_2 < \mu_{26}$, $x_R < \min\{x_4, x_8\}$.

Proposition 11

When both firms choose to manipulate online reviews, the platform will choose to govern if and only if $\pi_{PBG} > \pi_{PBU}$. According to the profits we calculated in Eqs. (45-3) and (40-3), there is one case.

(1) $\frac{m^2\lambda}{6\omega} < \mu_1 < \frac{\lambda + m^2\lambda}{12\omega}$, $\max\{0, \mu_{27}\} < \mu_2 < \mu_{28}$.

The values of μ_2 are as follows in Eqs. (A.78) and (A.79).

$$\mu_{27} = 2\left(ktz(1-\lambda) - 2\sqrt{\frac{k^2(1-m^2)t^2x_R^2\omega^2(1-\lambda)^2\lambda\mu_1^2(\lambda+m^2\lambda-12\omega\mu_1)}{(\lambda-6\omega\mu_1)^2(m^2\lambda-6\omega\mu_1)^2}} \right) \quad \text{(A.78)}$$

$$\mu_{28} = 2\left(ktz(1-\lambda) - 2\sqrt{\frac{k^2(1-m^2)t^2x_R^2\omega^2(1-\lambda)^2\lambda\mu_1^2(\lambda+m^2\lambda-12\omega\mu_1)}{(\lambda-6\omega\mu_1)^2(m^2\lambda-6\omega\mu_1)^2}} \right) \quad \text{(A.79)}$$

A.4 Proof of Proposition 12

When there is no governance, we summarize four scenarios in the following Table 2.

Table 2. Four scenarios when there is no governance.

		A	
		No manipulation	manipulation
B	No manipulation	(π_{ANU}, π_{BNU})	(π_{ASU}, π_{BSU})
	manipulation	(π_{AIU}, π_{BIU})	(π_{ABU}, π_{BBU})

This paper is interested in cases where both firms play a role in the equilibrium. Therefore, the conditions $\mu_1 > \frac{\lambda}{6\omega - 2x_R}$ and $0 < x_R < 3\omega$ are to ensure that the equilibrium prices and demands are positive in different scenarios. In this case, if firm A does not choose to manipulate, firm B is more likely to choose manipulation. This is because π_{BIU} is always higher than π_{BNU}. Similarly, if firm A chooses to manipulate, B is also better off choosing manipulation because π_{BBU} is greater than π_{BSU}. Then, this paper checks firm A's best response to firm B's manipulation. If firm B does not manipulate online reviews, firm A's best response is to manipulate online reviews because π_{ASU} is always larger than π_{ANU}. If firm B manipulates online reviews, firm A will always choose to manipulate online reviews because π_{ABU} is always higher than π_{AIU}. Therefore, both firms will always choose to manipulate online reviews in equilibrium.

A.5 Proof of Corollary 1

For firms A and B, they will fall into a prisoner's dilemma where both choosing manipulation is a suboptimal choice for them. That is, under the manipulation by both firms, both firms are worse off only if $\pi_{ABU} < \pi_{ANU}$ and $\pi_{BBU} < \pi_{BNU}$. By calculating $\pi_{ABU} < \pi_{ANU}$ and $\pi_{BBU} < \pi_{BNU}$, we have $\mu_1 > \frac{\lambda}{6\omega}$ and $0 < x_R < \frac{3}{4}\left(-\omega + 2\sqrt{2}\sqrt{-\frac{\omega(\lambda - 8\omega\mu_1)(\lambda - 6\omega\mu_1)^2}{\mu_1(2\lambda - 15\omega\mu_1)^2} + \frac{3\omega^2\mu_1}{-2\lambda + 15\omega\mu_1}}\right)$. There is one case where prisoner's dilemma exists.

A.6 Proof of Corollary 2

Even though effective platform governance can enable firms to refrain from manipulation within certain scopes when their competitors are manipulating. However, we find that when there is platform governance and firms A and B choose to manipulate for higher benefits ($\pi_{ABG} > \pi_{AIG}$ and $\pi_{BBG} > \pi_{BSG}$), they may also fall into a prisoner's dilemma. By calculating $\pi_{ABG} < \pi_{ANG}$ and $\pi_{BBG} < \pi_{BNG}$, we obtain this result. When the unit governance cost is large (i.e. $\mu_2 > \frac{4kt\omega(1-\lambda)}{2 + kmt\lambda}$) and the perceived quality difference revealed by unmanipulated reviews is small (i.e. $0 < x_R < x_9$), firms A and B choosing to manipulate will lead to a lower income for them compared to when both do not manipulate.

The values of x_R are as follows in Eqs. (A.80).

$$x_9 = \frac{3\left(m^2\lambda\mu_2 - 6\omega\mu_1\mu_2\right)}{4kt(-1+\lambda)\left(2m^2\lambda - 15\omega\mu_1\right)} + \frac{3\left(m^2\lambda - 6\omega\mu_1\right)^2 J}{2\sqrt{2}k^4m^2t^4\omega(1-\lambda)^2\lambda^2\mu_1\left(2m^2\lambda - 15\omega\mu_1\right)^2} \quad \text{(A.80)}$$

where
$$\begin{aligned} J =& 8k^2t^2\omega^2(1-\lambda)^2\left(2m^2\lambda - 15\omega\mu_1\right) - 8kt\omega(1-\lambda)\left(2m^2\lambda - 15\omega\mu_1\right)\mu_2 \\ &+ \left(-30\omega\mu_1 + m^2\lambda\left(4 + k^2t^2\lambda\left(-m^2\lambda + 8\omega\mu_1\right)\right)\right)\mu_2^2 \end{aligned}.$$