BM1 Final Project

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Purpose

We will be analyzing data from the "County Demographic Information" (CDI) data set, which contains characteristics of 440 counties in the United States collected from 1990-1992. The primary objective of this investigation is to develop insight relevant to predicting the crime rate in counties, namely to summarize as the crime rate per 1,000 population (CRM_1000).

Import the package we need

Data preprocessing

Transfer population variables to per capita variables

```
rm(list = ls())
cdi = read.csv("./data/cdi.csv") %>%
  mutate(crime_rate = crimes/pop,
         pcarea = area/pop,
         pcdocs = docs/pop,
         pcbeds = beds/pop,
         region = relevel(factor(region),ref = 3))
cdi_pc = cdi %>%
  dplyr::select(crime_rate, everything(), -id, -cty, -state, -area, -docs, -beds, -crimes, -pop, -totali
summary(cdi_pc)
```

```
##    crime_rate          pop18           pop65           hsgrad
##  Min.   :0.004601   Min.   :16.40   Min.   : 3.000   Min.   :46.60
##  1st Qu.:0.038102   1st Qu.:26.20   1st Qu.: 9.875   1st Qu.:73.88
##  Median :0.052429   Median :28.10   Median :11.750   Median :77.70
##  Mean   :0.057286   Mean   :28.57   Mean   :12.170   Mean   :77.56
##  3rd Qu.:0.072597   3rd Qu.:30.02   3rd Qu.:13.625   3rd Qu.:82.40
##  Max.   :0.295987   Max.   :49.70   Max.   :33.800   Max.   :92.90
##      bagrad         poverty           unemp           pcincome     region
##  Min.   : 8.10   Min.   : 1.400   Min.   : 2.200   Min.   : 8899   3:152
##  1st Qu.:15.28   1st Qu.: 5.300   1st Qu.: 5.100   1st Qu.:16118   1:103
##  Median :19.70   Median : 7.900   Median : 6.200   Median :17759   2:108
##  Mean   :21.08   Mean   : 8.721   Mean   : 6.597   Mean   :18561   4: 77
##  3rd Qu.:25.32   3rd Qu.:10.900   3rd Qu.: 7.500   3rd Qu.:20270
```

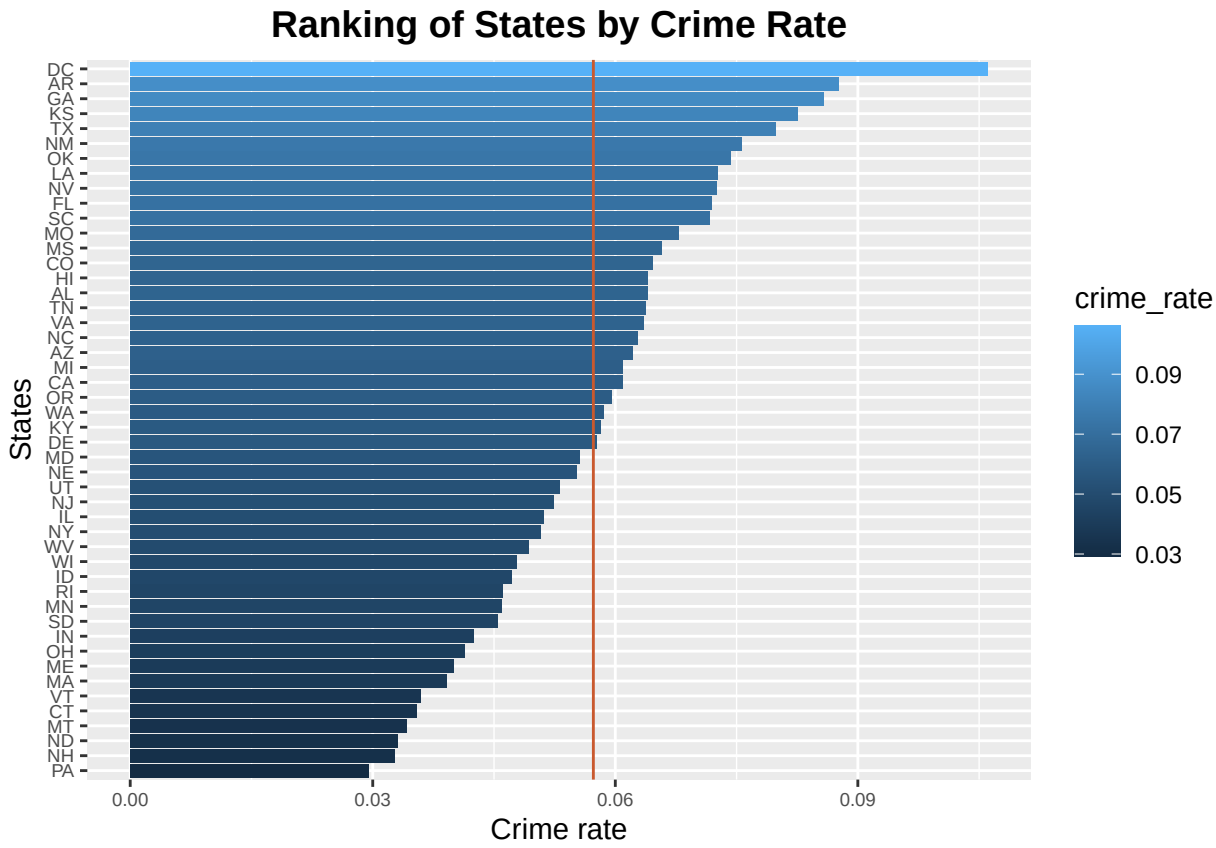
```
##  Max.   :52.30   Max.   :36.300   Max.   :21.300   Max.   :37541
##      pcarea              pcdocs             pcbeds
##  Min.   :3.086e-05   Min.   :0.0003559   Min.   :0.0001649
##  1st Qu.:1.323e-03   1st Qu.:0.0012127   1st Qu.:0.0021972
##  Median :2.977e-03   Median :0.0017509   Median :0.0033287
##  Mean   :4.760e-03   Mean   :0.0021230   Mean   :0.0036493
##  3rd Qu.:5.199e-03   3rd Qu.:0.0024915   3rd Qu.:0.0045649
##  Max.   :7.542e-02   Max.   :0.0170377   Max.   :0.0196982
```

Exploratory Data Analysis

Rank of the crime rate by state

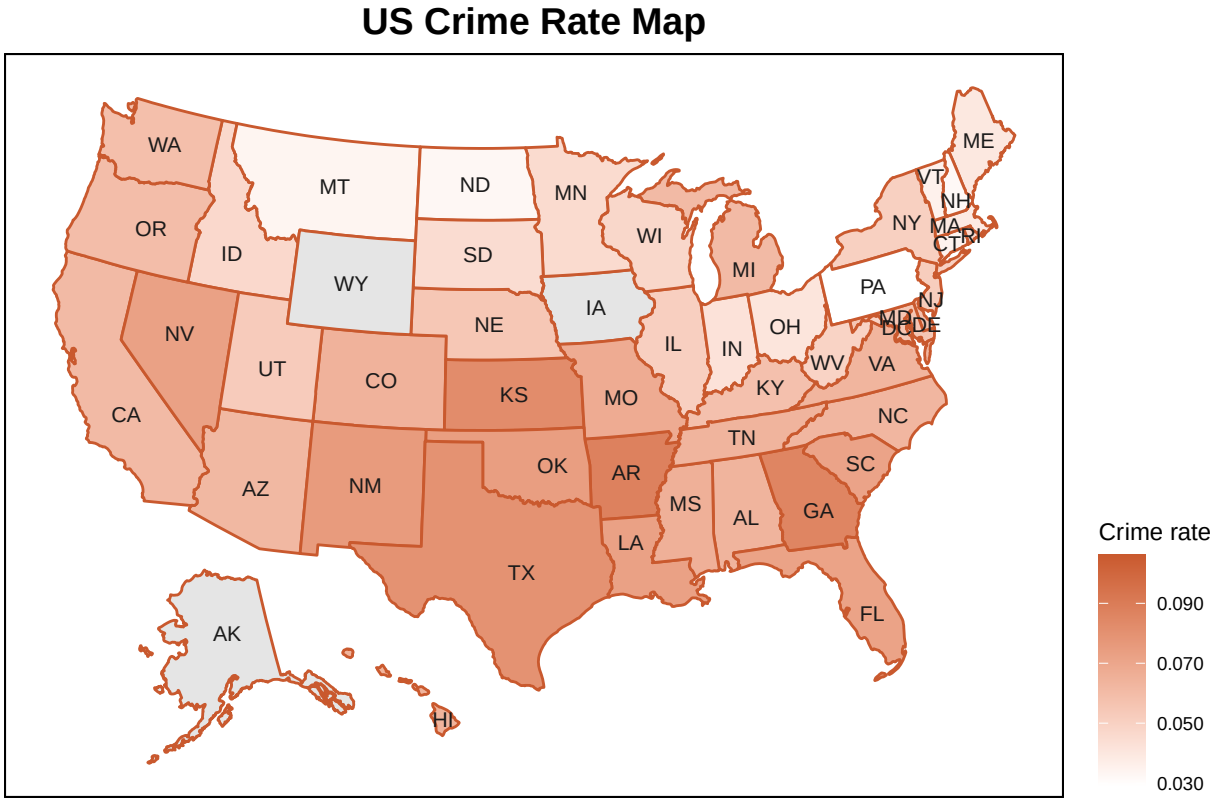
```
cdi_state <- cdi %>%
  group_by(state) %>%
  summarize(crime_rate = mean(crime_rate))
#cdi_state_rank <- cdi_state[order(-rank(cdi_state$crime_rate)),]

ggplot(cdi_state,aes(x=reorder(state,crime_rate),y=crime_rate,fill=crime_rate)) +
  geom_bar(stat ='identity')+
  coord_flip() +
  theme_grey() +
  labs(title = 'Ranking of States by Crime Rate',
       y='Crime rate',x='States') +
  geom_hline(yintercept = mean(cdi$crime_rate),color = "#C9592E")+
  theme(plot.title = element_text(hjust = 0.5,size = 14, face = "bold"),
        axis.text=element_text(size=6.5))
```



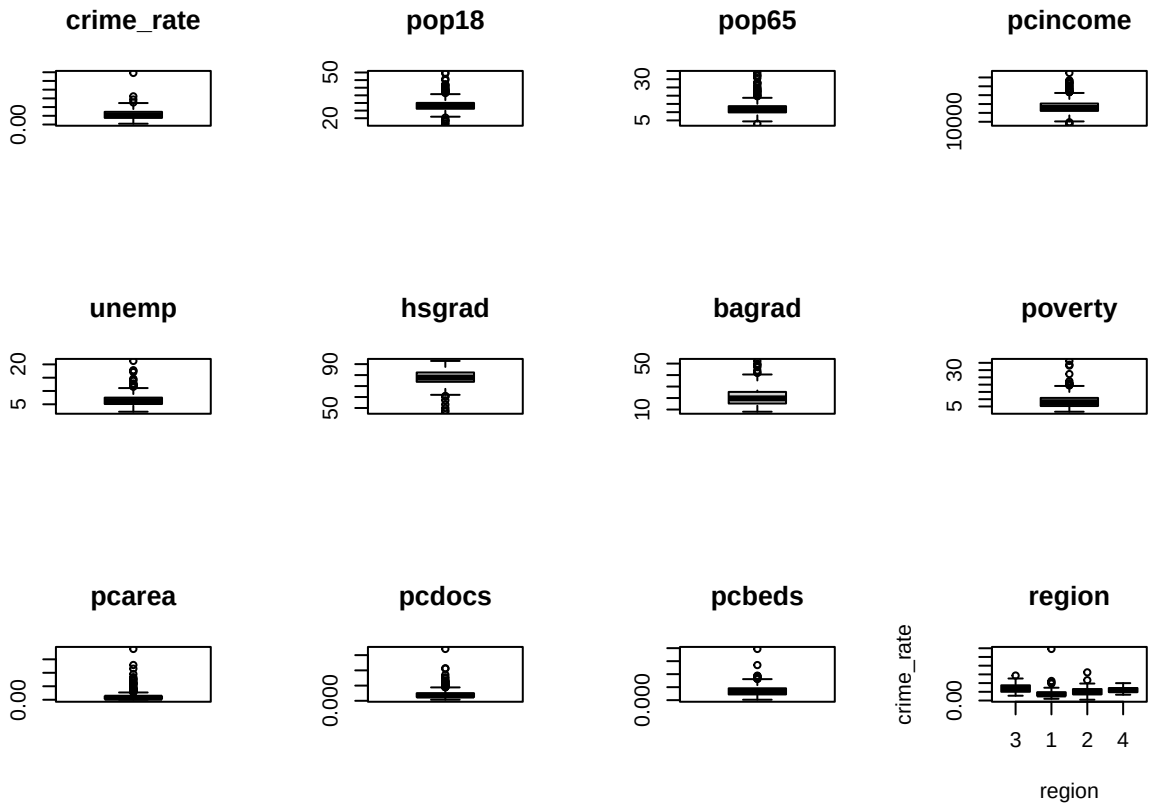
US crime rate map by state

```
p<-plot_usmap(data = cdi_state, values = "crime_rate", color = "#C9592E", size = 0.5,
              labels = TRUE, label_color = "grey10") +
   scale_fill_continuous(low = "white", high = "#C9592E", na.value = "grey90",
                         name = "Crime rate", label = scales::comma) +
   theme(legend.position = "right") +
   theme(panel.background = element_rect(colour = "black")) +
   labs(title = "US Crime Rate Map") +
   theme(plot.title = element_text(hjust = 0.5,size = 14, face = "bold"))
p$layers[[2]]$aes_params$size <- 2.8
print(p)
```



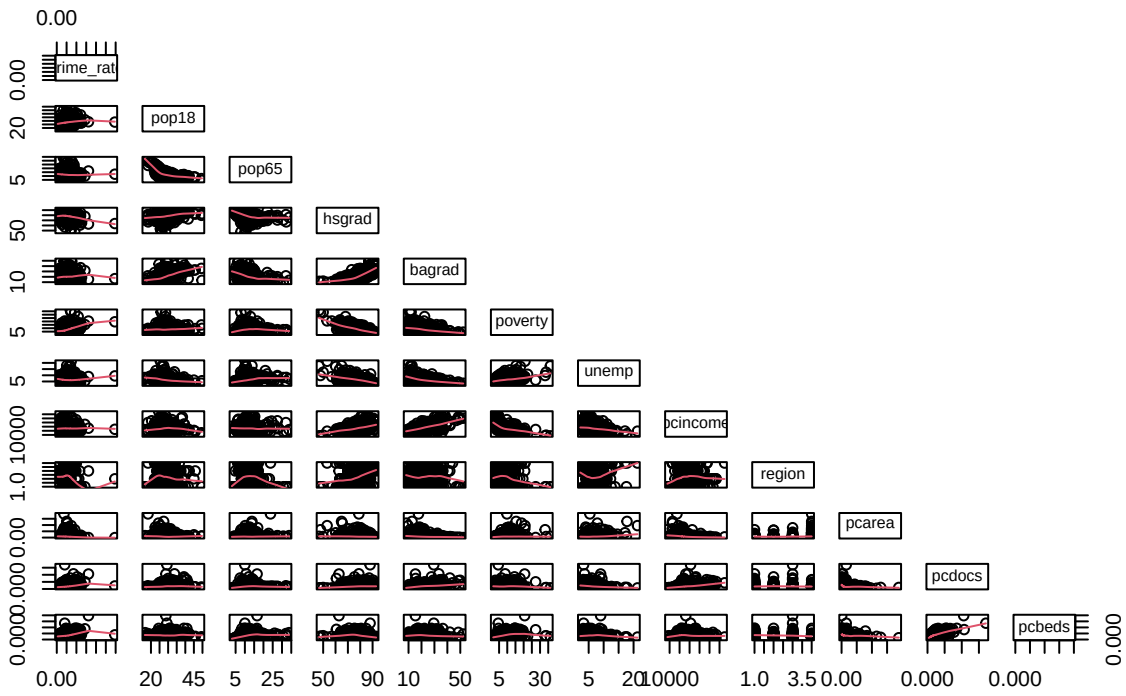
Boxplot for each variable

```
par(mfrow=c(3,4))
boxplot(cdi_pc$crime_rate,  main = "crime_rate")
boxplot(cdi_pc$pop18, main = "pop18")
boxplot(cdi_pc$pop65, main = "pop65")
boxplot(cdi_pc$pcincome, main = "pcincome")
boxplot(cdi_pc$unemp,main = "unemp")
boxplot(cdi_pc$hsgrad, main = "hsgrad")
boxplot(cdi_pc$bagrad, main = "bagrad")
boxplot(cdi_pc$poverty, main = "poverty")
boxplot(cdi_pc$pcarea, main = "pcarea")
boxplot(cdi_pc$pcdocs, main = "pcdocs")
boxplot(cdi_pc$pcbeds, main = "pcbeds")
boxplot(crime_rate ~ region, data = cdi_pc, main = "region")
```



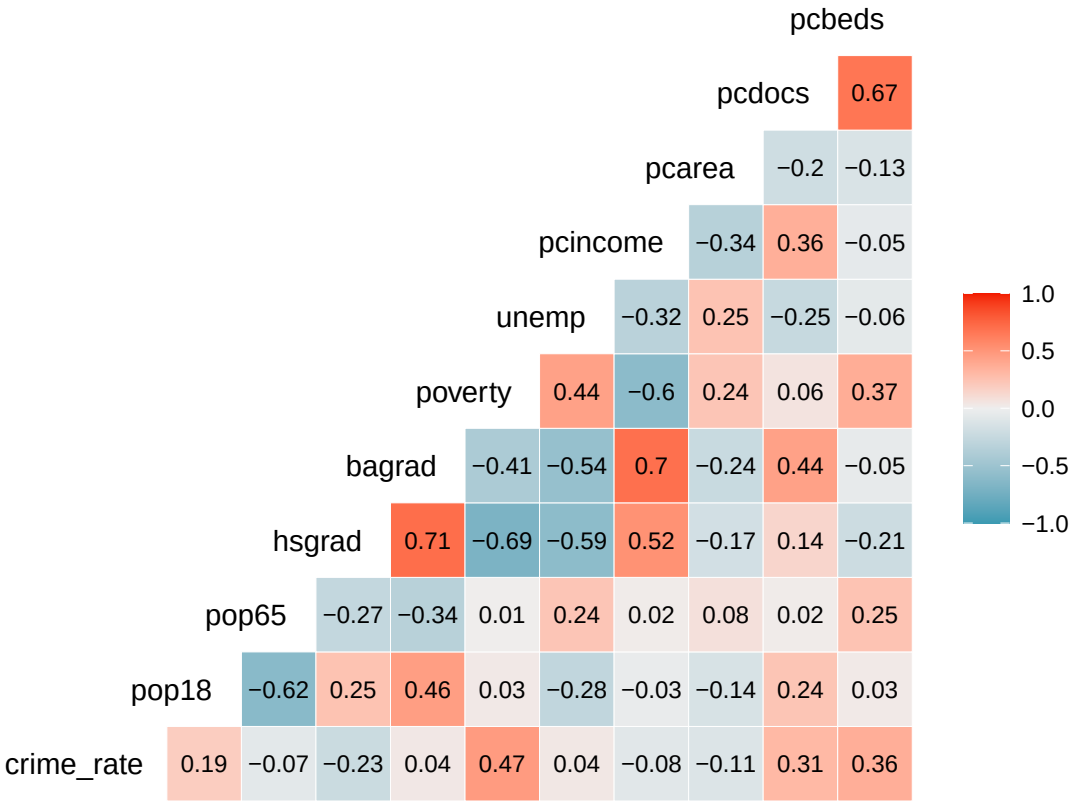
Scatterplot Matrix

```
pairs(~crime_rate +.,data=cdi_pc, panel = panel.smooth, upper.panel = NULL, main = "Scatterplot Matrix")
```



Correlation plot/ Heatmap

```
cdi_pc %>%
  dplyr::select(-region) %>%
  ggcorr(label = TRUE, hjust = 0.9, layout.exp = 2, label_size = 3, label_round = 2)
```



Modelling

Fit regression using all predictors

```
mult_fit = lm(crime_rate ~ ., data = cdi_pc)
summary(mult_fit)
```

```
##
## Call:
## lm(formula = crime_rate ~ ., data = cdi_pc)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.049868 -0.010696 -0.000262  0.009115  0.222726
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -5.110e-02  3.107e-02  -1.645 0.100775
## pop18        1.320e-03  3.721e-04   3.547 0.000433 ***
## pop65        2.234e-04  3.488e-04   0.640 0.522201
## hsgrad       1.678e-04  3.053e-04   0.549 0.582951
## bagrad      -5.330e-04  3.417e-04  -1.560 0.119567
## poverty      2.799e-03  4.323e-04   6.475 2.62e-10 ***
## unemp        3.280e-04  6.065e-04   0.541 0.588897
## pcincome     2.251e-06  5.292e-07   4.254 2.58e-05 ***
```

```
## region1     -2.352e-02  3.025e-03  -7.776 5.71e-14 ***
## region2     -1.491e-02  2.945e-03  -5.064 6.13e-07 ***
## region4      1.269e-03  3.452e-03   0.368 0.713311
## pcarea      -6.520e-01  1.695e-01  -3.847 0.000138 ***
## pcdocs       6.635e-01  1.157e+00   0.574 0.566606
## pcbeds       2.319e+00  9.046e-01   2.564 0.010699 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02033 on 426 degrees of freedom
## Multiple R-squared:  0.4628, Adjusted R-squared:  0.4464
## F-statistic: 28.23 on 13 and 426 DF,  p-value: < 2.2e-16
```

Backwards Elimination

```
mult_fit_back <- step(mult_fit, direction='backward')
```

```
## Start:  AIC=-3414.29
## crime_rate ~ pop18 + pop65 + hsgrad + bagrad + poverty + unemp +
##     pcincome + region + pcarea + pcdocs + pcbeds
##
##            Df Sum of Sq     RSS     AIC
## - unemp     1  0.000121 0.17624 -3416.0
## - hsgrad    1  0.000125 0.17624 -3416.0
## - pcdocs    1  0.000136 0.17625 -3416.0
## - pop65     1  0.000170 0.17629 -3415.9
## <none>                  0.17612 -3414.3
## - bagrad    1  0.001006 0.17712 -3413.8
## - pcbeds    1  0.002717 0.17883 -3409.6
## - pop18     1  0.005201 0.18132 -3403.5
## - pcarea    1  0.006118 0.18223 -3401.3
## - pcincome  1  0.007483 0.18360 -3398.0
## - poverty   1  0.017333 0.19345 -3375.0
## - region    3  0.033187 0.20930 -3344.3
##
## Step:  AIC=-3415.99
## crime_rate ~ pop18 + pop65 + hsgrad + bagrad + poverty + pcincome +
##     region + pcarea + pcdocs + pcbeds
##
##            Df Sum of Sq     RSS     AIC
## - hsgrad    1  0.000090 0.17633 -3417.8
## - pcdocs    1  0.000139 0.17638 -3417.6
## - pop65     1  0.000206 0.17644 -3417.5
## <none>                  0.17624 -3416.0
## - bagrad    1  0.001199 0.17744 -3415.0
## - pcbeds    1  0.002597 0.17883 -3411.6
## - pop18     1  0.005296 0.18153 -3405.0
## - pcarea    1  0.006032 0.18227 -3403.2
## - pcincome  1  0.008072 0.18431 -3398.3
## - poverty   1  0.020229 0.19647 -3370.2
## - region    3  0.034275 0.21051 -3343.8
##
```

```
## Step:  AIC=-3417.76
## crime_rate ~ pop18 + pop65 + bagrad + poverty + pcincome + region +
##     pcarea + pcdocs + pcbeds
##
##            Df Sum of Sq     RSS     AIC
## - pcdocs    1  0.000119 0.17645 -3419.5
## - pop65     1  0.000185 0.17651 -3419.3
## <none>                  0.17633 -3417.8
## - bagrad    1  0.001366 0.17769 -3416.4
## - pcbeds    1  0.002791 0.17912 -3412.9
## - pop18     1  0.005207 0.18153 -3407.0
## - pcarea    1  0.006153 0.18248 -3404.7
## - pcincome  1  0.008474 0.18480 -3399.1
## - region    3  0.034432 0.21076 -3345.3
## - poverty   1  0.032602 0.20893 -3345.1
##
## Step:  AIC=-3419.47
## crime_rate ~ pop18 + pop65 + bagrad + poverty + pcincome + region +
##     pcarea + pcbeds
##
##            Df Sum of Sq     RSS     AIC
## - pop65     1  0.000185 0.17663 -3421.0
## <none>                  0.17645 -3419.5
## - bagrad    1  0.001252 0.17770 -3418.4
## - pop18     1  0.005485 0.18193 -3408.0
## - pcarea    1  0.006170 0.18262 -3406.3
## - pcbeds    1  0.007404 0.18385 -3403.4
## - pcincome  1  0.009338 0.18578 -3398.8
## - poverty   1  0.032979 0.20943 -3346.1
## - region    3  0.034895 0.21134 -3346.1
##
## Step:  AIC=-3421.01
## crime_rate ~ pop18 + bagrad + poverty + pcincome + region + pcarea +
##     pcbeds
##
##            Df Sum of Sq     RSS     AIC
## <none>                  0.17663 -3421.0
## - bagrad    1  0.001396 0.17803 -3419.5
## - pop18     1  0.005894 0.18252 -3408.6
## - pcarea    1  0.006021 0.18265 -3408.3
## - pcincome  1  0.009446 0.18608 -3400.1
## - pcbeds    1  0.009589 0.18622 -3399.7
## - region    3  0.034710 0.21134 -3348.1
## - poverty   1  0.032938 0.20957 -3347.8
```

```
mult_fit_back
```

```
##
## Call:
## lm(formula = crime_rate ~ pop18 + bagrad + poverty + pcincome +
##     region + pcarea + pcbeds, data = cdi_pc)
##
## Coefficients:
## (Intercept)        pop18       bagrad      poverty     pcincome      region1
```

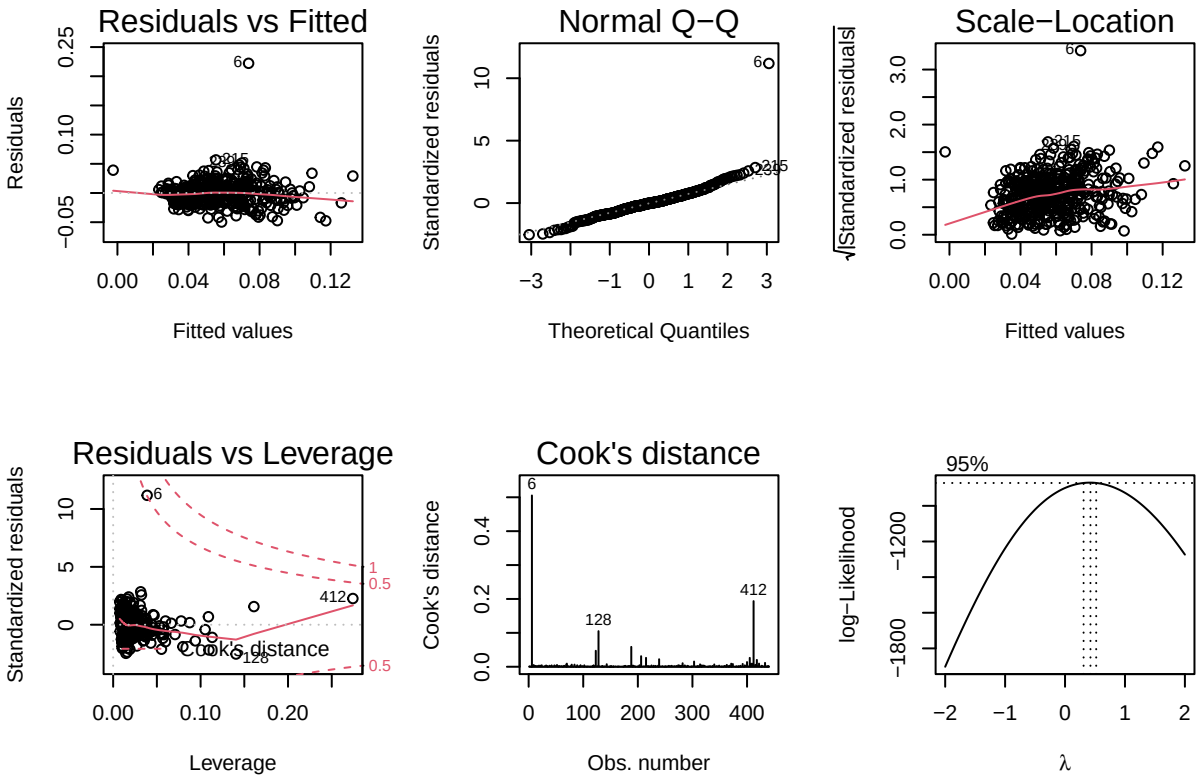
```
##  -3.242e-02    1.214e-03   -4.595e-04    2.727e-03    2.288e-06   -2.279e-02
##     region2      region4       pcarea       pcbeds
##  -1.458e-02    2.190e-03   -6.410e-01    2.750e+00
```

crime_rate ~ pop18 + bagrad + poverty + pcincome + region + pcarea + pcbeds

Model Diagnostics

Create Residuals vs Fitted plot & Normal Q-Q plot & Scale-Location plot & Residuals vs Leverage plot to detect the normality of residuals and outliers

```
par(mfrow=c(2,3))
plot(mult_fit_back)
plot(mult_fit_back, which = 4)
bc = boxcox(mult_fit_back)
```



Diagnose the model without outliers

```
# remove influential points
cdi_pc_out = cdi_pc[-c(6,128,412),]

# fit model with and without influential points
mult_fit_back_without = lm(crime_rate ~ pop18 + bagrad + poverty + pcincome + region + pcarea + pcbeds,
```

```
summary(mult_fit_back)
```

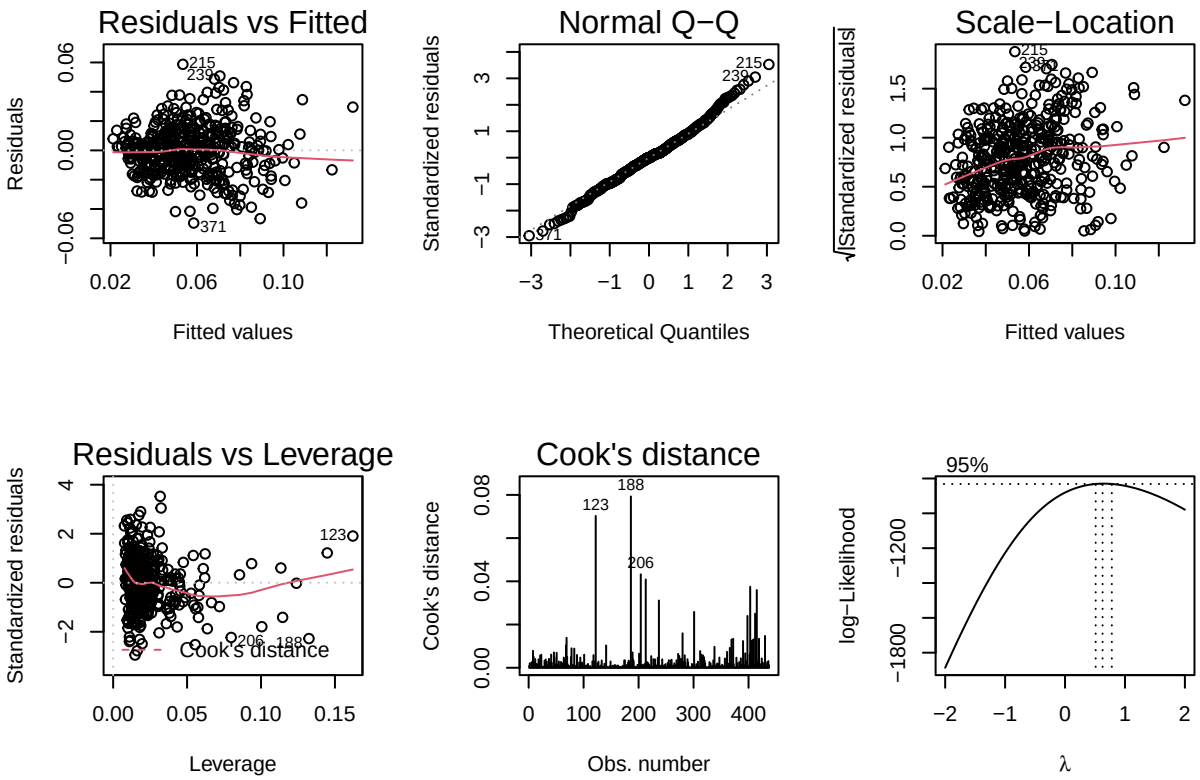
```
##
## Call:
## lm(formula = crime_rate ~ pop18 + bagrad + poverty + pcincome +
##     region + pcarea + pcbeds, data = cdi_pc)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.049729 -0.010702 -0.000564  0.009817  0.222166
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.242e-02  1.279e-02  -2.534 0.011638 *
## pop18        1.214e-03  3.206e-04   3.788 0.000174 ***
## bagrad      -4.595e-04  2.492e-04  -1.844 0.065937 .
## poverty      2.727e-03  3.046e-04   8.955  < 2e-16 ***
## pcincome     2.288e-06  4.770e-07   4.795 2.24e-06 ***
## region1     -2.279e-02  2.869e-03  -7.943 1.74e-14 ***
## region2     -1.458e-02  2.711e-03  -5.377 1.25e-07 ***
## region4      2.190e-03  3.170e-03   0.691 0.489979
## pcarea      -6.410e-01  1.674e-01  -3.828 0.000148 ***
## pcbeds       2.750e+00  5.691e-01   4.831 1.89e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.02027 on 430 degrees of freedom
## Multiple R-squared:  0.4612, Adjusted R-squared:   0.45
## F-statistic:  40.9 on 9 and 430 DF,  p-value: < 2.2e-16
```

```
summary(mult_fit_back_without)
```

```
##
## Call:
## lm(formula = crime_rate ~ pop18 + bagrad + poverty + pcincome +
##     region + pcarea + pcbeds, data = cdi_pc_out)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.049584 -0.010326  0.000031  0.010232  0.058723
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.833e-02  1.069e-02  -2.649  0.00837 **
## pop18        1.258e-03  2.701e-04   4.659 4.25e-06 ***
## bagrad      -4.979e-04  2.099e-04  -2.372  0.01813 *
## poverty      2.432e-03  2.755e-04   8.830  < 2e-16 ***
## pcincome     2.202e-06  3.985e-07   5.526 5.72e-08 ***
## region1     -2.607e-02  2.422e-03 -10.765  < 2e-16 ***
## region2     -1.563e-02  2.273e-03  -6.876 2.20e-11 ***
## region4      2.040e-03  2.673e-03   0.763  0.44568
## pcarea      -7.446e-01  1.614e-01  -4.613 5.24e-06 ***
```

```
## pcbeds       2.927e+00  4.883e-01   5.995 4.35e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01693 on 427 degrees of freedom
## Multiple R-squared:  0.5471, Adjusted R-squared:  0.5375
## F-statistic:  57.3 on 9 and 427 DF,  p-value: < 2.2e-16
```

```
# diagnose the model without outliers
par(mfrow=c(2,3))
plot(mult_fit_back_without)
plot(mult_fit_back_without, which = 4)
bc_without = boxcox(mult_fit_back_without)
```



Box-cox transformation

```
(lambda = bc_without$x[which.max(bc_without$y)])
```

```
## [1] 0.6262626
```

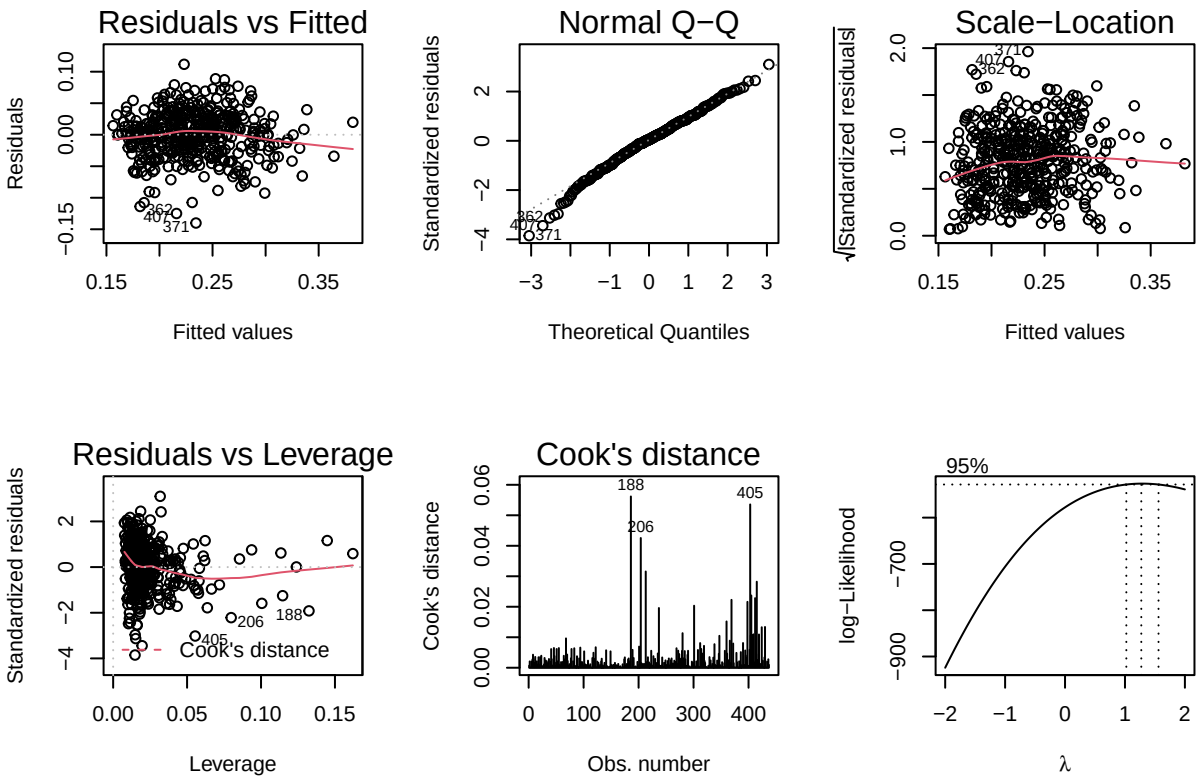
```
#mult_fit_back_without_trans = lm(((crime_rate^lambda-1)/lambda) ~ pop18 + bagrad + poverty + pcincome

mult_fit_back_without_trans = lm(crime_rate^0.5 ~ pop18 + bagrad + poverty + pcincome +
                                   region + pcarea + pcbeds, data = cdi_pc_out)

summary(mult_fit_back_without_trans)
```

```
##
## Call:
## lm(formula = crime_rate^0.5 ~ pop18 + bagrad + poverty + pcincome +
##     region + pcarea + pcbeds, data = cdi_pc_out)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.140088 -0.021520  0.001828  0.024375  0.111737
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.665e-02  2.314e-02   2.448   0.0148 *
## pop18        2.657e-03  5.844e-04   4.546 7.13e-06 ***
## bagrad      -1.087e-03  4.542e-04  -2.393   0.0171 *
## poverty      4.835e-03  5.961e-04   8.111 5.36e-15 ***
## pcincome     4.687e-06  8.624e-07   5.435 9.23e-08 ***
## region1     -5.867e-02  5.241e-03 -11.193  < 2e-16 ***
## region2     -3.469e-02  4.920e-03  -7.051 7.17e-12 ***
## region4      6.480e-03  5.784e-03   1.120   0.2632
## pcarea      -1.466e+00  3.493e-01  -4.197 3.29e-05 ***
## pcbeds       5.908e+00  1.057e+00   5.591 4.03e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03663 on 427 degrees of freedom
## Multiple R-squared:  0.5326, Adjusted R-squared:  0.5227
## F-statistic: 54.05 on 9 and 427 DF,  p-value: < 2.2e-16
```

```r
# Diagnose the model by square root transformation
par(mfrow = c(2,3))
plot(mult_fit_back_without_trans)
plot(mult_fit_back_without_trans, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_trans)
```



Compare the Adjusted R^2

```
rbind(mult_fit_back %>% broom::glance() %>% mutate(model_type = "mult_fit_back"),
      mult_fit_back_without %>% broom::glance() %>% mutate(model_type = "mult_fit_back_without"),
      mult_fit_back_without_trans %>% broom::glance()%>% mutate(model_type = "mult_fit_back_without_tra
        dplyr::select(model_type, everything())
```

```
## # A tibble: 3 x 13
##   model_type      r.squared adj.r.squared  sigma statistic  p.value    df logLik
##   <chr>               <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>  <dbl>
## 1 mult_fit_back       0.461         0.450 0.0203      40.9 1.57e-52     9  1096.
## 2 mult_fit_back_~     0.547         0.538 0.0169      57.3 5.69e-68     9  1167.
## 3 mult_fit_back_~     0.533         0.523 0.0366      54.1 4.34e-65     9   830.
## # ... with 5 more variables: AIC <dbl>, BIC <dbl>, deviance <dbl>,
## #   df.residual <int>, nobs <int>
```

```
mult_fit_back_without_trans %>% broom::glance() %>% as.data.frame()
```

```
##   r.squared adj.r.squared      sigma statistic      p.value df   logLik
## 1 0.5325634     0.5227111 0.03663442  54.05477 4.340109e-65  9 830.0392
##         AIC       BIC  deviance df.residual nobs
## 1 -1638.078 -1593.199 0.5730684         427  437
```

Assessing Multicollinearity

```
# Calculate the variance inflation factor (VIF)
check_collinearity(mult_fit_back_without)
```

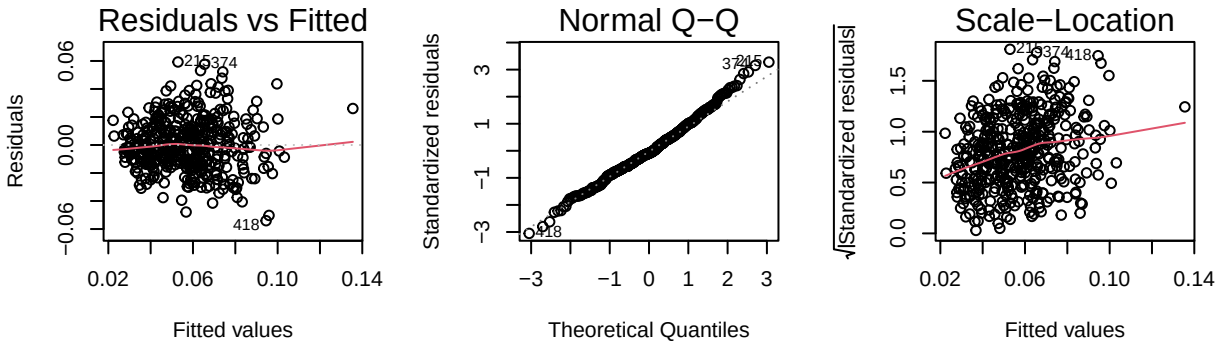
```
## # Check for Multicollinearity
##
## Low Correlation
##
##      Term  VIF Increased SE Tolerance
##  pcincome 1.00         1.00      1.00
##    pcarea 2.02         1.42      0.49
##
## Moderate Correlation
##
##    Term  VIF Increased SE Tolerance
##   pop18 7.83         2.80      0.13
##  bagrad 6.83         2.61      0.15
##  region 6.15         2.48      0.16
##  pcbeds 6.29         2.51      0.16
##
## High Correlation
##
##     Term   VIF Increased SE Tolerance
##  poverty 13.19         3.63      0.08
```

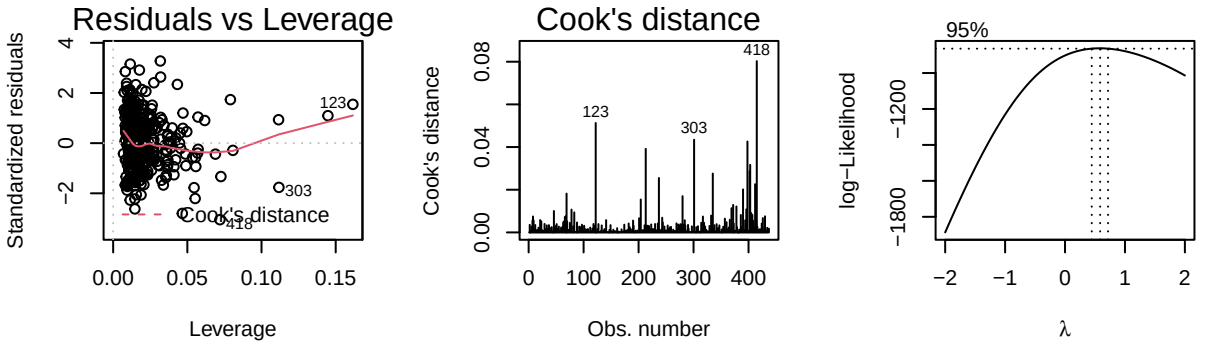
```
# Remove the variable whose vif is larger than 10
mult_fit_back_without_vif = lm(crime_rate ~ pop18 + bagrad + pcincome + region + pcarea + pcbeds, data =
summary(mult_fit_back_without_vif)
```

```
##
## Call:
## lm(formula = crime_rate ~ pop18 + bagrad + pcincome + region +
##     pcarea + pcbeds, data = cdi_pc_out)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.054061 -0.011870 -0.001622  0.010885  0.059278
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  4.686e-03  1.088e-02   0.431  0.66696
## pop18        1.442e-03  2.925e-04   4.932 1.17e-06 ***
## bagrad      -6.808e-04  2.269e-04  -3.001  0.00285 **
## pcincome     1.140e-06  4.127e-07   2.764  0.00596 **
## region1     -3.105e-02  2.559e-03 -12.137  < 2e-16 ***
## region2     -2.138e-02  2.366e-03  -9.034  < 2e-16 ***
## region4      1.904e-03  2.903e-03   0.656  0.51234
## pcarea      -5.425e-01  1.735e-01  -3.126  0.00189 **
## pcbeds       5.111e+00  4.573e-01  11.176  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 0.01839 on 428 degrees of freedom
## Multiple R-squared:  0.4644, Adjusted R-squared:  0.4543
## F-statistic: 46.38 on 8 and 428 DF,  p-value: < 2.2e-16
```

```
# Diagnose the model removing poverty term
par(mfrow = c(2,3))
plot(mult_fit_back_without_vif)
plot(mult_fit_back_without_vif, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_vif)
```



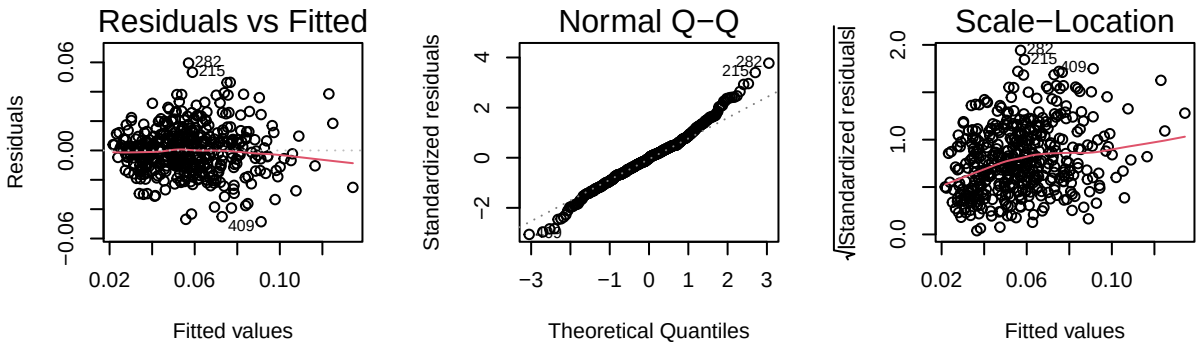


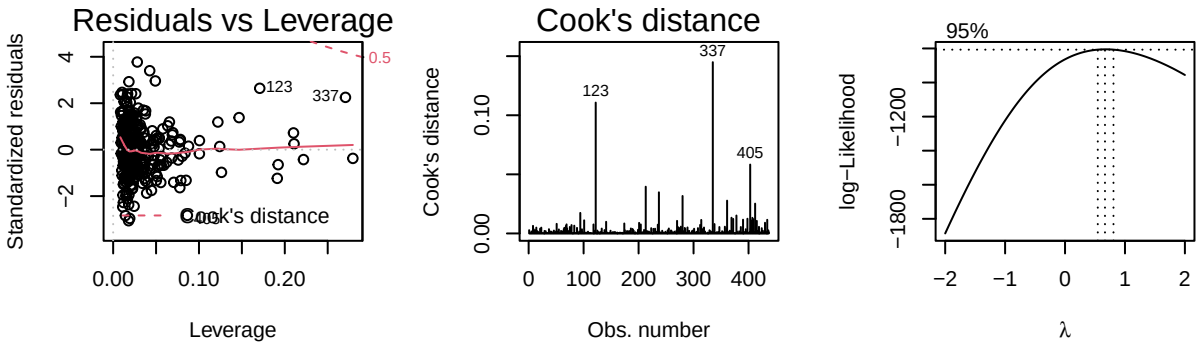

```
# Add the interaction
mult_fit_back_without_int = lm(crime_rate ~ pop18 + bagrad + pcincome + region + pcarea + pcbeds + pove
summary(mult_fit_back_without_int)
```

```
##
## Call:
## lm(formula = crime_rate ~ pop18 + bagrad + pcincome + region +
##     pcarea + pcbeds + poverty + pcincome * poverty + pcincome *
##     bagrad + pop18 * bagrad, data = cdi_pc_out)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.048555 -0.009658 -0.000259  0.007877  0.059566
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -4.851e-02  2.027e-02  -2.393 0.017154 *
```

```
## pop18              1.008e-03  5.271e-04   1.912 0.056516 .
## bagrad             1.243e-03  7.670e-04   1.621 0.105802
## pcincome           3.169e-06  9.379e-07   3.378 0.000796 ***
## region1           -2.550e-02  2.321e-03 -10.983  < 2e-16 ***
## region2           -1.526e-02  2.161e-03  -7.061 6.82e-12 ***
## region4           -2.490e-03  2.616e-03  -0.952 0.341857
## pcarea            -4.545e-01  1.582e-01  -2.872 0.004278 **
## pcbeds             1.327e+00  5.209e-01   2.547 0.011210 *
## poverty           -4.214e-04  7.964e-04  -0.529 0.597008
## pcincome:poverty   2.295e-07  5.077e-08   4.521 7.99e-06 ***
## bagrad:pcincome   -7.267e-08  2.288e-08  -3.176 0.001600 **
## pop18:bagrad      -3.205e-06  2.123e-05  -0.151 0.880068
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01601 on 424 degrees of freedom
## Multiple R-squared:  0.5978, Adjusted R-squared:  0.5864
## F-statistic: 52.52 on 12 and 424 DF,  p-value: < 2.2e-16
```

```r
# Diagnose the model with interaction term
par(mfrow = c(2,3))
plot(mult_fit_back_without_int)
plot(mult_fit_back_without_int, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_int)
```





Compare the Adjusted R^2 again

```
rbind(mult_fit_back %>% broom::glance() %>% mutate(model_type = "mult_fit_back"),
      mult_fit_back_without %>% broom::glance() %>% mutate(model_type = "mult_fit_back_without"),
      mult_fit_back_without_trans %>% broom::glance()%>% mutate(model_type = "mult_fit_back_without_tra
      mult_fit_back_without_vif %>% broom::glance() %>% mutate(model_type = "mult_fit_back_without_vif"
      mult_fit_back_without_int %>% broom::glance() %>% mutate(model_type = "mult_fit_back_without_int"
      dplyr::select(model_type, everything())
```

```
## # A tibble: 5 x 13
##   model_type      r.squared adj.r.squared  sigma statistic  p.value    df logLik
##   <chr>               <dbl>         <dbl>  <dbl>     <dbl>    <dbl> <dbl>  <dbl>
## 1 mult_fit_back       0.461         0.450 0.0203      40.9 1.57e-52     9  1096.
## 2 mult_fit_back_~     0.547         0.538 0.0169      57.3 5.69e-68     9  1167.
## 3 mult_fit_back_~     0.533         0.523 0.0366      54.1 4.34e-65     9   830.
## 4 mult_fit_back_~     0.464         0.454 0.0184      46.4 1.63e-53     8  1131.
## 5 mult_fit_back_~     0.598         0.586 0.0160      52.5 4.09e-76    12  1193.
## # ... with 5 more variables: AIC <dbl>, BIC <dbl>, deviance <dbl>,
## #   df.residual <int>, nobs <int>
```

Model Validation

Compute RMSE and adjusted R^2 by cross-validation

```
set.seed(1234)
cv_df =
  crossv_kfold(cdi_pc_out, k = 10) %>%
  mutate(
    train = map(train, as_tibble),
    test = map(test, as_tibble)) %>%
  mutate(
    mult_fit_back_without = map(train, ~lm(crime_rate ~ pop18 + bagrad + poverty +
                                pcincome + region + pcarea + pcbeds, data = .x)),
    mult_fit_back_without_trans = map(train, ~lm(sqrt(crime_rate) ~ pop18 + bagrad + poverty +
                                      pcincome + region + pcarea + pcbeds, data = .x)),
    mult_fit_back_without_vif = map(train, ~lm(crime_rate ~ pop18 + bagrad +
                                    pcincome + region + pcarea + pcbeds, data = .x)),
    mult_fit_back_without_int = map(train, ~lm(crime_rate ~ pop18 + bagrad + pcincome + region + pcarea
  mutate(
    rmse_model2 = map2_dbl(mult_fit_back_without, test, ~rmse(model = .x, data = .y)),
    rmse_model3 = map2_dbl(mult_fit_back_without_trans, test, ~rmse(model = .x, data = .y)),
    rmse_model4 = map2_dbl(mult_fit_back_without_vif, test, ~rmse(model = .x, data = .y)),
    rmse_model5 = map2_dbl(mult_fit_back_without_int, test, ~rmse(model = .x, data = .y))) %>%
  mutate(
    res_model2 = map(mult_fit_back_without, broom::glance %>% as.data.frame),
    res_model3 = map(mult_fit_back_without_trans, broom::glance %>% as.data.frame),
    res_model4 = map(mult_fit_back_without_vif, broom::glance %>% as.data.frame),
    res_model5 = map(mult_fit_back_without_int, broom::glance %>% as.data.frame))%>%
  unnest(res_model2, res_model3, res_model4, res_model5) %>%
```

```
  dplyr::select(rmse_model2,rmse_model3,rmse_model4,rmse_model5,
                value.adj.r.squared,value.adj.r.squared1,value.adj.r.squared2,value.adj.r.squared3) %>%
  rename(adjR_model2 = value.adj.r.squared,
         adjR_model3 = value.adj.r.squared1,
         adjR_model4 = value.adj.r.squared2,
         adjR_model5 = value.adj.r.squared3)

cv_df %>%
  summarise_each(funs(mean( .,na.rm = TRUE))) %>%
  mutate(
    AIC_model2 = nrow(cdi_pc_out)*log(rmse_model2^2) + 2*7,
    AIC_model3 = nrow(cdi_pc_out)*log(rmse_model3^2) + 2*7,
    AIC_model4 = nrow(cdi_pc_out)*log(rmse_model4^2) + 2*6,
    AIC_model5 = nrow(cdi_pc_out)*log(rmse_model5^2) + 2*10
  ) %>%
  mutate(
    BIC_model2 = nrow(cdi_pc_out)*log(rmse_model2^2) + log(nrow(cdi_pc_out))*7,
    BIC_model3 = nrow(cdi_pc_out)*log(rmse_model3^2) + log(nrow(cdi_pc_out))*7,
    BIC_model4 = nrow(cdi_pc_out)*log(rmse_model4^2) + log(nrow(cdi_pc_out))*6,
    BIC_model5 = nrow(cdi_pc_out)*log(rmse_model5^2) + log(nrow(cdi_pc_out))*10
  ) %>%
  t()
```

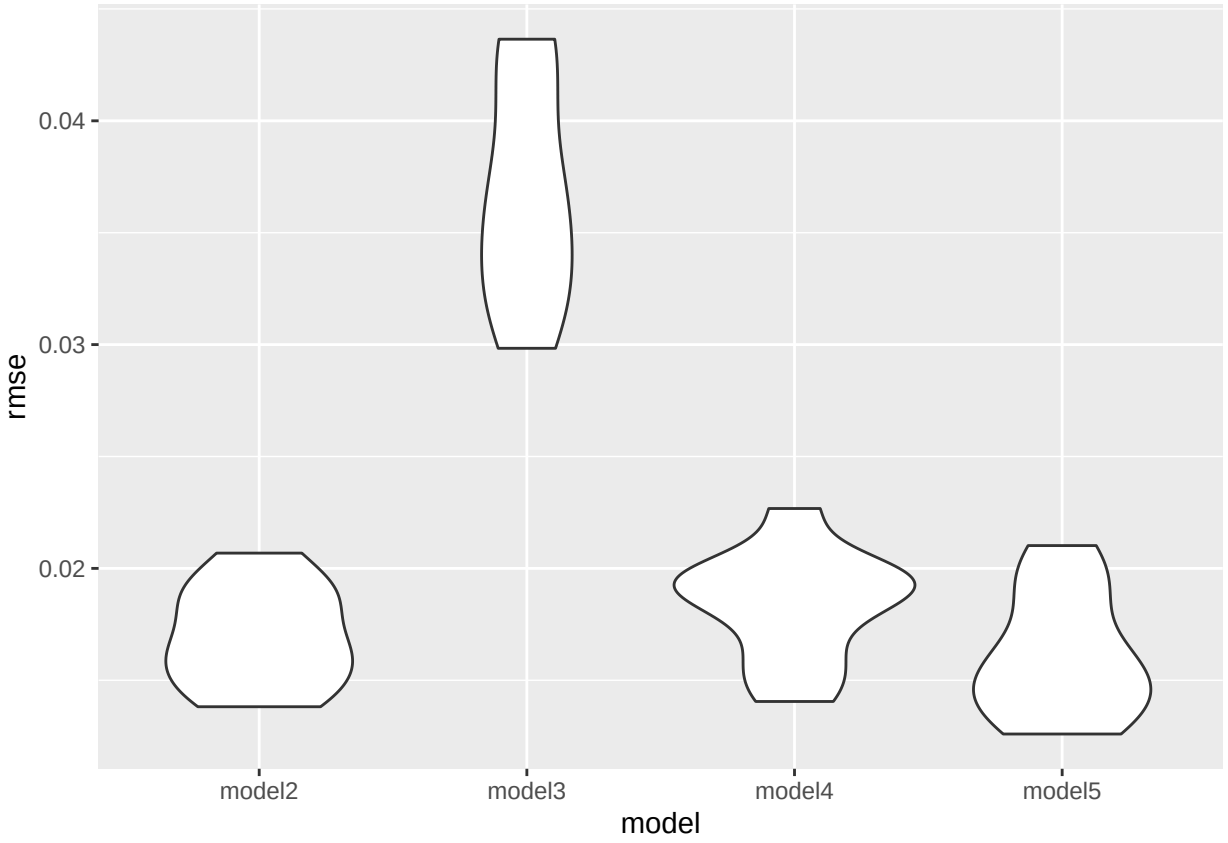
```
##                       [,1]
## rmse_model2   1.695206e-02
## rmse_model3   3.665591e-02
## rmse_model4   1.837443e-02
## rmse_model5   1.611934e-02
## adjR_model2   5.373646e-01
## adjR_model3   5.225545e-01
## adjR_model4   4.540860e-01
## adjR_model5   5.866515e-01
## AIC_model2   -3.549618e+03
## AIC_model3   -2.875602e+03
## AIC_model4   -3.481199e+03
## AIC_model5   -3.587641e+03
## BIC_model2   -3.521058e+03
## BIC_model3   -2.847042e+03
## BIC_model4   -3.456719e+03
## BIC_model5   -3.546842e+03
```

Plot the violin plot

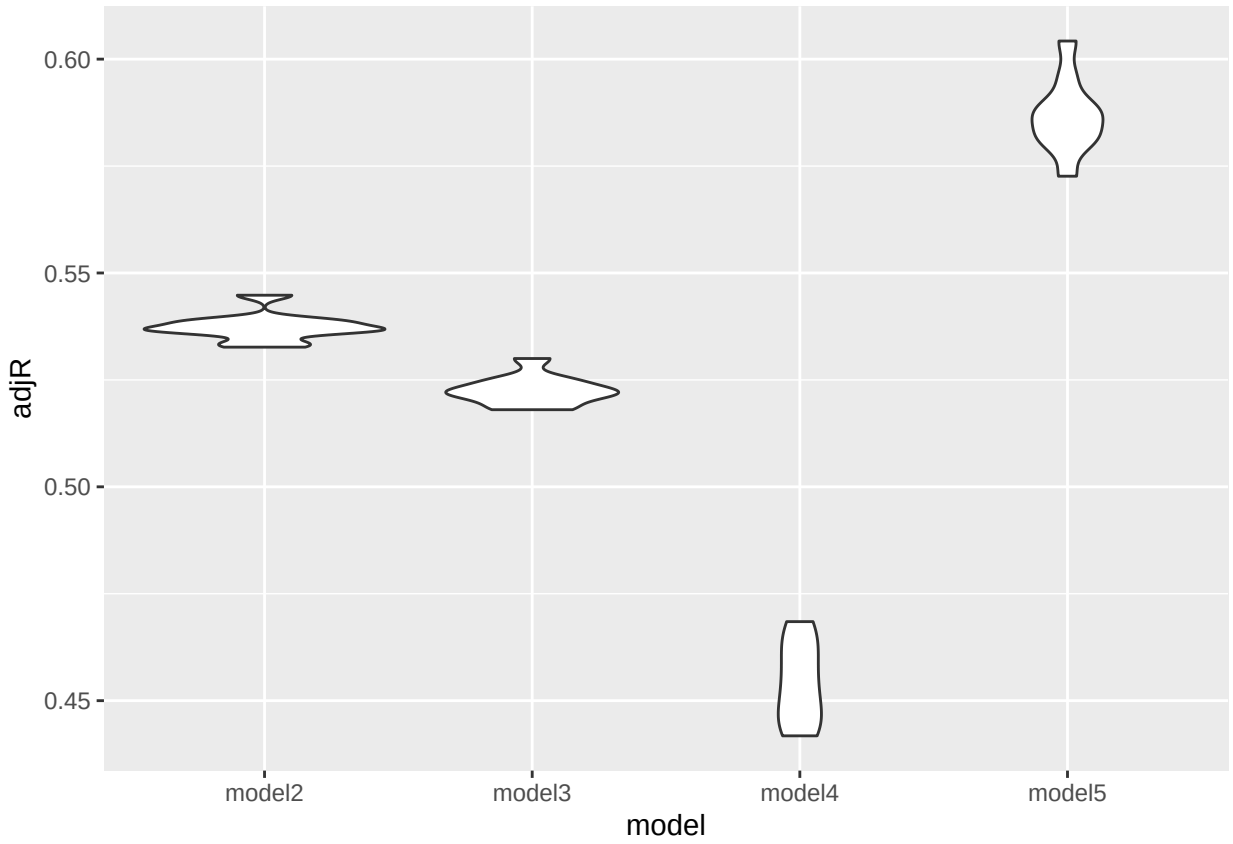
```
unnest_cd_df = cv_df %>%
  pivot_longer(rmse_model2:adjR_model5,
               names_pattern = "(.*)(......)$",
               names_to = c("limit", "model")) %>%
  mutate(limit=ifelse(limit=="", "value", limit)) %>%
  pivot_wider(id_cols = model,
              names_from = limit,
              values_from = value,
```

```
              names_repair = "check_unique") %>%
  unnest(c(rmse_, adjR_)) %>%
  rename(rmse = rmse_,
         adjR = adjR_)

unnest_cd_df %>% ggplot(aes(x = model, y = rmse)) + geom_violin()
```




```
unnest_cd_df %>% ggplot(aes(x = model, y = adjR)) + geom_violin()
```



Model Validation

Compute RMSE, R-squared, MAE by cross-validation

```
set.seed(1234)

#specify the cross-validation method
train <- trainControl(method = "cv", number = 5)
#fit a regression model and use k-fold CV to evaluate performance
model2 <- train(crime_rate ~ pop18 + bagrad + poverty + pcincome + region +
                pcarea + pcbeds, data = cdi_pc_out, method = "lm", trControl = train)
model3 <- train(sqrt(crime_rate) ~ pop18 + bagrad + poverty + pcincome + region +
                pcarea + pcbeds, data = cdi_pc_out, method = "lm", trControl = train)
model4 <- train(crime_rate ~ pop18 + bagrad + pcincome + region +
                pcarea + pcbeds, data = cdi_pc_out, method = "lm", trControl = train)
model5 <- train(crime_rate ~ pop18 + bagrad + pcincome + region + pcarea + pcbeds + poverty + pcincome*
                pcincome*bagrad + pop18*bagrad, data = cdi_pc_out, method = "lm", trControl = train)
#view summary of k-fold CV
print(model2)
```

```
## Linear Regression
##
## 437 samples
```

```
##   7 predictor
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 349, 350, 350, 349, 350
## Resampling results:
##
##   RMSE        Rsquared   MAE
##   0.01715508  0.5225531  0.01316356
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
print(model3)
```

```
## Linear Regression
##
## 437 samples
##   7 predictor
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 349, 349, 350, 349, 351
## Resampling results:
##
##   RMSE        Rsquared  MAE
##   0.03729014  0.507163  0.0286451
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
print(model4)
```

```
## Linear Regression
##
## 437 samples
##   6 predictor
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 351, 349, 349, 350, 349
## Resampling results:
##
##   RMSE        Rsquared   MAE
##   0.01861143  0.4416731  0.01452816
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
print(model5)
```

```
## Linear Regression
##
## 437 samples
##   7 predictor
```

```
##
## No pre-processing
## Resampling: Cross-Validated (5 fold)
## Summary of sample sizes: 349, 349, 349, 352, 349
## Resampling results:
##
##   RMSE       Rsquared   MAE
##   0.0162433  0.5787499  0.01229859
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
resamps <- resamples(list(model2 = model2,
                          model3 = model3,
                          model4 = model4,
                          model5 = model5))
summary(resamps)
```

```
##
## Call:
## summary.resamples(object = resamps)
##
## Models: model2, model3, model4, model5
## Number of resamples: 5
##
## MAE
##              Min.    1st Qu.     Median       Mean    3rd Qu.       Max. NA's
## model2 0.01224694 0.01248456 0.01296632 0.01316356 0.01388779 0.01423217    0
## model3 0.02556856 0.02859783 0.02955848 0.02864510 0.02962702 0.02987362    0
## model4 0.01315513 0.01408897 0.01423643 0.01452816 0.01547324 0.01568701    0
## model5 0.01169004 0.01198610 0.01201409 0.01229859 0.01218413 0.01361860    0
##
## RMSE
##              Min.    1st Qu.     Median       Mean    3rd Qu.       Max. NA's
## model2 0.01574334 0.01676428 0.01719324 0.01715508 0.01781078 0.01826378    0
## model3 0.03449311 0.03671838 0.03737053 0.03729014 0.03841245 0.03945623    0
## model4 0.01666671 0.01864254 0.01887495 0.01861143 0.01931267 0.01956027    0
## model5 0.01447099 0.01553911 0.01659153 0.01624330 0.01679110 0.01782378    0
##
## Rsquared
##             Min.   1st Qu.    Median      Mean   3rd Qu.      Max. NA's
## model2 0.4923859 0.4941192 0.5181387 0.5225531 0.5319339 0.5761876    0
## model3 0.4290804 0.5065050 0.5173170 0.5071630 0.5268753 0.5560373    0
## model4 0.3278797 0.4286584 0.4287951 0.4416731 0.4937760 0.5292565    0
## model5 0.5233889 0.5598982 0.5805630 0.5787499 0.6024655 0.6274341    0
```

Thanks for reading!