BM1 Final Project

Yijia Jiang, Yifei Xu, Xinyi Zhou, Hengxuan Ma, Chao Gao

11/16/2021

Purpose

We will be analyzing data from the "County Demographic Information" (CDI) data set, which contains characteristics of 440 counties in the United States collected from 1990-1992. The primary objective of this investigation is to develop insight relevant to predicting the crime rate in counties.

Import the package we need

Data preprocessing

:92.90

Max.

:52.30

##

Max.

Transfer population variables to per capita variables

```
rm(list = ls())
cdi = read.csv("./data/cdi.csv") %>%
  mutate(crime_rate = crimes/pop,
         pcarea = area/pop,
         pcdocs = docs/pop,
         pcbeds = beds/pop,
         region = relevel(factor(region), ref = 3))
cdi_pc = cdi %>%
  dplyr::select(crime_rate, everything(), -id, -cty, -state, -area, -docs, -beds, -crimes, -totalinc)
summary(cdi pc)
##
      crime_rate
                                              pop18
                                                               pop65
                             pop
##
           :0.004601
                       Min. : 100043
                                                  :16.40
                                                                  : 3.000
    Min.
                                          Min.
                                                           Min.
                       1st Qu.: 139027
                                          1st Qu.:26.20
##
    1st Qu.:0.038102
                                                           1st Qu.: 9.875
##
   Median :0.052429
                       Median : 217280
                                          Median :28.10
                                                           Median :11.750
##
           :0.057286
                       Mean
                               : 393011
                                          Mean
                                                  :28.57
                                                           Mean
                                                                   :12.170
                       3rd Qu.: 436064
                                          3rd Qu.:30.02
##
    3rd Qu.:0.072597
                                                           3rd Qu.:13.625
##
    Max.
           :0.295987
                       Max.
                               :8863164
                                          Max.
                                                  :49.70
                                                           Max.
                                                                   :33.800
                                        poverty
##
        hsgrad
                         bagrad
                                                           unemp
##
           :46.60
                          : 8.10
                                                              : 2.200
   Min.
                    Min.
                                     Min.
                                            : 1.400
                                                       Min.
                    1st Qu.:15.28
##
   1st Qu.:73.88
                                     1st Qu.: 5.300
                                                       1st Qu.: 5.100
##
   Median :77.70
                    Median :19.70
                                     Median : 7.900
                                                       Median : 6.200
##
  Mean
           :77.56
                    Mean
                            :21.08
                                     Mean
                                            : 8.721
                                                       Mean
                                                              : 6.597
    3rd Qu.:82.40
                    3rd Qu.:25.32
                                     3rd Qu.:10.900
                                                       3rd Qu.: 7.500
```

Max.

:36.300

Max.

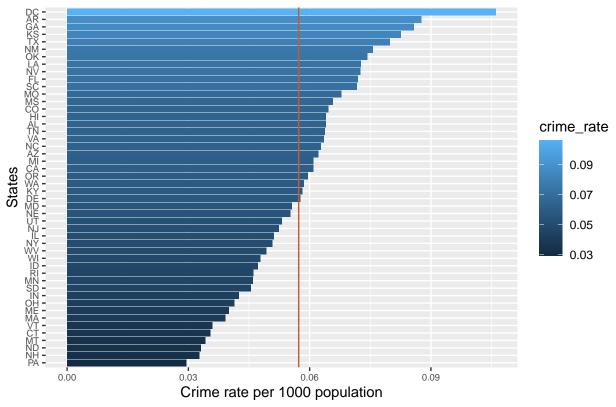
:21.300

```
pcarea
                                                 pcdocs
##
      pcincome
                  region
## Min.
         : 8899
                  3:152 Min.
                                 :3.086e-05 Min.
                                                    :0.0003559
                                             1st Qu.:0.0012127
   1st Qu.:16118
                  1:103
                          1st Qu.:1.323e-03
## Median :17759
                  2:108
                          Median :2.977e-03
                                             Median :0.0017509
## Mean
          :18561
                  4: 77
                          Mean
                                 :4.760e-03
                                             Mean
                                                    :0.0021230
   3rd Qu.:20270
                          3rd Qu.:5.199e-03
                                             3rd Qu.:0.0024915
##
## Max.
                          Max. :7.542e-02
                                             Max.
                                                    :0.0170377
          :37541
##
       pcbeds
## Min.
          :0.0001649
##
  1st Qu.:0.0021972
## Median :0.0033287
## Mean
          :0.0036493
## 3rd Qu.:0.0045649
          :0.0196982
## Max.
```

Exploratory Data Analysis

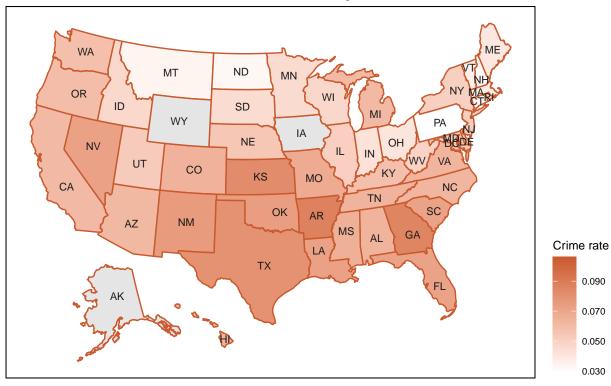
Rank of the crime rate by state

Ranking of States by Crime Rate



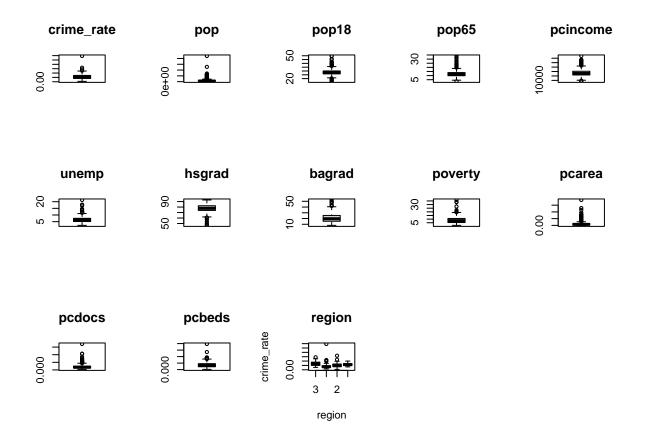
US crime rate map by state

US Crime Rate Map



Boxplot for each variable

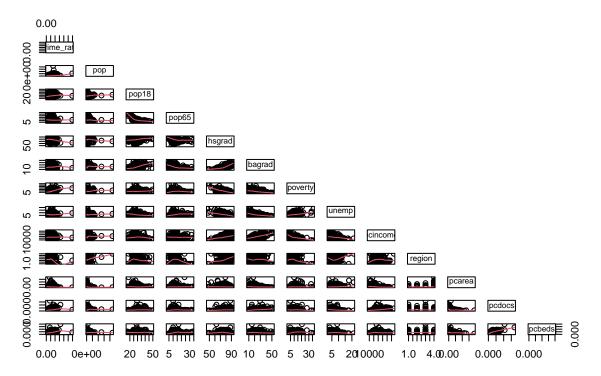
```
par(mfrow=c(3,5))
boxplot(cdi_pc$crime_rate, main = "crime_rate")
boxplot(cdi_pc$pop, main = "pop")
boxplot(cdi_pc$pop18, main = "pop18")
boxplot(cdi_pc$pop65, main = "pop65")
boxplot(cdi_pc$pcincome, main = "pcincome")
boxplot(cdi_pc$unemp,main = "unemp")
boxplot(cdi_pc$hsgrad, main = "hsgrad")
boxplot(cdi_pc$bagrad, main = "bagrad")
boxplot(cdi_pc$poverty, main = "poverty")
boxplot(cdi_pc$pcarea, main = "poverty")
boxplot(cdi_pc$pcdocs, main = "pcdocs")
boxplot(cdi_pc$pcbeds, main = "pcbeds")
```



Scatterplot Matrix

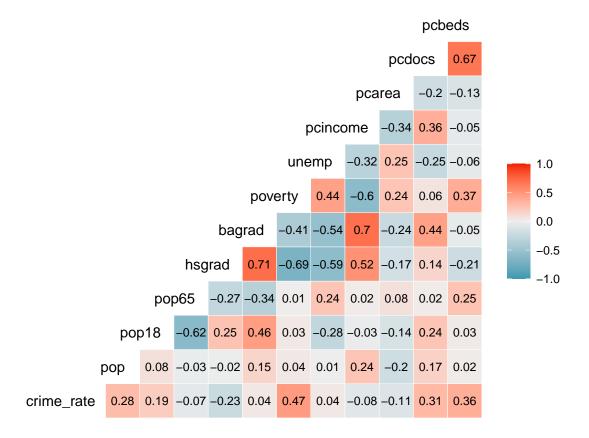
pairs(~crime_rate +.,data=cdi_pc, panel = panel.smooth, upper.panel = NULL, main = "Scatterplot Matrix"

Scatterplot Matrix



Correlation plot/ Heatmap

```
cdi_pc %>%
  dplyr::select(-region) %>%
  ggcorr(label = TRUE, hjust = 0.9, layout.exp = 2, label_size = 3, label_round = 2)
```



Modelling

Fit regression using all predictors

```
mult_fit = lm(crime_rate ~ ., data = cdi_pc)
summary(mult fit)
##
## lm(formula = crime_rate ~ ., data = cdi_pc)
##
## Residuals:
                         Median
        Min
                   1Q
                                        3Q
                                                 Max
## -0.066728 -0.010908 -0.000201 0.009418 0.211437
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.781e-02 3.043e-02
                                     -1.571 0.11690
## pop
                                      4.397 1.39e-05 ***
               7.796e-09 1.773e-09
## pop18
               1.111e-03 3.674e-04
                                      3.024 0.00264 **
## pop65
               1.204e-04
                          3.423e-04
                                      0.352 0.72518
## hsgrad
               2.926e-04
                          3.003e-04
                                      0.974
                                              0.33045
## bagrad
              -4.633e-04 3.350e-04 -1.383 0.16737
## poverty
               2.539e-03 4.273e-04
                                      5.942 5.86e-09 ***
```

```
## unemp
              4.462e-04 5.944e-04 0.751 0.45326
## pcincome
             1.746e-06 5.308e-07 3.289 0.00109 **
              -2.383e-02 2.963e-03 -8.042 8.87e-15 ***
## region1
## region2
              -1.601e-02 2.894e-03 -5.533 5.51e-08 ***
## region4
              -2.401e-03 3.482e-03 -0.689 0.49090
## pcarea
             -4.886e-01 1.701e-01 -2.873 0.00427 **
## pcdocs
              4.313e-01 1.134e+00 0.380 0.70392
              2.671e+00 8.893e-01 3.003 0.00283 **
## pcbeds
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Residual standard error: 0.01991 on 425 degrees of freedom
## Multiple R-squared: 0.4862, Adjusted R-squared: 0.4693
## F-statistic: 28.73 on 14 and 425 DF, p-value: < 2.2e-16
```

Backwards Elimination

```
mult_fit_back <- step(mult_fit, direction='backward')</pre>
## Start: AIC=-3431.87
## crime_rate ~ pop + pop18 + pop65 + hsgrad + bagrad + poverty +
##
       unemp + pcincome + region + pcarea + pcdocs + pcbeds
##
              Df Sum of Sq
                               RSS
##
                                       AIC
## - pop65
               1 0.0000490 0.16850 -3433.7
               1 0.0000573 0.16851 -3433.7
## - pcdocs
## - unemp
               1 0.0002234 0.16867 -3433.3
## - hsgrad
               1 0.0003763 0.16883 -3432.9
## - bagrad
               1 0.0007582 0.16921 -3431.9
## <none>
                           0.16845 -3431.9
## - pcarea 1 0.0032712 0.17172 -3425.4
## - pcbeds
               1 0.0035749 0.17203 -3424.6
## - pop18
               1 0.0036248 0.17208 -3424.5
## - pcincome 1 0.0042869 0.17274 -3422.8
## - pop
               1 0.0076647 0.17612 -3414.3
## - poverty
               1 0.0139957 0.18245 -3398.8
               3 0.0312633 0.19972 -3363.0
## - region
##
## Step: AIC=-3433.74
## crime_rate ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
       pcincome + region + pcarea + pcdocs + pcbeds
##
##
##
              Df Sum of Sq
                               RSS
## - pcdocs
               1 0.0000561 0.16856 -3435.6
## - unemp
               1 0.0002518 0.16875 -3435.1
               1 0.0003617 0.16886 -3434.8
## - hsgrad
## - bagrad
               1 0.0007628 0.16926 -3433.8
## <none>
                           0.16850 -3433.7
## - pcarea
               1 0.0032245 0.17172 -3427.4
## - pcbeds
               1 0.0041665 0.17267 -3425.0
## - pop18
               1 0.0042054 0.17271 -3424.9
## - pcincome 1 0.0042628 0.17276 -3424.7
```

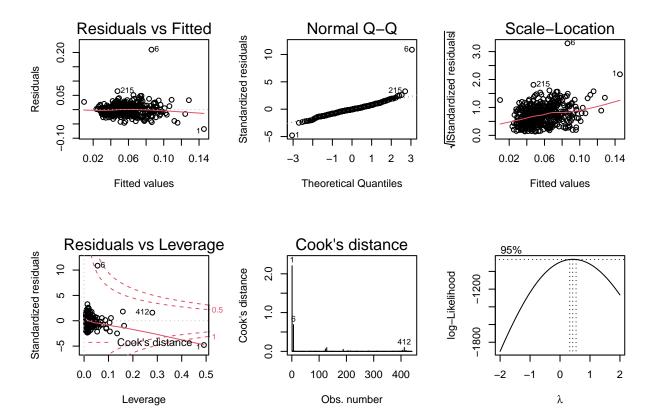
```
1 0.0077852 0.17629 -3415.9
## - pop
              1 0.0142181 0.18272 -3400.1
## - poverty
## - region
              3 0.0312244 0.19973 -3364.9
##
## Step: AIC=-3435.59
## crime_rate ~ pop + pop18 + hsgrad + bagrad + poverty + unemp +
      pcincome + region + pcarea + pcbeds
##
##
             Df Sum of Sq
                              RSS
                                      AIC
## - unemp
             1 0.0002552 0.16881 -3436.9
## - hsgrad
              1 0.0003389 0.16889 -3436.7
             1 0.0007073 0.16926 -3435.8
## - bagrad
                          0.16856 -3435.6
## <none>
## - pcarea
            1 0.0032310 0.17179 -3429.2
## - pop18 1 0.0043858 0.17294 -3426.3
## - pcincome 1 0.0045074 0.17306 -3426.0
              1 0.0078624 0.17642 -3417.5
## - pop
## - pcbeds
              1 0.0103601 0.17892 -3411.3
              1 0.0141806 0.18274 -3402.1
## - poverty
## - region
              3 0.0314610 0.20002 -3366.3
##
## Step: AIC=-3436.93
## crime_rate ~ pop + pop18 + hsgrad + bagrad + poverty + pcincome +
##
      region + pcarea + pcbeds
##
             Df Sum of Sq
                              RSS
             1 0.000250 0.16906 -3438.3
## - hsgrad
                          0.16881 -3436.9
## <none>
            1 0.000935 0.16975 -3436.5
## - bagrad
            1 0.003127 0.17194 -3430.9
## - pcarea
              1 0.004390 0.17320 -3427.6
## - pop18
## - pcincome 1 0.005080 0.17389 -3425.9
## - pop
              1 0.007769 0.17658 -3419.1
              1 0.010289 0.17910 -3412.9
## - pcbeds
              1 0.016870 0.18568 -3397.0
## - poverty
## - region
              3 0.032233 0.20104 -3366.0
##
## Step: AIC=-3438.28
## crime_rate ~ pop + pop18 + bagrad + poverty + pcincome + region +
##
      pcarea + pcbeds
##
##
             Df Sum of Sq
                              RSS
                                      ATC
             1 0.000705 0.16977 -3438.4
## - bagrad
                          0.16906 -3438.3
## <none>
              1 0.003317 0.17238 -3431.7
## - pcarea
              1 0.004209 0.17327 -3429.5
## - pop18
## - pcincome 1 0.004897 0.17396 -3427.7
## - pop
              1 0.007570 0.17663 -3421.0
## - pcbeds
              1 0.010555 0.17962 -3413.6
              1 0.024140 0.19320 -3381.6
## - poverty
              3 0.032219 0.20128 -3367.5
## - region
## Step: AIC=-3438.45
## crime_rate ~ pop + pop18 + poverty + pcincome + region + pcarea +
```

```
##
      pcbeds
##
             Df Sum of Sq
##
                              RSS
## <none>
                          0.16977 -3438.4
## - pcarea
              1 0.003619 0.17339 -3431.2
## - pop18
             1 0.004100 0.17387 -3429.9
## - pcincome 1 0.005499 0.17527 -3426.4
              1 0.008260 0.17803 -3419.5
## - pop
## - pcbeds
              1 0.010408 0.18018 -3414.3
## - poverty 1 0.024541 0.19431 -3381.0
## - region
              3 0.032233 0.20200 -3368.0
mult_fit_back
##
## Call:
## lm(formula = crime_rate ~ pop + pop18 + poverty + pcincome +
##
      region + pcarea + pcbeds, data = cdi_pc)
##
## Coefficients:
                                  pop18
## (Intercept)
                       pop
                                             poverty
                                                       pcincome
                                                                      region1
## -1.034e-02 7.962e-09
                              7.451e-04
                                           2.423e-03
                                                       1.236e-06 -2.209e-02
                                 pcarea
                                             pcbeds
##
      region2
                 region4
## -1.442e-02 -1.124e-03
                             -5.063e-01
                                           2.868e+00
crime_rate ~ pop + pop18 + poverty + pcincome + region + pcarea + pcbeds
```

Model Diagnostics

Create Residuals vs Fitted plot & Normal Q-Q plot & Scale-Location plot & Residuals vs Leverage plot to detect the normality of residuals and outliers

```
par(mfrow=c(2,3))
plot(mult_fit_back)
plot(mult_fit_back, which = 4)
bc = boxcox(mult_fit_back)
```



Diagnose the model without outliers

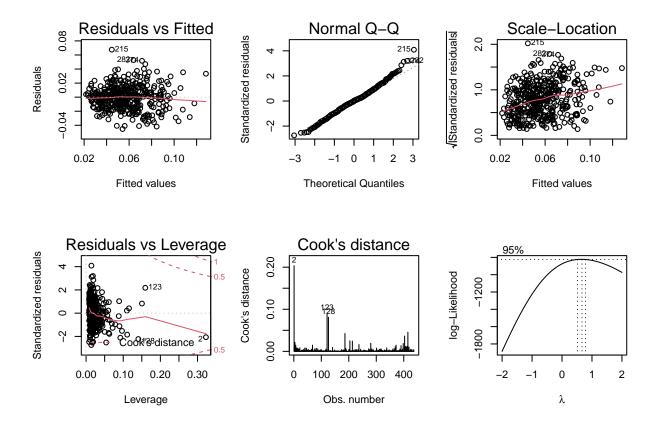
1.236e-06 3.312e-07

pcincome

```
# remove influential points
cdi_pc_out = cdi_pc[-c(1,6,412),]
# fit model with and without influential points
mult_fit_back_without = lm(crime_rate ~ pop + pop18 + poverty + pcincome + region + pcarea + pcbeds, da
summary(mult_fit_back)
##
## Call:
  lm(formula = crime_rate ~ pop + pop18 + poverty + pcincome +
       region + pcarea + pcbeds, data = cdi_pc)
##
##
## Residuals:
                          Median
##
                    1Q
                                                  Max
## -0.067811 -0.010690 -0.000725 0.010118
                                           0.209881
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.034e-02 1.060e-02
                                      -0.976 0.329595
## pop
                7.962e-09
                           1.741e-09
                                       4.574 6.26e-06 ***
                7.451e-04 2.312e-04
                                       3.223 0.001367 **
## pop18
## poverty
                2.423e-03 3.073e-04
                                       7.884 2.63e-14 ***
```

3.732 0.000216 ***

```
## region1
              -2.209e-02 2.730e-03 -8.093 6.01e-15 ***
## region2
              -1.442e-02 2.616e-03 -5.511 6.15e-08 ***
## region4
              -1.124e-03 3.188e-03 -0.352 0.724665
## pcarea
              -5.063e-01 1.672e-01 -3.027 0.002615 **
## pcbeds
               2.868e+00 5.586e-01 5.134 4.30e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.01987 on 430 degrees of freedom
## Multiple R-squared: 0.4822, Adjusted R-squared: 0.4713
## F-statistic: 44.49 on 9 and 430 DF, p-value: < 2.2e-16
summary(mult_fit_back_without)
##
## Call:
## lm(formula = crime_rate ~ pop + pop18 + poverty + pcincome +
      region + pcarea + pcbeds, data = cdi_pc_out)
##
## Residuals:
##
        Min
                   1Q
                         Median
                                      3Q
                                               Max
## -0.045291 -0.010145 0.000322 0.009630 0.067563
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.857e-03 8.932e-03 -0.432 0.666122
## pop
               1.068e-08 2.077e-09
                                    5.144 4.11e-07 ***
## pop18
               7.779e-04 1.940e-04 4.009 7.20e-05 ***
## poverty
             1.907e-03 2.635e-04
                                    7.237 2.14e-12 ***
              9.829e-07 2.839e-07 3.463 0.000589 ***
## pcincome
## region1
              -2.542e-02 2.306e-03 -11.019 < 2e-16 ***
## region2
              -1.569e-02 2.197e-03 -7.140 4.04e-12 ***
## region4
              -9.152e-04 2.711e-03 -0.338 0.735789
              -5.121e-01 1.642e-01 -3.119 0.001935 **
## pcarea
## pcbeds
              3.262e+00 4.696e-01 6.946 1.41e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.01665 on 427 degrees of freedom
## Multiple R-squared: 0.5614, Adjusted R-squared: 0.5521
## F-statistic: 60.73 on 9 and 427 DF, p-value: < 2.2e-16
# diagnose the model without outliers
par(mfrow=c(2,3))
plot(mult fit back without)
plot(mult_fit_back_without, which = 4)
bc_without = boxcox(mult_fit_back_without)
```



Box-cox transformation

```
(lambda = bc_without$x[which.max(bc_without$y)])
## [1] 0.6666667
mult_fit_back_without_trans = lm(crime_rate^0.5 ~ pop + pop18 + poverty + pcincome +
                                 region + pcarea + pcbeds, data = cdi_pc_out)
summary(mult_fit_back_without_trans)
##
## Call:
  lm(formula = crime_rate^0.5 ~ pop + pop18 + poverty + pcincome +
##
##
       region + pcarea + pcbeds, data = cdi_pc_out)
##
## Residuals:
##
                    1Q
                          Median
                                        3Q
                                                 Max
  -0.131052 -0.020616 0.001528 0.023104
                                           0.130713
##
## Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                1.087e-01 1.934e-02
                                       5.621 3.43e-08 ***
## pop
                2.226e-08 4.496e-09
                                       4.951 1.07e-06 ***
                1.608e-03 4.201e-04
                                       3.827 0.000149 ***
## pop18
```

```
## poverty
                   3.798e-03 5.705e-04
                                              6.658 8.57e-11 ***
## pcincome
                   2.093e-06
                                6.146e-07
                                              3.405 0.000724 ***
## region1
                  -5.709e-02
                                4.994e-03 -11.432 < 2e-16 ***
## region2
                  -3.463e-02
                                4.756e-03
                                             -7.282 1.59e-12 ***
## region4
                   2.682e-04
                                5.868e-03
                                              0.046 0.963566
## pcarea
                  -9.919e-01
                                3.554e-01
                                             -2.791 0.005490 **
## pcbeds
                                1.017e+00
                                              6.435 3.31e-10 ***
                   6.543e+00
##
## Signif. codes:
                      0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.03605 on 427 degrees of freedom
## Multiple R-squared: 0.547, Adjusted R-squared: 0.5374
## F-statistic: 57.28 on 9 and 427 DF, p-value: < 2.2e-16
# Diagnose the model by square root transformation
par(mfrow = c(2,3))
plot(mult_fit_back_without_trans)
plot(mult_fit_back_without_trans, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_trans)
        Residuals vs Fitted
                                               Normal Q-Q
                                                                                 Scale-Location
                                                                       (Standardized residuals)
                                   Standardized residuals
                                                              2150
    0.10
                                        0
Residuals
    0.00
                                                                           0.1
                                        0
                                        7
    -0.15
                                                                           0.0
      0.15
                         0.35
                                                   -1 0
                                                         1
                                                             2
                                                                              0.15
                                                                                       0.25
                                                                                                 0.35
                0.25
              Fitted values
                                              Theoretical Quantiles
                                                                                      Fitted values
      Residuals vs Leverage
                                             Cook's distance
Standardized residuals
                                   Cook's distance
                                                                       log-Likelihood
                                                                           -700
                                        0.10
    0
    7
              <sup>40</sup>℃ook's distance
                                        0.00
                          0.30
                                            0
                                                100
                                                    200 300
                                                                                                    2
       0.00
              0.10
                    0.20
                                                                                          0
                Leverage
                                                  Obs. number
                                                                                          λ
```

Compare the Adjusted R²

```
## # A tibble: 3 x 13
##
   model_type r.squared adj.r.squared sigma statistic p.value
                                                                      df logLik
    <chr>
                       <dbl>
                                    <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
                       0.482
                                     0.471 0.0199
                                                      44.5 3.64e-56
                                                                      9 1105.
## 1 mult_fit_back
## 2 mult_fit_back_~
                       0.561
                                     0.552 0.0167
                                                      60.7 6.54e-71
                                                                       9 1175.
## 3 mult_fit_back_~
                                     0.537 0.0360
                                                      57.3 6.01e-68
                                                                       9 837.
                       0.547
## # ... with 5 more variables: AIC <dbl>, BIC <dbl>, deviance <dbl>,
## # df.residual <int>, nobs <int>
```

region1

```
Assessing Multicollinearity
# Calculate the variance inflation factor (VIF)
check_collinearity(mult_fit_back_without)
## # Check for Multicollinearity
##
## Low Correlation
##
##
       Term VIF Increased SE Tolerance
##
        pop 1.00
                        1.00
                                   1.00
      pop18 4.82
                         2.20
                                   0.21
##
                                   1.00
##
  pcincome 1.00
                         1.00
##
     region 2.08
                         1.44
                                   0.48
                         1.40
                                   0.51
##
     pcarea 1.96
##
     pcbeds 3.92
                         1.98
                                   0.26
##
## Moderate Correlation
##
##
      Term VIF Increased SE Tolerance
  poverty 9.39
                        3.06
# Remove the variable with high VIF
mult_fit_back_without_vif = lm(crime_rate ~ pop + pop18 + pcincome + region + pcarea + pcbeds, data = c
summary(mult_fit_back_without_vif)
##
## lm(formula = crime_rate ~ pop + pop18 + pcincome + region + pcarea +
      pcbeds, data = cdi_pc_out)
##
## Residuals:
                         Median
                   1Q
## -0.051836 -0.010389 -0.000806 0.010247 0.070996
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) 2.984e-02 8.067e-03 3.699 0.000245 ***
               1.470e-08 2.118e-09 6.943 1.44e-11 ***
## pop
## pop18
              7.500e-04 2.053e-04 3.653 0.000291 ***
## pcincome
              -2.241e-07 2.431e-07 -0.922 0.357136
```

-2.898e-02 2.385e-03 -12.154 < 2e-16 ***

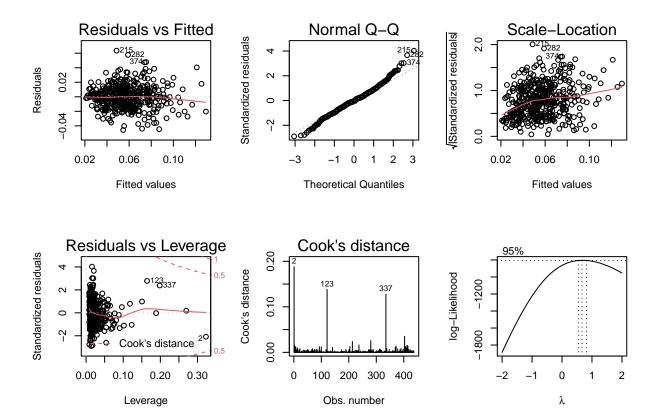
```
## region2
                  -2.015e-02 2.231e-03 -9.030 < 2e-16 ***
## region4
                  -2.505e-03
                                2.859e-03
                                             -0.876 0.381423
## pcarea
                                              -1.750 0.080901 .
                  -2.990e-01
                                1.709e-01
                                4.390e-01
                                                        < 2e-16 ***
## pcbeds
                   4.854e+00
                                              11.057
##
                       0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
## Signif. codes:
## Residual standard error: 0.01762 on 428 degrees of freedom
## Multiple R-squared: 0.5076, Adjusted R-squared: 0.4984
## F-statistic: 55.15 on 8 and 428 DF, p-value: < 2.2e-16
# Diagnose the model removing poverty term
par(mfrow = c(2,3))
plot(mult_fit_back_without_vif)
plot(mult_fit_back_without_vif, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_vif)
        Residuals vs Fitted
                                                Normal Q-Q
                                                                                   Scale-Location
                                                                        (Standardized residuals)
                                    Standardized residuals
                                                                             2.0
                                         4
Residuals
                                        \alpha
    0.00
                                         0
    -0.06
                                         7
       0.02
                       0.10
                                                              2
                                                                                0.02
               0.06
                                                           1
                                                                  3
                                                                                        0.06
                                                                                               0.10
              Fitted values
                                               Theoretical Quantiles
                                                                                       Fitted values
                                              Cook's distance
      Residuals vs Leverage
Standardized residuals
                                        0.30
                                    Cook's distance
                                                                        log-Likelihood
                                        0.15
    0
    7
                                                                             -1800
              o<sub>4</sub>Cook's distance
                                         0.00
                           0.30
              0.10
                                             0
                                                 100 200
       0.00
                    0.20
                                                          300
                                                                                 -2
                                                                                            0
                Leverage
                                                   Obs. number
                                                                                            λ
```

Add the interaction terms

```
mult_fit_back_without_int = lm(crime_rate ~ pop + pop18 + pcincome + region + pcarea + pcbeds + poverty
summary(mult_fit_back_without_int)
```

```
##
## Call:
## lm(formula = crime_rate ~ pop + pop18 + pcincome + region + pcarea +
## pcbeds + poverty + pcincome * poverty, data = cdi_pc_out)
```

```
##
## Residuals:
                         Median
##
                   1Q
## -0.045731 -0.009495 -0.000758 0.008857 0.063537
## Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
                    9.987e-03 8.802e-03
                                         1.135 0.25717
## (Intercept)
                                         3.825 0.00015 ***
## pop
                    7.785e-09 2.035e-09
                    7.308e-04 1.856e-04 3.938 9.61e-05 ***
## pop18
## pcincome
                    8.776e-08 3.048e-07
                                         0.288 0.77353
                   -2.427e-02 2.211e-03 -10.977 < 2e-16 ***
## region1
## region2
                   -1.504e-02 2.102e-03 -7.157 3.63e-12 ***
## region4
                   -3.649e-03 2.625e-03 -1.390 0.16523
## pcarea
                   -3.749e-01 1.583e-01 -2.368 0.01834 *
## pcbeds
                   1.575e+00 5.196e-01
                                          3.030 0.00259 **
                   -1.525e-03 5.891e-04 -2.589 0.00997 **
## poverty
## pcincome:poverty 2.707e-07 4.201e-08
                                         6.444 3.16e-10 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01591 on 426 degrees of freedom
## Multiple R-squared: 0.6003, Adjusted R-squared: 0.591
## F-statistic: 63.99 on 10 and 426 DF, p-value: < 2.2e-16
anova(mult_fit_back_without,mult_fit_back_without_int)
## Analysis of Variance Table
##
## Model 1: crime_rate ~ pop + pop18 + poverty + pcincome + region + pcarea +
##
      pcbeds
## Model 2: crime_rate ~ pop + pop18 + pcincome + region + pcarea + pcbeds +
      poverty + pcincome * poverty
               RSS Df Sum of Sq
##
    Res.Df
                                   F
                                      Pr(>F)
## 1
       427 0.11838
## 2
       426 0.10787 1 0.010513 41.52 3.16e-10 ***
## ---
## Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
# Diagnose the model with interaction term
par(mfrow = c(2,3))
plot(mult_fit_back_without_int)
plot(mult_fit_back_without_int, which = 4)
bc_without_trans = boxcox(mult_fit_back_without_int)
```



Compare the Adjusted R² again

```
rbind(mult_fit_back %>% broom::glance() %>% mutate(model_type = "mult_fit_back"),
      mult_fit_back_without %>% broom::glance() %% mutate(model_type = "mult_fit_back_without"),
      mult_fit_back_without_trans %% broom::glance()%>% mutate(model_type = "mult_fit_back_without_trans")
      mult_fit_back_without_vif %>% broom::glance() %>% mutate(model_type = "mult_fit_back_without_vif"
      mult_fit_back_without_int %>% broom::glance() %% mutate(model_type = "mult_fit_back_without_int"
      dplyr::select(model_type, everything())
## # A tibble: 5 x 13
##
     model_type
                     r.squared adj.r.squared sigma statistic
                                                                p.value
                                                                           df logLik
                                               <dbl>
     <chr>
                         <dbl>
                                        <dbl>
                                                         <dbl>
                                                                  <dbl> <dbl>
                                                                                <dbl>
##
                         0.482
## 1 mult_fit_back
                                        0.471 0.0199
                                                          44.5 3.64e-56
                                                                            9
                                                                               1105.
## 2 mult fit back ~
                         0.561
                                        0.552 0.0167
                                                          60.7 6.54e-71
                                                                            9
                                                                               1175.
                                                                                837.
## 3 mult_fit_back_~
                         0.547
                                        0.537 0.0360
                                                          57.3 6.01e-68
                                                                            9
```

55.1 3.21e-61

64.0 1.71e-78

8

1149.

10 1195.

0.600 ## # ... with 5 more variables: AIC <dbl>, BIC <dbl>, deviance <dbl>,

0.508

df.residual <int>, nobs <int>

4 mult_fit_back_~

5 mult_fit_back_~

0.498 0.0176

0.591 0.0159

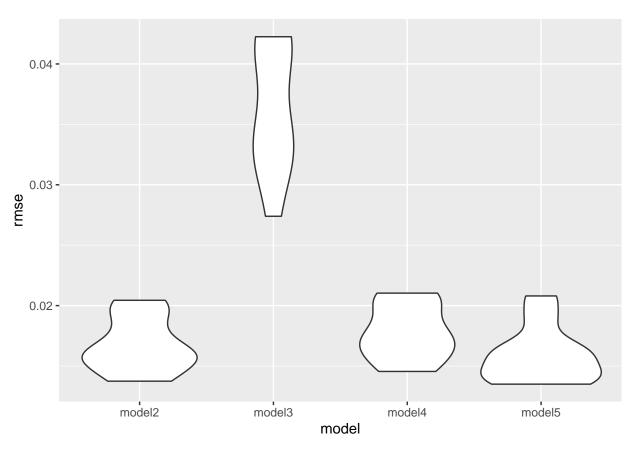
Model Validation

adjR_model4 0.49859335 ## adjR model5 0.59781245

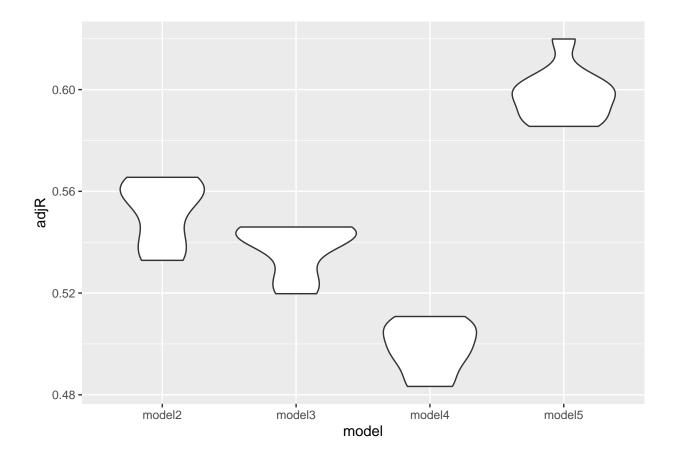
Compute RMSE and adjusted R² by cross-validation

```
set.seed(1234)
cv_df =
  crossv kfold(cdi pc out, k = 10) %>%
  mutate(
   train = map(train, as_tibble),
   test = map(test, as tibble)) %>%
  mutate(
   mult_fit_back_without = map(train, ~lm(crime_rate ~ pop + pop18 + poverty +
                                pcincome + region + pcarea + pcbeds, data = .x)),
   mult_fit_back_without_trans = map(train, ~lm(sqrt(crime_rate) ~ pop + pop18 + poverty +
                                      pcincome + region + pcarea + pcbeds, data = .x)),
   mult_fit_back_without_vif = map(train, ~lm(crime_rate ~ pop + pop18 +
                                    pcincome + region + pcarea + pcbeds, data = .x)),
   mult_fit_back_without_int = map(train, ~lm(crime_rate ~ pop + pop18 + pcincome + region + pcarea +
  mutate(
   rmse_model2 = map2_dbl(mult_fit_back_without, test, ~rmse(model = .x, data = .y)),
   rmse_model3 = map2_dbl(mult_fit_back_without_trans, test, ~rmse(model = .x, data = .y)),
   rmse_model4 = map2_dbl(mult_fit_back_without_vif, test, ~rmse(model = .x, data = .y)),
   rmse_model5 = map2_dbl(mult_fit_back_without_int, test, ~rmse(model = .x, data = .y))) %>%
   res_model2 = map(mult_fit_back_without, broom::glance %>% as.data.frame),
   res_model3 = map(mult_fit_back_without_trans, broom::glance %>% as.data.frame),
   res_model4 = map(mult_fit_back_without_vif, broom::glance %>% as.data.frame),
   res_model5 = map(mult_fit_back_without_int, broom::glance %>% as.data.frame))%>%
  unnest(res_model2, res_model3, res_model4, res_model5) %>%
  dplyr::select(rmse_model2,rmse_model3,rmse_model4,rmse_model5,
                value.adj.r.squared, value.adj.r.squared1, value.adj.r.squared2, value.adj.r.squared3) %>%
  rename(adjR_model2 = value.adj.r.squared,
         adjR_model3 = value.adj.r.squared1,
         adjR_model4 = value.adj.r.squared2,
         adjR_model5 = value.adj.r.squared3)
cv_df %>%
  summarise_each(funs(mean( .,na.rm = TRUE))) %>%
##
                     [,1]
## rmse_model2 0.01673048
## rmse_model3 0.03621843
## rmse_model4 0.01777271
## rmse_model5 0.01593793
## adjR_model2 0.55207433
## adjR_model3 0.53745725
```

Plot the violin plot



```
unnest_cd_df %>% ggplot(aes(x = model, y = adjR)) + geom_violin()
```



Thanks for reading!