Range-Based Subsidies and Product Upgrading of Battery Electric Vehicles in China

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April 23, 2023

The Latest Version

Abstract

This paper estimates the impact of notched driving-range-based (DRB) subsidies to consumers on Chinese battery-electric-vehicle (BEV) manufacturers' incentives to reduce their production costs of driving ranges. Chinese consumers received generous subsidies if the driving ranges were above certain thresholds. Using a dynamic structural model to infer unobserved investment decisions on the cost reduction of driving ranges by manufacturers, I find that the discontinuous incentives around the range thresholds of Chinese DRB subsidies increased Chinese BEV manufacturers' probability of investing in reducing production costs of driving ranges in 2019 by 20 to 35 percentage points. Compared with counterfactual DRB subsidies that are linear in driving ranges and provide the same total amount of subsidy for a BEV model, the notched scheme is especially effective in incentivizing investment by manufacturers with high production costs of ranges. This dynamic impact on production costs implies that the environmental benefits and welfare gains of notched DRB subsidies are very likely larger than the estimates of existing literature. It also implies that notched subsidies can be effective in inducing technological adoption or product upgrading.

Keywords: Industrial policy, product subsidy, technological catch-up, electric vehicle, dynamic discrete choice model

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1 Introduction

This paper estimates the impact of driving-range-based (DRB) subsidies to consumers on Chinese battery-electric-vehicle (BEV)[1] manufacturers' incentives to reduce their production costs of driving ranges. Under the pressure of negative growth of automobile sales due to the 2008 financial crisis, the Chinese government implemented a package of policies in 2009 to revitalize its automobile industry by promoting innovation by and competitive advantages of its domestic EV manufacturers.[2] The DRB subsidies to EV consumers are a crucial part of this ambition.[3] However, little is known whether and how the DRB subsidies contribute to the 14 times increase in Chinese EV sales in 2016-2022 (Chinese EV sales accounts for 65% of the global EV sales in 2022) and its domestic producers' newly-acquired presence in the global market in 2022.[4] More specifically, it is unclear whether the Chinese DRB subsidies induced Chinese EV manufacturers to actively reduce their production costs of driving ranges, such as building more energy-efficient models or improving their supply chains of batteries, or whether the manufacturers only responded by installing more batteries and building smaller cars to increase driving ranges.

Using a dynamic structural model to explain and infer firms' unobserved decisions regarding the cost reduction of driving ranges, I find that Chinese DRB subsidies, which offer a generous amount of subsidies to consumers once the driving ranges are above certain range thresholds, i.e. a notched subsidy scheme, increased Chinese BEV manufacturers' probability of investing in reducing production costs of ranges in 2019 by 20 to 35 percentage points relative to the scenarios of no subsidies. Compared with counterfactual DRB subsidies that are linear in driving ranges and provide the same total amount of subsidy for a BEV model, the notched scheme is especially effective in incentivizing investment by manufacturers with high production costs of ranges. This result implies that the environmental benefits and welfare gains of notched DRB subsidies are very likely larger than the estimates in existing literature that ignores this channel of reducing future production costs of ranges. The machinery built to estimate the impact on the production costs of ranges offers a possible tool for future research on devising better or even optimal DRB subsidies to incentivize cost reductions over ranges.

[1]BEVs are a type of electric vehicles who relies exclusively on rechargeable battery packs, without a secondary source of propulsion such as an internal combustion engine.

[2]The announcement is aimed at new energy vehicles, which are mainly BEVs and plug-in hybrid electric vehicles. Source: http://www.gov.cn/zwgk/2009-03/20/content_1264324.htm

[3]Source: https://www.chinanews.com.cn/ny/2010/10-13/2583130.shtml

[4]Sources: https://www.marklines.com, https://www.ev-volumes.com/, and http://ex.chinadaily.com.cn/exchange/partners/82/rss/channel/language/columns/v0m20b/stories/WS6389a97da31057c47eba25e3.html

From 2016 to 2020, BEVs in China became cheaper with longer ranges and used higher-density batteries, but there was no clear trend of a decline in BEV model sizes. Furthermore, there is large variation in prices and driving ranges in each given year, and the trend of changing prices and ranges varied across models and manufacturers. Underlying changes in battery technology alone cannot explain the variation. Possible explanations for the variation are different BEV model characteristics such as whether it is a high-end model, manufacturers' capability of exercising market power, and manufacturers' decisions to reduce their production costs of ranges. Manufacturers can reduce their production costs by designing more energy efficient models, so that the same BEV model with the same battery capacity can travel a longer distance. Manufacturers can also reduce their production costs by improving their exposure to the technological progress in batteries, such as sourcing better battery suppliers, vertically integrating with battery suppliers, or carrying out in-house R&D. I use a dynamic structural model to disentangle the channel of a reduction in the production costs of ranges from the remaining channels so that I can quantify the impact of Chinese DRB subsidies on manufacturers' decisions to reduce the production costs of ranges. I focus on BEVs because they are the most targeted EVs by Chinese DRB subsidies and have the largest sales among all the EVs.

In my structural model, homogeneous consumers make their purchase decisions in each period based on observed and unobserved model characteristics, and unobserved consumer-model-period-specific taste shocks. They can also choose not to buy a car. Manufacturers maximize their expected sum of current and discounted future profits by choosing the optimal prices and ranges in each period and make dynamic decisions on whether to reduce the production costs of ranges in the next period under an adaptive expectation about future prices and ranges of other BEV models and future subsidy schemes. If they decide to reduce the production costs, they pay an investment cost in this period. Their production costs of ranges are lower in the next period and will stay at this value until further investments are made in future periods. I assume that firms have rational expectations about the current prices and ranges and observe the current subsidy scheme, but have adaptive expectations about future prices, ranges, and subsidy schemes because predicting what happens in the current year is likely easier and more reliable than predicting what will happen in all future years with an infinite horizon. In the rapidly growing BEV industry, it is probably already difficult to predict what happens next year.

I follow Berry et al. (1995) and Crawford et al. (2019) by assuming that unobserved consumer-model-period-specific taste shocks are independent and identically distributed so that I can use BEV-model-level market shares in each year to estimate parameters of consumer preferences. One period in the structural model is one year in the data. I parameterize manufacturers' production costs as linear in exogenous observed and unobserved model characteristics and a polynomial of the endogenous control variable, the driving range. The first order conditions of manufacturers' profit maximization together with BEV model prices and consumers' preference parameters give the estimated production cost functions. To account for endogeneity caused by the unobserved model characteristics in both demand and supply, I construct instruments following Berry et al. (1995).

Since the driving range is an endogenous control variable, my estimation produces BEV-model-year-specific cost parameters of ranges like Crawford et al. (2019). I infer manufacturers' investment decisions from the changes in the cost parameters of ranges. A reduction in a BEV models' cost parameters this period means that its manufacturer invested in reducing the production cost of ranges in the previous period for this BEV model. A manufacturer invests when the increase in its expected discounted sum of future profits is larger than the investment cost. If a manufacturer has multiple BEV models, the manufacturer makes individual investment decisions for each of its BEV models not taking into account the impact of the investment decisions on its other BEV models. The investment cost is unobserved, and is estimated from the inferred investment decisions and that manufacturers maximize their expected sum of discounted profits under their beliefs about the future. The investment cost is assumed to be constant across firms and time.

Estimating the investment cost requires solving a non-stationary dynamic problem with an infinite time horizon for each BEV model. It's non-stationary because the subsidy scheme changes over time. It has an infinite time horizon because firms are assumed to live forever. To solve these dynamic problems, I discretize the estimated cost parameters of ranges and assume that the space of the cost parameters is a finite set with a known lower bound, that the cost parameters of ranges decrease by 1 unit when invest and otherwise stay constant, and that there are only a finite amount of possible subsidy schemes. These assumptions allow me to solve for the optimal investment policies in the dynamic problems by essentially recursion over the state space of the cost parameters from the lowest value to the highest, similar to the solution for finite-state directional dynamic games proposed by Iskhakov et al. (2016). To simplify the dynamic problem, I assume firms to have an extreme type of adaptive expectations about the future in the sense that the prices and ranges of other models and the subsidy scheme are believed to stay forever at their current values. This simplifies the optimal investment policies for each model in each period to the solution to an auxiliary single-agent stationary dynamic problem. Using these optimal investment policies and the inferred investment decisions, I can estimate the investment cost using maximum likelihood estimation, similar to Rust (1987)'s estimation for the bus engine replacement problem. However, Rust (1987) observes the dynamic decisions, i.e. whether an engine is replaced, while I infer the dynamic decisions from the estimation of the static part of the problem. I set the lower bound of the state space loose enough so that firms won't be able to reach it in the near future. The intuition behind this known and loose lower bound is that predicting the technology frontier over 50 or 100 years is difficult and, most of the time, manufacturers cannot foresee new technology in the distant future. It can also reflects technology bottlenecks that manufacturers in the current period can not credibly foresee themselves overcoming.

Applying my structural model to the data, which is a newly constructed rich dataset that contains national-level vehicle model sales in 2010-2021 and the technical description of all vehicle model variations available or once available on the market,[5] I find that the 400-kilometer threshold in the existing notched investment scheme in 2019 increased all the BEV manufacturers' investment probabilities by 20 to 35 percentage points. The investment probabilities for BEV models that currently do not qualify for subsidies because their ranges are below the threshold are also higher compared to the scenario of no subsidy scheme because of the perspective of a large jump in profits. For these BEV models, the notched subsidy scheme performs especially well when compared with linear subsidy schemes because linear subsidy schemes cannot incentivize investment unless the BEV models receive some positive amount of subsidy while, under the notched scheme, manufacturers are incentivized to invest when receiving no subsidy in the current period.

I assume that consumers are homogeneous. Although consumers' heterogeneous tastes and price sensitivity may contribute to the dispersion in prices and ranges in the sense that different models may target different types of consumers, assuming away consumer heterogeneity allows for analytical theoretical results, provide better intuition, and reduces the computational burden in the dynamic part. I argue that compared with the technological progress in electric vehicles, consumers' tastes are generally stable over time. Therefore, consumer heterogeneity cannot explain the variation in the model-specific trends in ranges and prices. They are used for identifying the changes in the production costs of ranges and the investment cost.

This paper provides the first empirical evidence that notched attribute-based subsidies can produce efficiency gains. Existing literature on attribute-based subsidies, such as Ito and Sallee (2018) and Jia et al. (2022), almost universally criticizes notched subsidy schemes for the distortions they create around the thresholds and therefore recommend against notched ones for the sake of efficiency. My results show that the discontinuity in firms' profits around the thresholds can incentivize firms to put more effort in reducing production costs

[5]The sources are Chezhu Home (https://www.16888.com) and Auto Home (https://www.autohome.com.cn)

and consequently creates efficiency gains.

This paper contributes to the literature on the effects of industrial policies on innovation and technological progress by providing empirical evidence on whether and how subsidies on the demand side can encourage innovation and technological upgrading. Studies have shown that reducing the costs of investment can raise R&D (Takalo et al. (2013) and Criscuolo et al. (2019)) because this increases the net returns to investment. In theory, such an increase in net returns can also be achieved by promoting demand. The field experiments in Bold et al. (2022) show that higher demand can encourage farmers' technological adoptions, suggesting that policies targeting the demand side can be effective. However, there is no empirical evidence that the same holds in the EV market and this paper fills that gap.

The methodological contribution of this paper is to combine Berry et al. (1995) and Crawford et al. (2019)'s model for structurally estimating demand and supply using aggregate data, such as automobile models' annual sales at the national level rather than individual consumer's decisions, with the literature of dynamic discrete choice models (Rust (1987)) where structural model primitives of dynamic decisions are estimated using data on observed individual decisions, so that one can do counterfactual analysis that involves dynamic decision making without observing the individual dynamic decisions.

The remainder of the paper is organized as follows. I present historical background and stylized facts in Section 2. I then introduce the model and demonstrate theoretical results in Section 3. I explain the estimation procedure in Section 4 and describe the dataset in Section 5. Results are provided in Section 6. Section 7 concludes.

2 Historical background and stylized facts

China's ambition of improving its global presence in automobile manufacturing can date back to 1980s. In spite of large amount of resources spent in the automobile industry, China is far from gaining prominence in the global automobile manufacturing till 2020s. In 2012, China decided to pursue its ambition by promoting electric vehicle (EV) manufacturing and regard its EV industry as a way to dominate the world automobile industry. To achieve this goal, the Chinese government unleashed a series of industrial policies and one import part of these policies is generous range-based subsidies to consumers to guide the domestic EV manufacturers' product upgrading. The result is Chinese domestic EV sales increased from 8,200 in 2011 to 3.5 million in 2021. Chinese domestic EV manufacturers also start to gain global presence in recent years, for example BYD's EV export increased by about 10 times from November, 2020 to November, 2021 (China Association of Automobile Manufacturers).

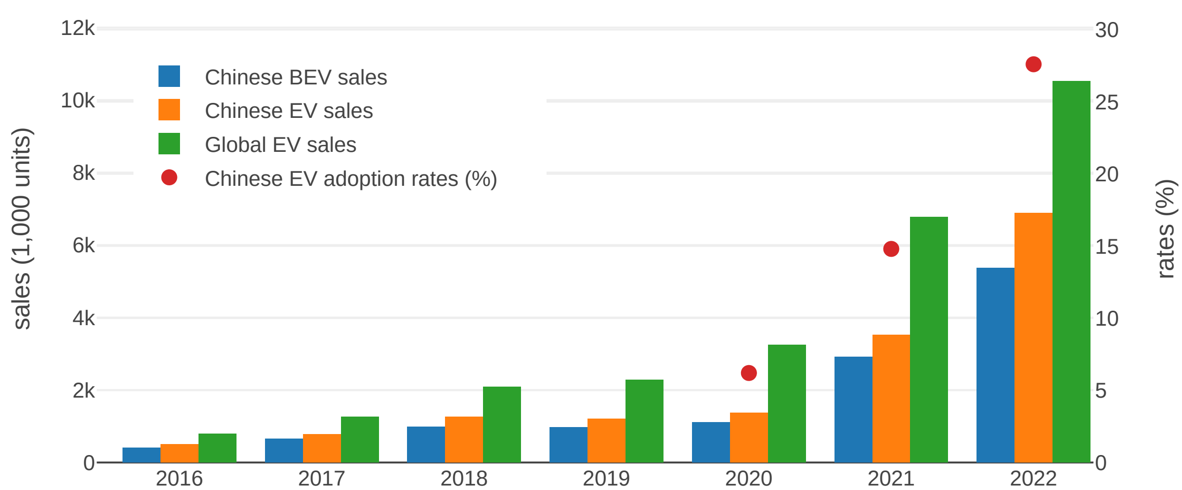
Table 1 demonstrates the range-based subsidies to consumers purchasing battery electric

Table 1: Range-based subsidy for BEV (10,000 RMB)

range (km)	2016	2017	2018	2019	2020	2021
$(-\infty, 100)$	0	0	0	0	0	0
$[100, 150)$	2.5	2	0	0	0	0
$[150, 200)$	4.5	3.6	1.5	0	0	0
$[200, 250)$	4.5	3.6	2.4	0	0	0
$[250, 300)$	5.5	4.4	3.4	1.8	0	0
$[300, 400)$	5.5	4.4	4.5	1.8	1.62	1.3
$[400, \infty)$	5.5	4.4	5	2.5	2.25	1.8

vehicles (BEVs). Here I only list the subsidies of BEVs because they are the focus of this paper. It is a notched subsidy scheme because the amount of subsidies consumers receive is a discontinuous function of the driving ranges. Both the thresholds of the driving ranges and the amount of subsidies for a given threshold change over time. In general, the thresholds become higher and the subsidies become smaller. While the subsidies that consumers receive decline in 2016-2021, the annual sales of BEV in China increases as shown in Figure 1.

Figure 1: Trends of BEV and EV sales

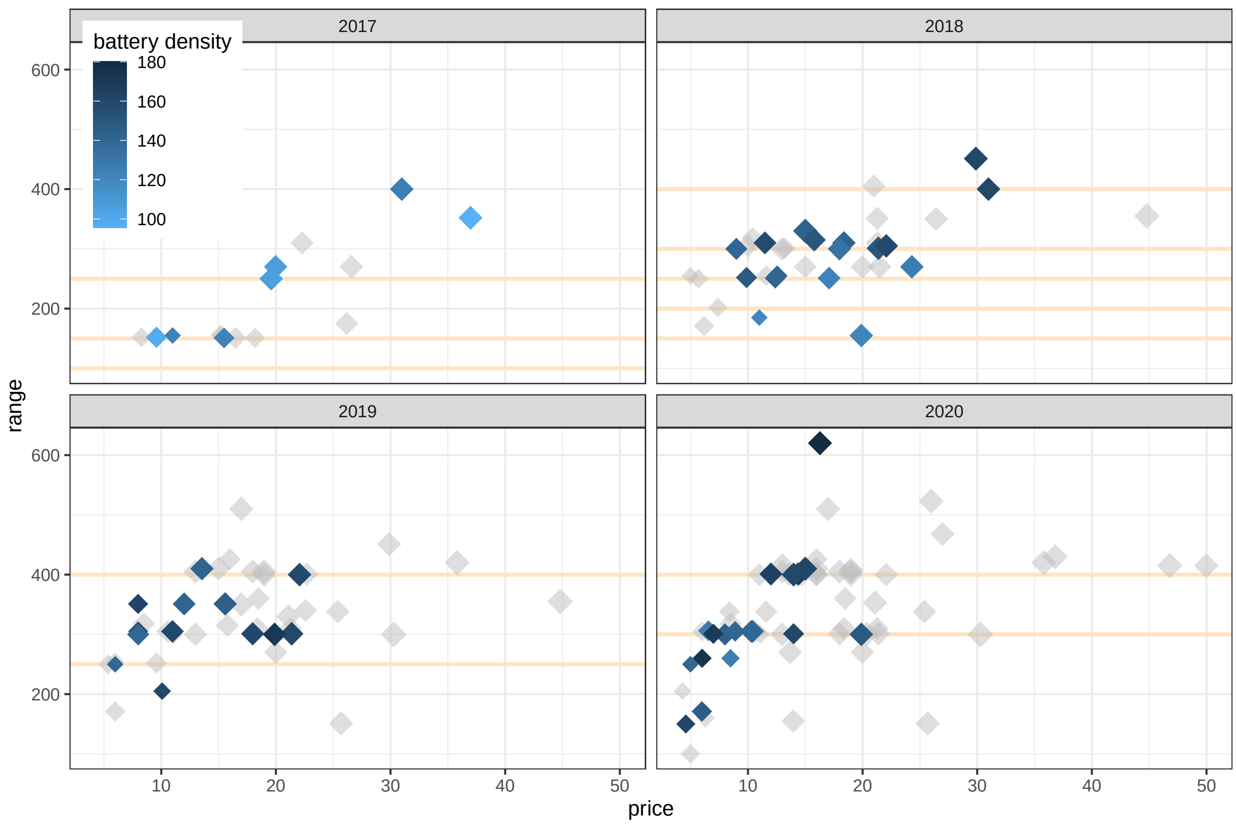


Source: The Chinese annual sales of BEVs and EVs are from www.marklines.com. Click on the link and then select the relevant years. Chinese adoption rates in 2021 and 2022 are from CPCA. Those in 2020 are from www.ce.cn. Global EV sales are from www.ev-volumes.com.

Figure 2 demonstrates prices (10,000 RMB), ranges (km), model sizes (length (m) ×

width (m) × height (m)), and battery density (battery capacity/battery weight) of new BEV models or new releases of existing BEV models in 2017-2020. I do not include 2016 because there were too few BEV models. The orange lines are the thresholds introduced in that year. Each diamond represents a BEV model. This figure shows that BEV models become cheaper with longer ranges, higher-density batteries, but almost no change in sizes, and that there is large variation in the trend of prices and ranges over the years.

FIGURE 2: Trends of BEV attributes in China



Notes: the sizes of the diamonds are the sizes of the BEV model. Color gray means no information on battery density. Orange lines are the range thresholds of subsidies.

There are several possible reasons behind the large variation: differences in BEV model characteristics other than ranges, different markups in the model prices, and different production costs of ranges. Examples of different production costs of ranges are how energy efficient are model designs, different access to existing battery technology, or varying level of difficulties in adopting higher density battery. In the next two sections, I will build a model that takes into account all these channels and disentangle the channel of differences in production costs of ranges from the remaining channels and then quantify how firms' production costs of ranges respond to the consumer subsidies.

3 Model

We start with the static part of the structural model which is similar to the setup in Berry et al. (1995) and Crawford et al. (2019). Starting from this section, a manufacturer is address as a firm. To separate the usage of model as a vehicle model from the usage as a structural model, a vehicle model is addressed as a product.

3.1 Demand

Consumers maximize their indirect utility by deciding whether to purchase a product, i.e. a car, and which product to purchase if they decide to purchase one. If consumer i chooses to buy a product j from firm f in the year t, the indirect utility from such purchase is U_{ijft}, which consists of the mean utility of purchasing this product δ_{jft} and the consumer i's taste shock for this product in the year t, ϵ_{ijft}. Since consumers are homogeneous, the mean utility is product-year specific and is the same for all the consumers. If the consumer chooses not to purchase a product or, in other words, to choose an outside option, the indirect utility is U_{i0t}, whose mean utility is normalized to 0 and the taste shock for the outside option is ϵ_{i0t}. ϵ_{ijft} and ϵ_{i0t} are independent and identically distributed Type-I extreme values with mean zero.

$$\begin{cases} U_{ijft} = \delta^{jft} + \epsilon_{ijft} & \text{for product } j \text{ from manufacturer } f \text{ at time } t \\ U_{i0t} \ \ = \epsilon_{i0t} & \text{otherwise} \end{cases}$$

where δ_{jft} is assumed to be:

$$\delta_{jft} = \alpha * p_{jft} + \beta * x_{jft} + \xi_{jft}$$

α and β are parameters of consumers' price sensitivity and tastes for observed model characteristics, x_{jft}. x_{jft} is a vector that includes sizes, power-weight ratios, torques, luxury levels, whether a model is released this year, and years since the model is released. Model-and-year two-way fixed effects are included in x_{jft} during estimation but are omitted in the formula here for the ease of demonstration. For EV cars, x_{jft} also include driving ranges and consumers' preference parameter of ranges is β^R. ξ_{jft} is consumers' utility derived from product j's unobserved characteristics in the year t.

Demand for product j from firm f in the year t is

$$N_t \cdot \frac{\exp(\delta_{jft})}{1 + \sum_{j,f} \exp(\delta_{jft})}$$

where N_t is the total number of households in the year t, which is my measure of the number of consumers considering whether to purchase a car in the year t and, if so, which model to purchase. This measure of the total number of consumers is the same as Berry et al. (1995). This product's market share is

$$s_{jft} = \frac{\exp(\delta_{jft})}{1 + \sum_{j,f} \exp(\delta_{jft})}$$

3.2 Supply

Firm f's profits at time t, π_{ft}, are the sum of all its products' profits. The set of firm f's products at time t is $\mathcal{J}_{ft}$.

$$\pi_{ft} = \sum_{j \in \mathcal{J}_{ft}} N_t s_{jft}(p_{jft} - mc_{jft} + \tau(R_{jft}))$$

where N_t is the total amount of household, s_{jft} is product j's market share, p_{jft} is the price, and $\tau(R_{jft})$ is the subsidy product j receives. $\tau(\cdot)$ represents the subsidy scheme which firms face and it specifies the amount of subsidy a product can receive based on its range R_{jft}. The marginal cost functions are parametrized as linear in exogenous model characteristics and the endogenous range squared:

$$mc_{jft} = c_{jft} R_{jft}^2 + \mu * w_{jft} + \eta_{jft}$$

If a model is a gasoline vehicle, then range R_{jft} does not enter the marginal cost functions. w_{jft} represents the observed covariates, which include power-weight ratio, size, luxury level, mpg(e) and torque. η_{jft} is the unobserved model characteristics that affect production costs.

Firms maximize the sum of their current and discounted expected future profits by choosing the optimal prices and ranges in each period, and by deciding whether to pay a fixed amount of investment cost λ for each of its product this year, i.e. period t, to reduce the cost parameter of ranges, $c_{jf,t+1}$, of the products next year. The cost parameter of range c_{jft} takes value from a known finite set $\mathcal{C} = \{c_1, c_2, \cdots, c_L\}$ that satisfies $c_{l-1} < c_l$ and $\log(c_l) - \log(c_{l-1})$ is constant for $l \in \{2, 3, ..., L\}$. This means it is more difficult to reduce c_l when c_l is small. If a model's current c_{jft} takes the value c_l and this firm invests, then in the next period, $c_{jf,t+1} = c_{l-1}$. Because c_{jft} can either stay constant or become smaller, this assumption of $\mathcal{C}$ being a finite set allows me to solve the dynamic problem using backward induction over the state space $\mathcal{C}$ following Iskhakov et al. (2016). Since prices and ranges are set in every period, they are static optimization problems. The first order conditions (FOCs) of prices and ranges are:

$$\frac{\partial \pi_{ft}}{\partial p_{jft}} = s_{jft} + \sum_{k \in \mathcal{J}_f} (p_{kft} - mc_{kft} + \tau(R_{kft})) \frac{\partial s_{kft}}{\partial p_{jft}} = 0 \tag{1}$$

$$\frac{\partial \pi_{ft}}{\partial R_{jft}} = s_{jft} \left(-\frac{\partial mc_{jft}}{\partial R_{jft}} + \frac{\partial \tau(R_{jft})}{R_{jft}} \right) + \sum_{k \in \mathcal{J}_f} (p_{kft} - mc_{kft} + \tau(R_{kft})) \frac{\partial s_{kft}}{\partial R_{jft}} = 0 \tag{2}$$

The FOCs can be written using matrix notation:

$$0 = \vec{s}_t + \Omega_t^P * (\vec{p}_t - \vec{mc}_t + \tau(\vec{R}_t))$$

$$0 = \text{diag} \left\{ -\frac{\partial mc_{jt}}{\partial R_{jt}} + \frac{\partial \tau(R_{jt})}{R_{jt}} \right\}_{j \in \mathcal{J}_{ft} \text{ ,for } f \in \mathcal{F}_t} * \vec{s}_t + \Omega_t^R * (\vec{p}_t - \vec{mc}_t + \tau(\vec{R}_t))$$

where $\vec{s}_t$, $\vec{p}_t$, $\vec{mc}_t$, and $\vec{R}_t$ are vectors of all the models' market shares, prices, marginal costs, and ranges at time t. $\mathcal{F}_t$ is the set of all the firms at time t. $\Omega_t^P(j,k) = \Omega_t^*(j,k) * \frac{\partial s_{kt}}{\partial p_j t}$, $\Omega_t^R(j,k) = \Omega_t^*(j,k) * \frac{\partial s_{kt}}{\partial R_j t}$. Ω_t^* is an ownership matrix and is defined as

$$\Omega_t^*(j,k) = \begin{cases} 1 \text{ , if } j,k \in \mathcal{J}_{ft} \text{ for some } f \\ 0 \text{ , otherwise} \end{cases}$$

The FOCs contain Ω_t^P and Ω_t^R because firms take into account the impact on all of its own products when setting the prices and ranges. Using the market shares from the demand model, I can derive the responses of market shares to changes in prices and ranges are:

$$\frac{\partial s_{kjt}}{\partial d_{jft}} = \begin{cases} -s_{kft} s_{jft} \frac{\partial \delta_{jft}}{\partial d_{jft}} \text{ , if } k \neq j \\ (1 - s_{jft}) s_{jft} \frac{\partial \delta_{jft}}{\partial d_{jft}} \text{ , if } k = j \end{cases}$$

and $d_{ift} \in \{p_{ift}, R_{ift}\}$, $\frac{\partial \delta_{ift}}{\partial p_{ift}} = \alpha$, $\frac{\partial \delta_{ift}}{\partial R_{ift}} = \beta^R$.

Plug the formulas of $\frac{\partial s_{kjt}}{\partial p_{jft}}$ and $\frac{\partial s_{kjt}}{\partial R_{jft}}$ into Equation (1) and (2):

$$
\begin{aligned}
0 =& 1 + \sum_{k \in \mathcal{J}_{ft}, k \neq j} (p_{kft} - mc_{kft} + \tau(R_{jft})) \cdot (-s_{kft}\alpha) \\
& + (p_{jft} - mc_{jft} + \tau(R_{jft})) \cdot (1 - s_{jft})\alpha \\
0 =& \left(-\frac{\partial mc_{jft}}{\partial R_{jft}} + \frac{\partial \tau(R_{jft})}{\partial R_{jft}}\right) + \sum_{k \in \mathcal{J}_{ft}, k \neq j} (p_{kft} - mc_{kft} + \tau(R_{jft})) \cdot (-s_{kft}\beta^R) \\
& + (p_{jft} - mc_{jft} + \tau(R_{jft})) \cdot (1 - s_{jft})\beta^R
\end{aligned}
$$

Combining these two equations and using the parametric form of marginal costs give:

$$
-2c_{jft}R_{jft} + \tau'(R_{jft}) - \frac{\beta^R}{\alpha} = 0
$$

where $\tau'(R^*_{jft}) = \frac{\partial \tau(R_{jft})}{\partial R_{jft}}$. When $\tau(\cdot)$ is discontinuous, such as the case of a notched subsidy scheme, the optimal range may not satisfy the FOCs because $\tau'(\cdot)$ does not exist at the thresholds. For notched schemes, $\tau'(R_{jft}) = 0$ when R_{jft} is not at the thresholds. Therefore, for notched scheme, I first find the ranges that satisfy the FOCs where $\tau'(R_{jft}) = 0$. If this interior solution's ranges are smaller than the thresholds, I calculate the profits at these interior solutions' ranges and the profits if firms set their ranges at the range thresholds, i.e. the corner solution. The one that gives higher profits is the optimal range.

If the market share of each individual product is very close to zero, i.e. $s_{ijt} \approx 0$, which turns out to be the case in my data, the right-hand side of the FOCs are approximated as:

$$
\begin{aligned}
0 \approx& 1 + (p_{jft} - mc_{jft} + \tau(R_{jft})) \cdot \alpha \\
0 \approx& -\left(\frac{\partial mc_{jft}}{\partial R_{jft}} + \frac{\partial \tau(R_{jft})}{R_{jft}}\right) + (p_{jft} - mc_{jft} + \tau(R_{jft}))\beta^R
\end{aligned}
$$

So firm j's optimal profits and optimal prices are approximately

$$
\pi^*_{ft} \approx -\frac{\sum_{j \in \mathcal{J}_{ft}} s^*_{jft}}{\alpha} \tag{3}
$$

$$
p^*_{jft} \approx -\frac{1}{\alpha} + c_{jft}(R^*_{jft})^2 + \mu * w_{jft} + \eta_{jft} - \tau(R^*_{jft}) \tag{4}
$$

where s^*_{jft} is the market share when all the firms choose their optimal prices p^*_{jft} and ranges R^*_{jft}. From these equations, it can be seen that firms optimal profits, prices, and ranges in each period are functions of $\vec{c}_t$ and all the remaining model characteristics. Since the remaining model characteristics are exogenous and taken as given, we express the optimal profits, prices, and ranges as $\pi_t^*(\vec{c}_t)$, $p_t^*(\vec{c}_t)$, and $R_t^*(\vec{c}_t)$.

In the next section, I will explain firms' investment decisions on reducing the cost parameters of ranges. Since all the profits, prices, ranges in the next section are the optimal values, I drop the $*$ in the notation.

3.3 Firms' dynamic investment problems

Firm f will invest in reducing c_{jft} for its product j if the expected returns to investment according to firm f's belief about the future market conditions are higher than the costs of investment. The returns to investment on product j are the increase in the sum of this product's expected discounted profits with an infinite time horizon. The investment decision on product j does not take into account the impact on the profits of f's other products. When considering investment for product j, firm f believes the prices and ranges of all the other products both inside and outside firm j in future periods will remain at their current values and that the exogenous product characteristics of all products including j will stay constant. Firm f also believes that the future subsidy scheme and the future total number of consumers will remain the same as the current one. In each period, firms update their product-specific beliefs after observing that period's prices, ranges, exogenous product characteristics, number of consumers, and subsidy scheme.

These firm-product-specific beliefs about future market conditions are a special case of adaptive expectations where future values are believed to be a linear function of historical values.[6] In my case, future values are believed by firms to equal the current values, so the weights on historical values in standard adaptive expectations are set to 0. I deviate from rational expectations in dynamic games to avoid equilibrium and identification problems of dynamic games among heterogeneous agents with rational expectations and because of the theoretical and empirical evidence showing that rational expectations have difficulties in rationalizing empirical data (Pesaran (1989), Manski (2004), and D'Haultfoeuille et al. (2018)). Putting the dynamic part and the static part of the structural model together, firms' beliefs about other products' prices and ranges in the current period are rational expectations, whereas beliefs about future prices and ranges are adaptive expectations. This reflects the critique about rational expectations that firms' usually lack information or capability to make accurate predictions. The intuitions behind my assumptions about firms' beliefs are that predicting what happens in the current year is likely easier and more reliable than predicting what will happen in all future years with an infinite horizon. In fact, it is sometimes

[6] Adaptive expectations are widely used in studies on inflation and monetary policies as well some applications for oligopoly (Okuguchi (1970) studies stability of oligopoly equilibrium under adaptive expectation) and firms' responses to technology shocks (Huang et al. (2009)).

difficult to even predict what happens next year in a rapidly growing industry. Besides, it has been reported that the changes in Chinese BEV subsidies came out as surprises to firms in some years. In addition, assuming rational expectations is a common practice in static empirical structural models and adaptive expectations are a widely-used alternative to rational expectations to guarantee existence of a unique equilibrium. The beliefs that all the other products' prices and ranges are constant means the investment decision for each model is a single-agent dynamic-discrete-choice problem.

Since all model primitives in firms' beliefs are constant, in each period and for each product, firms solve a stationary problem with an infinite horizon. In other words, there is a stationary dynamic problem for each product in each period. The observed investment decisions are the decisions in the first periods of all these auxiliary stationary dynamic problems. Denote the value of product j from firm f at time t under f's belief formed at time t in the relevant auxiliary stationary dynamic problem by $v_{jf}^{jft}(c_l)$ where $c_{jft} = c_l$. The superscript jft indicate the time when the belief is formed by firm f and for product j. There is no time subscript because of stationarity. The subscript jf indicates it is the value of product j from firm f. The Bellman equation of this auxiliary stationary problem is:

$$v_{jf}^{jft}(c_l) = \mathbb{E}[\max\{-\lambda + \pi_{jf}^{jft}(c_l) + \epsilon_{jf}(1) + \rho v_{jf}^{jft}(c_{l-1}), \pi_{jf}^{jft}(c_l) + \epsilon_{jf}(0) + \rho v_{jf}^{jft}(c_l) \tag{5}$$

The first part of Equation (5) is the value of investment and the second part if that of no investment. λ is the investment cost. $\epsilon_{jf}(0)$ and $\epsilon_{jf}(1)$ are Type-I extreme-value shocks with mean zero and are independent and identically distributed across time and products. $\mathbb{E}$ is taken over these choice-specific shocks. ρ is the discount factor. I define $U_{jf}^{jft}(c_l, 0)$ and $U_{jft}^{jft}(c_l, 1)$ as the choice-specific values that satisfy:

$$v_{jf}^{jft}(c_l) = \mathbb{E}[\max\{U_{jf}^{jft}(c_l, 1) + \epsilon_{jf}(1), U_{jf}^{jft}(c_l, 0) + \epsilon_{jf}(0)\}] \tag{6}$$

so

$$U_{jf}^{jft}(c_l, 1) = -\lambda + \pi_{jf}^{jft}(c_l) + \rho * v_{jf}^{jft}(c_{l-1}) \tag{7}$$

$$U_{jf}^{jft}(c_l, 0) = \pi_{jf}^{jft}(c_l) + \rho * v_{jf}^{jft}(c_l) \tag{8}$$

The probability of investment is:

$$\begin{aligned} \mathbb{P}(a_{jf}^{jft}(c_l) = 1) =& \mathbb{P}(U_{jf}^{jft}(c_l, 1) + \epsilon_{jf}(1) > U_{jf}^{jft}(c_l, 0)) + \epsilon_{jf}(0) \\ =& \frac{\exp(U_{jf}^{jft}(c_l, 1))}{\exp(U_{jf}^{jft}(c_l, 1)) + \exp(U_{jf}^{jft}(c_l, 0))} \end{aligned} \tag{9}$$

When $c_{jft} = c_l$, the investment probability of product j at time t in the data is $\mathbb{P}(a^{jft}_{jf}(c_l) = 1)$. Each period t's auxiliary stationary problem can be solved by backwards induction from $c_{jft} = c_1$ as explained in Iskhakov et al. (2016).[7] This also produces the optimal investment decisions, which are used to calculate the investment probabilities in the auxiliary stationary problem and the investment probabilities in the data.

At c_1, since there is no further reduction possible and prices and ranges of all the other products are constant according to firms' beliefs, value of c_1 under belief jft is:

$$v^{jft}_{jf}(c_1) = \frac{\pi^{jft}_{jf}(c_1)}{1-\rho}$$

For $1 < l \leq L$, $v^{jft}_{jf}(c_l)$, $U^{jft}_{jf}(c_l, 0)$, $U^{jft}_{jf}(c_l, 1)$, and$\mathbb{P}(a^{jft}_{jf}(c_l) = 1)$ can be solved by backward recursion according to Equation (6), (7), (8), and (9).

3.4 Investment probability under notched schemes, linear schemes, and no subsidies

Proposition 1 *All the firms' investment probabilities are higher under a notched subsidy scheme compared to the scenario of no subsidy scheme. The difference is larger for firms currently receiving 0 subsidies under the notched subsidy scheme.*

Proof. By construction, the lower bound of $\{c_1, \cdots, c_L\}$, i.e. c_1 is low enough that the optimal range is above the threshold of the notched subsidy scheme. Denote a notched scheme with range threshold a and subsidy τ^n as (a, τ^n). The profits at c_l for $l \in \{1, 2, \cdots, L\}$ under the notched scheme (a, τ^n) are denoted as $\pi^n(c_l)$, and the profits without subsidies are denoted as $\pi^0(c_l)$. I abstract away from the product-firm-time subscript because the proof here applies to all the products in all the periods. When $\tau^n > 0$, $\pi^n(c_1) > \pi^0(c_1)$.

Because $c_l < c_{l+1}$ for $l \in \{1, 2, \cdots, L-1\}$, there exists a value m such that $\pi^n(c_l) = \pi^0(c_l)$ when $l \geq m$ and $\pi^n(c_l) > \pi^0(c_l)$ when $l < m$. In other words, when $l \geq m$, c_l is so high that the firm decides to produce this product with range below the threshold and receives no subsidies under (a, τ^n). Denote the value function according to the Bellman equation defined in Equation (5) under (a, τ^n) as $v^n(c_l)$ and under no subsidy scheme as $v^0(c_l)$. Using backward recursion, it can be shown that $v^n(c_l) > v^0(c_l)$ for $l \in \{1, 2, \cdots, L\}$ and that $v^n(c_l) - v^n(c_{l+1}) > v^0(c_l) - v^0(c_{l+1})$ for $l \in \{1, 2, \cdots, L-1\}$. Therefore, the investment probabilities under the notched scheme are higher than the case of no subsidy scheme. ■

[7] There is no investment decision to be made at c_1 but the continuation value of c_1 is needed to solve the optimal investment decisions at c_2. The backward induction is over the space of the state variables because this is a non-stationary infinite-horizon problem and the transition is unidirectional.

This proposition means that even for products whose c is so high that they are currently not receiving subsidies under the notched scheme (a, τ^n), firms' investment probabilities will be larger than the case of no subsidy scheme. In other words, the government does not need to spend money on subsidizing these products in the current period and possibly over several future periods because the perspective of receiving subsidies in the future incentivizes investment.

Under a linear scheme where the amount of subsidies is a given linear function of ranges, firms' investment probabilities are higher than the case of no subsidy scheme only when the amount of subsidy products can receive is strictly positive in all the periods including the current one. Denote a linear subsidy by μ, and μR is the amount of subsidies a product with range R can receive. If I use the Bellman equation in Equation 5 and denote the value function under the linear scheme as $v^{\mu}(c_l)$ for $l \in \{1, 2, \cdots, L\}$, $v^{\mu}(c_l) > v^0(c_l)$ only when $\mu > 0$. A positive μ means all the products with a positive range will receive subsidies. Therefore, a linear scheme is more expensive to incentivize firms with high cost parameters to invest. When holding the amount of subsidies each product receives constant, notched scheme induce higher investment probability among high-cost firms than a linear scheme.

In Section 6, I will compare the investment probability under the existing notched scheme in China to a counterfactual linear scheme where the amount of subsidies a product receive in each period is the same as the notched scheme. Since firms believe the subsidies scheme does not change over time when making investment decisions, my counterfactual compares, in each period, a time-invariant notched scheme and a time-invariant linear scheme. The notched scheme is the scheme implemented in that period, i.e. year, in my data. The linear scheme's μ is set to a product-specific value, denoted by μ_{jft}, so that the amounts of subsidy a product receives under the notched scheme and under the counterfactual linear scheme in the current period are the same. This does not mean that the expected total amount of subsidies for a product over all future periods according firms' beliefs are the same. Using the auxiliary stationary problem defined in the previous section, the observed investment decisions in each period are the investment decisions of the first period of the relevant auxiliary stationary problem. Denote the period where the investment decision takes place in the data as t and the periods in the auxiliary stationary problem for period t as r^t. By construction, $r^t \geq t$. The first period of the auxiliary stationary problem is the period that overlaps with the data. μ_{jft} equalizes the amounts of subsidy at $r^t = t$ in the auxiliary station problem. Due to the linear incentives, the amount of subsidy increases faster than a notched scheme as c_l decreases. Therefore, the amount of subsidy under μ_{jft} in the auxiliary stationary problem at $r^t > t$ is likely larger than the one under the notched scheme. However, those periods are never realized. So I choose the value μ_{jft} using only the first period of the auxiliary stationary problem.

4 Estimation

The setup of the model allows me to first estimate the static part of model, i.e. the demand side and the static part of the supply side. This estimates all the demand parameters and the marginal cost functions, and it closely follows Berry et al. (1995) and Crawford et al. (2019). I then discretize the cost parameter of ranges, c_{jft} which is already estimated when estimating the marginal cost functions, and infer firms' investment decisions using changes in the discretized c_{jft}, in the sense that firms invest when the discretized c_{jft} decreases. Solving the dynamic problem in Equation (5), the conditional choice probability of investing is a function of the unknown investment cost λ because all the other parameters have been estimated in the static part. I then use maximum likelihood estimation to find the value of the investment cost that maximizes the inferred investment decisions. The first part of this section describes the estimation procedure in the static part and the second part explains the dynamic part.

4.1 The static part

According to the structural model, the mean utility of product j at time t is:

$$\delta_{jft} = \alpha * p_{jft} + \beta * x_{jft} + \xi_{jft}$$

Then the market share of model j from firm f at time t is:

$$s_{jft} = \frac{\exp(\delta_{jft})}{1 + \sum_{j,f} \exp(\delta_{jft})}$$

The market shares in logarithms relative to the outside option is:

$$\ln(s_{jft}) - \ln(s_{0t}) = \alpha * p_{jft} + \beta * x_{jft} + \xi_{jft}$$

The left-hand side and p_{jft} and x_{jft} from the right-hand side are known from data. I use the instrument variables constructed by Berry et al. (1995) to account for the endogeneity of prices and estimate α and β using the general method of moments (GMM). x_{jft} includes size, power-weight ratio, torque, luxury level, whether released this year, years the model exists in the data, model and year two-way fixed effects. The luxury level is a numerical indicator that sum over several dummy variables, such as wheter a product contains a rain sensor or has a key-less start. For EV cars, x_{jft} also include driving ranges and consumer preference parameter of ranges is β^R.

From the first order conditions of firms' profits maximization, i.e. Equation (1) and (2),

$$0 = s_{jft} + \sum_{k \in \mathcal{J}_f} (p_{kft} - mc_{kft} + \tau(R_{kft})) \frac{\partial s_{kft}}{\partial p_{jft}} \tag{10}$$

$$0 = -s_{jft} \left(\frac{\partial mc_{jft}}{\partial R_{jft}} + \frac{\partial \tau(R_{jft})}{R_{jft}} \right) + \sum_{k \in \mathcal{J}_f} (p_{kft} - mc_{kft} + \tau(R_{kft})) \frac{\partial s_{kft}}{\partial R_{jft}} \tag{11}$$

where

$$\frac{\partial s_{kjt}}{\partial d_{jft}} = \begin{cases} -s_{kft} s_{jft} \frac{\partial \delta_{jft}}{\partial d_{jft}} \text{ , if } k \neq j \\ (1 - s_{jft}) s_{jft} \frac{\partial \delta_{jft}}{\partial d_{jft}} \text{ , if } k = j \end{cases}$$

and $d_{ift} \in \{p_{ift}, R_{ift}\}$, $\frac{\partial \delta_{ift}}{\partial p_{ift}} = \alpha$, $\frac{\partial \delta_{ift}}{\partial R_{ift}} = \beta^R$. Therefore, with the estimated $\hat{\alpha}$ and $\hat{\beta}^R$, only mc_{jft} and $\frac{\partial mc_{jft}}{\partial R_{jft}}$ are unknown in Equation (10) and (11). I follow Crawford et al. (2019) to calculate $\widehat{mc}_{jft}$ and $\widehat{\frac{\partial mc_{jft}}{\partial R_{jft}}}$ using matrix inversion. Since the marginal cost functions is

$$mc_{jft} = c_{jft} R_{jft}^2 + \mu * w_{jft} + \eta_{jft}$$

Therefore:

$$\frac{\partial mc_{jft}}{\partial R_{jft}} = 2 c_{jft} R_{jft}$$

Thus, the estimated $\hat{c}_{jft}$ is:

$$\hat{c}_{jft} = \left(\widehat{\frac{\partial mc_{jft}}{\partial R_{jft}}} \right) * \frac{1}{2 R_{jft}}$$

The remaining parameters in the marginal cost function can be are estimated also using GMM with Berry et al. (1995) style of instruments to account for endogenous prices and ranges. However, I skip this step because I only need $\hat{c}_{ift}$.

4.2 The dynamic part

As explained in Section 3.3, the investment decisions for model j in period t under firm f's belief formed at t for model j is the optimal choice in the first period of its auxiliary stationary dynamic problem as defined Equation (5). Solving this problem following the step explained in Section 3.3, gives the investment probability for model j at time t in the data. Denote the choice of investing for product j by firm f at time t in the data as $a_{jft}(c_{jft})$. When $c_{jft} = c_l$, the investment probability $\mathbb{P}(a_{jft}(c_l) = 1) = \mathbb{P}(a_{jf}^{jft}(c_l) = 1)$ is given in

Equation (9), which is a function of the unknown investment cost.

I infer that there is an investment for product j at time t, i.e. $\hat{a}_{jft} = 1$ whenever $\hat{c}_{jft} > \hat{c}_{jft+1}$. Then the likelihood function of $\hat{a}_{jft} = 1$ for all the products from all the firms in all the periods available in the data is:

$$\sum_{j,f,t} \ln(\mathbb{P}[\hat{a}_{jft}(\hat{c}_{jft}) = 1|\lambda])$$

The estimated investment cost $\hat{\lambda}$ is:

$$\hat{\lambda} = \arg\max_{\lambda} \sum_{j,f,t} \ln(\mathbb{P}[\hat{a}_{jft}(\hat{c}_{jft}) = 1|\lambda])$$

5 Data

I collect sales data from Chezhu Home (https://www.16888.com) and technical description from both Chezhu Home (https://www.16888.com) and Auto Home (https://www.autohome.com.cn). Additional variables about BEVs, such as battery energy density (the ratio between battery capacity and battery weight), are collected using the Lists of Recommended New Energy Vehicles (hereafter the NEV Lists) published by the Chinese government in 2017-2020, which are in total 49 lists. These variables are required when calculating each product's subsidy value.

In addition to direct purchase subsidy, the exemption of purchasing tax is also an important factor for consumers' purchase decisions. I use the Lists of NEVs Eligible for Purchasing Tax Exemption (hereafter the NEVPTE List) and the Lists of NEVs Removed from the NEVPTE Lists (hereafter the RNEVPTE List) published by the Chinese government in 2016-2020 to decide whether a product receives tax exemption in a year. There are 32 NEVPT Lists and 10 RNEVPTE Lists in 2016-2020. The tax exemption is considered when estimating the demand parameters.

I do not observe the exact amount of subsidy per transaction but I collect the formula for calculating subsidies announced by the Chinese government in 2010-2020 and then calculate the subsidy using product characteristics accordingly. The number of hourseholds and consumer price index in each year in 2010-2020 are collected using Chinese Year Books.

According to the State Grid Corporation of China, the monopolistic electricity supplier in China, the electricity price remains at 0.542 RMB/kwh during my sample. I use the prices of 92 and 95 gasoline, and diesel in Beijing 2010-2021 from https://data.eastmoney.com/cjsj/oil_default.html to approximate the national average prices.

I limit my sample to only passenger vehicles with no more than 5 doors and include only

BEVs and gasoline vehicles. Table 2 shows the summary statistics of all the products in my sample in 2010-2020. The observation unit is a model in a year. Table 3 shows the summary statistics of BEVs. Since there were no passenger BEV before 2016, Table 3 only covers (2016-2020). The data described in Table 2 and Table 3 is used for the estimation in the static part.

Table 2: Summary statistics of the entire sample (2010-2020)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
price[1]	2,138	11.57	6.75	3.04	7.37	13.85	53.80
power/weight	2,138	0.10	0.01	0.05	0.09	0.11	0.17
cost per km (RMB)	2,138	0.40	0.11	0.05	0.33	0.46	0.91
size ($L \times H \times W$)[2]	2,138	8.24	0.71	5.30	7.79	8.72	10.27
torque	2,138	200.61	61.88	88	150	240	553
luxury level[3]	2,138	13.57	20.24	1	4.0	14.4	151

Each observation is model × year.
[1] Prices are deflated by the annual consumer index and are in units of 10,000 RMB.
[2] L, H, W are in units of meters.
[3] Luxury level is an index constructed as the sum of several dummy covariates such as whether the vehicle model has a rain sensor or a key-less start.

Table 3: Summary statistics of BEVs (2016-2020)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
range (km)	39	108.90	106.96	50	51	75	410
price[1]	39	14.71	3.37	5.11	12.87	17.40	21.90
price-τ[1]	39	14.27	3.76	3.99	12.71	16.28	21.90

[1] Prices and subsidies (τ) are deflated by the annual consumer index and are in units of 10,000 RMB. Purchasing tax is also deducted in price-τ if the product is eligible for tax exemption.

Table 4 is the summary statistics of BEVs in 2019. There were 14 BEV products in 2019. The investment cost is estimated using only these 14 BEVs and therefore the results about investment probabilities are for these 14 BEVs.

Table 4: Summary statistics of BEVs (2019)

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
range (km)	14	112.00	124.66	51	51	75	410
price [1]	14	14.23	2.07	10.85	12.89	15.37	17.83
price-τ[1]	14	13.79	2.67	6.44	12.89	15.37	17.83

[1] Prices and subsidies (τ) are deflated by the annual consumer index and are in units of 10,000 RMB. Purchasing tax is also deducted in price-τ if the product is eligible for tax exemption.

6 Results

Table 5 displays the consumer tastes parameters. The parameters excluding β^R, i.e. consumer's tastes for ranges, are estimated using the entire data, including both gasoline vehicles and BEVs. β^R is estimated using only BEVs. Cost per km measures the expenditure on fuel per kilometer travelled, which reflects the fuel efficiency. Luxury level is a numerical index that sums over several dummy variables including whether the vehicle model contains a rain sensor or has a key-less start. Instrumental variables are constructed following Berry et al. (1995).

The first two rows in Table 6 display the distributions of the BEVs' range cost parameters in logs in 2019 and the investment cost estimated using the inferred investment decision by BEVs in 2019, i.e. whether the range cost parameters decreases in 2020. The investment cost is reported as the ratio between model-specific profits and the investment cost. The model-specific profits are estimated using the estimated marginal costs. On average, the investment cost is fairly small compared to model-specific annual profits. This is likely due to profits are calculated using price minus marginal cost of production while the total cost for each model should also include fixed costs. The third row reports the markups of the BEVs, showing a fairly high markups.

Figure 3 demonstrates the investment probabilities of for all possible values of $\log(c_{jf,2019})$, i.e. $\{c_1, c_2, \cdots, c_L\}$. The majority of vehicle models are between -9 and -7. The notched scenario is under the subsidy scheme $(a, \tau^n) = (400, 2.5)$, i.e. I only use the highest threshold in 2019 (as shown in Table 1 and Figure 2). τ^n is in units of 10,000 RMB. When estimating the model primitives, all the range thresholds and their relevant subsidies are consider. To demonstrate the impact of notched scheme, I take the estimated model primitives and calculate the investment probabilities with only one threshold, i.e. the 400 km threshold. The counterfactual linear scenario is also compared to this notched scheme with one threshold.

The linear scenario chooses a product specific μ for each product so that the amounts of subsidy a product receives in 2019 under the notched scheme and the linear scheme are the

Table 5: Estimated consumer taste parameters on prices and ranges

		estimated values
α		0.57
		[0.14]
β	R	0.02
		[0.01]
	power-weight ratios	23.28
		[10.10]
	cost per km	0.66
		[1.11]
	size (L×H×W)	0.99
		[0.53]
	torque	0.01
		[0.004]
	luxury level [1]	0.035
		[0.01]

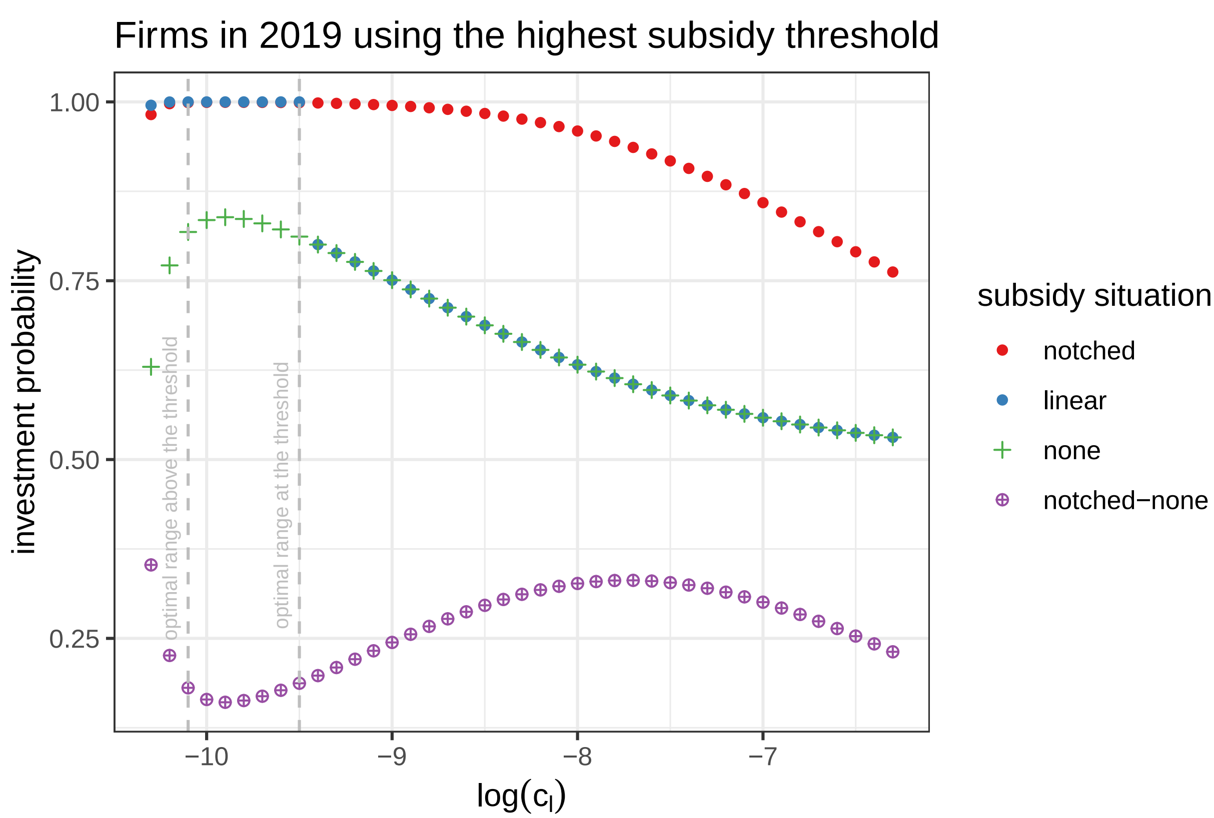
Standard errors are in the brackets. Model-year two-way fixed effects are included.
All the parameters except β^R is estimated using the entire dataset, i.e. including both gasoline vehicle models and BEVs. β^R only uses BEVs.
[1] Luxury level is an index constructed as the sum of several dummy covariates such as whether the vehicle model has a rain sensor or a key-less start.

Table 6: Summary statistics of cost parameters of ranges and investment-cost-profit ratios (BEVs, 2019)

	Mean	St. Dev.	min	Pctl(10)	Pctl(25)	Median	Pctl(75)	Pctl(90)	max
$\log(c_{jft})$	-7.94	0.57	-8.90	-8.70	-8.40	-7.90	-7.47	-7.23	-6.80
π_{jft}/λ	958.95	325.01	602.81	656.26	701.93	829.47	$1,123.59$	$1,476.21$	$1,863.86$
p_{jft}/mc_{jft}	3.38	1.35	1.78	1.95	2.28	3.12	4.24	5.13	7.35

same. After abstracting a way from the other product characteristics, this product-specific μ is a function of the value of c_{jft} which can take values in $\{c_1, c_2, \cdots, c_L\}$, and therefore denote the product-specific counterfactual linear scheme by μ_l for $l = 1, 2, \cdots, L$. In Figure 3, the scenario named "none" means no subsidy scheme. "Notched-none" indicates the differences in investment probabilities between the notched scenario and the scenario of no subsidy scheme. The vertical grey line marks the highest $\log(c_l)$ where a product's optimal range is equal to the 400 kilometer threshold. The grey line on the left marks the place where a product's optimal range is above the 400 kilometer threshold. In other words, products between the two grey lines all have their ranges equal to 400, which reflects clustering around the thresholds induced by the notched scheme. Products to the right of the right grey line do not receive the 25,000 RMB subsidies in 2019. Since in my analysis, $(400, 2.5)$ is the only threshold imposed. These products do not receive any subsidies, which means the product-model specific counterfactual linear scheme μ_l for these product are 0.

FIGURE 3: Investment probabilities of BEVs in 2019 under three subsidy scenarios



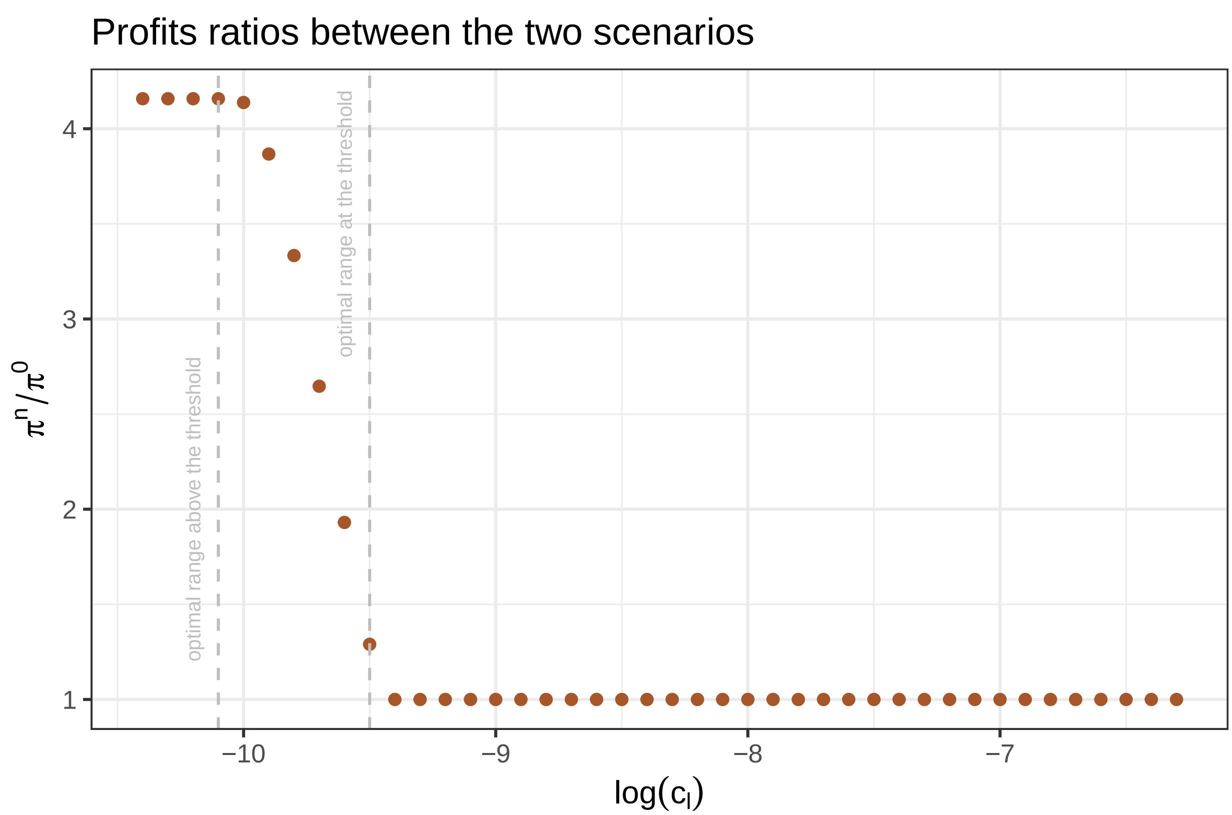
Moving from $\log(c_L)$ to $\log(c_1)$, i.e. moving from the right side to the left side of Figure 3, the cost parameter decreases and the investment probability under the notched scheme first increases and then decreases slightly, but it is well above the one without a subsidy scheme.

The investment probabilities under no subsidy scheme has a similar shape but they increase much slower at high $\log(c_l)$ and drop much faster later at low $\log(c_l)$. The investment probability increases because the increase in profits is higher for lower $\log(c_l)$. On the other hand, since $\log(c_l) - log(c_{l-1})$ is constant for $l \in \{2, 3, ..., L\}$, which means $c_l - c_{l-1}$ declines when c_l decreases. The improvement in c_l as the result of investment becomes smaller for smaller c_l, which reduces the increase in profits. Therefore, eventually investing becomes less attractive and the investment probability declines. The investment probability under the notched scheme is higher even when firms currently do not receive subsidies, i.e. their profits and optimal prices and ranges are the same as the no-subsidy scenario. This is because the perspective of investing themselves above the range threshold and consequently receiving higher profits induces a higher investment probability. The investment probability under the notched scheme increases faster at high $\log(c_l)$ and declines slower than the case of no scenario because BEV models start to receive subsidies when their optimal ranges reach the threshold. The profit ratios between notched scenario and no-subsidy scenario consequently increase when c_l decreases (Figure 4). In Figure 4, π^n denotes the profits under the notched scheme and π^0 denotes the profits under the no-subsidy scenario.

In the linear-subsidy scenario, the investment probability is the same as those in the no-subsidy scenario when the optimal range is below the threshold. Because these products receive no subsidy under the notched scheme and therefore has $\mu = 0$ in the linear scenario. For products with low enough range cost parameters, the investment probability becomes almost the same as those under the notched scheme and becomes slightly higher for very lower $\log(c_l)$. This is because firms believe the linear scheme to remain the same forever and because firms will receive more subsidies in future period when their c_{jft} becomes lower under a time-invariant linear scheme than the time-invariant notched scheme. Since this phenomenon is more salient for lower $\log(c_l)$, the investment probability under the linear scheme is slightly higher than the notched scheme for the smallest value, $\log(c_1)$.

The majority BEVs' ranges in 2019 are below the 400 km threshold, therefore we are mainly in the domain that the notched scheme induces a 20 to 35 percentage points increase in investment probability compared to the scenario of linear subsidy or no subsidy.

FIGURE 4: Profits differences between the notched-scheme scenario and the no-subsidy scenario



7 Conclusion

This paper models and estimates manufacturers' incentives to reduce the production cost of ranges in response to range-based subsidies to consumers purchasing battery electric vehicles (BEVs) and finds that the notched scheme increases the investment probabilities by 20 to 35 percentage points in 2019. Comparing to a linear scheme where each BEV's total amount of subsidy is constant, notched scheme is especially efficient in incentivizing manufacturers with high production costs of ranges to invest.

The numerical results in this paper is under several assumptions such as homogeneous consumers, adaptive expectations about other BEV models' future prices and ranges, adaptive expectations about future subsidy scheme, investment leads to one unit of decrease in $\log(c_l)$ for sure. The assumption of homogeneous consumer can be relaxed by allowing the consumer taste parameters to differ across consumers and then estimate the demand side of the structural model using random-coefficient estimation as it is implemented in Berry et al. (1995). This will improve the accuracy of the consumer taste parameters and therefore may affect the magnitude of the empirical results. The adaptive expectation about other BEVs' prices and ranges can be relaxed by using adaptive expectations about other BEV models' range cost parameters, i.e. c_{jft}. This implies that investment decisions on one BEV takes into account the price and range responses of the other BEVs. This adaptive expectation about prices and ranges can also be replaced by rational expectation so that the investment decision is a dynamic game. Under the belief that subsidy scheme stays constant, the auxiliary dynamic problem in each period is still a stationary dynamic game and can still be solved using Iskhakov et al. (2016). The assumption that $\log(c_l)$ decreases by 1 unit for sure if invest can be relaxed by allowing the change in $\log(c_{jft})$ to follow a distribution where $\log(c_{jft})$ may decreases by more than 1 unit. Although most BEVs' $\log(c_{jft})$ either decrease by one unit or do not change in 2019, there are some BEVs whose $\log(c_{jft})$ decreases by more than 1 unit. Allowing stochastic improvements over c_{jft} can improve the fitness of the structural model to the data on this regard. Instead of assuming that manufacturers believe the subsidy scheme never change, firms can be assumed to have perfect foresight about future subsidy schemes. This is probably the most difficult extension because the problem of investment decisions is then non-stationary with infinite time horizon. If the adaptive expectation about other BEVs' prices and ranges is kept, the investment decision problem is still a single-agent problem. Since the subsidy scheme in China will eventually disappear, the periods after the subsidy scheme disappear can be modeled as stationary and the periods with subsidy scheme can be solved by backward recursion.

In all these extensions, as long as profits are higher under a notched scheme at certain point in the future compared to the scenario of no subsidy, the notched scheme increases investment probability. Furthermore, the notched scheme will still be more efficient in increasing investment probability than a linear scheme for BEVs who currently do not qualify for the scheme.

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Appendix

A Data preparation

We need to merge all the data collected from various source into one panel data. Each observation is a model-year-month. The variables including monthly sales, technical descriptions such as horse power, whether eligible for subsidy, the value of subsidy if it is eligible, whether eligible for purchase tax exemption, the price of electricity and gasoline, the number of charging poles or charging stations. The biggest challenge is merge sales data, subsidy info, purchase tax info, and technical description. In this section, we will talk about how this merge is carried out.

A.1 Merge NEVPTE List, RNEVPTE List, and the NEV List

We first convert the lists published by government into data format. These lists share the same model identity. For each of three list groups, we treat observations as redundant if the same model ID in the same year-month appears more than once. Among the redundant observations, we keep the observation with the least number of missing variables, the smallest size, and the smallest weight. We merge based on model ID and use variables shared by the three groups of lists to check the merging is correct. More specifically, the lists all report manufacture name, model type, weight. We first check whether the manufacture names and model types are the same. Next we check whether weight are close enough, i.e. less than 10% difference. We call the merged data subsidy-purchase-tax data (SPT).

A.2 Merge sales, SPT, and the technical description data

We first take the sales data collected from Chezhu Home. Using model names to match the technical description data collected from Chezhu Home and Auto Home. We first use exact matching and the apply fuzzy matching for those unmatched by the exact matching. We then repeat the same process to match with SPT. Sales data is at the model level but the technical description data is at the model-variation level. Meaning, for each model, there are multiple variation in our technical description data. We keep the variation that has the least amount of missing variables, the lowest price, and the smallest sizes, weight, and horse power, following the practice of Berry et al. (1995). For the unmatched observation, we check whether the unmatching is due to incorrectly recorded names and correct them manually if needed.

A.3 Summary stats

Table 7

year	sales Merged	sales Scrapped	sales web	gap merged and scrapped	gap scrapped and web
2010	7,763	11,040	13,758	0.300	0.200
2011	12,019	14,316	14,473	0.160	0.010
2012	12,844	16,455	15,494	0.220	-0.060
2013	20,481	21,135	17,929	0.030	-0.180
2014	21,856	22,424	19,701	0.030	-0.140
2015	21,755	22,322	21,146	0.030	-0.060
2016	23,304	23,788	24,377	0.020	0.020
2017	20,237	20,689	24,718	0.020	0.160
2018	19,523	19,858	23,710	0.020	0.160
2019	17,403	17,834	21,444	0.020	0.170
2020	15,761	16,311	20,178	0.030	0.190
2021	13,612	13,843	21,482	0.020	0.360

B A dynamic game

B.1 An illustrative example of two single-product firms for solving the dynamic problem

This section demonstrates how to solve the auxiliary dynamic programming for firms at time t in a simple setup of two single-product firms, j and m. Since firms do not internalize the impact of investing in one of its product on the profits of its other products, the solution explained in this section is applied to multiple multi-product firms. I drop the model subscript i because firm j and m are single-product.

Following Iskhakov et al. (2016), I use backward recursion over the space of firms range production cost c_{jr} and c_{mr}, in the sense that I start with $(c_{jr}, c_{mr}) = (c_1, c_1)$ and then move backwards to $(c_{jr}, c_{mr}) = (c_2, c_1)$ and $(c_{jr}, c_{mr}) = (c_1, c_2)$, and then move backward to $(c_{jr}, c_{mr}) = (c_2, c_2)$. This process continues until all the possible realization of (c_{jr}, c_{mr}) for $c_{jr}, c_{mr} \in \mathcal{C}$ are covered.

When $(c_{jr}, c_{mr}) = (c_1, c_1)$, there is no investment decision possible because it is at the boundary of $\mathcal{C}$. Because firms expect subsidy scheme and exogenous model characteristics to stay constant, the value for firm j and m are:

$$v_j(c_1, c_1) = \frac{\pi_j^{*t}(c_1, c_1)}{1-\rho} \approx -\frac{s_j^{*t}(c_1, c_1)}{(1-\rho)\alpha} \tag{12}$$

$$v_m(c_1, c_1) = \frac{\pi_m^{*t}(c_1, c_1)}{1-\rho} \approx -\frac{s_m^{*t}(c_1, c_1)}{(1-\rho)\alpha} \tag{13}$$

The approximation follows from Equation (3). The superscript t means the equilibrium market shares $s_j^{*t}(c_1, c_1)$ and $s_m^{*t}(c_1, c_1)$ are calculated using the subsidy scheme and exogenous model characteristics at time t. There is no subscript r because the problem is stationary under my assumption about firms' expectations.

Move one step backwards, I will solve the situation where $(c_{jr}, c_{mr}) = (c_1, c_2)$ and $(c_{jr}, c_{mr}) = (c_2, c_1)$, and then solve the situation where $(c_{jr}, c_{mr}) = (c_2, c_2)$.

When $(c_{jr}, c_{mr}) = (c_1, c_2)$, j doesn't invest because j is at the boundary but m can still invest.

$$v_m(c_1, c_2) = \pi_m(c_1, c_2) + \max\{\epsilon_m(0) + \rho v_m(c_1, c_2), -\lambda + \epsilon_m(1) + \rho v_m(c_1, c_1)\} \tag{14}$$

The optimal strategy for m at (c_1, c_2) is:

$$a_m(c_1, c_2) = \begin{cases} 1, \text{ if } -\lambda + \epsilon_m(1) + \rho v_m(c_1, c_1) \geq \epsilon_m(0) + \rho v_m(c_1, c_2) \\ 0 \text{ , otherwise} \end{cases}$$

Firm j does not observe firm m's $\epsilon_m(0)$ and $\epsilon_m(1)$ in the current period.

$$\begin{aligned} v_j(c_1, c_2) =& \pi_j(c_1, c_2) + \rho \mathbb{E}[v_j(c_1, c_m')] \\ =& \pi_j(c_1, c_2) + \rho(\mathbb{P}[a_m(c_1, c_2) = 1] \cdot v_j(c_1, c_1) + \mathbb{P}[a_m(c_1, c_2) = 0] \cdot v_j(c_1, c_2)) \qquad (15) \\ =& \pi_j(c_1, c_2) + \rho \mathbb{P}[\epsilon_m(1) - \epsilon_m(0) \geq \rho v_m(c_1, c_2) + \lambda - \rho v_m(c_1, c_1)] \cdot v_j(c_1, c_1) + \qquad (16) \\ & \rho \mathbb{P}[\epsilon_m(1) - \epsilon_m(0) < \rho v_m(c_1, c_2) + \lambda - \rho v_m(c_1, c_1)] \cdot v_j(c_1, c_2) \end{aligned}$$

$\mathbb{P}[\cdot]$ indicates the probability of the statement inside is true. I first solve $v_m(c_1, c_2)$ as a fixed point to Equation (14) using $v_m(c_1, c_1)$ calculated in Equation (13). Use $v_m(c_1, c_2)$, I calculate the probability of $a_m(c_1, c_2) = 1$ and $a_m(c_1, c_2) = 0$ before $\epsilon_m(0)$ and $\epsilon_m(1)$ are realized. These together with $v_j(c_1, c_1)$ from Equation 12 allow me to solve $v_j(c_1, c_2)$ as a fixed point of Equation 15.

Similarly, I can solve $v_j(c_2, c_1)$, $a_j(c_2, c_1)$, and $v_m(c_2, c_1)$. When $(c_j, c_m) = (c_2, c_2)$, both firms can invest.

$$
\begin{aligned}
v_j(c_2,c_2) =& \pi_j(c_2,c_2)+\max\{\epsilon_j(0)+\rho\mathbb{E}_j[v_j(c_2,c_m')],-\lambda+\epsilon_j(1)+\rho\mathbb{E}_j[v_j(c_1,c_m')]\} \\
=& \pi_j(c_2,c_2)+\max\{\epsilon_j(0)+\rho\cdot(v_j(c_2,c_1)\cdot\mathbb{P}[a_m(c_2,c_2)=1]+v_j(c_2,c_2)\cdot(1-\mathbb{P}[a_m(c_2,c_2)=1])), \\
& -\lambda+\epsilon_j(1)+\rho\cdot(v_j(c_1,c_1)\cdot\mathbb{P}[a_m(c_2,c_2)=1]+v_j(c_1,c_2)\cdot(1-\mathbb{P}[a_m(c_2,c_2)=1]))\} \\
v_m(c_2,c_2) =& \pi_m(c_2,c_2)+\max\{\epsilon_m(0)+\rho\mathbb{E}_m[v_m(c_j',c_2)],-\lambda+\epsilon_m(1)+\rho\mathbb{E}_m[v_m(c_j',c_1)]\} \\
=& \pi_m(c_2,c_2)+\max\{\epsilon_m(0)+\rho\cdot(v_m(c_1,c_2)\cdot\mathbb{P}[a_j(c_2,c_2)=1]+v_m(c_2,c_2)\cdot(1-\mathbb{P}[a_m(c_2,c_2)=1])), \\
& -\lambda+\epsilon_m(1)+\rho\cdot(v_m(c_1,c_1)\cdot\mathbb{P}[a_j(c_2,c_2)=1]+v_m(c_2,c_1)\cdot(1-\mathbb{P}[a_j(c_2,c_2)=1]))\}
\end{aligned}
$$

and

$$
a_j(c_2,c_2)=\begin{cases}1, \text{ if } -\lambda+\epsilon_j(1)+\rho\mathbb{E}_j[v_j(c_1,c_m')]\geq\epsilon_j(0)+\rho\mathbb{E}_j[v_j(c_2,c_m')] \\ 0\text{ , otherwise}\end{cases}
$$

$$
a_m(c_2,c_2)=\begin{cases}1, \text{ if } -\lambda+\epsilon_m(1)+\rho\mathbb{E}_m[v_m(c_j',c_1)]\geq\epsilon_m(0)+\rho\mathbb{E}_m[v_m(c_j',c_2)] \\ 0\text{ , otherwise}\end{cases}
$$

so

$$
\begin{aligned}
\mathbb{P}[a_j(c_2,c_2)=1] =& \mathbb{P}[\epsilon_j(1)-\epsilon_j(0)\geq\rho\mathbb{E}_j[v_j(c_2,c_m')]+\lambda-\rho\mathbb{E}_j[v_j(c_1,c_m')]] \\
\mathbb{P}[a_m(c_2,c_2)=1] =& \mathbb{P}[\epsilon_m(1)-\epsilon_m(0)\geq\rho\mathbb{E}_m[v_m(c_j',c_2)]+\lambda-\rho\mathbb{E}_m[v_m(c_j',c_1)]]
\end{aligned}
$$

The solution is a fixed point to this system of equations.

The backward induction then move on to solve the case of (c_1, c_3), (c_3, c_1), then (c_2, c_3) and (c_3, c_2), and then (c_3, c_3). I continue this process till all the possible realization of (c_j, c_m) are considered.

B.2 Changes in profits due to investment under notched and linear subsidy schemes

It is difficult to analytically derive the explicit formula of changes in the investment probability because of the strategic interactions among firms. However, if the increase in profits from invest is lower under one scheme, the incentive for investment should also be lower under this scheme. The differences in incentives between a notched scheme and a linear scheme should be the largest when firms invest out of the binding threshold constraint under the notched scheme. For the ease of demonstration, I continue using the 2-firm setup as in the section above and drop the covariates x and w. To capture the biggest differences in investment incentives, I will look at changes in profits between two time periods that are not adjacent, i.e. there may be multiple periods between these two periods. The notched scheme, denoted as (a, τ_n), has a threshold a so that models with driving range above a is qualified for subsidies τ_n. I compare it to a linear scheme where a model with range R receives subsidies $\mu_t R$. μ_t is the linear parameter under the linear scheme and is set to the value so that the total amount of subsidies is the same as the notched scheme in each period.

In the first period, denoted as t_1, both firms' state variables, c_j and c_m, are so low that their optimal choices is to choose a range that is below the threshold of the notched scheme, a. They receive zero subsidies and the optimal ranges R^*_{j1} and R^*_{m1} are:

$$
\begin{aligned}
R^*_{j1} &= -\frac{\beta^R}{2c_{j1}\alpha} \\
R^*_{m1} &= -\frac{\beta^R}{2c_{m1}\alpha}
\end{aligned}
$$

Since neither firm receives subsidies under the notched scheme, the linear scheme's μ_1 at t_1 is set to $\mu_1 = 0$.

When both of the firms' market shares are 0, their optimal prices and profits, as derived in Equation (4) and Equation (3), are approximately:

$$
\begin{aligned}
p^*_{j1} &\approx -\frac{1}{\alpha} + c_{j1} \cdot (R^*_{ji})^2 \\
\pi^*_{j1} &\approx -\frac{s^*_{j1}}{\alpha} \\
p^*_{m1} &\approx -\frac{1}{\alpha} + c_{m1} \cdot (R^*_{mi})^2 \\
\pi^*_{m1} &\approx -\frac{s^*_{m1}}{\alpha}
\end{aligned}
$$

The equilibrium market shares are approximately:

$$
\begin{aligned}
s^*_{j1} \approx & \frac{\exp\left(-1 + \alpha \cdot c_{j1}\left(\frac{\beta^R}{2c_{j1}\alpha}\right)^2 - \frac{(\beta^R)^2}{2c_{j1}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 + \alpha \cdot c_{f1}\left(\frac{\beta^R}{2c_{f1}\alpha}\right)^2 - \frac{(\beta^R)^2}{2c_{f1}\alpha}\right)} = \frac{\exp\left(-1 - \frac{(\beta^R)^2}{4c_{j1}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\beta^R)^2}{4c_{f1}\alpha}\right)} \\
s^*_{m1} \approx & \frac{\exp\left(-1 + \alpha \cdot c_{m1}\left(\frac{\beta^R}{2c_{m1}\alpha}\right)^2 - \frac{(\beta^R)^2}{2c_{m1}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 + \alpha \cdot c_{f1}\left(\frac{\beta^R}{2c_{f1}\alpha}\right)^2 - \frac{(\beta^R)^2}{2c_{f1}\alpha}\right)} = \frac{\exp\left(-1 - \frac{(\beta^R)^2}{4c_{m1}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\beta^R)^2}{4c_{f1}\alpha}\right)}
\end{aligned}
$$

Since $\mu_1 = 0$, the optimal prices, ranges, and profits under the linear scheme is the same.

In the period t_2, which may be several periods after t_1, c_{j2} and c_{m2} are small enough so that both firms' optimal ranges under the notched scheme (a, τ_n) are the interior solution

$$R_{j2}^{n*} = -\frac{\beta^R}{2c_{j2}\alpha}$$
$$R_{m2}^{n*} = -\frac{\beta^R}{2c_{m2}\alpha}$$

and $R_{j2}^{n*}, R_{m2}^{n*} > a$. Without loss of generality, I assume $c_{j2} < c_{m2}$. The total amount of subsidies firms receive are

$$\tau_n \sum_{f\in\{j,m\}} s_{f2}^{n*}$$

where s_{f2}^{n*} for $f \in \{j, m\}$ denotes the equilibrium market share. The linear scheme's μ_2 in t_2 satisfies:

$$\mu_2 \sum_{f\in\{j,m\}} R_{f2}^{l*} s_{f2}^{l*} = \tau_n \sum_{f\in\{j,m\}} s_{f2}^{n*}$$

where s_{f2}^{l*} and R_{f2}^{l*} is the equilibrium market share under the linear scheme μ_2, and s_{f2}^{n*} is the equilibrium market share under the notched scheme (a, τ_n).

The equilibrium market shares under the notched scheme (a, τ_n) are approximately:

$$s_{j2}^{n*} \approx \frac{\exp\left(-1 - \frac{(\beta^R)^2}{4c_{j2}\alpha} - \alpha\tau_n\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\beta^R)^2}{4c_{f2}\alpha} - \alpha\tau_n\right)}$$
$$s_{m2}^{n*} \approx \frac{\exp\left(-1 - \frac{(\beta^R)^2}{4c_{m2}\alpha} - \alpha\tau_n\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\beta^R)^2}{4c_{f2}\alpha} - \alpha\tau_n\right)}$$

and the profits are approximately:

$$\pi_{j2}^{n*} \approx -\frac{s_{j2}^{n*}}{\alpha}$$
$$\pi_{m2}^{n*} \approx -\frac{s_{m2}^{n*}}{\alpha}$$

Under the linear scheme μ_2, i.e. models with range R is qualified for subsidies $\mu_2 R$ in period t_2, the optimal ranges are:

$$R_{j2}^{l*} = \frac{\mu_2\alpha - \beta^R}{2c_{j2}\alpha}$$
$$R_{m2}^{l*} = \frac{\mu_2\alpha - \beta^R}{2c_{m2}\alpha}$$

The market shares are approximately:

$$s_{j2}^{l*} \approx \frac{\exp\left(-1 - \frac{(\mu_2\alpha - \beta^R)^2}{4c_{j2}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\mu_2\alpha - \beta^R)^2}{4c_{f2}\alpha}\right)} \tag{17}$$

$$s_{m2}^{l*} \approx \frac{\exp\left(-1 - \frac{(\mu_2\alpha - \beta^R)^2}{4c_{m2}\alpha}\right)}{1 + \sum_{f\in\{j,m\}} \exp\left(-1 - \frac{(\mu_2\alpha - \beta^R)^2}{4c_{f2}\alpha}\right)} \tag{18}$$

and profits under the linear scheme are approximately:

$$\pi_{j2}^{l*} \approx -\frac{s_{j2}^{l*}}{\alpha}$$
$$\pi_{m2}^{l*} \approx -\frac{s_{m2}^{l*}}{\alpha}$$

The differences in the increase of profits for firm j and m between t_1 and t_2 under the two schemes are:

$$(\pi_{j2}^{n*} - \pi_{j1}^{*}) - (\pi_{j2}^{l*} - \pi_{j1}^{*}) = \pi_{j2}^{n*} - \pi_{j2}^{l*} \approx -\frac{s_{j2}^{n*} - s_{j2}^{l*}}{\alpha}$$
$$(\pi_{m2}^{n*} - \pi_{m1}^{*}) - (\pi_{m2}^{l*} - \pi_{m1}^{*}) = \pi_{m2}^{n*} - \pi_{m2}^{l*} \approx -\frac{s_{m2}^{n*} - s_{m2}^{l*}}{\alpha}$$

Proposition 2 *If an individual model i whose cost parameter of ranges is c_i faces a subsidy scheme (a, τ_n), i.e. the model is qualified for subsidy τ if the range is no less than a, holding the choices for all the other models in the market fixed, there exists a unique linear subsidy scheme so that the total amount of subsidy this model receive is the same as under the notched scheme (a, τ_n). Denote the optimal market share and range choice under the linear scheme as s_i^{l*} and R_i^{l*} and under the notched scheme as s_i^{m*} and R_i^{n*}, then it means there exists a $\tilde{\mu}_i$ such that $s_i^{l*} * \tilde{\mu}_i R_i^{l*} = s_i^{n*}\tau_n \mathbb{1}(R_i^{n*} \geq a)$. Denote the optimal profits under the notched scheme and the linear scheme as π_i^{n*} and π_i^{l*}, then*

$$\pi_i^{n*} \geq \pi_i^{l*}$$

The strict inequality holds when $R_i^{n*} \geq a$. Furthermore, $\tilde{\mu}_i$ increases as c_i increases.

Proof. Denote the production cost parameter of ranges for this model as c_i. When $R_i^{n*} < a$, this firm receives 0 subsidy under (a, τ_n). So the unique solution to $s_i^{l*} * \mu R_i^{l*} = s_i^{n*}\tau_n \mathbb{1}(R_i^{n*} \geq a)$ is $\tilde{\mu}_i = 0$ and $\pi_i^{n*} = \pi_i^{l*}$.

When $R_i^{n*} \geq a$, I will first show the existence and uniqueness of μ. The market share of this model i is:

$$s_i^* \approx \frac{\exp(-1+\alpha c_i(R_i^*)^2+\beta^R R_i^*-\alpha\tau(R_i^*))}{1+\sum_{m\in\mathcal{M}}\exp(-1+\alpha c_m(R_m^*)^2+\beta^R R_m^*-\alpha\tau(R_m^*))}$$

where $\tau(R)=\tau_n\mathbb{1}(R^{n*}\geq a)$ under (a,τ_n) and $\tau(R)=\mu R$ under the linear scheme μ. $\mathcal{M}$ is the set of all the models available on the market. When the range and price choices of the remaining models are held fixed, the model i's market share under a linear scheme at its optimal range and price increases in μ because $\exp(-1+\alpha c_i(R_i^*)^2+\beta^R R_i^*-\alpha\tau(R_i^*))=\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_i\alpha}\right)$ under the linear scheme μ, similar to the derivation in Equation (17) and (18), and is monotonically increasing in μ. Therefore, when $\mu=\infty$, $R^{l*}=\frac{\mu\alpha-\beta^R}{2c_i\alpha}=\infty$, so the left-hand side of the equation $s_i^{l*}*\mu R_i^{l*}=s_i^{n*}\tau_n\mathbb{1}(R_i^{n*}\geq a)$ is ∞. When $\mu=0$, the left-hand side is 0. Furthermore, the left-hand side is monotonically increasing in μ because both s^{l*} and R^{l*} are monotonically increasing in μ and is a continuous function of μ on the space of non-negative real numbers $\mathbb{R}^+$. Therefore, there exists a unique $\tilde{\mu}_i>0$ such that $s_i^{l*}*\mu R_i^{l*}=s_i^{n*}\tau_n$ holds.

Next, I will show that $\pi^{n*}>\pi^{l*}$ when $R_i^{n*}\geq a$. Suppose $\tilde{\mu}_i R^{l*}=\tau_n$, then according to $s_i^{l*}*\mu R_i^{l*}=s_i^{n*}\tau_n$, $s_i^{l*}=s_i^{n*}$. However, $\exp(-1+\alpha c_i(R_i^{l*})^2+\beta^R R_i^{l*}-\alpha\mu R_i^{l*})<\exp(-1+\alpha c_i(R_i^{n*})^2+\beta^R R_i^{n*}-\alpha\tau_n)$. The proof of this inequality is as follow.

$\frac{\mu\alpha-\beta^R}{2c_i\alpha}=R_i^{l*}>R_i^{n*}=-\frac{\beta^R}{2c_i\alpha}$ because $\tilde{\mu}_i>0$. $\alpha c_i(R_i^{n*})^2+\beta^R R_i^{n*}<\alpha c_i(R_i^{n*})^2+\beta^R R_i^{n*}$ because $\alpha c_i R^2+\beta^R R$ is decreasing in R when $R\geq-\frac{\beta^R}{2c_i\alpha}$. When $\tilde{\mu}_i R^{l*}=\tau_n$, then it must hold that $\exp(-1+\alpha c_i(R_i^{l*})^2+\beta^R R_i^{l*}-\alpha\mu R_i^{l*})<\exp(-1+\alpha c_i(R_i^{n*})^2+\beta^R R_i^{n*}-\alpha\tau_n)$. Therefore, when $\tilde{\mu}_i R^{l*}=\tau_n$, $s_i^{l*}<s_i^{n*}$, so the amount of subsidies model i receives under $\tilde{\mu}_i$ is smaller than under (a,τ_n). Therefore, $\tilde{\mu}_i R^{l*}>\tau_n$ because the amount of subsidy received under μ increases in μ. The equality $s_i^{l*}*\tilde{\mu}_i R_i^{l*}=s_i^{n*}\tau_n$ implies $s_i^{l*}<s^{n*}$ under $\tilde{\mu}_i$. Under the setup that firms market shares are small, i.e. Equation (3), this implies $\pi_i^{n*}=\pi_i^{l*}$.

Since s_i^{l*} increases as c_i decreases or $\tilde{\mu}_i$ increases, therefore $\tilde{\mu}_i$ increases in c_i. ■

Lemma 1 *In an economy of two single-product firms, j and m face a notched scheme (a,τ_n), if both firms have a small market share, i.e. there is a large share of consumers choosing the outside option, then there exists a unique linear scheme with $\tilde{\mu}$ such that $\sum_{f\in\{j,m\}}s_f^{l*}*\tilde{\mu}R_f^{l*}=\sum_{f\in\{j,m\}}s_f^{n*}\tau_n\mathbb{1}(R_f^{n*}\geq a)$.*

Proof. Without loss of generality, I assume $c_j>c_m$ therefore $R_j^{l*}<R_m^{l*}$ and $R_j^{n*}<R_m^{n*}$.

The market shares are approximately:

$$s_j^* \approx \frac{\exp(-1+\alpha c_j(R_j^*)^2 + \beta^R R_j^* - \alpha\tau(R_j^*))}{1+\sum_{f\in\{j,m\}}\exp(-1+\alpha c_f(R_f^*)^2+\beta^R R_f^* - \alpha\tau(R_f^*))}$$
$$s_m^* \approx \frac{\exp(-1+\alpha c_m(R_m^*)^2 + \beta^R R_m^* - \alpha\tau(R_m^*))}{1+\sum_{f\in\{j,m\}}\exp(-1+\alpha c_f(R_f^*)^2+\beta^R R_f^* - \alpha\tau(R_f^*))}$$

The approximation is due to approximation of the optimal prices as in Equation (4). According to Equation (17) and (18), the market shares under a linear scheme μ are approximately:

$$s_j^{l*} \approx \frac{\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_j\alpha}\right)}{1+\sum_{f\in\{j,m\}}\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_f\alpha}\right)}$$
$$s_m^{l*} \approx \frac{\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_m\alpha}\right)}{1+\sum_{f\in\{j,m\}}\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_f\alpha}\right)}$$

s_j^{l*} and s_m^{l*} increase as μ increases. Because R_j^{l*} and R_m^{l*} also increase with μ, so the left hand side of $\sum_{f\in\{j,m\}} s_f^{l*} * \mu R_f^{l*} = \sum_{f\in\{j,m\}} s_f^{n*}\tau_n \mathbb{1}(R_f^{n*} \geq a)$ increases with μ. The left-hand side is 0 when $\mu = 0$ and goes to infinity when $\mu \to \infty$. The left-hand side is also a continuous function of μ. Therefore, there exists a unique $\tilde{\mu} \in \mathbb{R}^+$ such that the equality holds. Further more, when $a \leq R_m^{n*}$, i.e. the right-hand side is positive, $\tilde{\mu}$ is also positive. ■

Proposition 3 *In an economy of two single-product firms j and m face a notched scheme (a, τ_n) and the notched scheme (a, τ_n) and the linear scheme $\tilde{\mu}$ are the same as in Lemma 1, if both firms have a small market share, i.e. there is a large share of consumers choosing the outside option and that $c_j \geq c_m$ and if a is low enough that $a \leq R_f^{n*}$ for $f \in \{j,m\}$, then there exists a threshold Δ_c such that when $\Delta_c < \infty$,*

$$\pi_f^{l*} < \pi_f^{n*}, \textit{ for } f \in \{j,m\}\ , \textit{ if } |c_j - c_m| < \Delta_c$$
$$\pi_j^{l*} < \pi_j^{n*} \textit{ but } \pi_m^{l*} > \pi_f^{m*}\ , \textit{ if } |c_j - c_m| > \Delta_c$$

When $\Delta = \infty$,

$$\pi_f^{l*} < \pi_f^{n*}, \textit{ for } f \in \{j,m\}$$

Proof. Since both firms' market share are small, changes in their market share under a linear scheme μ in response to changes in μ is dominated by the numerator of their market shares:

$$
\begin{aligned}
s_j^{l*} \approx & \frac{\exp\left(-1 - \frac{(\mu\alpha-\beta^R)^2}{4c_j\alpha}\right)}{1+\sum_{f\in\{j,m\}}\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_f\alpha}\right)} \\
s_m^{l*} \approx & \frac{\exp\left(-1 - \frac{(\mu\alpha-\beta^R)^2}{4c_m\alpha}\right)}{1+\sum_{f\in\{j,m\}}\exp\left(-1-\frac{(\mu\alpha-\beta^R)^2}{4c_f\alpha}\right)}
\end{aligned}
$$

Therefore s_f^{l*} for $f \in \{j, m\}$ increases in μ. Define $\tilde{\mu}_j$ and $\tilde{\mu}_m$ as the values that satisfy the following equations

$$
\begin{aligned}
\tilde{\mu}_j s_j^{l*}(\tilde{\mu}_j) R_j^{l*}(\tilde{\mu}_j) = \tau_n s_j^{n*} \\
\tilde{\mu}_m s_m^{l*}(\tilde{\mu}_m) R_m^{l*}(\tilde{\mu}_m) = \tau_n s_m^{n*}
\end{aligned}
$$

Where $s_f^{l*}(\mu)$ and $R_f^{l*}(\mu)$ denote the equilibrium market share and optimal ranges of firm f under the linear scheme μ and s_j^{n*} denotes the market share of firm f under the given notched scheme (a, τ_n).

Following a similar argument as in Proposition 2, $\tilde{\mu}_m \leq \tilde{\mu} \leq \tilde{\mu}_j$. $\tilde{\mu}_m \leq \tilde{\mu}_j$ comes from a similar argument for Proposition 2's results that $\tilde{\mu}_i$ increases in c_i. One can show that when $\tilde{\mu} < \tilde{\mu}_m$, the total amount of subsidies under $\tilde{\mu}$ is smaller than that under (a, τ_n); when $\tilde{\mu} > \tilde{\mu}_j$, the total amount of subsidies under $\tilde{\mu}$ is greater than that under (a, τ_n).

Since $\tilde{\mu} \leq \tilde{\mu}_j$, $s_j^{l*}(\tilde{\mu}) < s_j^{n*}$ (Proposition 2). The relation between $s_m^{l*}(\tilde{\mu})$ and s_m^{n*} depends on the distance between c_j and c_m. If $c_j = c_m$, $\tilde{\mu} = \tilde{\mu}_j = \tilde{\mu}_m$. If $c_j > c_m$, the sign of $s_m^{l*}(\tilde{\mu}) - s_m^{n*}$ depends on the sign of

$$
\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m - \tau_n)
$$

where μ_m is defined as the value that satisfy $s_m^{l*}(\mu_m) = s_m^{n*}$, i.e. the value of μ that equates m's market share under μ to its market share under (a, τ_n). Following the argument in Proposition 2, $\tilde{\mu} < \mu_m$. If $\mu_m < \tilde{\mu}$, i.e. μ_m is not large enough in the sense that the total amount of subsidy under $\tilde{\mu}$ is smaller than that under (a, τ_n), then $\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m - \tau_n) > 0$ and $s_m^{l*}(\tilde{\mu}) > s_m^{n*}$.

If I hold c_m constant but increase c_j, i.e. increase the distance Δ_c between c_j and c_m, μ_m almost doesn't change because share of outside option is close to 1. This also implies that $R_m^{l*}(\mu_m)$ barely changes. However, $\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m)$ increases because $\tilde{\mu}_j$ increases in c_j. Therefore, $\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m - \tau_n)$ increases in c_j when holding c_m constant. When $c_j = c_m$, since $\tilde{\mu} = \tilde{\mu}_m < \mu_m$, $\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m -$

$\tau_n) < 0$.

If there exists a $\tilde{c}_j$ such that

$$\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m - \tau_n) = 0$$

and that $R_j^{n*} >\geq a$, then $\Delta_c \equiv \tilde{c}_j - c_m$. If for all the $c_j >= c_m$ and that $R_j^{n*} >\geq a$, $\tau_n s_j^n - \mu_m R_j^{l*}(\mu_m) s_j^{l*}(\mu_m) - s_m^{n*}(R_m^{l*}(\mu_m)\mu_m - \tau_n) < 0$, then $\Delta_c \equiv \infty$. Then,

$$\begin{aligned} s_f^{l*} < s_f^{n*}, \text{ for } f \in \{j, m\} &, \text{ if } |c_j - c_m| < \Delta_c \\ s_j^{l*} < s_j^{n*} \text{ but } \pi_m^{l*} > \pi_f^{m*} &, \text{ if } |c_j - c_m| > \Delta_c \end{aligned}$$

When $\Delta_c = \infty$, then $s_f^{l*} < s_f^{n*}$ for $f \in \{j, m\}$. According to Equation (3), this establishes the desired results.

■

Proposition 3 implies that when holding the total amount of subsidies constant, firms' increases in profits are higher under the notched scheme if they are sufficiently close technologically and that both firms have stronger incentives to invest out of the binding threshold constraints of a notched scheme. If the technology gap between two firms is sufficiently large, the laggard's incentive to invest is still larger under the notched scheme while the leader's incentives are larger under the linear scheme because the linear scheme allows the leader to claim the vast majority of the total subsidy using its superior technology.