Geo Problems

XYZYZL

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§1 EGMO

§1.1 Chapter 1

§1.2 Chapter 2

§1.3 Chapter 3

Problem 1.1 (APMO 2004/2)**.** 27. Let O be the circumcenter and H the orthocenter of an acute triangle ABC. Prove that the area of one of the triangles AOH, BOH, and COH is equal to the sum of the areas of the other two.

Solution 1.2. Because the base of each of the triangles is equal, it suffices to prove that the heights of the triangles from OH satisfy this property. For simplicity, we claim that $[AOH] = [BOH] + [COH]$; the other directions can be proven similarly.

It is well known that O, G, H are collinear on the Euler Line. Note that we can make the base any line that passes through G, including the one t that is parallel to BC. Let the median from A intersect $\overline{BC}$ at M. Then $AG = 2GM$, so the height from A to t is twice the height from B to t, which is equal to the height from C to t as $t||BC$. Hence, the height from A to t is equal to the sum of the height from B to t and the height from C to t, which completes the proof.