GOV 2001/ 1002/ E-200 Section 11 Choice Modeling and Future Directions in Methods[1]

Anton Strezhnev

Harvard University

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[1] These section notes are heavily indebted to past Gov 2001 TFs for slides and R code.

LOGISTICS

Reading Assignment- UPM Chapter 8, Glasgow et. al., 2012

Final Paper Due April 27th at 5:00 pm. But with automatic extensions to May 5th at 5:00 pm.

Final Exam (for E-school students not doing the paper) Released on April 27th. You will "check out" the exam on Canvas any time during exam period. After check-out, you will have 1 week to finish. The final deadline is May 14th at 5:00 pm.

Fill out the RSVP for the party on May 7th! We only have 4 respondents so far!

Overview

- In this section you will...
  - learn how to model choice data
  - learn how latent space models work.
  - learn how to generalize from an unrepresentative sample.
  - learn how to think about learning methods beyond this course!

OUTLINE

Choice Models

Ideal Point Models

Modern Survey Sampling

Learning more methods

MODELING CHOICES

- We want to model some choice among a set of unordered outcomes...
  - ...vote choice in multiparty elections.
  - ...choices among potential coalition partners in government.
  - ...patients choosing different types of medications.
  - ...consumer purchasing decisions.

Multinomial logit

- Can intuitively generalize our friendly logit model to multiple outcomes – the multinomial logit!
- Stochastic component: Y_i is a J-length vector -

  $$Y_i \sim \text{Multinomial}(1, \vec{\pi}_i)$$

  where $\vec{\pi}_i$ is a J-length vector of choice probabilities for each of J choices: $\{\pi_{i1}, \pi_{i2}, \ldots, \pi_{ij}\}$
- Systematic component:

  $$\pi_{ij} = \frac{\exp(\eta_{ij})}{\sum_{k=1}^{K} \exp(\eta_{ik})}$$
  $$\eta_{ij} = X_i \beta_j$$

Multinomial logit

- Identification: Need to fix one category as “baseline”. For notation, that’s J. So let $\eta_{iJ} = 0$ and therefore $\exp(\eta_{iJ}) = 1$.
- How many parameters are we estimating? $J - 1 \times$ length of β.
- Likelihood $L(\beta|X, Y)$:

$$\propto \prod_{i=1}^{N} \prod_{j=1}^{J} \pi_{ij}^{Y_{ij}}$$

$$\propto \prod_{i=1}^{N} \prod_{j=1}^{J} \left[\frac{\exp(\eta_{ij})}{\sum_{k=1}^{K} \exp(\eta_{ik})} \right]^{Y_{ij}}$$

$$\propto \prod_{i=1}^{N} \left[\prod_{j=1}^{J-1} \left[\frac{\exp(X_i \beta_j)}{1 + \sum_{k=1}^{K-1} \exp(X_i \beta_k)} \right]^{Y_{ij}} \times \left[\frac{1}{1 + \sum_{k=1}^{K-1} \exp(X_i \beta_k)} \right]^{Y_{iJ}} \right]$$

ASSUMPTION: INDEPENDENCE OF IRRELEVANT ALTERNATIVES

- Likelihood massively simplified by assuming logit form for each observation.
- However, has implicit assumption about choice behavior: Independence of Irrelevant Alternatives (IIA).
- Ratio of choice probability of category 1 to 2 does not depend on any other category:

$$\frac{\pi_{ij}}{\pi_{ik}} = \frac{\frac{\exp(\eta_{i1})}{\sum \exp(\eta_{ik})}}{\frac{\exp(\eta_{i2})}{\sum \exp(\eta_{ik})}} = \frac{\exp(\eta_{i1})}{\exp(\eta_{i2})} = \frac{\exp(X_i\beta_1)}{\exp(X_i\beta_2)}$$

Violations of IIA

- What does it mean for IIA to be violated? Adding or removing a third option should not affect the ratio of choice probabilities between the other categories.
- Commonly violated when choices are *substitutes*.
- Red Bus/Blue Bus problem:
  - A person chooses between commuting by Car or a Red Bus. They're indifferent so $Pr(\text{Car}) = Pr(\text{Red Bus}) = .5$ and $\frac{Pr(\text{Car})}{Pr(\text{Red Bus})} = 1$.
  - Suppose a third option is introduced - a Blue Bus. Let's assume that the color doesn't really matter to the person, so *given* that they take a bus, they'll take either with equal probability.
  - New probs: $Pr(\text{Car}) = .5$, $Pr(\text{Red Bus}) = Pr(\text{Blue Bus}) = .25$. $\frac{Pr(\text{Car})}{Pr(\text{Red Bus})} = \frac{.5}{.25} = 2 \neq 1$

Conditional Logit

- The multinomial model only considers attributes of individuals. But we might want to know how characteristics of alternatives/choices affect behavior?
  - Market research: What's the probability of buying a red car vs. a grey car
  - Appointments: Given that a president picks a Supreme Court candidate, how does experience/background affect probability of appointment.
- "Conditional" because we are conditioning on a choice being made among a set of alternatives.
- Systematic component changes slightly - same logit form, but η_{ij} changes

$$\eta_{ij} = Z_j\gamma$$

Z_j are covariates for alternative j and γ are estimated coefficients.

Combining Multinomial and Conditional Logit

- Can combine the two to estimate *both* individual and alternative specific attributes (and interactions!)

$$\eta_{ij} = X_i \beta_j + Z_{ij} \gamma$$

Outline

Latent Space Modeling

European Parliament. Photo by David Iliff. License: CC-BY-SA 3.0

- We have high dimensional data
  - ... M votes in Congress by N legislators
  - ... M exam questions by N students
- We want to summarize patterns in a meaningful way.
  - ... which legislators are the most liberal/conservative
  - ... which students perform the best on exams.

Why model?

- Intuitive summary - look at % voting agreement between two legislators.
- Problem! What does 90% agreement mean?
  - What if those 90% were unanimous votes? What if those were votes where *only* those two legislators were on the same side? How should we interpret % agreement?
- Exam Analogy:
  - Year 1 - student gets 70% on an exam. Year 2 - student gets 90%. Did the student improve? Or did the exam get easier?
- Simple metrics like % agreement miss important variation in agenda.

Item Response Theory (IRT)

- Developed in educational testing! But huge expansion into psychology, sociology and political science.
- **Goal:** Infer latent ability/preferences from observed outcomes (test questions/votes).
- Starting point for poli-sci: Clinton, Jackman, Rivers (2004) "The Statistical Analysis of Roll Call Data" APSR.

Simple 2-Parameter, 1-Dimensional Model

- We observe a N by M matrix of roll call votes $\mathbf{Y}$. N legislators. M votes.
- Assume each legislator i has a single latent unobserved "ideal point" $\mathbf{x_i}$. For each vote j, the observed outcome Y_{ij} is

$$Y_{ij} = \begin{cases} 1 & \text{if } z_{ij} > 0 \\ 0 & \text{if } z_{ij} \leq 0 \end{cases}$$

- and z_{ij} is a combination of ideal point, roll call characteristics, and random error.

$$z_{ij} = \alpha_j + \beta_j \mathbf{x_i} + \epsilon_{ij}$$

- Possible to justify this from a "utility maximization" model

Simple 2-Parameter, 1-Dimensional Model

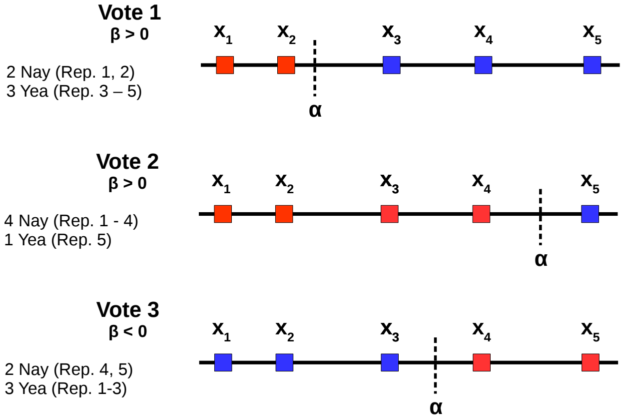
- If we assume $\epsilon_{ij} \sim \mathcal{N}(0,1)$, then we can write

$$Pr(Y_{ij} = 1) = \Phi(\beta_j \mathbf{x_i} - \alpha_j)$$

- What does that remind us of? A probit model!
- What do the parameters mean?
  - α_{ij}: "difficulty" parameter – For roll calls: if close to 0, then vote is probably evenly split. If large, then vote is probably lopsided.
  - β_j: "discrimination" parameter – For roll calls: How well does this vote reflect latent preferences? Positive β_j: high $\mathbf{x_i}$ = high $Pr(Y_{ij} = 1)$. Negative β_j: high $\mathbf{x_i}$ = low $Pr(Y_{ij} = 1)$.

IRT Example

Figure: Example of latent space model with no voting error



IDENTIFICATION

- We can write the likelihood as the product of Y_{ij} over i and j (assuming independence between votes).
- What's the issue with ML estimates? Not identified! Likelihood depends only on distances between ideal points. Invariant to scale or rotation!
- Solutions:
  - Constrain scale
  - Fix some legislators' locations
- Even then, ML estimates are inconsistent. As N gets large, the number of parameters also grows!
- More simply - it's just a hard likelihood to maximize!

Bayesian Estimation

- Most modern ideal point estimation techniques rely on Bayesian approaches (with priors on the ideal point and roll call parameters to constrain the estimates).
- "Markov Chain Monte Carlo" (MCMC) techniques allow us to simulate draws from the posterior and obtain point estimates/credible intervals.
- **Intuition**
  - Posterior $f(\mathbf{x}, \alpha, \beta|\mathbf{Y})$ hard to calculate!
  - But $f(\alpha, \beta|\mathbf{x}, \mathbf{Y})$ is just probit regression!
  - And $f(\mathbf{x}|\alpha, \beta, \mathbf{Y})$ is also a regression problem!
  - MCMC methods repeatedly take draws from these conditionals. Markov chain theory tells us that this converges to drawing from the true posterior!

What IRT models can show us.

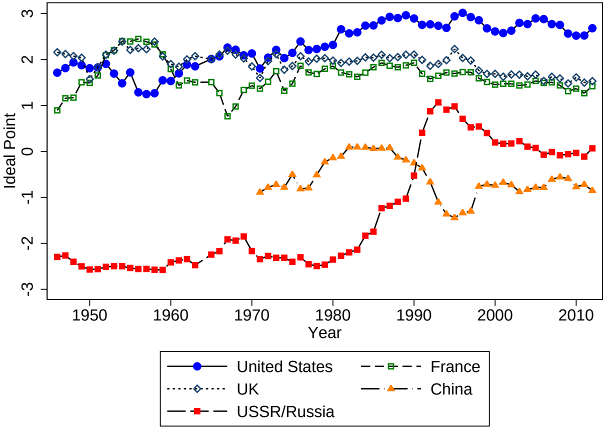


Figure: Dynamic ideal point estimates of P5 countries from UNGA voting - Voeten et. al. (2015)

OUTLINE

The Crisis in Polling

- Traditional landline-based surveys no longer get close to a representative sample of the population.
  - Growth of cell-phone use
  - Abysmal response rates (5% to 15% for Pew)
- Pretty much all pollsters work with non-random samples.
- True of researchers as well. Why do we talk about random samples then? Theoretical exposition!

Generalizing from non-random samples

- How do we get an unbiased estimated of the population using a non-random sample? Statistical modelling! Meaning: Assumptions!
- Ideally, we'd know $Pr(\text{Person Selected})$. When we do by design, can weight by $\frac{1}{Pr(\text{Person Selected})}$. This is rare though (especially for internet convenience samples).
- One common approach – "Multilevel Regression and Post-stratification" (MRP) (or Mr. P!)
  - Multilevel Regression: Make a model predicting individual response using *individual* and *group* (e.g. county/state) variables.
  - Post-stratification: Use what we know about population-level covariate distribution to reweight data for predictions.

Post-Stratification

- Why post-stratification?
  - "Stratification" because we're trying to match our sample proportions to population proportions for certain strata (age, gender, etc...)
  - "Post-" because we do it *after* sampling.
- Example: Using a convenient internet sample, predict % that would vote for Obama in Hennepin County, MN.
- How? Re-weight our sample to match known population characteristics of Hennepin County.

POST-STRATIFICATION

- Basic post-stratification weighting
  - Step 1: Identify the population of interest - e.g. whole U.S. population, state of NY, Cuyahoga County.
  - Step 2: Identify covariates that are correlated with the outcome you care about. E.g. For vote choice: Party ID, gender, race, income, etc...
  - Step 3: Get data on the distributions of covariates for your population of interest. Often proportions (e.g. % registered Dem, % White, etc...). Often using census data.
  - Step 4: Calculate weights for your observations such that the (weighted) sample distributions of covariates match the population distributions.

Calculating PS Weights

- When the full distribution of strata is known, weight for observation i in stratum h:

$$w_i = \frac{n}{n_h} \times P_h$$

  - n is the sample size
  - n_h is the number of sample obs. in stratum h
  - P_h is the population proportion in stratum h
- Can also think of it as

$$w_i = \frac{P_h}{p_h}$$

  where p_h is the sample proportion in stratum h.
- **Intuition:** Upweight observations that are rare relative to population. Downweight observations that are common.

Raking

- However, you rarely have the full joint distribution for lots of covariates. Just the marginals.
  - Ex. We know % White, % Women, % Age 18-35 from census. But we don't know % White Women Age 18-35.
- Solution: "raking" - iteratively reweight to match the population marginals as closely as possible
  - Implemented in the `R` package `survey`
- Raking procedure:
  - Step 1: Calculate PS weights for the first variable.
  - Step 2: Using those weights, calculate the new in-sample proportions of the second variable.
  - Step 3: Re-calculate the PS weights for the second variable given the previously calculated weighting.
  - Step 4: Repeat across all of the variables in sequence until convergence (no change in weights).

OUTLINE

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Learning more methods

WHY METHODS?

- Research is about persuasion.
  - Given some data, why should I believe your story about the world?
- Research methods provide a *common* language for arguing from data.
- Statistics gives us a coherent set of rules for drawing inferences given data.
  - Still have to argue for the assumptions.
  - But statistics gives us tools to adjudicate. And different methods entail different arguments – they rely on different (sometimes weaker/more flexible) assumptions.
  - Alternatively, can justify assumptions via *design* (e.g. randomization/natural experiments).
- Perusading researchers requires you to make arguments that make sense to both *you* and them. Statistical methods lay out one useful method of argumentation.

WHAT METHODS?

- Don't think only in terms of learning about specific methods.
  - You'll be learning fancy new methods by reading papers. Instead, focus on how to understand those papers and how to fit those methods into your repertoire.
  - E.g.: Don't just learn "text analysis," learn how to think about high-dimensional data where $p >> n$.

What Methods?

- What questions does a method help you answer better?
  - **Modeling.** How can I parsimoniously summarize my data in a meaningful way?
    - Primary topic of this course. Choosing among models is often challenging.
    - Trade-off between flexibility and parsimony.
    - Simple example: Logit models for binary outcomes
    - Complex example: Latent space models for roll call voting.
  - **Estimation.** How can I get a good estimate of a quantity using my data?
    - Bias/Variance trade-offs. Frequentist vs. Bayesian approaches.
  - **Identification.** How can I connect quantities I can estimate to quantities that I care about (e.g. causal effects)?
    - Often contributions in terms of research design.
    - Simple example: randomization for causal effects.
    - More complex examples: instrumental variables, regression discontinuity

WHAT METHODS?

- All of these are elements of almost every methods paper!
  - I can write down a really complicated model... but it's useless if I can't estimate it!
  - I can get a really efficient estimate of some regression parameter... but if I want to claim causality, it's useless if I can't also argue that it identifies a causal parameter of interest.
- **Main Takeaway:** Think first in terms of what you need to better argue from your data, then go out and find what you don't know.

Questions

Questions?