Advanced Quantitative Research Methodology, Lecture Notes: Single Equation Models[1]

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Binary Variable Regression Models

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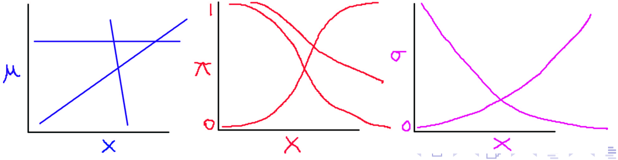
(The graph in the middle:)



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What do we do with this?

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   (d) Average effects: compute effects for every observation and average

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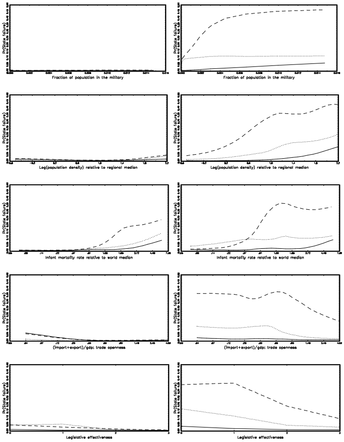

Example Marginal Effect Plot of a neural network model (about which more later). Full democracies (dotted), partial democracies (dashed), and autocracies (solid). From Gary King and Langche Zeng. "Improving Forecasts of State Failure," *World Politics*, Vol. 53, No. 4 (July, 2001): 623-58.

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For any quantity but a probability, always also include a measure of uncertainty (standard error, confidence interval, etc.)

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Variable	From		To	First Difference
Sex	Male	$\rightarrow$	Female	.05
Age	65	$\rightarrow$	75	$-.10$
Home	NYC	$\rightarrow$	Madison, WI	.26
Income	\$35,000	$\rightarrow$	\$75,000	.14

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(a) Max value of logit derivative: $\hat{\beta} \times 0.5(1 - 0.5) = \hat{\beta}/4$

(b) Max value for probit $[\pi_i = \Phi(X_i\beta)]$ derivative: $\hat{\beta} \times 0.4$

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Since Y^* is unobserved anyway, define the threshold as $\tau = 0$. (Plus the same independence assumption, which from now on is assumed implicit.)

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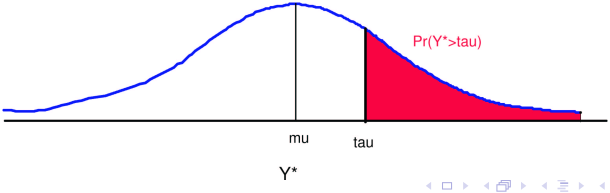

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5. $\implies$ interpret β as regression coefficients of Y^* on X: $\hat{\beta}_1$ is what happens to Y^* on average (or μ_i) when X_1 goes up by one unit, holding constant the other explanatory variables (and conditional on the model). In probit, one unit of Y^* is one standard deviation.

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- Which would enable you to choose logit over probit?

How Not to Present Statistical Results

TABLE 1

Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
Nagelkerke R^2	.295
Overall correct classification (%)	73.96

SOURCE: *Times Mirror* polls from September 1992, January 1993, September 1993, January 1994, and June 1995.

NOTE: Unstandardized coefficients for logistic regression. Dependent variable is knowledge of which group conquered most of Bosnia.

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1. This one is typical of current practice, not that unusual.

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How Not to Present Statistical Results

TABLE 1
Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
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Overall correct classification (%)	73.96

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4. Reading assignment: (at my web page) the handout on papers, and King, Tomz, Wittenberg, "Making the Most of Statistical Analyses: Improving Interpretation and Presentation" American Journal of Political Science, Vol. 44, No. 2 (March, 2000): 341-355.

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Denote the draw $\tilde{\gamma} = \text{vec}(\tilde{\beta}, \tilde{\alpha})$, which has *k* elements.

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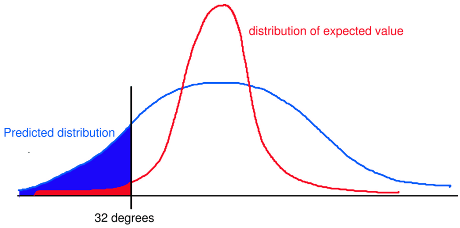
Repeat algorithm say $M = 1000$ times, to produce 1000 predicted values. Use these to compute a histogram for the full posterior, the average, variance, percentile values, or others.

The Distribution of Expected v. Predicted Values



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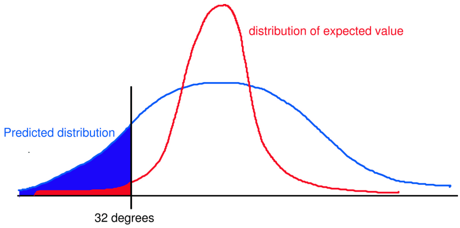

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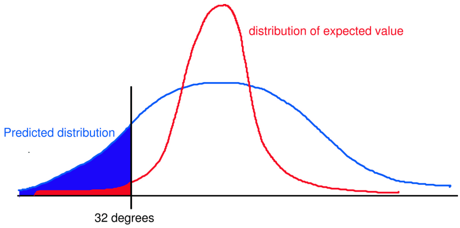

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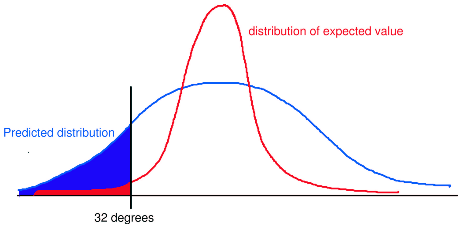

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3. Predicted values: include estimation and fundamental uncertainty.

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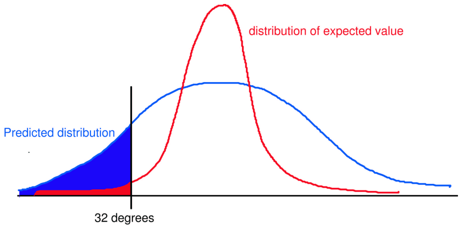

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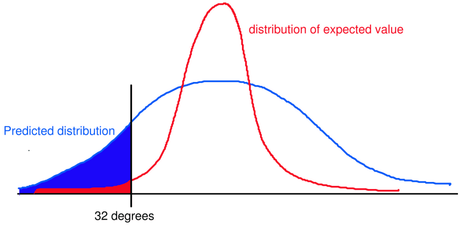
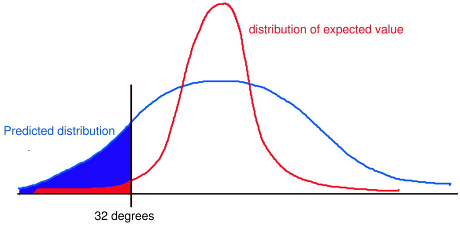
4. Expected values: average away fundamental uncertainty


distribution of expected value
Predicted distribution
32 degrees



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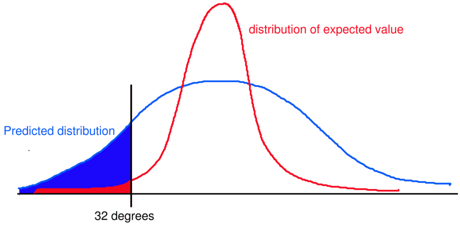

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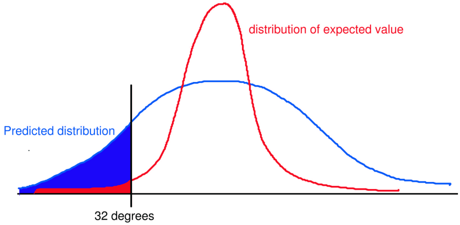

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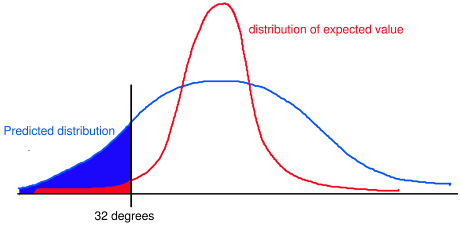

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8. Which to use for causal effects & first differences?

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5. Average over the fundamental uncertainty by calculating the mean of the m simulations to yield one simulated expected value $\tilde{E}(Y_c) = \sum_{k=1}^{m} \tilde{Y}_c^{(k)}/m$.

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5. When $E(Y_c) = \theta_c$, we can skip the last two steps. E.g., in the logit model, once we simulate π_i, we don't need to draw Y and then average to get back to π_i. (If you're unsure, do it anyway!)

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4. (To save computation time, and improve approximation, use the same simulated β in each.)

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   In all 3 cases, η is unbounded: estimate it, simulate from it, and reparameterize back to the scale you care about.

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5. Clarify does this all automatically in Stata. Zelig does the same and more in R.

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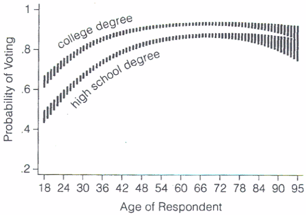
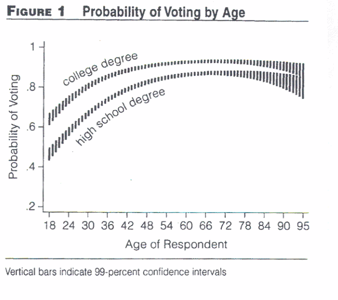
3. Use $M = 1000$ and compute 99% CI:

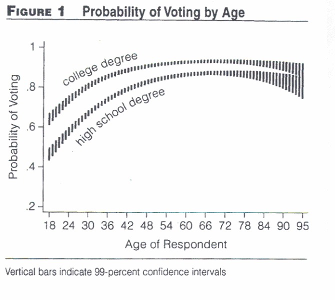
FIGURE 1 Probability of Voting by Age



Vertical bars indicate 99-percent confidence intervals



To create this graph, simulate:



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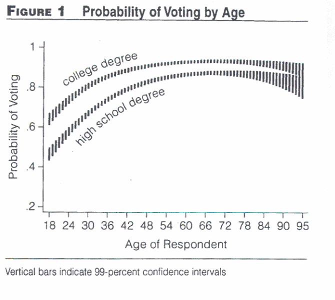

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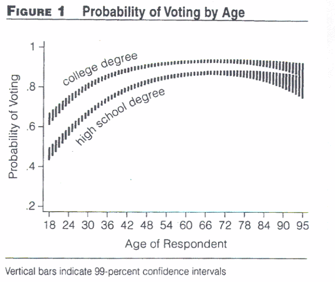

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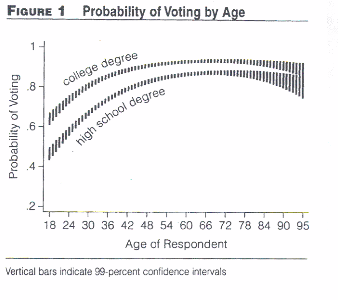

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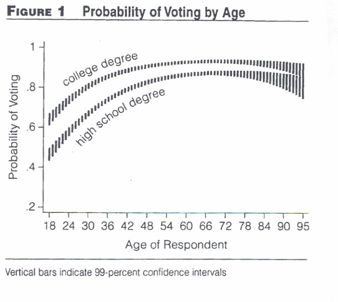

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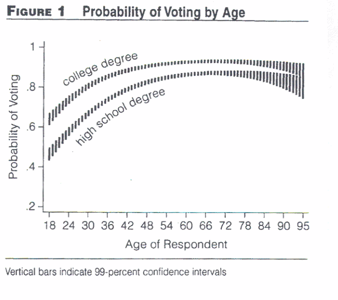

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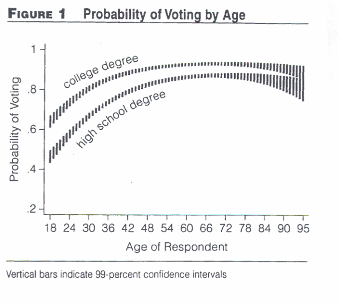

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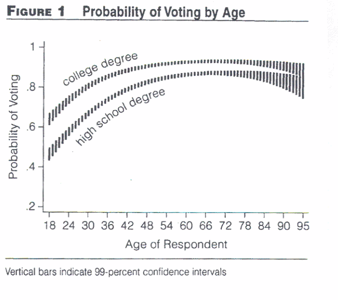

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8. Repeat for other ages and for college degree.

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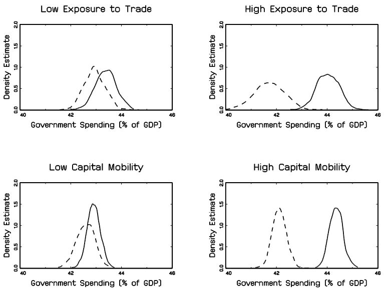

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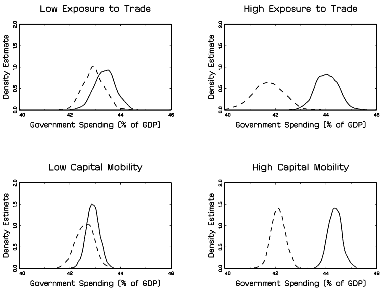

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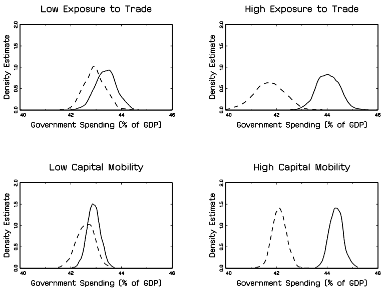

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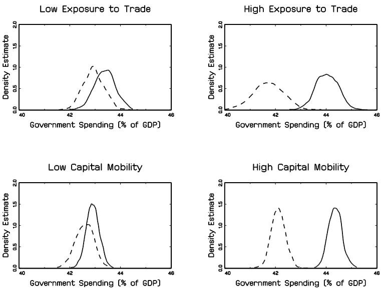

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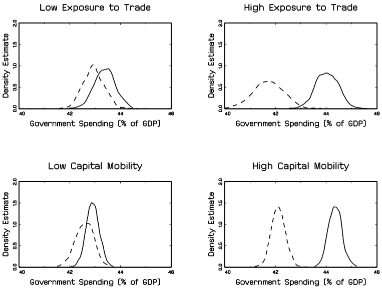

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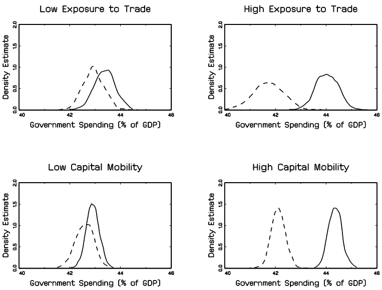

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- How could we summarize this information with less real estate?

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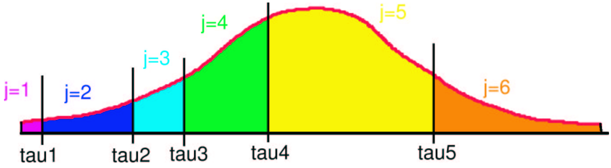

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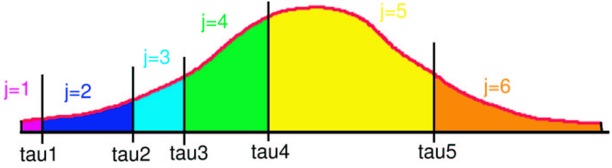

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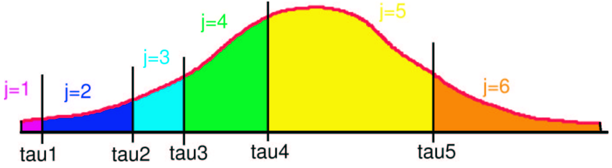

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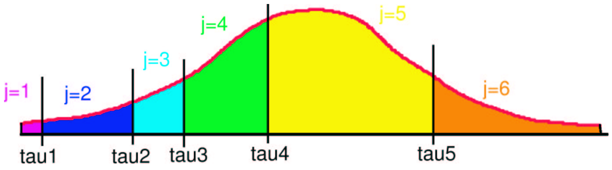

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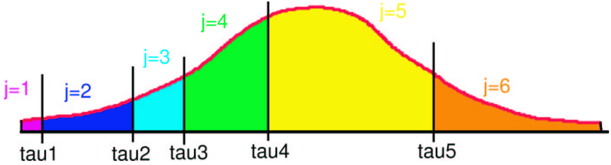

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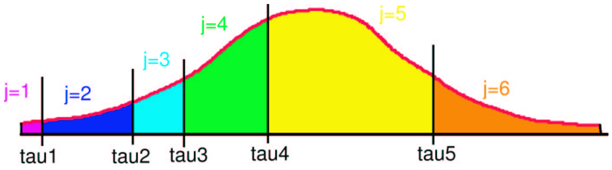
5. This is the same model, and the same parameters are being estimated; only the observation mechanism differs.



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Bracketed portion has only one factor for each i.

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\ln L(\beta, \tau|y) &= \sum_{i=1}^{n} \sum_{j=1}^{J} y_{ij} \ln \Pr(Y_{ji} = 1) \\
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Deriving the likelihood function

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(Careful of constraints during optimization: $\tau_{j-1} < \tau_j$, $\forall j$)

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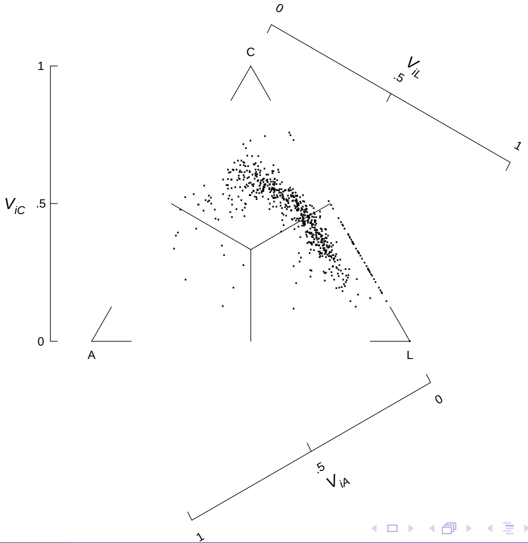

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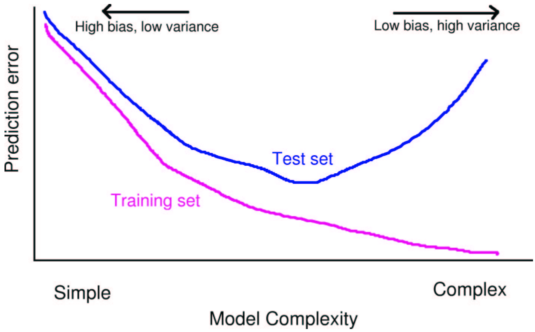

(See Trevor Hastie et al. 2001. *The Elements of Statistical Learning*, Springer, Chapter 7: Fig 7.1.)

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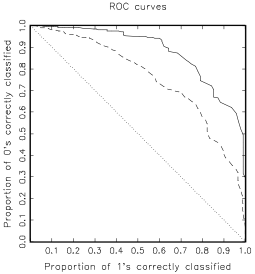
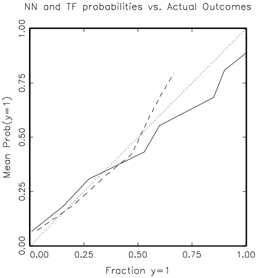
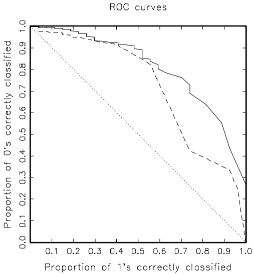
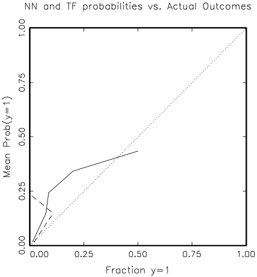
- Otherwise, one model is better than the other in only given specificed ranges of C (i.e., for only some normative perspectives).





In-sample ROC, on left (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)





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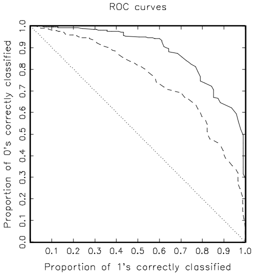
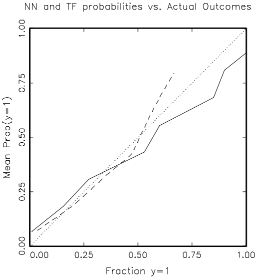
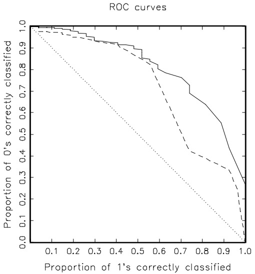
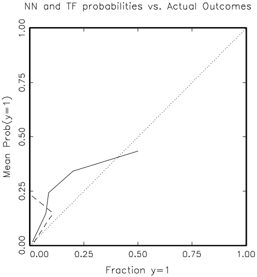




In-sample calibration graph on right (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)





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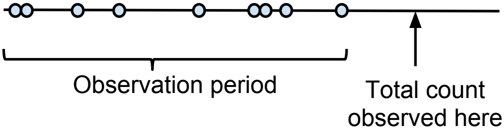

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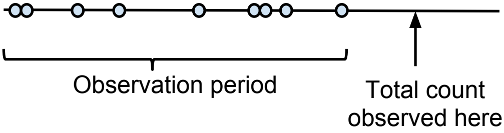

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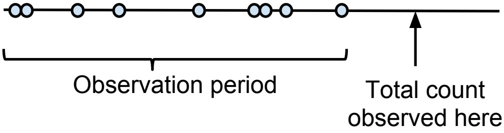

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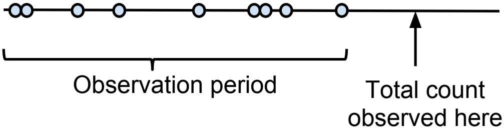

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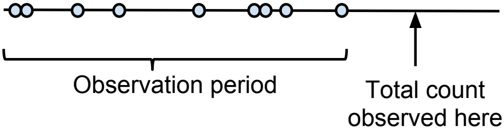

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Recall Poisson distribution's first principles:

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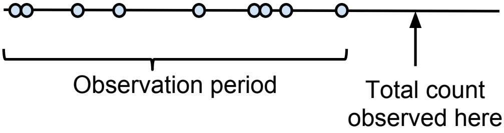

2. Assumptions are about: events occurring between start and count observation. The process of event generation is not observed.
3. 0 events occur at the start of the period
4. Observe only: number of events at end of the period
5. No 2 events can occur at the same time
6. Pr(event at time t | all events up to time $t-1$) is constant for all t.

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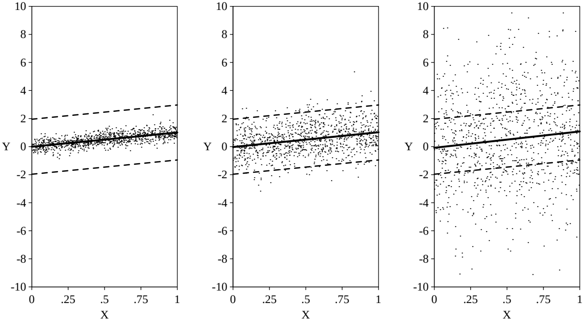
What happens without an extra parameter? Stylized Normal.



(a) (b) (c)

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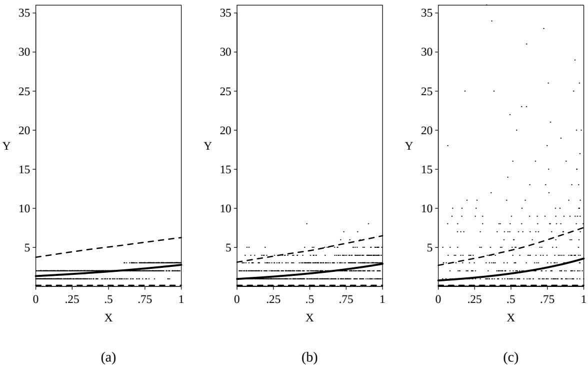

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Negative Binomial Event Count Model

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4. Careful of off-the-shelf programs: maybe $V(Y|X) = \phi(1 + \sigma^2\phi)$

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$$x^{(m,\delta)}=\begin{cases}\prod_{i=0}^{m-1}(x+\delta i)=x(x+\delta)(x+2\delta)\cdots[x+\delta(m-1)] & m\geq 1\\ 1 & m=0\end{cases}$$

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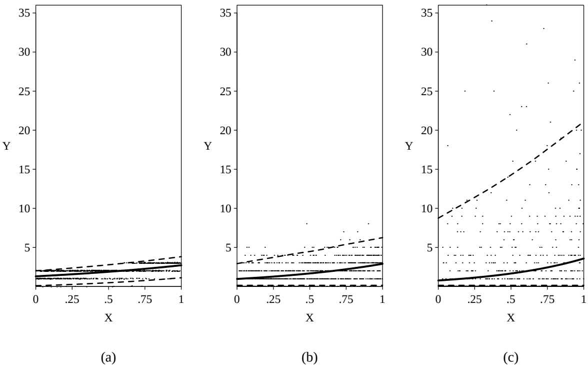
- Can simulate in three parts, indexed by σ^2

What happens with the extra parameter? GEC



$E(Y|X)$ and 95% CI.

King, Gary and Curtis S. Signorino. "The Generalization in the Generalized Event Count Model" in *Political Analysis*, 6 (1996): 225-252.

King, Gary. "Variance Specification in Event Count Models: From Restrictive Assumptions to a Generalized Estimator," *American Journal of Political Science*, 33, 3 (August, 1989): 762-784.

Duration Models and Censoring: The Exponential Model

King, Gary; James Alt; Nancy Burns; and Michael Laver. "A Unified Model of Cabinet Dissolution in Parliamentary Democracies," American Journal of Political Science, Vol. 34, No. 3 (August, 1990): Pp. 846-871; Errata Vol. 34, No. 4 (November, 1990): P. 1168. (replication dataset: ICPSR s1115).

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Log-likelihood:

$$\ln L(\beta|y) = \sum_{i=1}^{n} \{\ln \lambda_i - \lambda_i y_i\}$$

Duration Models and Censoring: The Exponential Model

A density with the same first principles as the Poisson, except that we observe Y, the duration between events. Then

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  - Include censoring information in the likelihood

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$$Y_i^* \sim \text{expon}(y_i^*|\lambda_i)$$

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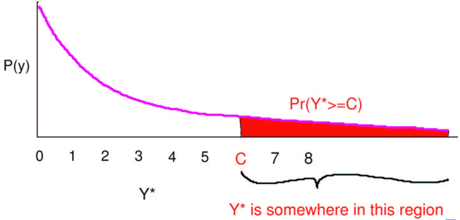

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Thus, the full likelihood:

$$L(\beta|y) = \left[\prod_{y_i^* < y_i^c} \text{expon}(y_i|\lambda_i)\right] \left[\prod_{y_i^* \geq y_i^c} \Pr(Y_i^* \geq y_i^c)\right]$$