Gov2001 Practice Final Exam - Solutions

Extension School

Spring 2016

Problem 1

In this problem, we will work with the data from Martin (1992) to predict when countries cooperate in enforcing sanctions. Load the `Zelig` package and then load the data as:

```
data(sanction)
```

The dependent variable - `coop` - is the number of countries that cooperated in enforcing sanctions.

A)

Recode the dependent variable to `coop1`, which will be 0 when only one country cooperated (`coop`$== 1$) and 1 if more than one country cooperated (`coop`> 1).

```
sanction$coop1 <- ifelse(sanction$coop>1,1,0)
```

B)

Write out the stochastic and systematic component for a probit model.

Stochastic:
$Y_i \sim f_{\text{bern}}(y_i|\pi_i)$

Systematic:

$\pi_i = \Phi(X_i\beta)$ where Φ is the CDF of the standard normal distribution.

C)

Derive the log-likelihood and write a function that evaluates the log-likelihood in R.

$$
\begin{aligned}
L(\beta|\mathbf{y}) &\propto \prod_{i=1}^{n} f_{\text{bern}}(y_i|\pi_i) \\
&= \prod_{i=1}^{n} (\pi_i)^{y_i}(1-\pi_i)^{(1-y_i)}
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
\ln L(\beta|\mathbf{y}) &\propto \sum_{i=1}^{n} y_i \ln(\pi_i) + (1-y_i)\ln(1-\pi_i) \\
&= \sum_{i=1}^{n} y_i \ln(\Phi(X_i\beta)) + (1-y_i)\ln(1-\Phi(X_i\beta))
\end{aligned}
$$

In R:

```
ll.probit <- function(beta, y=y, X=X){
  if(sum(X[,1]) != nrow(X)) X <- cbind(1,X)
  phi <- pnorm(X%*%beta, log = TRUE)
  opp.phi <- pnorm(X%*%beta, log = TRUE, lower.tail = FALSE)
  logl <- sum(y*phi + (1-y)*opp.phi)
  return(logl)
}
```

D)

The independent variables we will examine are:

- `import`: an indicator of whether the leading sender imported from the target country
- `export`: an indicator of whether the leading sender exported to the target country
- `target`: the political stability and health of the target country
- `cost`: a measure of the cost of sanctions to countries imposing them - 1 being least costly, and 4 being most costly

Use the likelihood function you coded in Part C and `optim()` to run a probit model of `coop1` on the above four variables and an intercept. Report your coefficients and standard errors in a nicely formatted table.

```
y <- sanction$coop1
```

```
X <- sanction[,c(''import'', ''export'', ''target'', ''cost'')]

yX <- na.omit(cbind(y,X))

y <- yX[,1]
X <- as.matrix(yX[,-1])

opt <- optim(par = rep(0,5), fn = ll.probit, y=y, X=X,
             method = ''BFGS'', control = list(fnscale = -1,
                                   maxit = 1000), hessian = TRUE)

#Check with zelig
out <- zelig(coop1 ~ import + export + target+ cost, data=sanction, model=''probit'')
```

	Coefficient	SE
Intercept	-0.87	0.50
import	-0.07	0.38
export	0.27	0.35
target	-0.22	0.23
cost	0.66	0.30

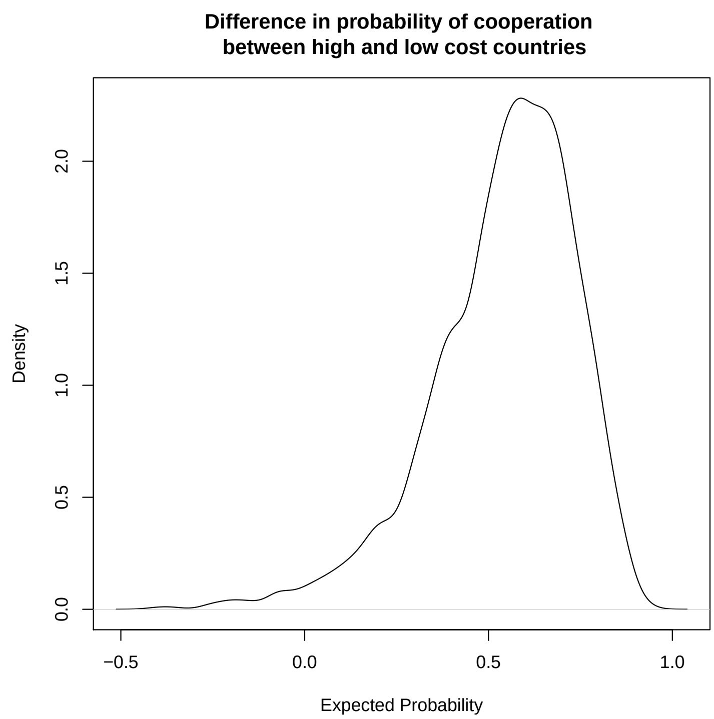
E)

Simulate the distributions of the expected probabilities for two types of countries: high cost (`cost=4`) and low cost (`cost=1`). Keep all other covariates at their means. Plot a density plot of the distribution of the first differences. What is the mean first difference?

The mean first difference is .54.

```
highcost <- c(1,apply(X,2,mean))
highcost[5] <- 4

lowcost <- c(1, apply(X,2,mean))
lowcost[5] <- 1

library(MASS)
beta.draws <- mvrnorm(10000, mu = opt$par, Sigma
                      = solve(-opt$hessian))
p.highcost <- c()
for(i in 1:10000){
  p.ass.att <- pnorm(highcost%*%beta.draws[i,])
  outcomes <- rbinom(10000, 1, p.ass.att)
```

```
  p.highcost[i] <- mean(outcomes)
}
p.lowcost <- c()
for(i in 1:10000){
  p.ass.att <- pnorm(lowcost%*%beta.draws[i,])
  outcomes <- rbinom(10000, 1, p.ass.att)
  p.lowcost[i] <- mean(outcomes)
}
pdf("FirstDiffProbit.pdf")
plot(density(p.highcost-p.lowcost), main="Difference in probability of cooperation \n between high
dev.off()

mean(p.highcost-p.lowcost)
```



Problem 2

In this problem, we will work with the data from King, Alt, Burns, and Laver (1990) to model the duration of parliamentary coalitions. Load the `Zelig` package and then load the data as:

```
data(coalition)
```

The dependent variable - `duration` - is the observed survival time of a coalition. The data is right-censored because in some cases, the constitutionally-mandated election period forces the parliamentary coalition to dissolve. In this data, `ciep12=1` if the observation is not censored and `ciep12=0` if the observation is right-censored.

The independent variables we will examine are:

- `invest`: dummy variable for legal requirement for legislative investiture
- `fract`: index of the number and size of parties in parliament
- `polar`: polarization (higher values mean more support for extremist parties)
- `numst2`: dummy variable for a majority government (1=majority, 0=minority)
- `crisis`: length in days of crisis preceding government formation

You will use the Weibull distribution as the stochastic component to model this data. The Weibull is a generalization of the exponential distribution that allows for non-flat, monotonic hazard functions. The Weibull model nests the exponential model (as will be seen below). To help you get started, we are providing you with the PDF, the CDF, and a generic likelihood for censored data.

The PDF is:

$$f(t_i) = \left(\frac{\alpha}{\lambda_i^\alpha}\right) t_i^{\alpha-1} \exp\left[-\left(\frac{t_i}{\lambda_i}\right)^\alpha\right]$$

The CDF is:

$$F(t_i) = 1 - e^{-(t_i/\lambda_i)^\alpha}$$

Note that the Weibull distribution has two parameters, λ (scale) and α (shape), where λ, $\alpha > 0$. When $\alpha = 1$, the Weibull model reduces to an exponential model.

The expected value of the Weibull distribution is:

$$E(T_i) = \lambda_i \Gamma\left(1 + \frac{1}{\alpha}\right)$$

Note that Γ is the `gamma` function in R.

Letting $c_i = 1$ when observation i is not censored and $c_i = 0$ when it is censored, the generic likelihood function for censored data is:

$$
\begin{aligned}
\mathcal{L} &= \prod_{i=1}^{n} [f(t_i)]^{c_i} \; [P(T_i \geq t_i^c)]^{1-c_i} \\
&= \prod_{i=1}^{n} [f(t_i)]^{c_i} \; [1 - F(t_i)]^{1-c_i}
\end{aligned}
$$

The systematic component is:

$$\lambda_i = \exp(x_i \beta),$$

where x_i contains all 5 of the independent variables noted above as well as an intercept. You do not need to parametrize α as a function of covariates.

A)

Derive the log-likelihood and program it in R. (Hint: remember to reparametrize the α parameter so as to make optimization easier.)

Derivation of the log-likelihood:

$$
\begin{aligned}
\mathcal{L} &= \prod_{i=1}^{n} [f(t_i)]^{c_i} \; [1 - F(t_i)]^{1-c_i} \\
&= \prod_{i=1}^{n} \left[\left(\frac{\alpha}{\lambda}\right) \left(\frac{t}{\lambda}\right)^{\alpha-1} e^{-(t/\lambda)^{\alpha}} \right]^{c_i} \; \left[e^{-(t/\lambda)^{\alpha}} \right]^{1-c_i} \\
&= \prod_{i=1}^{n} \left[\left(\frac{\alpha}{\lambda^{\alpha}}\right) t^{\alpha-1} e^{-(t/\lambda)^{\alpha}} \right]^{c_i} \; \left[e^{-(t/\lambda)^{\alpha}} \right]^{1-c_i} \\
\ln \mathcal{L} &= \sum_{i=1}^{n} c_i \left[\ln \alpha - \alpha \ln \lambda + (\alpha - 1) \ln t_i \right] - \left(\frac{t_i}{\lambda}\right)^{\alpha} \\
&= \sum_{i=1}^{n} c_i \left[\ln \alpha - \alpha \, \mathbf{x}_i \beta + (\alpha - 1) \ln t_i \right] - \left(\frac{t_i}{e^{\mathbf{x}_i \beta}}\right)^{\alpha}
\end{aligned}
$$

In R:

```
ll.weibull <- function(theta,y,x,c){
beta <- theta[1:ncol(x)]
alpha <- exp(theta[(ncol(x)+1)])
sum(c*(log(alpha)-alpha*x%*%beta+(alpha-1)*log(y))-(y/exp(x%*%beta))^(alpha))
}
```

B)

Run `optim()` to find the MLE values of β and α. Use the `BFGS` optimization method. Report the estimates along with the standard errors in a nicely formatted table.

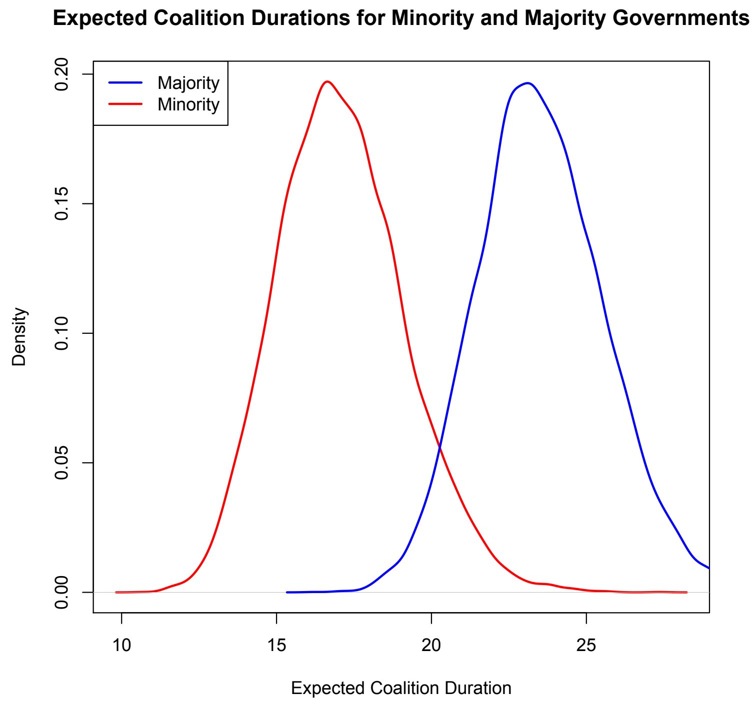
```
c <- coalition$ciep12
x <- as.matrix(cbind(1,coalition[,c("invest","fract","polar","numst2","crisis")]))
y <- coalition$duration
opt.out <- optim(par=rep(0,ncol(x)+1),fn=ll.weibull,y=y,x=x,c=c,
method="BFGS",control=list(fnscale=-1), hessian=TRUE)
```

	Coefficient	SE
Intercept	-0.531	0.530
invest	-0.467	0.161
fract	0.006	0.001
polar	-0.056	0.006
numst2	0.325	0.150
crisis	0.004	0.002
log(alpha)	-0.112	0.053

C)

Simulate the distributions of *expected* coalition duration times for two types of countries: majority (`numst2=1`) and minority (`numst2=0`). Keep all other covariates at their means. Plot the two distributions on the same plot and provide a legend. Do 10000 simulations. (Hint: use the formula for the expected value of the Weibull distribution provided to you above.)

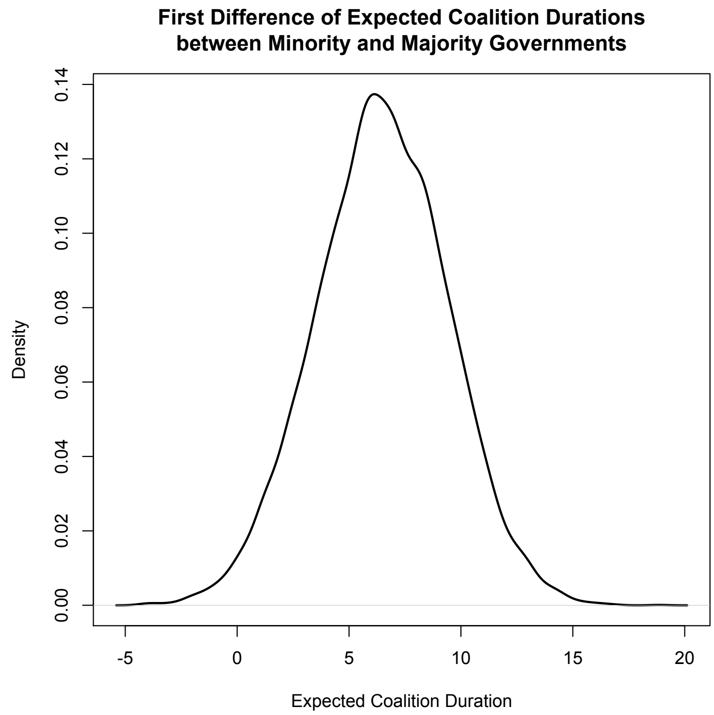
```
pars.sim <- mvrnorm(n=10000,mu=opt.out$par,-solve(opt.out$hessian))
x.mean <- colMeans(x)
x.low <- x.high <- x.mean
x.low["numst2"] <- 0
x.high["numst2"] <- 1

expect.low <- exp(pars.sim[,1:ncol(x)] %*% x.low) *gamma(1+1/exp(pars.sim[,6]))
expect.high <- exp(pars.sim[,1:ncol(x)] %*% x.high)
                                %*% *gamma(1+1/exp(pars.sim[,6]))

plot(density(expect.low),col="red",ylab="Density",xlab="Expected
Coalition Duration", main="Expected Coalition Durations for Minority
and Majority Governments", lwd=2)
lines(density(expect.high),col="blue", lwd=2)
legend("topleft",c("Majority","Minority"),col=c("blue","red"),lty=c(1,1),lwd=c(2,2))
```



D)

Simulate the distribution of the first difference between *expected* coalition duration times of majority (`numst2=1`) and minority (`numst2=0`) governments. Again, keep all other covariates at their means. Do 10000 simulations. Provide a density plot.

```
plot(density(expect.high-expect.low),main="First Difference of
Expected Coalition Durations\nbetween Minority and Majority
Governments", lwd=2,ylab="Density",xlab="Expected Coalition Duration")
```



E)

What is the 95% confidence interval for the first difference from Part D? Do you reject the null hypothesis that there is no difference between expected duration times for majority and minority governments at the 5% significance level?

```
quantile(expect.high-expect.low,0.025)
quantile(expect.high-expect.low,0.975)
```

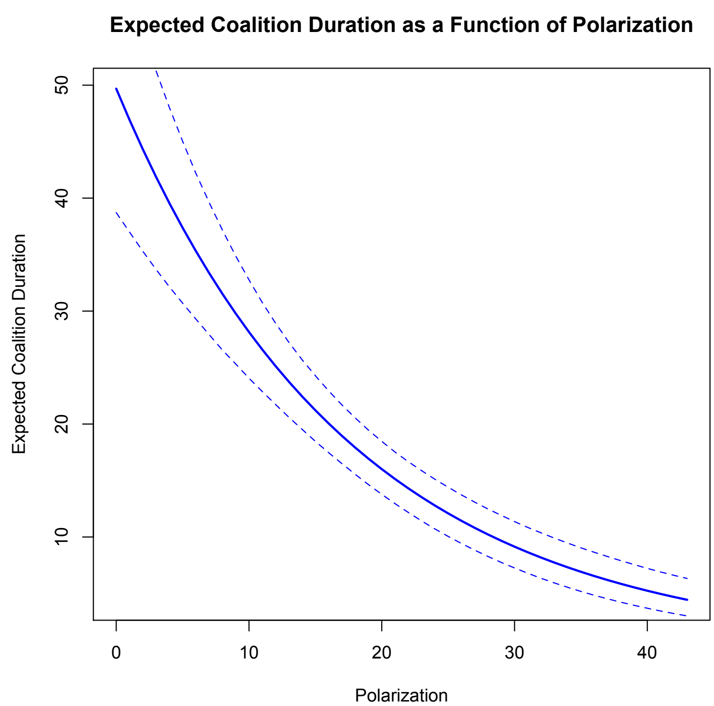
Since the 95% confidence interval does not include 0, we reject the null at the 5% significance level.

F)

Now, we want to simulate the expected duration values as a function of polarization (`polar`) (using 10000 simulations). Keep all other covariates at their means. You want to create a plot where the x-axis ranges from the minimum value of `polar` to the maximum value of `polar`. The y-axis should have the expected duration of the coalition. Please include 95% confidence intervals on the plot. Make sure the plot is well-labeled.

```
x.mean <- colMeans(x)
polar.range <- seq(range(coalition$polar)[1],range(coalition$polar)[2],by=1)
holder <- matrix(NA,ncol=length(polar.range),nrow=10000)
```

```
for(i in 1:length(polar.range)){
x.mean["polar"] <- polar.range[i]
holder[,i] <- exp(pars.sim[,1:ncol(x)] %*% x.mean) *gamma(1+1/exp(pars.sim[,6]))
}

plot(polar.range,colMeans(holder), type="l", col="blue",lwd=2,
main="Expected Coalition Duration as a Function of Polarization",
xlab="Polarization",ylab="Expected Coalition Duration")
lines(polar.range,apply(holder,MARGIN=2,function(x)
quantile(x,0.975)),col="blue",lty=2)
lines(polar.range,apply(holder,MARGIN=2,function(x)
quantile(x,0.025)),col="blue",lty=2)
```



G)

We want to test whether an exponential model can be used for our data. Run a restricted model where you fix $\alpha = 1$. Compare the likelihood from the restricted model from the unrestricted model using a likelihood ratio test. Do you reject the null that the restricted and unrestricted models are the same at the 5% significance level? What is the p-value?

The likelihood ratio statistic you obtain is 13.34. We can compare this against a χ^2_1 distribution to obtain a p-value of 0.00026. We can reject the null hypothesis.

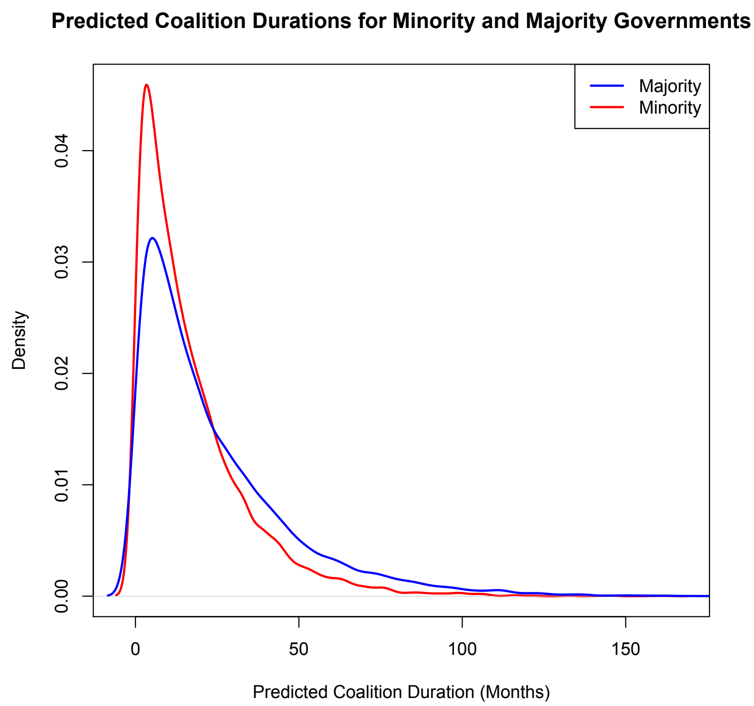
```
ll.weibull.res <- function(theta,y,x,c){
beta <- theta[1:ncol(x)]
alpha <- 1
sum(c*(log(alpha)-alpha*x%*%beta+(alpha-1)*log(y))-
(y/exp(x%*%beta))^(alpha))
}

opt.out.res <- optim(par=rep(0,ncol(x)),fn=ll.weibull.res,y=y,x=x,c=c,
method="BFGS",control=list(fnscale=-1), hessian=TRUE)
opt.out.res$par
opt.out.res$val

r <- 2*(opt.out$val-opt.out.res$val)
1-pchisq(r,df=1)
```

F) Extra Credit

We want to plot the *predicted* durations for majority and minority governments. The setup is similar to Part C, but now we want to also simulate fundamental uncertainty. Use the `rweibull()` function. Note that the `shape` parameter is α and the `scale` parameter is λ_i. Create a plot similar to the one in Part C with two densities, but now with predicted values.

The likelihood ratio statistic you obtain is 13.34. We can compare this against a χ^2_1 distribution to obtain a p-value of 0.00026. We can reject the null hypothesis.

```
draw.low <- sapply(1:10000,function(i)
rweibull(1,shape=exp(pars.sim[i,6]),scale=exp(pars.sim[i,1:ncol(x)] %*% x.low)))
draw.high <-sapply(1:10000,function(i)
rweibull(1,shape=exp(pars.sim[i,6]),scale=exp(pars.sim[i,1:ncol(x)] %*% x.high)))

plot(density(draw.low),col="red",ylab="Density",xlab="Predicted
Coalition Duration (Months)", main="Predicted Coalition Durations for
 Minority and Majority Governments",lwd=2)
lines(density(draw.high),col="blue",lwd=2)
legend("topright",c("Majority","Minority"),col=c("blue","red"),lty=c(1,1),lwd=c(2,2))
```


Predicted Coalition Durations for Minority and Majority Governments
Majority
Minority
Density
0.00
0.01
0.02
0.03
0.04
0
50
100
150
Predicted Coalition Duration (Months)