GOV 2001/ 1002/ E-200 Section 3
Inference and Likelihood

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February 10, 2016

Logistics

Reading Assignment- Unifying Political Methodology ch 4 and Eschewing Obfuscation

Problem Set 3- Due by 6pm, 2/24 on Canvas.

Assessment Question- Due by 6pm, 2/24 on on Canvas. You must work alone and only one attempt.

Replication Paper

1. Read *Publication, Publication*

2. Find a coauthor. See the Canvas discussion board to help with this.

3. Choose a paper based on the crieria in *Publication, Publication*.

4. Have a classmate sign-off on your paper choice.

Overview

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  - learn how to calculate a Bayesian posterior distribution and generate quantities of interest from it.
  - learn about common pitfalls in hypothesis testing and think about how to interpret p-values more critically.
  - learn that Frequentists and Bayesians aren't really that different after all!

Outline

Likelihood Inference

Bayesian Inference

Hypothesis Testing

Likelihood inference

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- This week we're talking about inference – Given the data, what can we say about the parameters.
- Likelihood approaches to inference ask "What parameters make our data most likely?"

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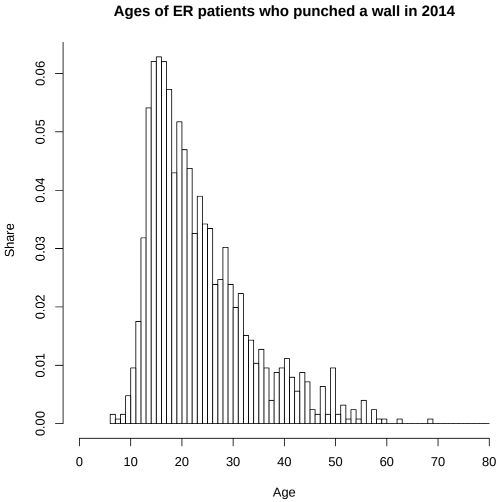
- Let's take a look at one injury category – wall punching. We're interested in modelling the distribution of the ages of individuals who visit the ER having punched a wall.
- To do this, we write down a probability model for the data.

Empirical distribution of wall-punching ages



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The density of the log-normal distribution is given by

$$f(Y_i|\mu,\sigma^2) = \frac{1}{Y_i\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln(Y_i)-\mu)^2}{2\sigma^2}\right)$$

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Basically the same as saying $\ln(Y_i)$ is normally distributed!

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- Unfortunately, $f(\mathbf{Y}|\mu, \sigma^2)$ is an n-dimensional density, and n is huge! How do we simplify this? The *i.i.d.* assumption lets us factor the density!

$$\mathcal{L}(\mu, \sigma^2 | \mathbf{Y}) \propto \prod_{i=1}^{N} f(Y_i | \mu, \sigma^2)$$

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- It's also often analytically easier to work with sums over products.
- This is why we typically work with the log-likelihood (often denoted ℓ). Because taking the log is a monotonic transformation, it retains the proportionality!

$$\mathcal{L}(\mu, \sigma^2 | \mathbf{Y}) \propto \ell(\mu, \sigma^2 | \mathbf{Y})$$

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  - $\log(e) = \ln(e) = 1$
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- Notational note: log in math is almost always used as short-hand for the natural log (ln) as opposed to the base-10 log.

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Writing the log-likelihood in R

- We can often make use of the built-in PDF functions in R for distributions to write a function that takes as input μ, σ^2 and the data. Here, we want to use `dlnorm` (the density of the log-normal).

```
### Log-Likelihood function
log.likelihood.func <- function(mu, sigma, Y){
  # Return the sum of the log of dnorm evaluated for all Y with fixed mu and
      sigma
  return(sum(dlnorm(Y, meanlog=mu, sdlog=sigma, log=T))) ## Set log=T to return
       the log-density
}
```

Plotting the log-likelihood

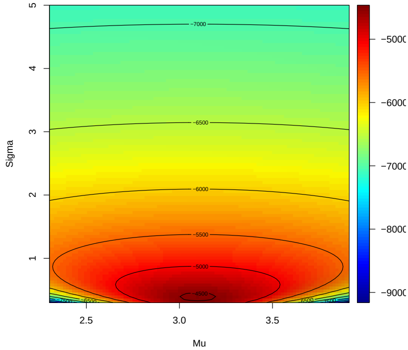

Figure : Contour plot of the log-likelihood for different values of μ and σ

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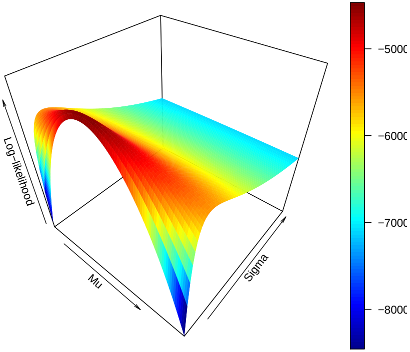

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PLOTTING THE LIKELIHOOD

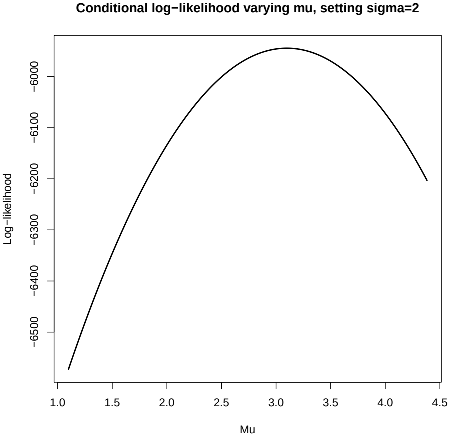

Figure : Plot of the conditional log-likelihood of μ given $\sigma = 2$

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- Example 2: $\mu = 3.099$, $\sigma = 0.379$: Log-likelihood = -4461.054 (actually the MLE)!
- Let's plot the implied distribution of Y_i for each parameter set over the empirical histogram!

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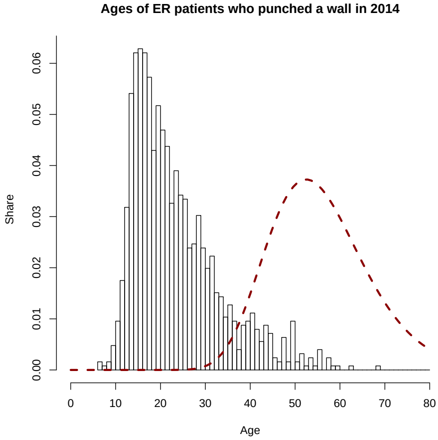

Figure : Empirical distribution of ages vs. log-normal with $\mu = 4$ and $\sigma = .2$

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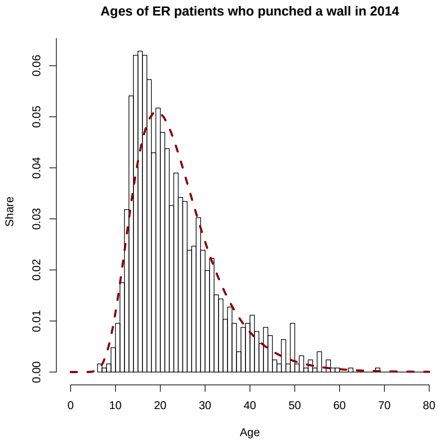

Figure : Empirical distribution of ages vs. log-normal using MLEs of parameters

OUTLINE

Likelihoods vs. Bayesian Posteriors

Likelihood:

$$\begin{aligned} p(\lambda|y) &= \frac{p(\lambda)p(y|\lambda)}{p(y)} \\ L(\lambda|y) &= k(y)p(y|\lambda) \\ &\propto p(y|\lambda) \end{aligned}$$

There is a fixed, true value of λ. We use the likelihood to estimate λ with the MLE.

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λ is a random variable and therefore has fundamental uncertainty. We use the posterior density to make probability statements about λ.

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- $p(\lambda)$ is the prior density

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In Bayesian inference, we have a prior *subjective* belief about λ , which we update with the data to form posterior beliefs about λ.

$$p(\lambda|y) \quad \propto \quad p(\lambda)p(y|\lambda)$$

- $p(\lambda|y)$ is the posterior density
- $p(\lambda)$ is the prior density
- $p(y|\lambda)$ is proportional to the likelihood

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7. Summarize the posterior distribution. (posterior mean, posterior standard deviation, posterior probabilities)

EXAMPLE: WAITING TIME FOR A TAXI ON MASS AVE

If you randomly show up on Massachusetts Avenue, how long will it take you to hail a taxi?

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- Example: Beta distribution is conjugate to Binomial data.

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- By inspection, the posterior for λ is also the form of a Gamma. Here, it's Gamma$(\alpha+1, \beta+X_i)$
- We could also integrate the above form to get the normalizing constant and get an explicit density if we didn't recognize it as a known distribution.

Plotting the posterior

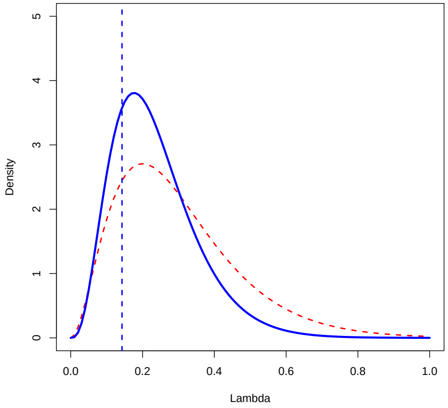

Figure : Prior and Posterior densities for λ (Red = Prior, Blue = Posterior). Vertical line denotes MLE). $\alpha = 3$, $\beta = 10$

OUTLINE

Likelihood Inference

Bayesian Inference

Hypothesis Testing

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- Answer: Your priors about effect size will affect how you interpret p-values.

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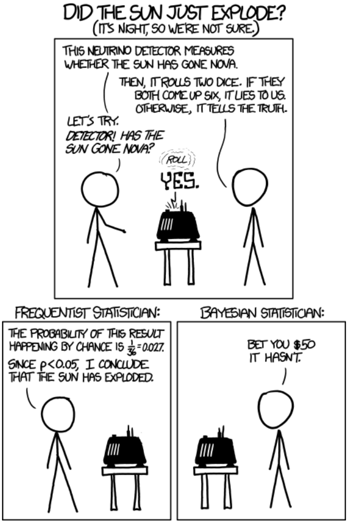

Figure : A misleading caricature - everyone uses priors

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- Don't forget what you learned in Intro to Probability!
- Classic example: A disease has a very low base rate (.1% of the population). A test for the disease has a 5% false positive rate and a 5% false negative rate. Given that you test positive, what's the probability you have the disease?
- Bayes' rule: $P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D)+P(+|\text{Not D})P(\text{Not D})}$
- $P(D|+) = \frac{.95\times.001}{.95\times.001+.05\times.999} = .01866 = 1.86\%$
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- The same principles apply to hypothesis testing! Always important to ask: given my decision to reject, how likely is it that my decision is misleading?

THINKING ABOUT P-VALUES

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- But false-positive rate can be misleading conditional on a positive result. Determining how "informative" our result is depends on additional design-related factors.
  - 1) The effect size
  - 2) The sample size

Type "M" and "S" Errors

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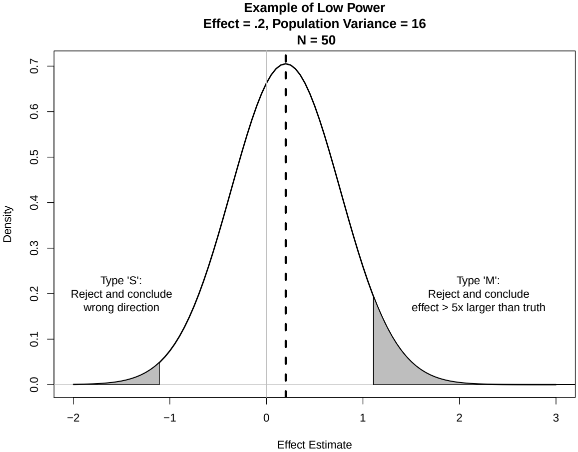
- Type "M" error: Given that you reject the null, what's the probability that your estimate is too extreme?
- Both depend not only on your sampling distribution's variance, but also on the effect size.

Calculating Type "M" and "S" error rates



$Pr(\text{Reject}) = .0644$. $Pr(\text{Wrong Sign}|\text{Reject}) = .16$.

$Pr(\text{Estimate 5x Truth}|\text{Reject}) = .84$

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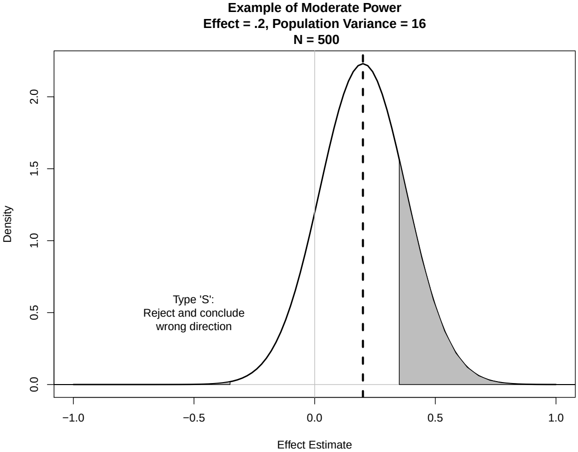

$Pr(\text{Reject}) = .200$. $Pr(\text{Wrong Sign}|\text{Reject}) = .005$.

Low probability of Type 'S' and our positive estimates are a lot more reasonable!

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Questions

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