BIO 226, Spring 2015: Lab 1

Contrasts in SAS

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Outline

- Guinea pigs ANOVA example
- How to change the reference level for class variables.
- Writing hypotheses in terms of β's.
- Writing contrasts.
- Testing contrasts in SAS.

Example: Dosages of four cardiotoxic drugs at death of infused guinea pigs

- Evaluating potencies of four cardiac treatments
- Observe dosage at which animals (guinea pigs) die for each treatment
- 10 guinea pigs per treatment (40 observations in all)

Example: Dosages of four cardiotoxic drugs at death of infused guinea pigs

- Assess any differences in toxicity of four treatments, i.e. differences in mean dosage required to kill animal

$$\bar{y}_1 = 25.9, \bar{y}_2 = 22.2, \bar{y}_3 = 20.0, \bar{y}_4 = 19.6$$

SAS Syntax for One-Way ANOVA

```
DATA toxic;
INFILE 'tox.txt';
INPUT y drug;
RUN;

PROC GLM DATA=toxic;
CLASS drug;
MODEL y=drug / solution;
RUN;
```

SAS Syntax for One-Way ANOVA

Parameter		Estimate		Standard Error	t Value	Pr > |t|
Intercept		19.60000000	B	0.98728021	19.85	<.0001
DRUG	1	6.30000000	B	1.39622507	4.51	<.0001
DRUG	2	2.60000000	B	1.39622507	1.86	0.0708
DRUG	3	0.40000000	B	1.39622507	0.29	0.7761
DRUG	4	0.00000000	B	.	.	.

Changing the reference level

```
proc sort data=toxic; by descending drug; run;

PROC GLM DATA=toxic order=data;
CLASS drug;
MODEL y=drug / solution;
RUN;
```

Recall from Lecture: SAS Syntax for One-Way ANOVA

```
The GLM Procedure

    Class Level Information

Class          Levels    Values
DRUG                4    1 2 3 4

Dependent Variable: Y
                                   Sum of
Source                     DF     Squares      Mean Square    F Value    Pr > F
Model                       3    249.8750000    83.2916667       8.55    0.0002
Error                      36    350.9000000     9.7472222
Corrected Total            39    600.7750000

R-Square     Coeff Var      Root MSE       Y Mean
0.415921      14.23970      3.122054     21.92500
```

Recall from Lecture: SAS Syntax for One-Way ANOVA

Parameter		Estimate		Standard Error	t Value	Pr > |t|
Intercept		25.90000000	B	0.98728021	26.23	<.0001
DRUG	4	-6.30000000	B	1.39622507	-4.51	<.0001
DRUG	3	-5.90000000	B	1.39622507	-4.23	0.0002
DRUG	2	-3.70000000	B	1.39622507	-2.65	0.0119
DRUG	1	0.00000000	B	.	.	.

The model

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \epsilon_i$$

$$X_{i2} = \begin{cases} 1 & DRUG\ 2 \\ 0 & otherwise \end{cases}$$

$$X_{i3} = \begin{cases} 1 & DRUG\ 3 \\ 0 & otherwise \end{cases}$$

$$X_{i4} = \begin{cases} 1 & DRUG\ 4 \\ 0 & otherwise \end{cases}$$

The model

$$Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \epsilon_i$$

$$\mu_1 = \beta_1$$

$$\mu_2 = \beta_1 + \beta_2$$

$$\mu_3 = \beta_1 + \beta_3$$

$$\mu_4 = \beta_1 + \beta_4$$

Hypotheses to test

H_0: Drug 2 is as potent as drug 3.

How would we write that hypothesis in terms of the β's?

$$\mu_1 = \beta_1$$

$$\mu_2 = \beta_1 + \beta_2$$

$$\mu_3 = \beta_1 + \beta_3$$

$$\mu_4 = \beta_1 + \beta_4$$

Hypotheses to test

H_0: Drug 2 is as potent as drug 3.

How would we write that hypothesis in terms of the β's?

$\beta_2 = \beta_3$

$$\mu_1 = \beta_1$$

$$\mu_2 = \beta_1 + \beta_2$$

$$\mu_3 = \beta_1 + \beta_3$$

$$\mu_4 = \beta_1 + \beta_4$$

Hypotheses to test

H_0: Drug 2 is as potent as drug 3.

H_0: $\beta_2 = \beta_3$

H_0: $\beta_2 - \beta_3 = 0$

$$\begin{aligned} \begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} &= 0 * \beta_1 + 1 * \beta_2 + -1 * \beta_3 + 0 * \beta_4 \\ &= \beta_2 - \beta_3 \end{aligned}$$

Hypotheses to test

H_0: Drug 2 is as potent as drug 3.

H_0: $\beta_2 = \beta_3$

H_0: $\beta_2 - \beta_3 = 0$

$$\underbrace{\begin{bmatrix} 0 & 1 & -1 & 0 \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}}_{\boldsymbol{\beta}} = 0 * \beta_1 + 1 * \beta_2 + -1 * \beta_3 + 0 * \beta_4$$

$$= \beta_2 - \beta_3$$

H_0: $\mathbf{L}\boldsymbol{\beta} = 0$

Hypotheses to test

H_0: Drugs 2, 3 and 4 are equally potent.

How would we write that hypothesis in terms of the β's?

$$\mu_1 = \beta_1$$

$$\mu_2 = \beta_1 + \beta_2$$

$$\mu_3 = \beta_1 + \beta_3$$

$$\mu_4 = \beta_1 + \beta_4$$

Hypotheses to test

H_0: Drugs 2, 3 and 4 are equally potent.

How would we write that hypothesis in terms of the β's?

$\beta_2 = \beta_3 = \beta_4$

$$\mu_1 = \beta_1$$

$$\mu_2 = \beta_1 + \beta_2$$

$$\mu_3 = \beta_1 + \beta_3$$

$$\mu_4 = \beta_1 + \beta_4$$

Hypotheses to test

H_0: Drugs 2, 3 and 4 are equally potent.

H_0: $\beta_2 = \beta_3 = \beta_4$

H_0: $\beta_2 = \beta_3$ and $\beta_3 = \beta_4$

H_0: $\beta_2 - \beta_3 = 0$ and $\beta_3 - \beta_4 = 0$

$$\underbrace{\begin{bmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}}_{\boldsymbol{\beta}} = \begin{bmatrix} 0 * \beta_1 + 1 * \beta_2 + -1 * \beta_3 + 0 * \beta_4 \\ 0 * \beta_1 + 0 * \beta_2 + 1 * \beta_3 + -1 * \beta_4 \end{bmatrix}$$

$$= \begin{bmatrix} \beta_2 - \beta_3 \\ \beta_3 - \beta_4 \end{bmatrix}$$

H_0: $\mathbf{L}\boldsymbol{\beta} = 0$

Hypotheses to test

H_0: All 4 drugs are equally potent.

H_0: $\beta_2 = \beta_3 = \beta_4 = 0$

H_0: $\beta_2 = 0$ and $\beta_3 = 0$ and $\beta_4 = 0$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{L}} \underbrace{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}}_{\boldsymbol{\beta}} = \begin{bmatrix} 0 * \beta_1 + 1 * \beta_2 + 0 * \beta_3 + 0 * \beta_4 \\ 0 * \beta_1 + 0 * \beta_2 + 1 * \beta_3 + 0 * \beta_4 \\ 0 * \beta_1 + 0 * \beta_2 + 0 * \beta_3 + 1 * \beta_4 \end{bmatrix}$$

$$= \begin{bmatrix} \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

H_0: $\mathbf{L}\boldsymbol{\beta} = 0$

How to test these Hypotheses

We know that the least squares (LS) estimate $\widehat{\boldsymbol{\beta}}$, which is also the maximum likelihood (ML) estimator, has distribution

$$\widehat{\boldsymbol{\beta}} \sim N\left(\boldsymbol{\beta}, \mathrm{Cov}(\widehat{\boldsymbol{\beta}})\right)$$

(see Lecture 2 slide 22)

then

$$\mathbf{L}\widehat{\boldsymbol{\beta}} \sim N\left(\mathbf{L}\boldsymbol{\beta}, \mathbf{L}\mathrm{Cov}(\widehat{\boldsymbol{\beta}})\mathbf{L}'\right)$$

Thus, we can perform inference to test H_0: $\mathbf{L}\boldsymbol{\beta} = 0$.

Luckily, SAS does this for us!

Contrast Statement

CONTRAST 'label' effect values / E;

label - identifies the contrast on the output.

effect - the variable in the model

values - the $\mathbf{L}$ matrix

E - displays the $\mathbf{L}$ matrix. This option is useful in confirming the ordering of parameters for specifying.

H_0: Drug 2 is as potent as drug 3.

H_0: $\beta_2 - \beta_3 = 0$

```
PROC GLM DATA=toxic order=data;
CLASS drug;
MODEL y=drug / solution;
CONTRAST 'Drug 2 vs Drug 3' DRUG 0 -1 1  0 / E;
RUN;
```

Contrast Statement

H_0: Drug 2 is as potent as drug 3.

H_0: $\beta_2 - \beta_3 = 0$

E - displays the $\mathbf{L}$ matrix. This option is useful in confirming the ordering of parameters for specifying .

```
Coefficients for Contrast Drug 2 vs Drug 3

                      Row 1

Intercept                 0

DRUG      4               0
DRUG      3              -1
DRUG      2               1
DRUG      1               0
```

Contrast Statement

H_0: Drug 2 is as potent as drug 3.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Drug 2 vs Drug 3	1	24.20000000	24.20000000	2.48	0.1238

Contrast Statement

H_0: Drugs 2, 3 and 4 are equally potent.

```
PROC GLM DATA=toxic order=data;
CLASS drug;
MODEL y=drug / solution;
CONTRAST 'Drug 2 vs Drug 3 vs Drug 4' DRUG 0 -1 1 0,
    DRUG -1 1 0 0 / E;
RUN;
```

Contrast Statement

H_0: Drugs 2, 3 and 4 are equally potent.

E - displays the displays the $\mathbf{L}$ matrix. This option is useful in confirming the ordering of parameters for specifying .

```
Coefficients for Contrast Drug 2 vs Drug 3 vs Drug 4

                      Row 1           Row 2

Intercept                 0               0

DRUG       4              0              -1
DRUG       3             -1               1
DRUG       2              1               0
DRUG       1              0               0
```

Contrast Statement

H_0: Drugs 2, 3 and 4 are equally potent.

Contrast	DF	Contrast SS	Mean Square	F Value	Pr > F
Drug 2 vs Drug 3 vs Drug 4	2	39.2000	19.600000	2.01	0.1486