Prediction of Insurance Policy Customers

by Machine Learning Techniques

Xiner Zhou
UNI: xz2315
xz2315@columbia.edu
M.A. in Statistics, Columbia University

Table of Contents

Abstract: Ability to predict potential customers is crucial for insurance company or any other companies to market campaign. Machine learning algorithms have been successfully shown to generate high predictive accuracy on real-world data. In this project, four different supervised learning algorithms are discussed and applied to predict potential caravan insurance policy buyers on the insurance company (TIC) benchmark dataset: Naïve Bayes, SVM, Tree, and AdaBoost. Instead of looking at accuracy of prediction, another set of statistics is used: sensitivity, specificity, precision, and F1-score. Results show that Naïve Bayes generates superior predictive performance than the others in terms of sensitivity and F1-score.

Keywords: Machine learning, supervised learning, Naïve Bayes, SVM, Tree, AdaBoost, Sensitivity, Specificity, Precision, F1-score

Part I: Preliminary Analysis

1. Introduction

Direct mailings to a company's potential customers—"junk mail" to many—can be a very effective way to market a product or service. Much of this junk mail is really of no interest to the majority of the people that receive it. Most of it ends up thrown away, not only wasting the money that the company spent on it, but also filling up landfill waste sites.
However, if the company had a better understanding of who their potential customers were, they would know more accurately who to send it to, so some of this waste and expense could be reduced. Therefore, the CoIL Challenge 2000 data mining competition provided The Insurance Company (TIC) Benchmark Dataset, and asked the question:
Can you predict who would be interested in buying a caravan insurance policy and give an explanation why?

2. Data Description

The Insurance Company (TIC) Benchmark Dataset was provided by the COIL Challenge 2000 which was organized by the Computational Intelligence and Learning (CoIL) cluster, a cooperation between four EU funded research networks. The problem is representative of an important class of real world learning problems: noisy, correlated, redundant and high dimensional data with a weak relationship between predictors and target. The homepage of TIC Benchmark is:
http://www.liacs.nl/~putten/library/cc2000/

Data files:

(1) TICDATA2000.txt:
Dataset to train prediction models and build a description (5822 customer records). Each record consists of 86 attributes, containing sociodemographic data (attribute 1-43) and insurance policy ownership (attributes 44-86).The sociodemographic data is derived from zip codes. All customers living in areas with the same zip code have the same sociodemographic attributes. Attribute 86, "CARAVAN:Number of mobile home policies" (i.e. whether or not having the caravan insurance policy), is the target variable. The detailed meaning of the attributes and attribute values is given in Appendix A.

(2) TICEVAL2000.txt:
Dataset for test or prediction (4000 customer records). It has the same format as TICDATA2000.txt, only the target is missing.

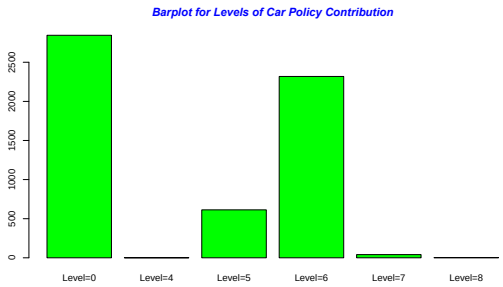
(3) TICTGTS2000.txt:
Targets for the test set.

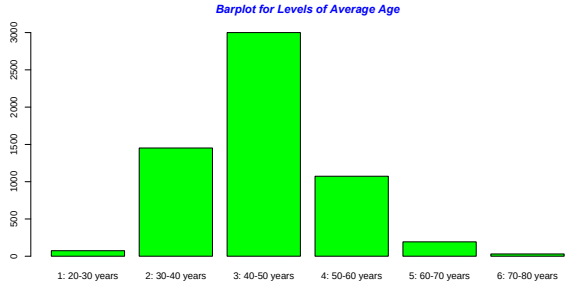
3. Exploratory Data Analysis

(1) Characteristic 1: Categorical Variables → Method of Feature Selection

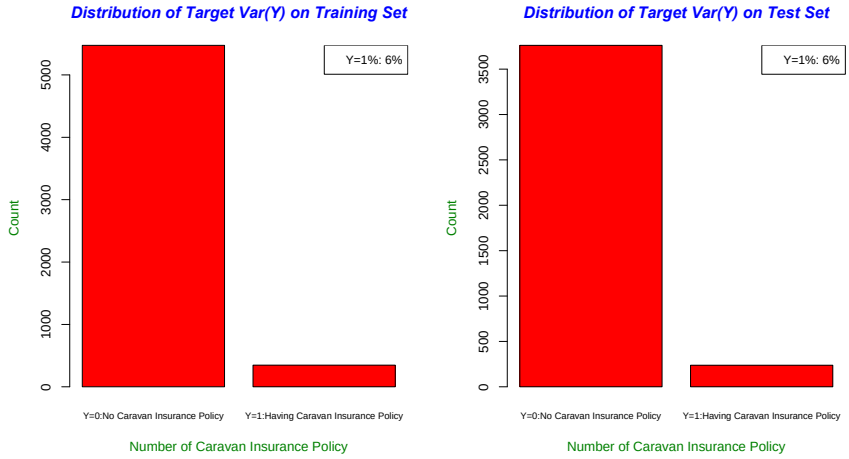
Since all the 86 variables are indicators of levels or number of insurance policy ownership, they are all categorical. For example, the 4th column (sociodemographic data) representing "Average Age" has 6 levels; the 47th column (insurance policy ownership data) representing "Car Policy Contribution" has levels 0-8.

The characteristic that all of the variables are categorical inspires the way to do feature selection, as explained in the *section "Feature Selection".*





(2) Characteristic 2: Only 6% Positive Response → Measures of Performance



In the supervised learning setting, the classification can be formulated as: let Y=1 if having caravan insurance policy and Y=0 if not, the goal is to classify observations into two classes Y=0 or Y=1. From the barplots above, we see that on both training and test sets, the positive response (Y=1) is an "rare event", that is, only 6% of the sample belongs to class 1. This characteristic makes the "accuracy" of classification non-optimal for measuring performance of different models, as discussed in the *section "Measure of Performance".*

4. Measure of Performance

Given that only 6% of customers on both training and test sets actually owns the caravan insurance policy, regular 0-1 loss classification "accuracy" or "error rate" is not appropriate measure of performance for different classification models. A "naïve" model that simply predicts no one will buy has a high classification accuracy of 94% but is useless.

		True Class		
		1=Positive	0=Negative	
Predicted Class	1 =Positive	True Positive	False Positive (Type I error)	Precision Rate=TP/(TP+FP)
	0 =Negative	False Negative (Type II error)	True Negative	

		Sensitivity=TP/(TP+FN)	Specificity=TN/(TN+FP)

Sensitivity, **Specificity**, and **Precision Rate**, are statistical measures of the performance of a *binary classification test*, also known in statistics as "*classification function*".

- **Sensitivity** (also called the *true positive rate*):
  measures the proportion of actual positive which are correctly identified as positive.

$$\begin{aligned} &Sensitivity \\ &= \frac{number\ of\ true\ positive}{number\ of\ true\ positive + number\ of\ false\ negative} \\ &= \frac{number\ of\ true\ positive}{total\ number\ of\ actual\ positive} \\ &= P(Positive\ result \mid Actual\ positive) \\ &= 1 - Type\ II\ error \end{aligned}$$

Sensitivity relates to the test's ability to identify positive results. For example, a sensitivity of 100% means that the test recognizes all actual positives. A test with a high sensitivity has a low type II error.

- **Specificity** (also called the *true negative rate*):
  measures the proportion of negatives which are correctly identified as negative

$$\begin{aligned} &Specificity \\ &= \frac{number\ of\ true\ negative}{number\ of\ true\ negative + number\ of\ false\ positive} \\ &= \frac{number\ of\ true\ negative}{total\ number\ of\ actual\ negative} \\ &= P(Negative\ result \mid Actual\ negative) \\ &= 1 - Type\ I\ error \end{aligned}$$

Specificity relates to the test's ability to identify negative results. A test with a high specificity has a low type I error.

- **Precision Rate**:
  the proportion of positive results that are true positive.

$$\begin{aligned} &Precision \\ &= \frac{number\ of\ true\ positive}{number\ of\ true\ positive + number\ of\ false\ positive} \\ &= \frac{number\ of\ true\ positive}{total\ number\ of\ positive\ results} \end{aligned}$$

We prefer the tests with high sensitivity, specificity, and precision. But there are intrinsic relationship among them and there is a trade-off. For example, a classifier simply predicts no one will buy caravan policy has specificity=100%, but sensitivity=0%; while another classifier predicts everyone will buy has sensitivity=100%, but sensitivity=0% and precision=6%. Both of the extreme cases are not ideal. Thus, we introduce the **F1-score**.

F1-Score

In statistics, the F1-score is a measure of a test's accuracy. It considers both the precision rate and the sensitivity to compute the score. The F1-score can be interpreted as a weighted average of the precision rate and sensitivity, where an F1-score reaches its best value at 1 and worst score at 0.

$$F1 = 2 \times \frac{Precision \times Sensitivity}{Precision + Sensitivity}$$

The F1-score is often used in machine learning for measuring classification performance. Hence, we will use the F1-score to measure model performance and do model selection.

5. Feature Selection and Feature Importance

The TIC dataset contains 85 features, possibly highly correlated. To find a reliable subset of features for predictive model is the first important task.

Initial Guess: Among the 85 features, containing sociodemographic data (feature 1-43) and insurance product ownership (feature 44-85). Some people would think the sociodemographic feature unimportant, since they're derived from zip codes and all customers living in the same zip code have the same sociodemographic features, thus can't reflect individual information. But afterwards we'll see some of them are really informative and others are truly uninformative. Since the target variable is the caravan insurance policy ownership, we would expect the features relating to car important.

Information Gain

The usual statistics *Pearson's correlation coefficient* is a measure of linear relationship for continuous variables, thus inappropriate for our case. We'll use "*information gain*" as a measure of predictive power for individual feature.

- Entropy:
  is a measure of the amount of uncertainty.

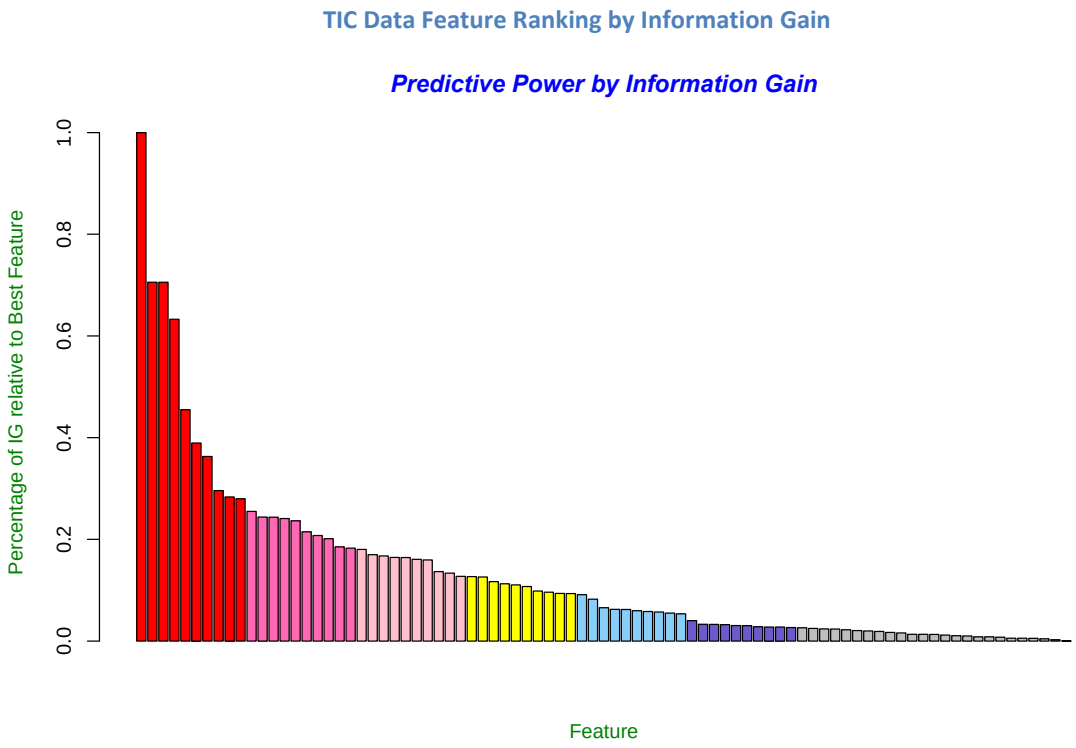
$$E(Y) = E\left[-log\left(P(Y)\right)\right] = -\sum_i P(Y = y_i) \times logP(Y = y_i)$$

When H(Y)=0, Y is perfectly classified, i.e. all points are of the same class. The higher the H(Y), the more uncertainty about Y.

- Information Gain IG(X):
  is a measure of the difference in entropy from before to after Y is split on X, i.e. how much uncertainty in Y was reduced after split on X.

$$IG(X) = H(Y) - H(Y|X)$$

If X and Y are independent, then knowing X does not provide any additional information about Y, i.e. H(Y|X)=H(Y) → IG(X)=0; If X and Y are identical, then knowing X perfectly determines Y, i.e. H(Y|X)=0 → IG(X)=H(Y). Thus, information gain is a measure of the inherent dependence of X and Y, the larger the IG(X) the greater predictive power of X.



The top-10 strongest features are:

1. Car policy contribution
2. Fire Policy contribution
3. Number of Car policies
4. Customer subtype
5. Customer main type
6. Average income
7. Purchasing power class
8. Income < 30
9. Lower level education
10. Private third party insurance contribution

Expected insurance ownership relating to car
Other insurance product ownership
Sociodemographic variable

The most and third-most important variables are truly the same as expected which are indicators relating to car; there are also some features relating to personal wealth, it's reasonable since they are related to purchasing power; but the fire policy contribution, lower level education, and private third party insurance contribution, are somewhat surprising here.

The question of feature selection is how many features should be included in our predictive models, and it is an issue of model complexity and will be choosen according to difference types of classification model.

Part II: Machine Learning Methods

1. NAÏVE BAYES CLASSIFIER

Bayesian Classification

In the supervised learning problem, let Y=1 if having caravan insurance policy and Y=0 if not, let X_i denote the ith feature. In the risk framework, the best classifier is the one which minimizes the empirical risk. The risk of a classifier f under 0-1 loss $L^{0-1}(y, f(x))$is defined as:

$$\begin{aligned} R(f) = E_{X,Y}\left[L^{0-1}(y, f(x))\right] &= E_X\left[E_Y L^{0-1}(y, f(x))|X = x\right] \\ &= \int E_{Y|X=x}\left[L^{0-1}(y, f(x))\right] \times p(x)dx \\ &= \int \left(\sum_{y\in\{0,1\}} L^{0-1}(y, f(x))p(y|x)\right)p(x)dx \\ &= \int \left(\sum_{y\in\{0,1\}} I(y \neq f(x))p(y|x)\right)p(x)dx \end{aligned}$$

Minimizing R(f) is equivalent to minimize $\sum_{y\in\{0,1\}} L^{0-1}(y, f(x))p(x)$ for every x, by monotonicity of the integral. Instead, minimizing:

$$\begin{aligned} 1 - \sum_{y\in\{0,1\}} I(y \neq f(x))p(y|x) \\ = \sum_{y\in\{0,1\}} p(y|x) - \sum_{y\in\{0,1\}} I(y \neq f(x))p(y|x) \\ = \sum_{y\in\{0,1\}} p(y|x)I(y = f(x)) = p(y = f(x)|x) \end{aligned}$$

Thus, the Bayes-optimal classifier under 0-1 loss is:

$$f_{Bayes}(x) = \arg max_{y\in\{0,1\}} P(y|x)$$

That is, the Bayes-optimal classification rule is "maximizing posterior probability". But what is posterior probability $P(y|x)$?

$$\begin{aligned} \text{Bayes Equation:} \quad P(x,y) &= P(y|x)P(x) = P(x|y)P(y) \\ \Rightarrow P(y|x) &= \frac{P(x|y)P(y)}{P(x)} \\ \Rightarrow f_{Bayes}(x) &= \arg max_{y\in\{0,1\}} P(y|x) \\ &= \arg max_{y\in\{0,1\}} \frac{P(x|y)P(y)}{P(x)} \\ &= \arg max_{y\in\{0,1\}} P(x|y)P(y) \end{aligned}$$

The rest problem is, how to model class-conditional distribution $P(x|y)$?

- Parametric method: Gaussian discriminant Analysis (LDA/QDA)
- Nonparametric method: Naïve Bayes Classification

Why Naïve Bayes and Its Assumption

Although the Gaussian discriminant analysis is a popular classification model, it's not appropriate for two reasons:

- GDA assumes features are continuous and normally distributed; but in our TIC case, features are all categorical;
- When features space is high-dimensional, it's hard to estimate Gaussian covariance matrix.

While Naïve Bayes makes a strong but elegant assumption.

NB assumption:
Features $X_1, \dots, X_n$ are conditional independent given class labels.

$$p(X_1, \dots, X_n|y) = \prod_{i=1}^{n} p(X_i|y)$$

Naïve Bayes Model

Naïve Bayes Classifier

$$f_{NB}(x) = argmax_{y\in\{0,1\}} p(y) \prod_{p=1}^{n} p(X_p|y)$$

We need to model the class-conditional probability distribution for each feature. Since all the features are categorical, the multinomial distribution is an ideal first choice. **We can think of the TIC data sampled following the Bayes model intuitively as two steps:**

1. **Randomly sample a customer from a Bernoulli process with probability $p(Y = 1)$, Y=1 if having caravan policy and Y=0 if not;**
2. **Then the customer decide each of his/her features X_p from a multinomial distribution:**

$$\boldsymbol{X_p \sim Multinomial}\ (1, \begin{pmatrix} p(X_p = x_{p,1}) \\ \cdots \\ p(X_p = x_{p,n_p}) \end{pmatrix})$$

Then, we can get the full Naïve Bayes Model for our problem:

Naïve Bayes Model

$$f_{NB}(x) = argmax_{y\in\{0,1\}} p(y) \prod_{p=1}^{n} p(X_p|y)$$

where: $Y \sim Bernoulli(p(Y = 1))$

$$X_p|Y = 0 \sim Multinomial\ (1, \begin{pmatrix} p_0(X_p = x_{p,1}) \\ \cdots \\ p_0(X_p = x_{p,n_p}) \end{pmatrix})$$

$$X_p|Y = 1 \sim Multinomial\ (1, \begin{pmatrix} p_1(X_p = x_{p,1}) \\ \cdots \\ p_1(X_p = x_{p,n_p}) \end{pmatrix})$$

Parameter Estimation

We're given a training set $\{x_i, y_i\}$, we use *Maximum Likelihood Estimation* (MLE) to train parameters.

Maximum Likelihood Estimators:

$$\hat{p}(Y = 1) = \frac{\sum_i I\{y_i = 1\}}{n}$$

$$\hat{p}_0(X_p = x_{p,j}) = \frac{\sum_i I\{X_p = x_{p,j} \wedge y_i = 0\} + 1}{\sum_i I\{y_i = 0\} + \mu_p}$$

$$\hat{p}_1(X_p = x_{p,j}) = \frac{\sum_i I\{X_p = x_{p,j} \wedge y_i = 1\} + 1}{\sum_i I\{y_i = 1\} + \mu_p}$$

where: $x_{p,j}$ is the j-th value of the p-th feature X_p
μ_p is the "phantom parameter", which is the number of values of the p-th feature

2-Fold Cross Validation and Model Selection

Tuning Parameter: number of features

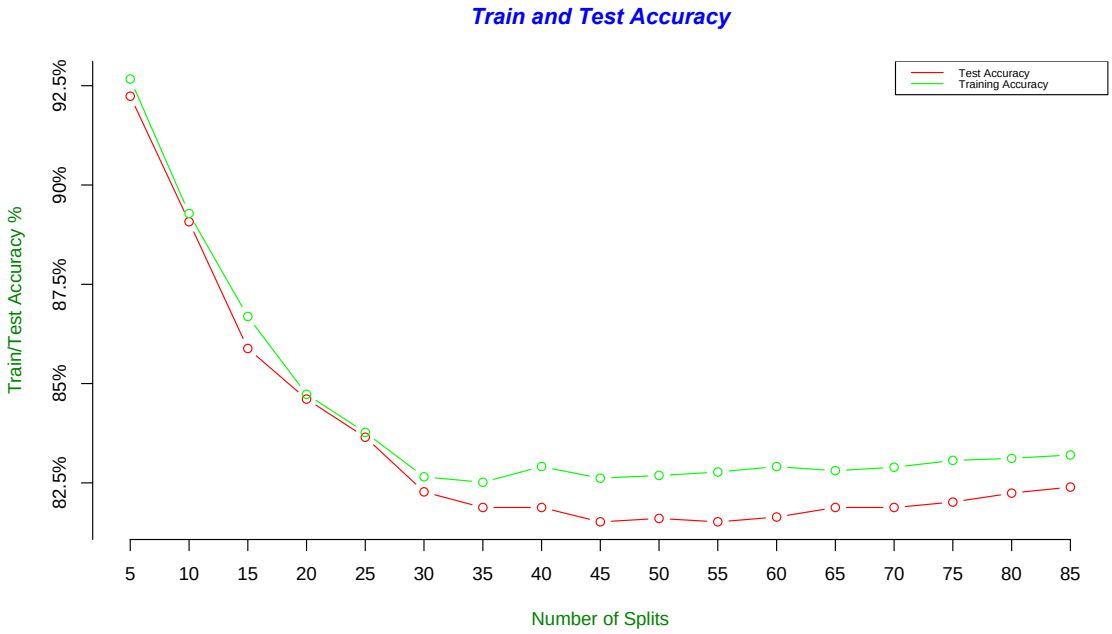
Purpose of Cross Validation: The training process optimizes the tuning parameters to make the model fit the training data as well as possible. If we then take an independent dataset from the same distribution as the training dataset, it will generally turn out the model does not fit as well as it fits the training set. This is called "*Overfitting*". Cross-validation is a way to trade-off between overfitting and underfitting.

Why 2-Fold Cross Validation:
The TIC training dataset contains 5822 observations but only 348 positive instances which is really small. So we can't make the number of folds too large. The 2-fold CV would be optimal. We simply split the training dataset into two equal size folds, and train on fold1 and test on fold2, followed by train on fold2 and test on fold1. This has the advantage that our **training and validation sets are both large enough, and each data points is used for both training and testing.**

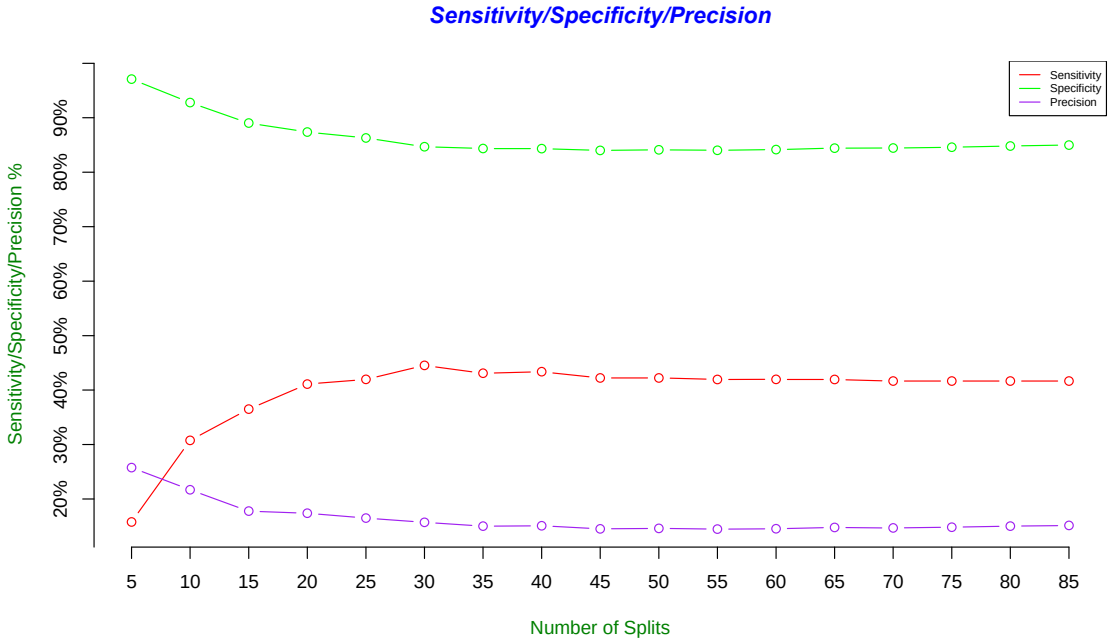
The TIC Training Dataset

fold 1	fold 2

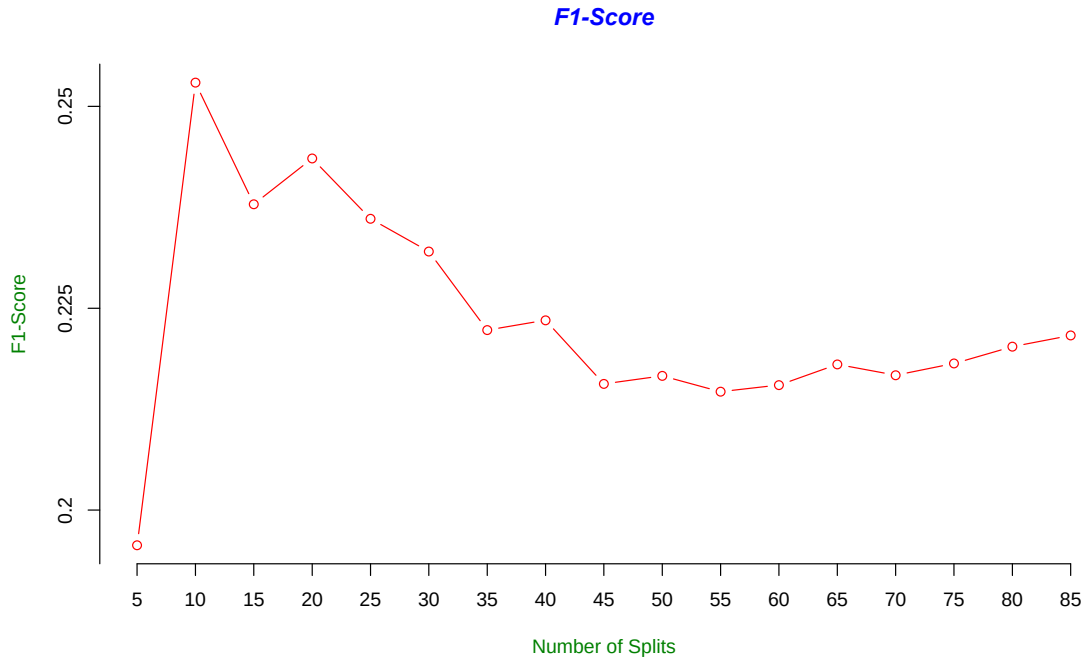
2-Fold CV Results on TIC Training Set



- Both training and test accuracy are very high (80%-92%). Interesting, the accuracy decays when number of features in the model increases.



- The green line shows that specificity are always very large, i.e., NB classifier always has good ability to identify class 0;
- The red line shows that sensitivity are generally small, i.e., NB classifier's ability to identify class 1 increases when the number of features included in the model increases;
- The purple line shows that precision decays with the number of features.



- **The F1-score reaches its maximum at number of features=10. Hence, the optimal number of feature in the NB model should be 10.**

NB with Top-10 Features on Test Dataset

F1= 0.2060811

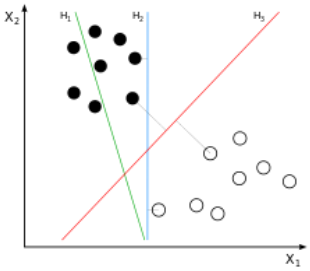
		True Class		
		1=Positive	0=Negative	
Predicted Class	1 =Positive	61	293	Precision=17.23%
	0 =Negative	177	3469	
		Sensitivity= 25.63%	Specificity= 92.21%	

- **The Naive Bayes model able to identify 25.63% of customers who actually having caravan insurance policy, 92.21% of those who actually don't have, and about 17.23% of those who are predicted to have caravan policy are actually policy owners.**

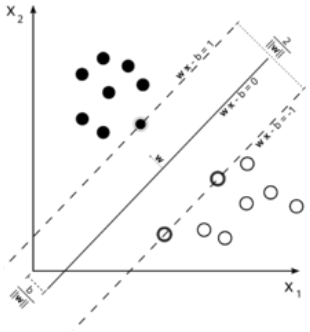
2. Support Vector Machine (SVM)

In machine learning, support vector machines (SVMs) are supervised learning models used for binary classification. It's a generalization of perceptron which only works for linearly separable problems, thus SVMs can efficiently perform a non-linear classification using what is called the kernel trick, implicitly mapping their inputs into high-dimensional feature spaces. The TIC data is apparently linear un-separable, SVMs are good candidate classifiers.

Maximum Margin Motivation



For linear separable data, there're multiple hyper-planes that might classify the data. But they have different predictive performance on another independent data. The maximum margin idea is, in order to achieve good generalization (low prediction error), place the hyper-plane in the "middle" between two classes; more precisely, choose hyperplane such that distances to the closest point in each class is maximal. This distance is called the "**margin**". Since without distribution assumption, best guess is symmetric.



The SVM classifier is obtained by solving the optimization problem.

SVM classifier:

$$f_{SVM}(x) = sgn(< v_H^*, x > -c^*)$$

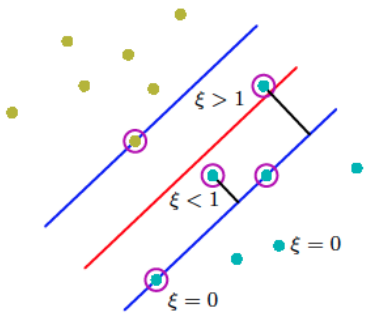
$$min_{v_H,c} \|v_H\|$$

$$s.t. \quad y_i(< v_H, x_i > -c) \geq 1$$
$$for\ i = 1, \dots, n$$

Soft-Margin SVM

- Motivation 1: non-separable data
  SVMs with maximum margin are linear classifiers without further modification, they can't be trained on a non-separable data. (The TIC data is non-separable)
- Motivation 2: robustness
  Suppose we have two training samples and train an SVM on each, if locations of support vectors vary significantly between samples, SVM classifiers vary significantly, thus SVM are not "robust".

The idea of soft-margin SVM is to introduce "**slack variables $\xi_i \geq 0$**", and permit training data to cross the margin or even of the hyperplane, but impose cost which increases the further beyond the margin.



Soft-margin SVM classifier:

$$f_{SVM}(x) = sgn(< v_H^*, x > -c^*)$$

$$min_{v_H,c}\|v_H\| + C\sum_i {\xi_i}^2$$

$$s.t. \quad y_i(< v_H, x_i > -c) \geq 1 - \xi_i$$
$$\xi_i \geq 0$$
$$for\ i = 1, \dots, n$$

where: **Cost parameter C** is a tuning parameter, which specifies the cost of allowing a point on the wrong side.

Non-Linear SVM with Kernel

The SVMs discussed so far uses the scalar product $< \cdot\,,\ \cdot >$ *on* R^d as a measure of similarity and distance between two points, since the scalar product is linear, the SVM is a linear classifier, i.e. the decision boundary is a hyperplane. But most empirical data can be separated more accurately by a non-linear classifier. By using kernel functions, we can make the decision boundary non-linear.

A function k: $R^d \times R^d \to R$ is called a "**Kernel**" on R^d,
if there is some function ϕ: $R^d \to \mathcal{F}$ such that

$$k(x, x') =< \phi(x), \phi(x') >_{\mathcal{F}}$$

i.e. **k is a kernel is it can be interpreted as a scalar product on some other space $\mathcal{F}$.**

If we substitute the kernel function $k(x, x')$ for $< x, x' >$ in all equations, we implicitly train a linear SVM on the higher-dimensional space $\mathcal{F}$ on which a linear SVM works well, but back to the original R^d space, the decision boundary is non-linear.

Soft-margin SVM with kernel:

$$f_{SVM}(x) = sgn(k(v_H, x_i) - c^*)$$

$$min_{v_H,c}\|v_H\| + C\sum_i {\xi_i}^2$$

$$s.t. \quad y_i(k(v_H, x_i) - c) \geq 1 - \xi_i$$
$$\xi_i \geq 0$$
$$for\ i = 1, \dots, n$$

where: **Cost parameter C** is a tuning parameter, which specifies the cost of allowing a point on the wrong side.

Full-Fledged SVM

Ingredient	**Purpose**
Maximum margin	Good generalization property
Slack variables	Overlapping classes, Robustness
Kernel	Non-linear decision boundary

2-Fold Cross Validation and Model Selection

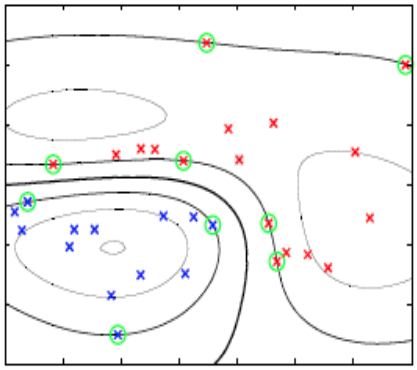
The effectiveness of SVM depends on the selection of kernel and soft margin parameter γ. A common choice is RBF kernel.

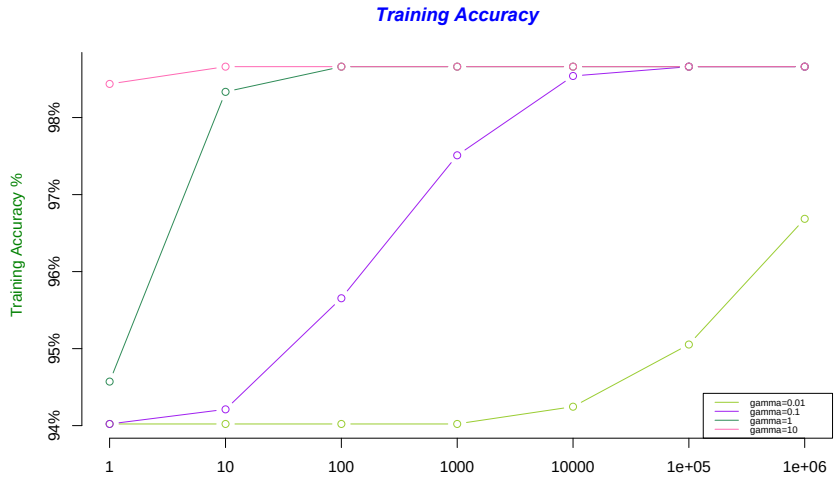
RBF Kernel: $k_{RBF}(x, x') = exp(-\gamma\|x - x'\|^2)$

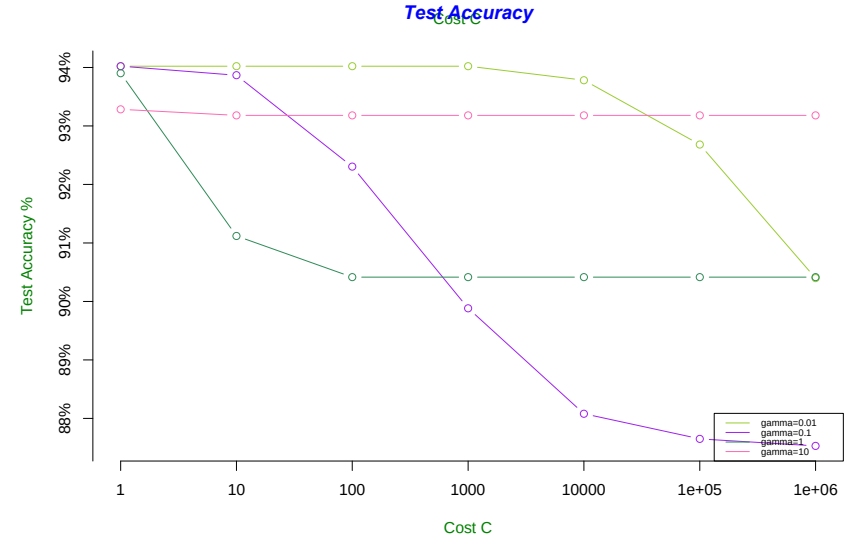
Two tuning parameters:

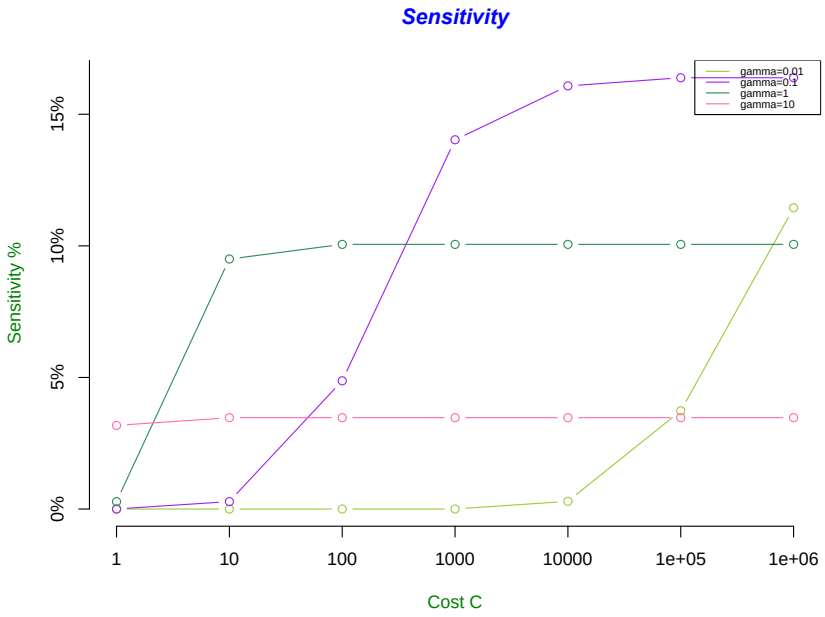
(1) Kernel parameter γ: Intuitively, the γdefines how far the influence of a single training data reaches, with low values meaning "far" and high values meaning "close".

(2) Cost parameter C: It trades off misclassification of training data against simplicity of the decision boundary. A low cost makes the decision boundary smooth, while a high cost makes the decision boundary rough.

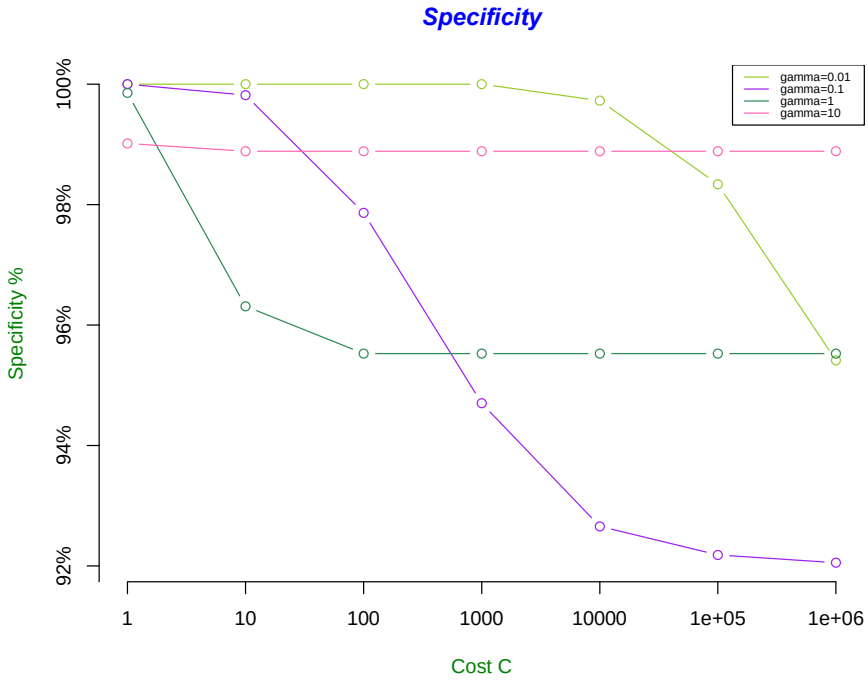




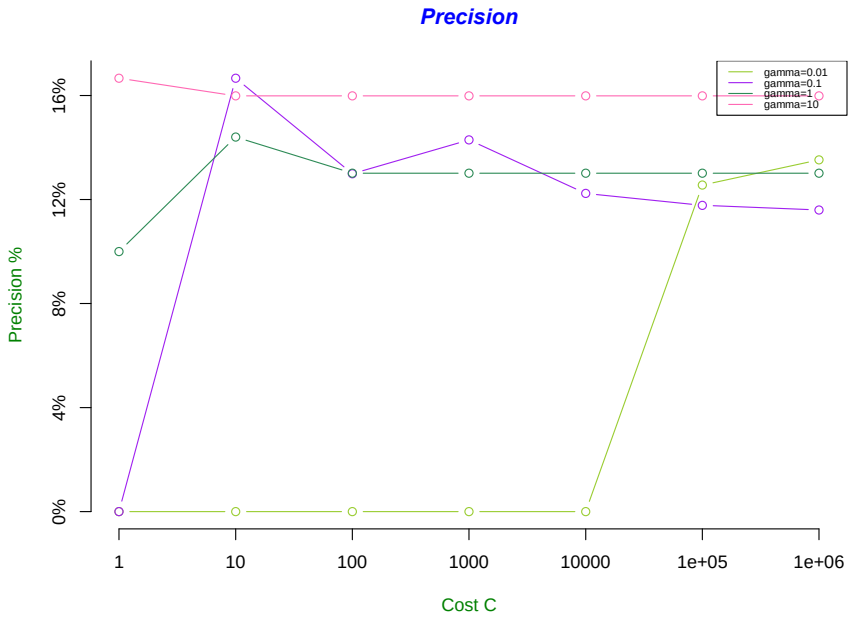


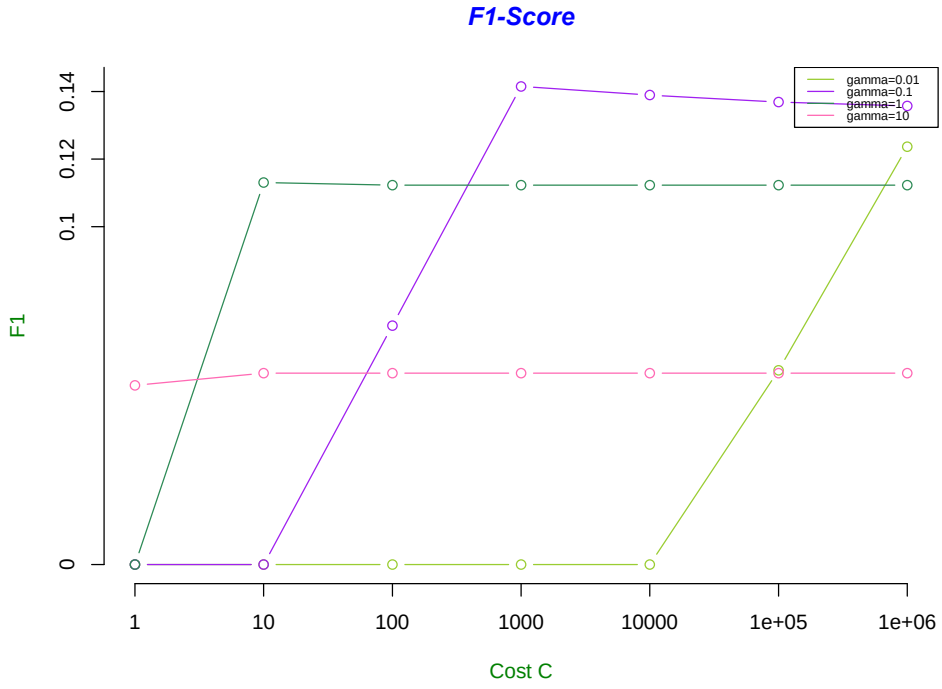


- No matter what the value of the kernel parameter gamma is, sensitivity always increases when the cost parameter increases.
- Sensitivity achieves its maximum at kernel parameter gamma=0.1 and cost >=10000.



- In contrast to sensitivity, specificity always decays, and achieves its maximum at Cost=1.





- **F1-score achieves its maximum when gamma=0.1 and cost=1000. Hence, the optimal SVM model is RBF kernel with gamma=0.1 and cost=1000.**

RBF Kernel with gamma=0.1 & C=1000 on Test Dataset

F1= 0.1409922

		True Class		
		1=Positive	0=Negative	
Predicted Class	1 =Positive	27	118	Precision=18.62%
	0 =Negative	211	3644	
		Sensitivity= 11.34%	Specificity= 96.86%	

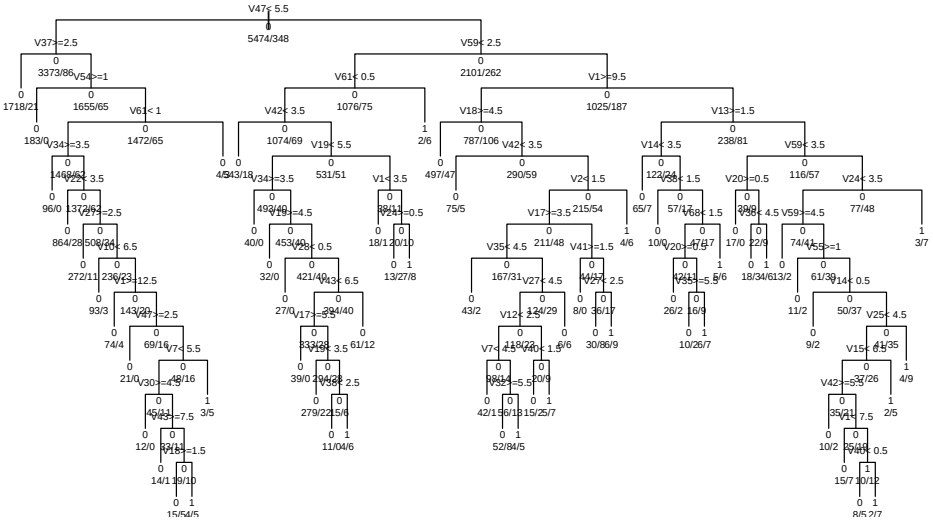
- **The SVM with RBF kernel model able to identify 11.34% of customers who actually having caravan insurance policy, 96.86% of those who actually don't have, and about 18.62% of those who are predicted to have caravan policy are actually policy owners.**

3. Classification Tree

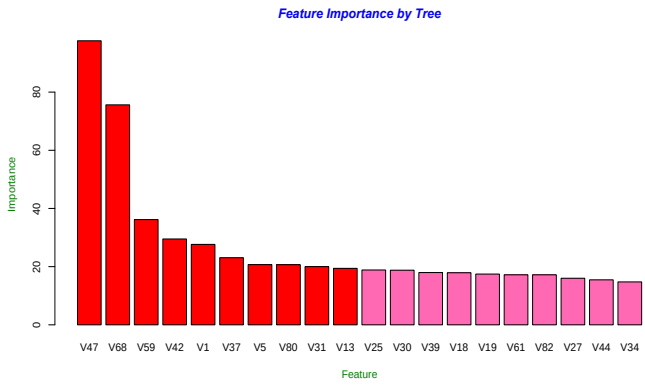
Unlike SVMs, Naïve Bayes we have discussed, a classification tree partitions the feature space in a recursive manner and fit local methods in each region instead of a global model in a large feature space. Its most attractive advantages are interpretability and built-in feature selection by the impurity measure "*information gain*".

Un-Pruned Tree

TIC Data using all features



Feature Importance by Tree



The top-10 strongest features are:

1. Car policy contribution
2. Number of Car policies
3. Fire Policy contribution
4. Average income
5. Customer subtype
6. Income < 30
7. Customer main type
8. Number of fire policy
9. Home owner
10. Singles

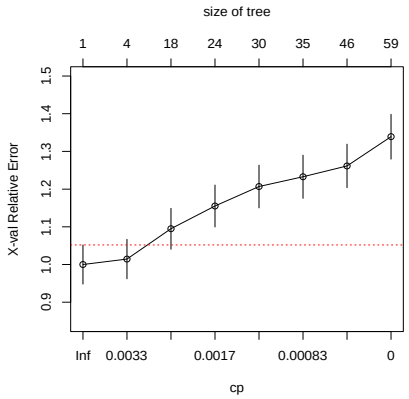
Expected insurance ownership relating to car
Other insurance product ownership
Sociodemographic variable

There're some difference of the most important features by tree and individual predictive power, although they're both based on "information gain". **Tree select subsets of features one after another rather than just evaluate features individually and independently.** There may be several reasons to look at subset instead of single features. A feature with high individual predictive power but also high correlation to variables that are already selected does not add much information to the model, so it should not be included. A feature with low individual predictive power may have some complex joint relationship with a selected variable that is highly predictive, in which case it may be advisable to include it.

Overfitting and Pruning

We have built a complete but too complex tree. To grow each branch of the tree just deeply enough to perfectly classify the training data, in fact it can lead to difficulties when there is noise in the training data, or when the number of trainings is too small to produce a representative sample of the true population. In either of these cases, this can produce trees that overfits the training data. **The accuracy of the tree over the trainings increases monotonically as the tree grows, however, the accuracy over the test set first increase then decreases. So we need to prune the tree.**

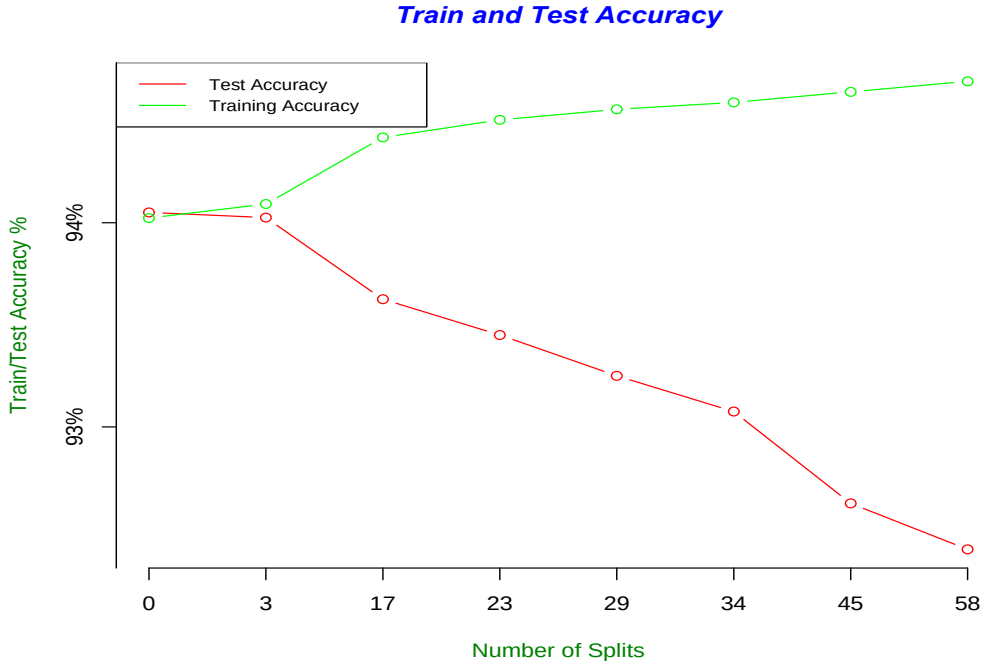
```
        CP nsplit rel error xerror     xstd
0.00383142      0   1.00000 1.0000 0.051979
0.00287356      3   0.98851 1.0144 0.052327
0.00191571     17   0.93391 1.0948 0.054223
0.00143678     23   0.91954 1.1552 0.055590
0.00095785     29   0.91092 1.2069 0.056727
0.00071839     34   0.90517 1.2328 0.057283
0.00057471     45   0.89655 1.2615 0.057893
0.00000000     58   0.88793 1.3391 0.059497
```



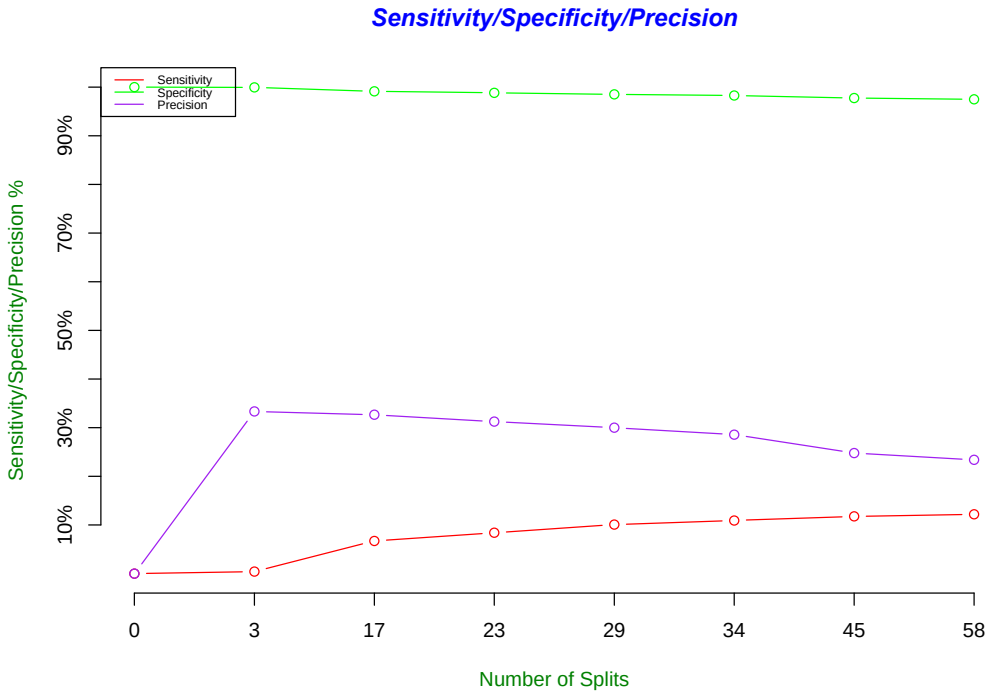
- The column labeled **CP** is the complexity parameter. Its serves as a penalty term to control tree size and is always monotonically decreases with the number of splits (**nsplit**). The smaller the CP, the more complex will be the tree (i.e. the larger the number of splits).
- The 10-fold cross-validation error (**xerror**) is the misclassification rate relative to the simplest tree with only the root node and no splitting. Thus, xerror=1 when nspli=0.
- From the misclassification perspective, for TIC data, since the xerror achieves minimum with no splitting, the best tree would be just assign every data to class 0.

Model Selection

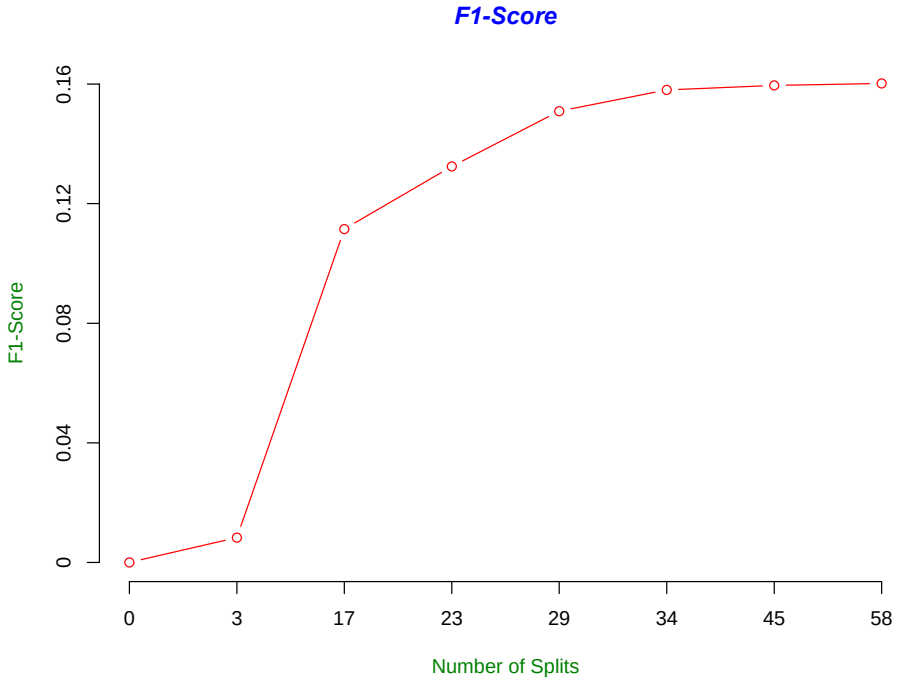
The effective of tree depends on how complex the tree grows, that is, how many split the tree contains. Thus, we use the F1-score to prune the tree and tune the parameter "**nplit**".



- The training accuracy increases monotonically as the tree grows; however, the testing accuracy decreases monotonically as the tree grows.



- The green line shows that specificity are always very large, i.e., tree always has good ability to identify class 0;
- The red line shows that sensitivity are generally small, i.e., tree ability to identify class 1 increases when the number of features included in the model increases;
- The purple line shows that precision achieves maximum at nsplit=3.



- **F1-score achieves its maximum when nsplit=58. Hence, the optimal classification tree is no pruning.**

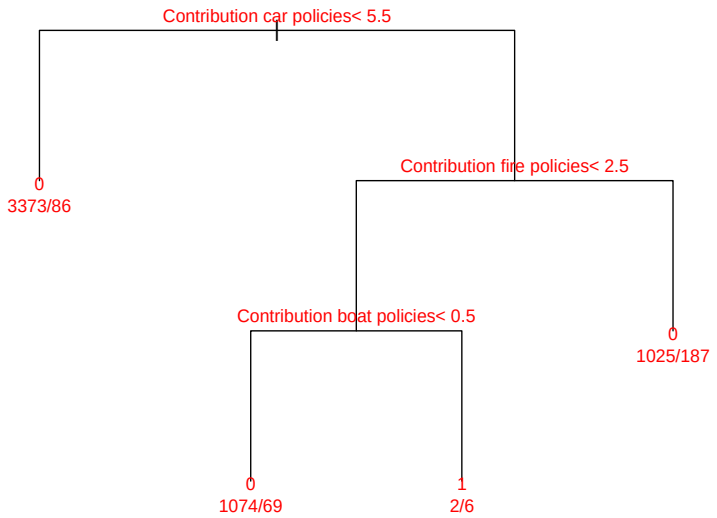
Best Classification Tree on Test Dataset

F1= 0.160221

		True Class		
		1=Positive	0=Negative	
Predicted Class	1 =Positive	29	95	Precision=23.39%
	0 =Negative	209	3667	
		Sensitivity= 12.18%	Specificity= 97.47%	

- **The classification tree able to identify 12.18% of customers who actually having caravan insurance policy, 97.47% of those who actually don't have, and about 23.39% of those who are predicted to have caravan policy are actually policy owners.**

A Simple Tree for Interpretation Task



- For the description task, **if a customer with Car policy contribution greater than 6, Fire policy contribution less than 2, and Boat policy contribution equal to 0, that is, very high level of Car policy contribution, very low level of Fire policy contribution, and no Boat policy contribution, then he/she is 33.3% likely to have or will buy the Caravan insurance policy**. This information provided by tree is extremely informative and intuitively interpretable, thus can be used in practice for the insurance company.

An **ensemble method** makes a prediction by combining the predictions of many classifiers into a single vote. The individual classifiers are usually required to perform only slightly better than random. This means slightly more than 50% of the data are classified correctly. Such a classifier is called a **weak learner**. If the weak learners are random and independent, the prediction accuracy of the majority vote will increase with the number of weak learners. Since the weak learners all have to be trained on the same training set, producing random and independent weak learners is difficult. Different ensemble methods (Bagging, Boosting, and Random Forest, etc) use different strategyes to train and combine weak learners that behave relatively independent.

Boosting is arguably the most popular (and historically the first) ensemble method. The independence of weak learners is obtained by modifying the training data using weights after training each weak learner. The **AdaBoost** algorithm of Freud and Schapire was the first practiceal boosting algorithm, and remains one of the most widely used and standard.

Pseudocode for AdaBoost

In AdaBoost, a weight value is assigned to each training data. At each step, data points which are correctly classified correctly are weighted down and which are misclassified are weighted up, i.e. the weight is smaller the more the weak learners already trained classify the data point correctly, and vice versa. So the next weak learner which is trained on the weighted dataset focuses more on data points which are "hard to classify" by previous weak learners. Roughly speaking, each weak learner tries to get those points right which are currently not classified correctly.

Input:

- Training data $(x_1, y_1), \dots, (x_n, y_n)$
- Algorithm parameter: number of weak learners T

Training Algorithm:

- Initialize weights: $w_1(i) = \frac{1}{n} \quad for\ i = 1, \dots, n$
- For t=1,...,T:

  Train a weak learner using weights $w_t(i)$: $f_t(x_i) \in \{-1, +1\}\ for\ i = 1, \dots, n$

  Weight Update:

  $\varepsilon_t = \frac{\sum_{i=1}^{n} I\{y_i \neq f_t(x_i)\}}{\sum_{i-1}^{n} w_t(i)}$

  $\alpha_t = \frac{1}{2} log(\frac{1-\varepsilon_t}{\varepsilon_t})$

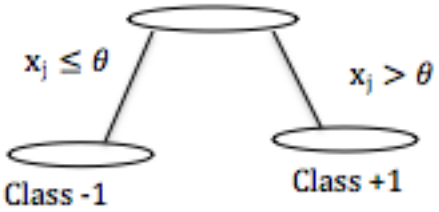
  $$w_{t+1}(i) = w_t(i) exp(-\alpha_t y_i f_t(x_i)) = \begin{cases} w_t(i)\sqrt{\frac{\varepsilon_t}{1-\varepsilon_t}} & if\ f_t\ classify\ x_i\ correctly \\ w_t(i)\sqrt{\frac{1-\varepsilon_t}{\varepsilon_t}} & if\ f_t\ misclassify\ x_i \end{cases}$$

- Output the ensemble classifier:

  $f(x) = sgn\left\{\sum_{t=1}^{T} \alpha_t f_t(x)\right\}$

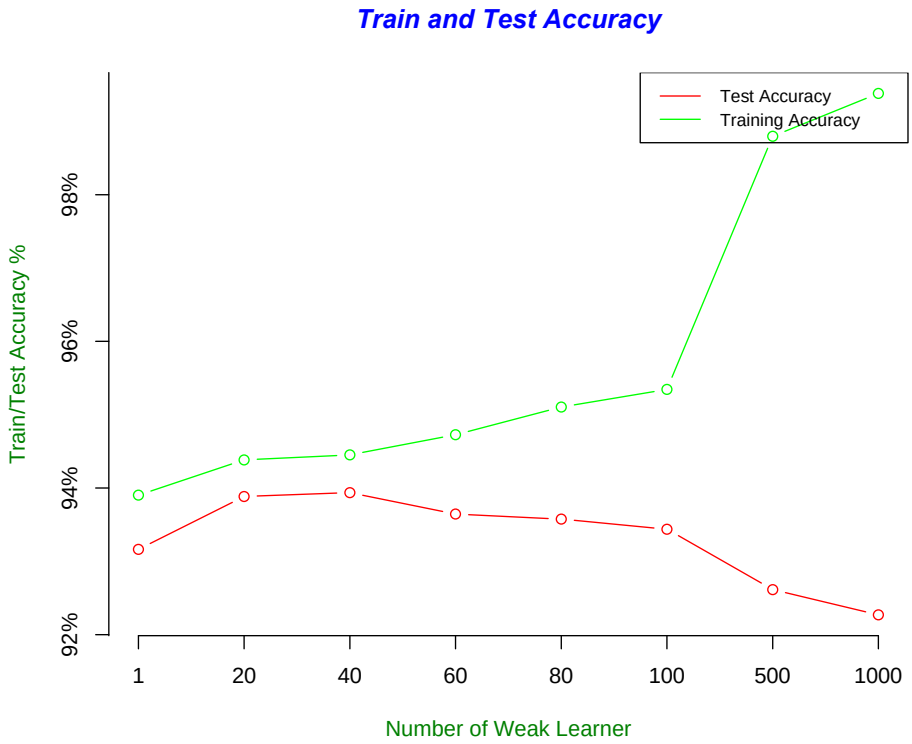
AdaBoost using Decision Stump as Weak Learner

The simplest tree is a tree of depth 1. Such a classifier is called a **decision stump**. A decision stump is parameterized by a pair (j, θ) of an axis j and a splitting point θ. Decision boundary is a hyperplane which is perpendicular to axis j and intersects the axis at θ.

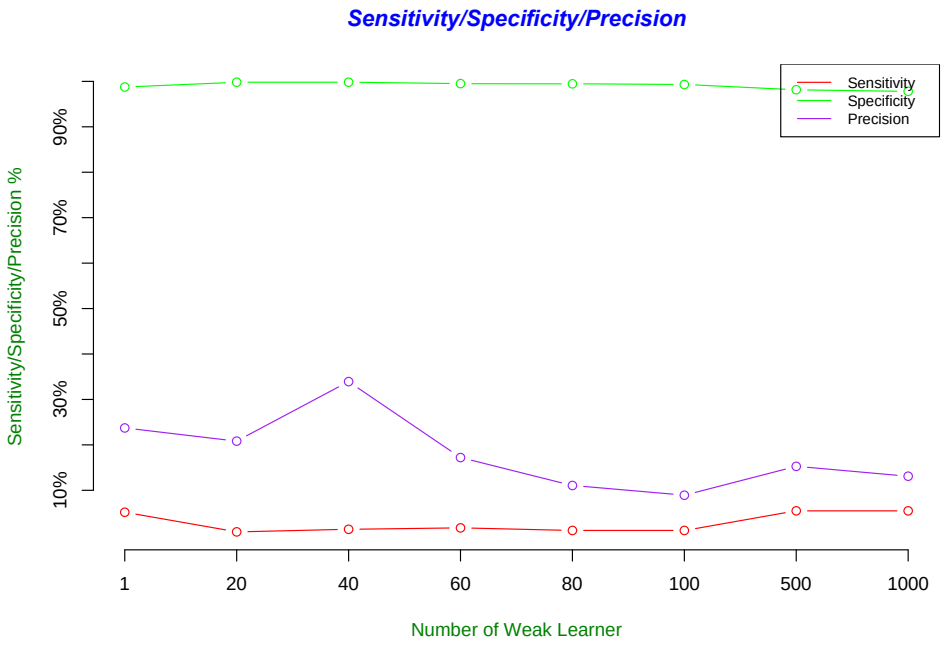


2-Fold CV and Model Selection

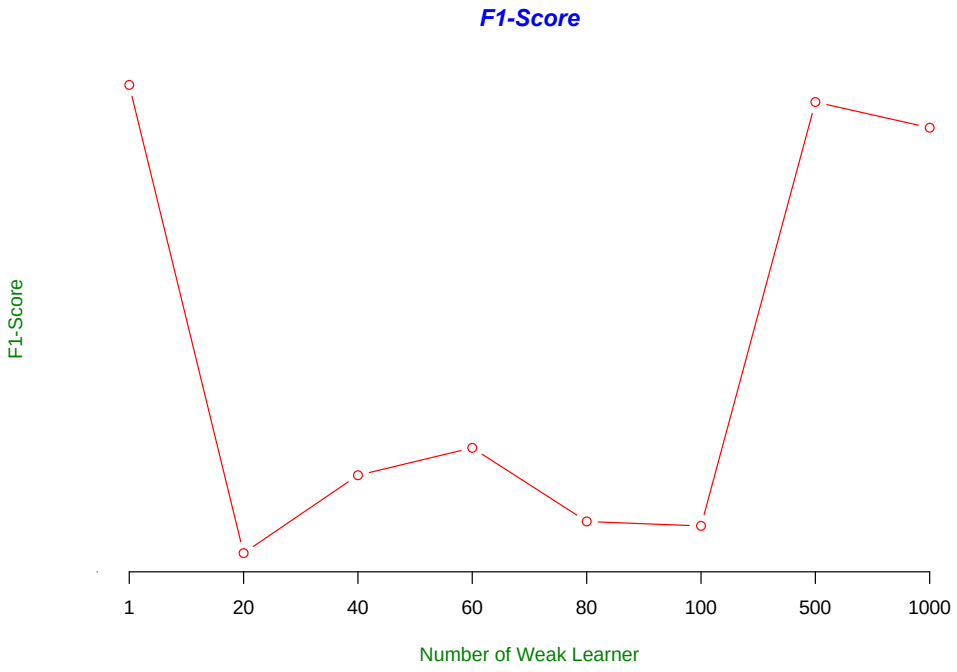
The effective of AdaBoost depends on the tuning parameter: number of weak learners. Thus, we use the F1-score to do cross validation and tune the parameter.



- The training accuracy increases when the number of weak learners increases, and when greater than 500, the training accuracy >99%.
- The test accuracy does not increase with the number of weak learners.



- Sensitivity, specificity, and precision do not vary significantly with the number of weak learners, this means that AdaBoost algorithm does not have good improvement regarding sensitivity, specificity, and precision. This is reasonable, since AdaBoost focuses on error rate.



- **F1-score achieves its maximum when combining 500 weak learners, but ensemble method does not have superior performance for the TIC dataset.**

AdaBoost with 500 Decision Stumps on Test Dataset

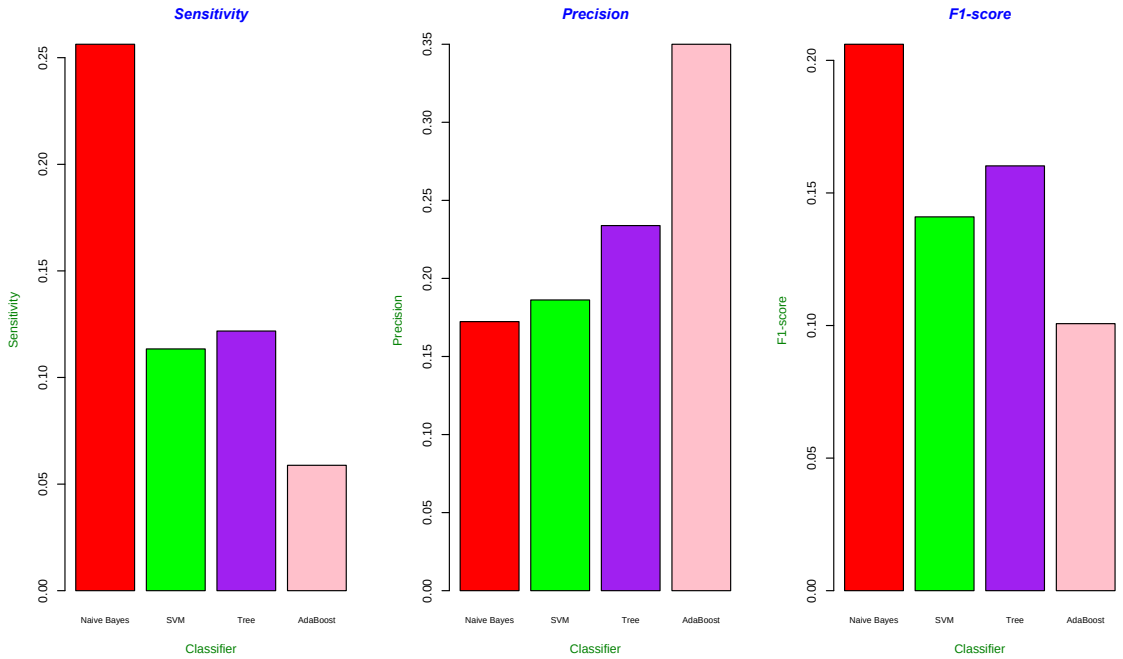
F1= 0.1007194

		True Class		
		1=Positive	0=Negative	
Predicted Class	1 =Positive	14	26	Precision= 35%
	0 =Negative	224	3636	
		Sensitivity= 5.88%	Specificity= 99.31%	

- **The classification tree able to identify 5.88% of customers who actually having caravan insurance policy, 99.31% of those who actually don't have, and about 35% of those who are predicted to have caravan policy are actually policy owners.**

Part III

Conclusion



The insurance company's main question is who would buy the product, thus we should focus on tests' ability to identify positive results—Sensitivity. The greater the sensitivity the more potential customers we would detect. **But we also need to look at the precision, since there is a trade-off between cost of mailing and profit of positive customers**, but in this case, the four algorithms' precision do not have major difference. Thus, specificity is not a major concern in this

case. **The F1-score is a weighted average of sensitivity and precision, and is the statistics we have used to do model selection.**
Both of sensitivity and F1-score give the same suggestion that Naïve Bayes is the best classifier for our case. The **Naïve Bayes model** with the top-10 most important features is able to:

- identify 25.63% of customers who actually having caravan insurance policy, so **about ¼ potential customers will recognized**;
- identify 92.21% of those who actually don't have, so **more than 9/10 of those won't buy the caravan policy will be excluded from the mailing list, which save a lot of unnecessary expense for the company**;
- about 17.23% of those who are predicted to have caravan policy are actually policy owners, so we have the confidence that **about 1/5 of those remain in the mailing list will buy caravan policy**.

Part IV Appendix

Appendix A: Data Dictionary

Nr Name Description Domain

1 MOSTYPE Customer Subtype see L0
2 MAANTHUI Number of houses 1 – 10
3 MGEMOMV Avg size household 1 – 6
4 MGEMLEEF Avg age see L1
5 MOSHOOFD Customer main type see L2
6 MGODRK Roman catholic see L3
7 MGODPR Protestant ...
8 MGODOV Other religion
9 MGODGE No religion
10 MRELGE Married
11 MRELSA Living together
12 MRELOV Other relation
13 MFALLEEN Singles
14 MFGEKIND Household without children
15 MFWEKIND Household with children
16 MOPLHOOG High level education
17 MOPLMIDD Medium level education
18 MOPLLAAG Lower level education
19 MBERHOOG High status
20 MBERZELF Entrepreneur
21 MBERBOER Farmer
22 MBERMIDD Middle management
23 MBERARBG Skilled labourers
24 MBERARBO Unskilled labourers
25 MSKA Social class A
26 MSKB1 Social class B1
27 MSKB2 Social class B2
28 MSKC Social class C
29 MSKD Social class D
30 MHHUUR Rented house
31 MHKOOP Home owners
32 MAUT1 1 car
33 MAUT2 2 cars
34 MAUT0 No car
35 MZFONDS National Health Service
36 MZPART Private health insurance
37 MINKM30 Income < 30.000
38 MINK3045 Income 30-45.000
39 MINK4575 Income 45-75.000
40 MINK7512 Income 75-122.000
41 MINK123M Income >123.000
42 MINKGEM Average income
43 MKOOPKLA Purchasing power class
44 PWAPART Contribution private third party insurance see L4
45 PWABEDR Contribution third party insurance (firms) ...
46 PWALAND Contribution third party insurane (agriculture)
47 PPERSAUT Contribution car policies
48 PBESAUT Contribution delivery van policies
49 PMOTSCO Contribution motorcycle/scooter policies
50 PVRAAUT Contribution lorry policies
51 PAANHANG Contribution trailer policies
52 PTRACTOR Contribution tractor policies
53 PWERKT Contribution agricultural machines policies
54 PBROM Contribution moped policies
55 PLEVEN Contribution life insurances
56 PPERSONG Contribution private accident insurance policies
57 PGEZONG Contribution family accidents insurance policies
58 PWAOREG Contribution disability insurance policies
59 PBRAND Contribution fire policies
60 PZEILPL Contribution surfboard policies
61 PPLEZIER Contribution boat policies
62 PFIETS Contribution bicycle policies
63 PINBOED Contribution property insurance policies
64 PBYSTAND Contribution social security insurance policies
65 AWAPART Number of private third party insurance 1 - 12
66 AWABEDR Number of third party insurance (firms) ...
67 AWALAND Number of third party insurane (agriculture)
68 APERSAUT Number of car policies
69 ABESAUT Number of delivery van policies

70 AMOTSCO Number of motorcycle/scooter policies
71 AVRAAUT Number of lorry policies
72 AAANHANG Number of trailer policies
73 ATRACTOR Number of tractor policies
74 AWERKT Number of agricultural machines policies
75 ABROM Number of moped policies
76 ALEVEN Number of life insurances
77 APERSONG Number of private accident insurance policies
78 AGEZONG Number of family accidents insurance policies
79 AWAOREG Number of disability insurance policies
80 ABRAND Number of fire policies
81 AZEILPL Number of surfboard policies
82 APLEZIER Number of boat policies
83 AFIETS Number of bicycle policies
84 AINBOED Number of property insurance policies
85 ABYSTAND Number of social security insurance policies
86 CARAVAN Number of mobile home policies 0 – 1

L0:

	Value	Label
1	1	High Income, expensive child
2	2	Very Important Provincials
3	3	High status seniors
4	4	Affluent senior apartments
5	5	Mixed seniors
6	6	Career and childcare
7	7	Dinki's (double income no kids)
8	8	Middle class families
9	9	Modern, complete families
10	10	Stable family
11	11	Family starters
12	12	Affluent young families
13	13	Young all american family
14	14	Junior cosmopolitan
15	15	Senior cosmopolitans
16	16	Students in apartments
17	17	Fresh masters in the city
18	18	Single youth
19	19	Suburban youth
20	20	Etnically diverse
21	21	Young urban have-nots
22	22	Mixed apartment dwellers
23	23	Young and rising
24	24	Young, low educated
25	25	Young seniors in the city
26	26	Own home elderly
27	27	Seniors in apartments
28	28	Residential elderly
29	29	Porchless seniors: no front yard
30	30	Religious elderly singles
31	31	Low income catholics
32	32	Mixed seniors
33	33	Lower class large families
34	34	Large family, employed child
35	35	Village families
36	36	Couples with teens 'Married with children'
37	37	Mixed small town dwellers
38	38	Traditional families
39	39	Large religous families
40	40	Large family farms
41	41	Mixed rurals

L1:

1 20-30 years
2 30-40 years
3 40-50 years
4 50-60 years
5 60-70 years
6 70-80 years

L2:

1 Successful hedonists
2 Driven Growers
3 Average Family
4 Career Loners
5 Living well
6 Cruising Seniors
7 Retired and Religeous
8 Family with grown ups
9 Conservative families
10 Farmers

L3:

0 0%
1 1 - 10%
2 11 - 23%
3 24 - 36%
4 37 - 49%
5 50 - 62%
6 63 - 75%
7 76 - 88%
8 89 - 99%
9 100%

L4:

0 f 0
1 f 1 – 49
2 f 50 – 99
3 f 100 – 199
4 f 200 – 499
5 f 500 – 999
6 f 1000 – 4999
7 f 5000 – 9999
8 f 10.000 - 19.999
9 f 20.000 - ?

```
# ============ Import Data ================================================================================
getwd()
setwd("/Users/cindy/Downloads")
ticdata2000<-read.table("ticdata2000.txt",header=FALSE,sep="\t") #training
ticeval2000<-read.table("ticeval2000.txt",header=FALSE,sep='\t') #test
tictgts2000<-read.table("tictgts2000.txt",header=FALSE,sep="\t") #test label

# =========== Exploratory Data Analysis ==================================================================

counts <- table(ticdata2000[,4])
barplot(counts,col="green",names.arg=c("1: 20-30 years", "2: 30-40 years", "3: 40-50 years", "4: 50-60 years", "5: 60-70 years",
"6: 70-80 years"))
title(main="Barplot for Levels of Average Age",col.main="blue",font.main=4)

counts <- table(ticdata2000[,47])
barplot(counts,col="green",names.arg=c("Level=0","Level=4","Level=5","Level=6","Level=7","Level=8"))
title(main="Barplot for Levels of Car Policy Contribution",col.main="blue",font.main=4)

counts <- table(ticdata2000[,68])
barplot(counts,col="green",names.arg=c("0 Car Policy","1 Car Policy","2 Car Policy","3 Car Policy","4 Car Policy","6 Car
Policy","7 Car Policy"))
title(main="Barplot for Number of Car Policy",col.main="blue",font.main=4)

par(mfrow=c(1,2))
counts1 <- table(ticdata2000[,86])
barplot(counts1,col="red",names.arg=c("Y=0:No Caravan Insurance Policy","Y=1:Having Caravan Insurance
Policy"),xlab="Number of Caravan Insurance Policy",ylab="Count",col.lab=rgb(0,0.5,0),cex.names=0.7)
title(main="Distribution of Target Var(Y) on Training Set",col.main="blue",font.main=4)
legend("topright", c("Y=1%: 6%"), cex=0.9)

counts2 <- table(tictgts2000)
barplot(counts2,col="red",names.arg=c("Y=0:No Caravan Insurance Policy","Y=1:Having Caravan Insurance
Policy"),xlab="Number of Caravan Insurance Policy",ylab="Count",col.lab=rgb(0,0.5,0),cex.names=0.7)
title(main="Distribution of Target Var(Y) on Test Set",col.main="blue",font.main=4)
legend("topright", c("Y=1%: 6%"), cex=0.9)


# ============Feature Selection============================================================================
library(infotheo)

# vector MI contains the mutual informations for each feature
MI<-rep(0,85)
for(i in 1:85){
        MI[i]<-mutinformation(ticdata2000[,i],ticdata2000[,86],method="emp")
}

MI.percent<-MI/MI[which.max(MI)]

# vector o contains the ordering according to MI
o<-order(MI,decreasing=TRUE)
var.name<-names(ticdata2000)[-86]

# data.frame MI.sort.df contains:feature names,mutual information and percentage,sorted from high to low
MI.sort.df<-data.frame(var=var.name[o],mutualinfo=MI[o],percent=MI.percent[o])

#plot feature importance
mycolor<-
c(rep("red",10),rep("hotpink",10),rep("pink",10),rep("yellow",10),rep("lightskyblue",10),rep("slateblue",10),rep("gray",25))
barplot(MI.sort.df$percent,col=mycolor,xlab="Feature",ylab="Percentage of IG relative to Best Feature ",col.lab=rgb(0,0.5,0))
title(main="Predictive Power by Information Gain",col.main="blue",font.main=4)

#top 10 features
MI.sort.df$var[1:10]
```

```
 # ===================Make Training and Test Sets=========================================================
tic.train.x<-ticdata2000[,o]
tic.train.y<-ticdata2000[,86]
tic.test.x<-ticeval2000[,o]
tic.test.y <-tictgts2000$V1

fold1 <- c(1:2911)
fold2 <- c(2912:5822)



# ========================== Naive Bayes Classification=========================================================
NaiveBayes <- function(Xtrain,Ytrain,Xtest,d){

                #class probability y=0
   prob.y0<-length(which(Ytrain==0))/length(Ytrain)

   #class probability y=1
   prob.y1<-length(which(Ytrain==1))/length(Ytrain)

   # predicted value vector for test set
   Ytest<-rep(0,dim(Xtest)[1])

   # log probability matrices for two class
   log.prob.y0 <- mat.or.vec(dim(Xtest)[1],d)
   log.prob.y1 <- mat.or.vec(dim(Xtest)[1],d)

                for(i in 1:d){
                                # extract possible values for each feature from both training and test sets
     values<-unique(c(Xtrain[,i],Xtest[,i]))

     # mulinomial parameters for feature i,row 1 for class y=0,row 2 for class y=1
     par.est<-mat.or.vec(2,length(values))

     for(j in 1:length(values)){
                  par.est[1,j]<-(1+length(which(Ytrain==0 & Xtrain[,i]==values[j])))/(length(values)+length(which(Ytrain==0)))
                  par.est[2,j]<-(1+length(which(Ytrain==1 & Xtrain[,i]==values[j])))/(length(values)+length(which(Ytrain==1)))
     }

     for(obs in 1:dim(Xtest)[1]){
     log.prob.y0[obs,i]<-log(par.est[1,which(values==Xtest[obs,i])])
     log.prob.y1[obs,i]<-log(par.est[2,which(values==Xtest[obs,i])])
                  }
                }

                 prob.class0<-rowSums(log.prob.y0)+log(prob.y0)
    prob.class1<-rowSums(log.prob.y1)+log(prob.y1)

    for(h in 1:dim(Xtest)[1]){if(prob.class1[h]>=prob.class0[h]){Ytest[h]<-1}}

                return(Ytest)
}

tr.accuracy <-rep(0,17)
te.accuracy <-rep(0,17)
sens <-rep(0,17)
spec <-rep(0,17)
prec<-rep(0,17)
F1 <-rep(0,17)

npar <-seq(from=5,to=85,by=5)

for(n in npar){

                y1 <- NaiveBayes(tic.train.x[fold1,],tic.train.y[fold1],tic.train.x,n)
```

```
        y2 <- NaiveBayes(tic.train.x[fold2,],tic.train.y[fold2],tic.train.x,n)

        # train accuracy
        tr.a1 <-
(length(which(y1[fold1]==1&tic.train.y[fold1]==1))+length(which(y1[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        tr.a2 <-
(length(which(y2[fold2]==1&tic.train.y[fold2]==1))+length(which(y2[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        tr.accuracy[n/5] <-(tr.a1+tr.a2)/2

        # test accuracy
        te.a1 <-
(length(which(y1[fold2]==1&tic.train.y[fold2]==1))+length(which(y1[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        te.a2 <-
(length(which(y2[fold1]==1&tic.train.y[fold1]==1))+length(which(y2[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        te.accuracy[n/5] <- (te.a1+te.a2)/2

        # sensitivity
        sens1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(tic.train.y[fold2]==1))
        sens2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(tic.train.y[fold1]==1))
        sens[n/5]<-(sens1+sens2)/2

        # specificity
        spec1 <- length(which(y1[fold2]==0&tic.train.y[fold2]==0))/length(which(tic.train.y[fold2]==0))
        spec2 <- length(which(y2[fold1]==0&tic.train.y[fold1]==0))/length(which(tic.train.y[fold1]==0))
        spec[n/5] <-(spec1+spec2)/2

        # precision
        prec1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(y1[fold2]==1))
        prec2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(y2[fold1]==1))
        prec[n/5]<-(prec1+prec2)/2

        # F1-score
        F1.1<- 2*(prec1*sens1)/(prec1+sens1)
        F1.2<- 2*(prec2*sens2)/(prec2+sens2)
        F1[n/5]<-(F1.1+F1.2)/2

}

# accuracy plot
plot(te.accuracy,type="b",col="red",xlab="Number of
Splits",axes=FALSE,ylim=range(c(te.accuracy,tr.accuracy)),ylab="Train/Test Accuracy %",col.lab=rgb(0,0.5,0))
lines(tr.accuracy,type="b",col="green")
title(main="Train and Test Accuracy",col.main="blue",font.main=4)
axis(1,at=1:length(npar),labels=npar)
axis(2,at=c(0.8,0.825,0.85,0.875,0.9,0.925,0.95),labels=c("80%","82.5%","85%","87.5%","90%","92.5%","95%"))
legend("topright", c("Test Accuracy","Training Accuracy"), cex=0.6,
  col=c("red","green"), lty=1)

# sensitivity,specificity,precision plot
plot(sens,type="b",col="red",xlab="Number of
Splits",axes=FALSE,ylim=range(c(sens,spec,prec)),ylab="Sensitivity/Specificity/Precision %",col.lab=rgb(0,0.5,0))
lines(spec,type="b",col="green")
lines(prec,type="b",col="purple")
title(main="Sensitivity/Specificity/Precision",col.main="blue",font.main=4)
axis(1,at=1:length(npar),labels=npar)
axis(2,at=seq(from=0.1,to=1,by=0.1),labels=c("10%","20%","30%","40%","50%","60%","70%","80%","90%","100%"))
legend("topright", c("Sensitivity","Specificity","Precision"), cex=0.6,
  col=c("red","green","purple"), lty=1)

# F1 plot
plot(F1,type="b",col="red",xlab="Number of Splits",axes=FALSE,ylim=range(c(F1)),ylab="F1-Score",col.lab=rgb(0,0.5,0))
title(main="F1-Score",col.main="blue",font.main=4)
axis(1,at=1:length(npar),labels=npar)
axis(2,at=seq(from=0.15,to=0.275,by=0.025),labels=seq(from=0.15,to=0.275,by=0.025))

# Test Set
y <- NaiveBayes(tic.train.x,tic.train.y,tic.test.x,10)
table(y,tic.test.y)
sensitivity<-length(which(y==1&tic.test.y==1))/length(which(tic.test.y==1))
```

```
specificity<-length(which(y==0&tic.test.y==0))/length(which(tic.test.y==0))
precision<-length(which(y==1&tic.test.y==1))/length(which(y==1))
F1<-2*(precision*sensitivity)/(precision+sensitivity)



# ======================SVM================================================================================
library(class)
library(e1071)

# =================2-Fold Cross Validation:RBF Kernels=====================================================
# Tuning Parameter: cost, gamma

tr.accuracy <-mat.or.vec(4,7)
te.accuracy <-mat.or.vec(4,7)
sens <-mat.or.vec(4,7)
spec <-mat.or.vec(4,7)
prec<-mat.or.vec(4,7)
F1 <-mat.or.vec(4,7)

for(g in c(0.01,0.1,1,10)){
for(c in c(1,10,100,1000,10000,100000,1000000)){

        model1<-svm(tic.train.x[fold1,1:10],tic.train.y[fold1],type="C",kernel="radial",cost=c,gamma=g)
        y1<-predict(model1,tic.train.x[,1:10])

        model2<-svm(tic.train.x[fold2,1:10],tic.train.y[fold2],type="C",kernel="radial",cost=c,gamma=g)
        y2<-predict(model2,tic.train.x[,1:10])

        # train accuracy
        tr.a1 <-
(length(which(y1[fold1]==1&tic.train.y[fold1]==1))+length(which(y1[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        tr.a2 <-
(length(which(y2[fold2]==1&tic.train.y[fold2]==1))+length(which(y2[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        tr.accuracy[log10(g)+3,log10(c)+1] <-(tr.a1+tr.a2)/2

        # test accuracy
        te.a1 <-
(length(which(y1[fold2]==1&tic.train.y[fold2]==1))+length(which(y1[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        te.a2 <-
(length(which(y2[fold1]==1&tic.train.y[fold1]==1))+length(which(y2[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        te.accuracy[log10(g)+3,log10(c)+1] <- (te.a1+te.a2)/2

        # sensitivity
        sens1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(tic.train.y[fold2]==1))
        sens2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(tic.train.y[fold1]==1))
        sens[log10(g)+3,log10(c)+1]<-(sens1+sens2)/2

        # specificity
        spec1 <- length(which(y1[fold2]==0&tic.train.y[fold2]==0))/length(which(tic.train.y[fold2]==0))
        spec2 <- length(which(y2[fold1]==0&tic.train.y[fold1]==0))/length(which(tic.train.y[fold1]==0))
        spec[log10(g)+3,log10(c)+1] <-(spec1+spec2)/2

        # precision
        prec1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(y1[fold2]==1))
        prec2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(y2[fold1]==1))
        prec[log10(g)+3,log10(c)+1]<-(prec1+prec2)/2

        # F1-score
        F1.1<- 2*(prec1*sens1)/(prec1+sens1)
        F1.2<- 2*(prec2*sens2)/(prec2+sens2)
        F1[log10(g)+3,log10(c)+1]<-(F1.1+F1.2)/2

}
}

# accuracy plot
plot(tr.accuracy[1,],type="b",col="yellowgreen",xlab="Cost C",axes=FALSE,ylim=range(tr.accuracy),ylab="'Training
Accuracy %",col.lab=rgb(0,0.5,0))
```

```
lines(tr.accuracy[2,],type="b",col="purple")
lines(tr.accuracy[3,],type="b",col="seagreen")
lines(tr.accuracy[4,],type="b",col="hotpink")
title(main="Training Accuracy",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0.94,0.95,0.96,0.97,0.98,0.99),labels=c("94%","95%","96%","97%","98%","99%"))
legend("bottomright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)

plot(te.accuracy[1,],type="b",col="yellowgreen",xlab="Cost C",axes=FALSE,ylim=range(te.accuracy),ylab="Test
Accuracy %",col.lab=rgb(0,0.5,0))
lines(te.accuracy[2,],type="b",col="purple")
lines(te.accuracy[3,],type="b",col="seagreen")
lines(te.accuracy[4,],type="b",col="hotpink")
title(main="Test Accuracy",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0.87,0.88,0.89,0.90,0.91,0.92,0.93,0.94,0.95),labels=c("87%","88%","89%","90%","91%","92%","93%","94%","95
%"))
legend("bottomright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)

# sensitivity,specificity,precision plot
plot(sens[1,],type="b",col="yellowgreen",xlab="Cost
C",axes=FALSE,ylim=range(sens),ylab="Sensitivity %",col.lab=rgb(0,0.5,0))
lines(sens[2,],type="b",col="purple")
lines(sens[3,],type="b",col="seagreen")
lines(sens[4,],type="b",col="hotpink")
title(main="Sensitivity",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0,0.05,0.10,0.15,0.20),labels=c("0%","5%","10%","15%","20%"))
legend("topright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)

plot(spec[1,],type="b",col="yellowgreen",xlab="Cost
C",axes=FALSE,ylim=range(spec),ylab="Specificity %",col.lab=rgb(0,0.5,0))
lines(spec[2,],type="b",col="purple")
lines(spec[3,],type="b",col="seagreen")
lines(spec[4,],type="b",col="hotpink")
title(main="Specificity",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0.9,0.92,0.94,0.96,0.98,1.00),labels=c("90%","92%","94%","96%","98%","100%"))
legend("topright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)

plot(prec[1,],type="b",col="yellowgreen",xlab="Cost  C",axes=FALSE,ylim=range(prec),ylab="Precision  %",col.lab=rgb(0,0.5,0))
lines(prec[2,],type="b",col="purple")
lines(prec[3,],type="b",col="seagreen")
lines(prec[4,],type="b",col="hotpink")
title(main="Precision",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0,0.04,0.08,0.12,0.16,0.20),labels=c("0%","4%","8%","12%","16%","20%"))
legend("topright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)



# F1 plot
plot(F1[1,],type="b",col="yellowgreen",xlab="Cost C",axes=FALSE,ylim=range(F1),ylab="F1",col.lab=rgb(0,0.5,0))
lines(F1[2,],type="b",col="purple")
lines(F1[3,],type="b",col="seagreen")
lines(F1[4,],type="b",col="hotpink")
title(main="F1-Score",col.main="blue",font.main=4)
axis(1,at=1:7,labels=c(1,10,100,1000,10000,100000,1000000))
axis(2,at=c(0,0.2,0.4,0.6,0.8,0.10,0.12,0.14,0.16),labels=c(0,0.2,0.4,0.6,0.8,0.10,0.12,0.14,0.16))
legend("topright", c("gamma=0.01","gamma=0.1","gamma=1","gamma=10"), cex=0.6,
  col=c("yellowgreen","purple","seagreen","hotpink"), lty=1)


# Test Set
```

```
model<-svm(tic.train.x[,1:10],tic.train.y,type="C",kernel="radial",cost=1000,gamma=0.1)
y<-predict(model,tic.test.x[,1:10])
table(y,tic.test.y)
sensitivity<-length(which(y==1&tic.test.y==1))/length(which(tic.test.y==1))
specificity<-length(which(y==0&tic.test.y==0))/length(which(tic.test.y==0))
precision<-length(which(y==1&tic.test.y==1))/length(which(y==1))
F1<-2*(precision*sensitivity)/(precision+sensitivity)
```

```
# ========================Classification Tree=========================================================
library(rpart)
library(cluster)
library(foreach)
library(lattice)
library(plyr)
library(reshape2)
library(caret)


# use all features to build a full tree
fit<-rpart(ticdata2000$V86~.,method="class", y=TRUE,control=rpart.control(cp=0,xval=2),
parms=list(split="information"),data=ticdata2000)

#plot tree
plot(fit, margin=0,uniform=T, branch=1)
text(fit, use.n=TRUE, all=TRUE, cex=.5)
title(main="TIC Data using all features",col.main="blue",font.main=4)

# Feature importance
var.imp<-fit$variable.importance
mycolor1<-c(rep("red",10),rep("hotpink",10))
barplot(var.imp[c(1:20)],col=mycolor1,xlab="Feature",ylab="Importance",col.lab=rgb(0,0.5,0))
title(main="Feature Importance by Tree",col.main="blue",font.main=4)

# CP table
printcp(fit)
plotcp(fit,upper="size")

# Prune the tree
num.split<-fit$cptable[,"nsplit"] #possible number of splits
tr.accuracy <-rep(0,length(num.split))
te.accuracy <-rep(0,length(num.split))
sens <-rep(0,length(num.split))
spec <-rep(0,length(num.split))
prec<-rep(0,length(num.split))
F1 <-rep(0,length(num.split))

for(i in 1:length(num.split)){
        fit.prune <- prune(fit,cp=fit$cptable[i,"CP"])
        y.tr<- predict(fit.prune,ticdata2000[,1:85],type="class")
        y.te <- predict(fit.prune,ticeval2000,type="class")

        # train accuracy
        tr.accuracy[i] <-length(which(y.tr==tic.train.y))/length(tic.train.y)

        # test accuracy
        te.accuracy[i] <- length(which(y.te==tic.test.y))/length(tic.test.y)

        # sensitivity
        sens[i]<-length(which(y.te==1&tic.test.y==1))/length(which(tic.test.y==1))

        # specificity
        spec[i] <-length(which(y.te==0&tic.test.y==0))/length(which(tic.test.y==0))

        # precision
        prec[i]<-length(which(y.te==1&tic.test.y==1))/length(which(y.te==1))
```

```
        # F1-score
        F1[i]<-2*(prec[i]*sens[i])/(prec[i]+sens[i])
}

prec[1]<-0
F1[1]<-0


# accuracy plot
plot(te.accuracy,type="b",col="red",xlab="Number of
Splits",axes=FALSE,ylim=range(c(te.accuracy,tr.accuracy)),ylab="Train/Test Accuracy %",col.lab=rgb(0,0.5,0))
lines(tr.accuracy,type="b",col="green")
title(main="Train and Test Accuracy",col.main="blue",font.main=4)
axis(1,at=1:length(num.split),labels=num.split)
axis(2,at=c(0.90,0.91,0.92,0.93,0.94,0.95),labels=c("90%","91%","92%","93%","94%","95%"))
legend("topleft", c("Test Accuracy","Training Accuracy"), cex=0.8,
  col=c("red","green"), lty=1)

# sensitivity,specificity,precision plot
plot(sens,type="b",col="red",xlab="Number of
Splits",axes=FALSE,ylim=range(c(sens,spec,prec)),ylab="Sensitivity/Specificity/Precision %",col.lab=rgb(0,0.5,0))
lines(spec,type="b",col="green")
lines(prec,type="b",col="purple")
title(main="Sensitivity/Specificity/Precision",col.main="blue",font.main=4)
axis(1,at=1:length(num.split),labels=num.split)
axis(2,at=seq(from=0.1,to=1,by=0.1),labels=c("10%","20%","30%","40%","50%","60%","70%","80%","90%","100%"))
legend("topleft", c("Sensitivity","Specificity","Precision"), cex=0.6,
  col=c("red","green","purple"), lty=1)

# F1 plot
plot(F1,type="b",col="red",xlab="Number of Splits",axes=FALSE,ylim=range(c(F1)),ylab="F1-Score",col.lab=rgb(0,0.5,0))
title(main="F1-Score",col.main="blue",font.main=4)
axis(1,at=1:length(num.split),labels=num.split)
axis(2,at=seq(from=0,to=0.16,by=0.04),labels=seq(from=0,to=0.16,by=0.04))

# Test Set
y <- predict(fit,ticeval2000,type="class")
table(y,tic.test.y)
sensitivity<-length(which(y==1&tic.test.y==1))/length(which(tic.test.y==1))
specificity<-length(which(y==0&tic.test.y==0))/length(which(tic.test.y==0))
precision<-length(which(y==1&tic.test.y==1))/length(which(y==1))
F1<-2*(precision*sensitivity)/(precision+sensitivity)


# Pruned tree with nsplit=3
colnames(ticdata2000)[c(47,59,61)] <- c("Contribution car policies","Contribution fire policies","Contribution boat policies")
fit<-rpart(ticdata2000$V86~.,method="class", y=TRUE,control=rpart.control(cp=0,xval=2),
parms=list(split="information"),data=ticdata2000)

pruned.tree <- prune(fit,cp=fit$cptable[2,"CP"])

# Plot Pruned Tree
plot(pruned.tree, margin=0.1,uniform=T, branch=1)
text(pruned.tree, use.n=TRUE, all=FALSE, cex=0.8,col="red")



# ================================ AdaBoost ============================================================
library(rpart)
library(ada)

T<-c(1,20,40,60,80,100,500,1000)
tr.accuracy <-rep(0,length(T))
te.accuracy <-rep(0,length(T))
sens <-rep(0,length(T))
spec <-rep(0,length(T))
prec<-rep(0,length(T))
F1 <-rep(0,length(T))
```

```
fold1 <- c(1:2911)
fold2 <- c(2912:5822)
data1<-ticdata2000[fold1,]
data2<-ticdata2000[fold2,]


for(t in 1:length(T)){
        model1 <- ada(data1$V86~.,data=data1,iter = T[t], loss = "e", type = "discrete")
        y1 <-predict(model1,ticdata2000[,1:85])
        model2 <- ada(data2$V86~.,data=data2,iter = T[t], loss = "e", type = "discrete")
        y2 <-predict(model2,ticdata2000[,1:85])


        # train accuracy
        tr.a1 <-
(length(which(y1[fold1]==1&tic.train.y[fold1]==1))+length(which(y1[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        tr.a2 <-
(length(which(y2[fold2]==1&tic.train.y[fold2]==1))+length(which(y2[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        tr.accuracy[t] <-(tr.a1+tr.a2)/2

        # test accuracy
        te.a1 <-
(length(which(y1[fold2]==1&tic.train.y[fold2]==1))+length(which(y1[fold2]==0&tic.train.y[fold2]==0)))/length(fold2)
        te.a2 <-
(length(which(y2[fold1]==1&tic.train.y[fold1]==1))+length(which(y2[fold1]==0&tic.train.y[fold1]==0)))/length(fold1)
        te.accuracy[t] <- (te.a1+te.a2)/2

        # sensitivity
        sens1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(tic.train.y[fold2]==1))
        sens2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(tic.train.y[fold1]==1))
        sens[t]<-(sens1+sens2)/2

        # specificity
        spec1 <- length(which(y1[fold2]==0&tic.train.y[fold2]==0))/length(which(tic.train.y[fold2]==0))
        spec2 <- length(which(y2[fold1]==0&tic.train.y[fold1]==0))/length(which(tic.train.y[fold1]==0))
        spec[t] <-(spec1+spec2)/2

        # precision
        prec1 <- length(which(y1[fold2]==1&tic.train.y[fold2]==1))/length(which(y1[fold2]==1))
        prec2 <- length(which(y2[fold1]==1&tic.train.y[fold1]==1))/length(which(y2[fold1]==1))
        prec[t]<-(prec1+prec2)/2

        # F1-score
        F1.1<- 2*(prec1*sens1)/(prec1+sens1)
        F1.2<- 2*(prec2*sens2)/(prec2+sens2)
        F1[t]<-(F1.1+F1.2)/2



}

# accuracy plot
plot(te.accuracy,type="b",col="red",xlab="Number of Weak
Learner",axes=FALSE,ylim=range(c(te.accuracy,tr.accuracy)),ylab="Train/Test Accuracy %",col.lab=rgb(0,0.5,0))
lines(tr.accuracy,type="b",col="green")
title(main="Train and Test Accuracy",col.main="blue",font.main=4)
axis(1,at=1:length(T),labels=T)
axis(2,at=seq(from=0.9,to=1,by=0.02),labels=c("90%","92%","94%","96%","98%","100%"))
legend("topright", c("Test Accuracy","Training Accuracy"), cex=0.8,
  col=c("red","green"), lty=1)

# sensitivity,specificity,precision plot
plot(sens,type="b",col="red",xlab="Number of Weak
Learner",axes=FALSE,ylim=range(c(sens,spec,prec)),ylab="Sensitivity/Specificity/Precision %",col.lab=rgb(0,0.5,0))
lines(spec,type="b",col="green")
lines(prec,type="b",col="purple")
title(main="Sensitivity/Specificity/Precision",col.main="blue",font.main=4)
axis(1,at=1:length(T),labels=T)
```

```
axis(2,at=seq(from=0.1,to=1,by=0.1),labels=c("10%","20%","30%","40%","50%","60%","70%","80%","90%","100%"))
legend("topright", c("Sensitivity","Specificity","Precision"), cex=0.8,
  col=c("red","green","purple"), lty=1)

# F1 plot
plot(F1,type="b",col="red",xlab="Number of Weak Learner",axes=FALSE,ylim=range(c(F1)),ylab="F1-
Score",col.lab=rgb(0,0.5,0))
title(main="F1-Score",col.main="blue",font.main=4)
axis(1,at=1:length(T),labels=T)
axis(2,at=seq(from=0,to=0.01,by=0.001),labels=seq(from=0,to=0.01,by=0.001))

# Test Set
model <- ada(ticdata2000$V86~.,data=ticdata2000,iter = 500, loss = "e", type = "discrete")
y <-predict(model,ticeval2000)

table(y,tic.test.y)
sensitivity<-length(which(y==1&tic.test.y==1))/length(which(tic.test.y==1))
specificity<-length(which(y==0&tic.test.y==0))/length(which(tic.test.y==0))
precision<-length(which(y==1&tic.test.y==1))/length(which(y==1))
F1<-2*(precision*sensitivity)/(precision+sensitivity)
```