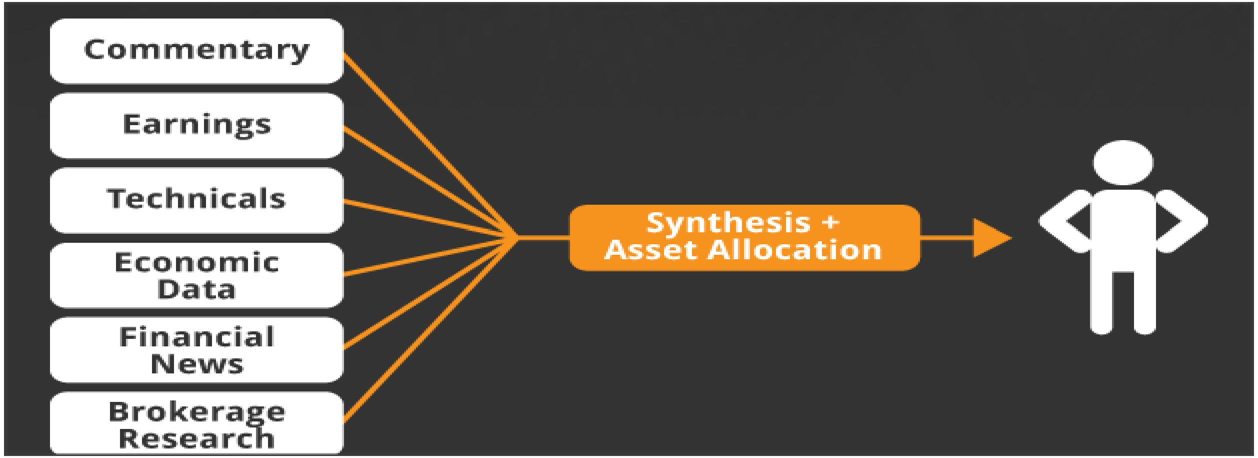
Stat 107: Introduction to Business and Financial Statistics

Class 8: Simulation and Portfolios

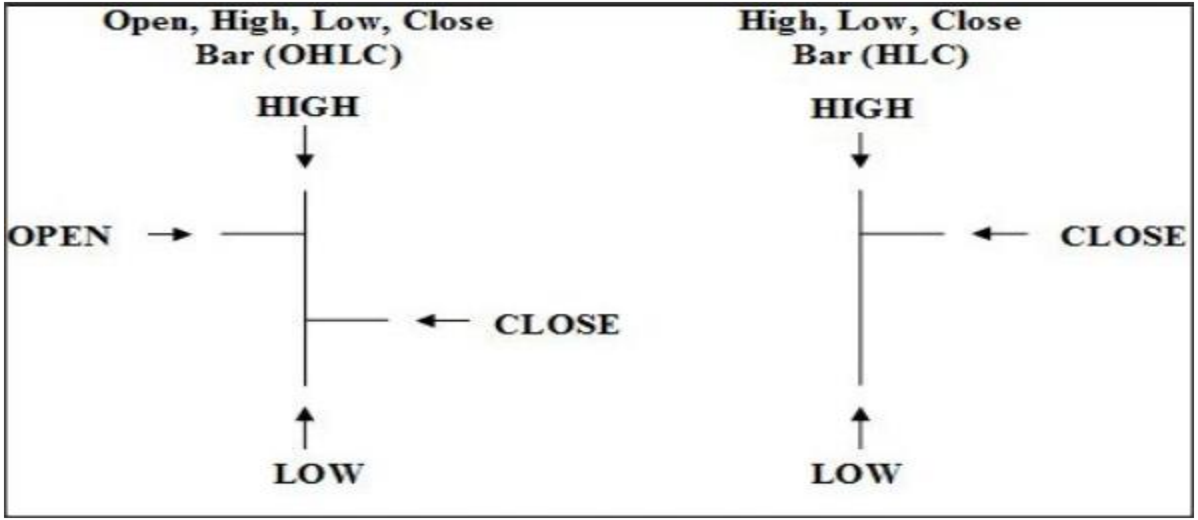
Trading is Complicated

- Can we make it algorithmic to make life easier? The search for the Holy Grail.



The OHLC Bar

- Could be daily, hourly, 15mins, 1 min, etc..



Internal Bar Strength

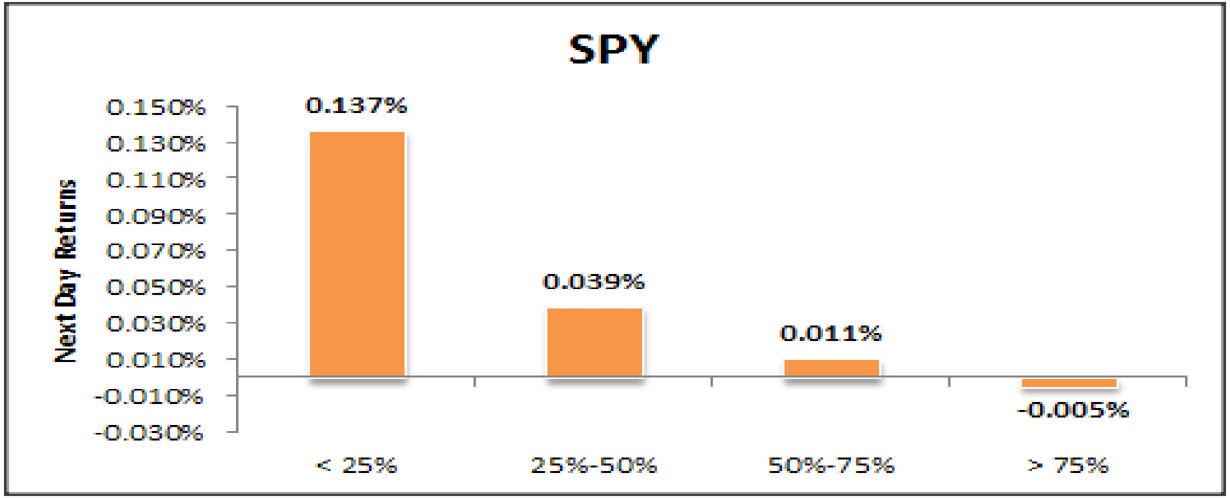
- The location of the closing price within the day's range is a surprisingly powerful predictor of next-day returns for equity indices.
- The closing price in relation to the day's range is called Internal Bar Strength and calculated as

$$IBS = \frac{Close - Low}{High - Low}$$

- It takes values between 0 and 1 and simply indicates at which point along the day's range the closing price is located.

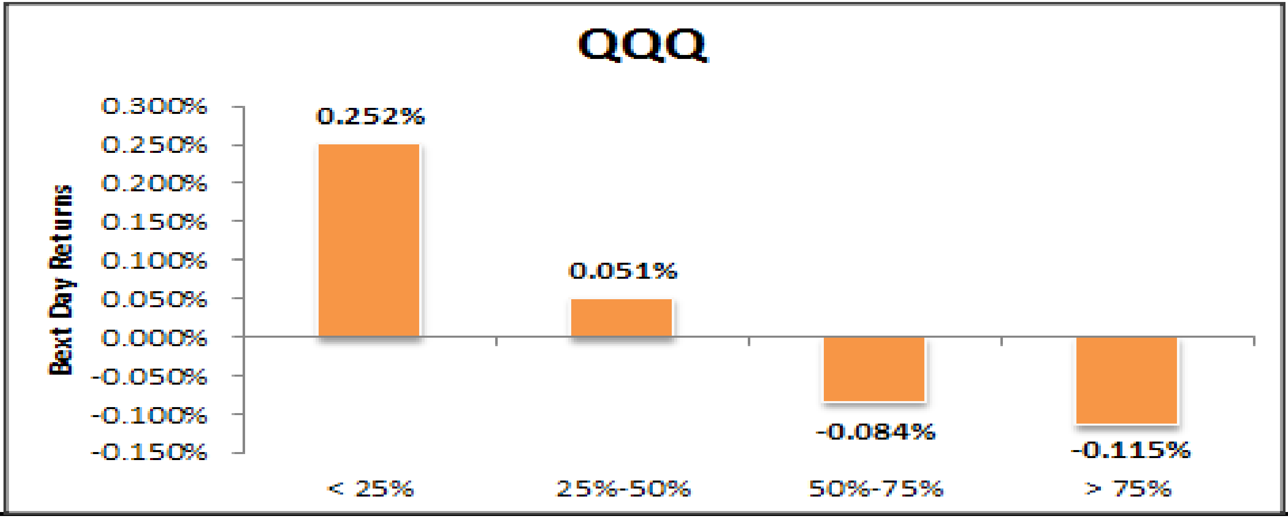
Looking for an Edge

- Check out tomorrow's SPY returns based on today's IBS:



Same Graph for QQQ

- Check out tomorrow's QQQ returns based on today's IBS:



The BUY and SELL Signals

$$IBS = \frac{Close - Low}{High - Low} < 0.25$$

We will buy at today's close if IBS<0.25

$$IBS = \frac{Close - Low}{High - Low} > 0.75$$

We will sell at today's close if IBS>0.75

The Results

- Time Period Matters! From 1/2005 to now

```
> myibs("SPY")
[1] "Ticker  SPY"
[1] "Time period =  2705  Days"
[1] "IBS time in market =  1320  Days"
[1] "IBS $1000 becomes =  2625.78085237085"
[1] "BH $1000 becomes =  2145.88955381009"
```

- From 1/2010 to now

```
[1] "Ticker  SPY"
[1] "Time period =  1677  Days"
[1] "IBS time in market =  754  Days"
[1] "IBS $1000 becomes =  1790.83633658444"
[1] "BH $1000 becomes =  2195.53779451378"
```

More Examples

```
> myibs("GOOG",startdate="2005-01-01")
[1] "Ticker  GOOG"
[1] "Time period =  2936  Days"
[1] "IBS time in market =  1791  Days"
[1] "IBS $1000 becomes =  3130.38166677285"
[1] "BH $1000 becomes =  7595.65673511053"

> myibs("TSLA",startdate="2005-01-01")
[1] "Ticker  TSLA"
[1] "Time period =  1555  Days"
[1] "IBS time in market =  944  Days"
[1] "IBS $1000 becomes =  20674.598504685"
[1] "BH $1000 becomes =  8846.37944103723"

> myibs("GE",startdate="2005-01-01")
[1] "Ticker  GE"
[1] "Time period =  2936  Days"
[1] "IBS time in market =  1868  Days"
[1] "IBS $1000 becomes =  1465.82701436556"
[1] "BH $1000 becomes =  1282.50943462539"
```

Change dates and different results
Can randomly select start and end dates and check performance
Can cycle through best buy and sell signals.
Lots of data mining to do

Simulating Two Assets

- In the previous examples, we assumed we were simply investing in one asset, the market index.
- But that was purely for ease of explanation; we can simulate (correlated) assets in R and thereby simulate investing in a portfolio.

Multivariate Normality

- Suppose you want to simulate a portfolio where part of your money is in the S&P500, and part of your money is in bonds.
- Suppose you believe the bond returns are normal with mean 4% and standard deviation 7%, and that the correlation between the S&P 500 and bonds is -0.2.
- We assume the index returns are normal with mean 0.113 and standard deviation 0.2021.

Multivariate Normality

- Because we believe the Stock and Bond returns are correlated, we can’t just simulate each one by drawing from two different normal distributions.
- To allow for the correlation between stocks and bonds, we need to simulate their returns by drawing from a multivariate (bivariate in this case) normal distribution.

The Covariance matrix

- Recall that Cor(X,Y) = Cov(X,Y)/[sd(X)sd(Y)]
- We need to create the covariance matrix

$$\begin{bmatrix} \sigma_S^2 & \sigma_{SB} \\ \sigma_{SB} & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} .2021^2 & -.2(.2021)(.07) \\ -.2(.2021)(.07) & .07^2 \end{bmatrix}$$

Covariance of stocks and bonds=
cor(stocks,bonds)*sd(stocks)*sd(bonds)

Simulating Multivariate Normals

```
sigma=matrix(c(.2021^2,-.2*.2021*.07,
-.2*.2021*07,.07^2),nrow=2,ncol=2)
> sigma
            [,1]     [,2]
[1,]  0.04084441 -0.28294
[2,] -0.00282940  0.00490
 > rmvnorm(1,c(.1131,.04),sigma)
           [,1]       [,2]
 [1,] 0.2738058 0.08095085
 > rmvnorm(1,c(.1131,.04),sigma)
            [,1]      [,2]
 [1,] 0.07291433 0.1187463
 > rmvnorm(1,c(.1131,.04),sigma)
           [,1]         [,2]
 [1,] 0.3845906 -0.002318950
```

The means

The variance-covariance matrix

Random stock return

Random bond return

Simulated Correlated Returns

Simulate 100000 returns

```
> example=rmvnorm(100000,c(.1131,.04),sigma)
```

First column is stocks, second column is bonds

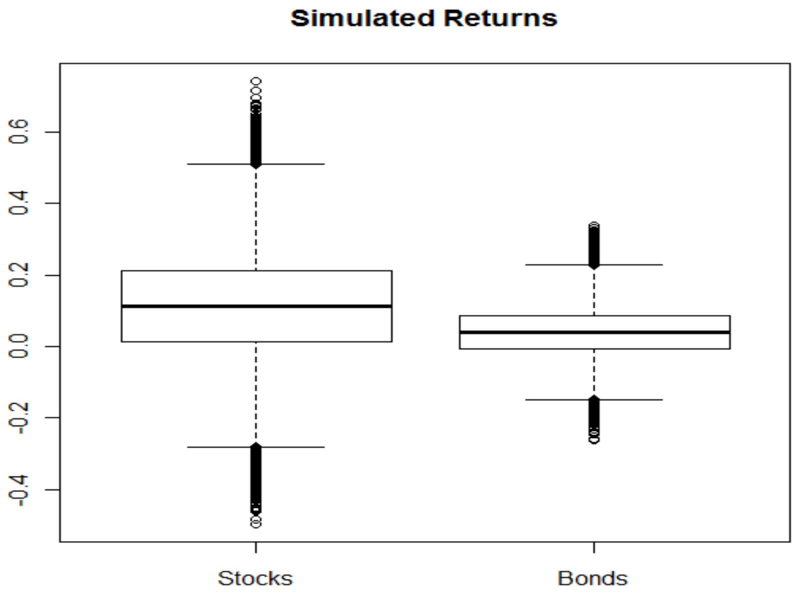
```
> stock.rets=example[,1]
> bond.rets=example[,2]
```

Hey, they are correlated

```
> cor(stock.rets,bond.rets)
[1] -0.02063216

> boxplot(stock.rets,bond.rets)
>
```

The Boxplot

```
boxplot(stock.rets,bond.rets,
names=c("Stocks","Bonds"),
main="Simulated Returns")
```



Portfolio Math

The return on a portfolio of two assets 1 and 2 is given by

$$R_p = w_1 R_1 + (1 - w_1) R_2$$

(a weighted sum of the two different returns)

Generate the Returns

- We first draw two returns from a bivariate normal

```
sigma=matrix(c(.2021^2,-.0028294,
          -.0028294,.07^2),nrow=2,ncol=2)
ret=rmvnorm(1,c(.1131,.04),sigma)
```

- Then the 50% stocks/50% bonds portfolio return will be

```
.5*ret[1]+.5*ret[2]
```

The 50-50 Stock/Bond Portfolio

```
simul=function(howmany) {
values = 1:howmany
for(j in 1:howmany) {

   money = 1000
   for(i in 1:30) {

     sigma=matrix(c(.2021^2,-.0028294,
         -.0028294,.07^2),nrow=2,ncol=2)
     ret=rmvnorm(1,c(.1131,.04),sigma)

      money = money*(1+.5*ret[1]+.5*ret[2])
}
values[j] = money
}

return(values)
}
```

The Output

- The 50/50 portfolio

```
Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
1181    5688    8161    9146   11350   70460
```

CAGR = 7.2%

CAGR = 15.2%

- How does this portfolio compare to being 100% in the S&P 500?

```
> summary(out)
    Min.  1st Qu.   Median      Mean  3rd Qu.     Max.
   249.1   7229.0  15370.0  24810.0  30870.0 489900.0
>
```

Bonds are boring

- Here are the results of being 100% in bonds:

```
> summary(port3)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   1014    2370    3077    3264    3903    8493
>
```

I Forgot Something!

- We don’t need the multivariate normal distribution!
- If X and Y are normal, X+Y is normal.
- All we care about is the distribution of the sum of the two returns, we don’t need to draw from a multivariate normal for that!
- Good thing, since drawing from a multivariate normal is slow! (computational issues)

Quick Review

- If Z = aX+bY (X,Y random variables, a,b constants)

$$E(Z) = aE(X) + bE(Y)$$

$$Var(Z) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X,Y)$$

- IF X and Y are normal then Z is normal too.

Portfolio Math

The return on a portfolio of two assets 1 and 2 is given by

$$R_p = w_1 R_1 + (1 - w_1) R_2$$

(a weighted sum of the two different returns)

The formulas

If we invest w_1% in stock 1 and $(1-w_1)$% in stock 2, we obtain the following portfolio results:

Average return of portfolio

$$w_1\overline{r_1} + (1 - w_1)\overline{r_2}$$

Variance of return of portfolio

$$w_1^2 s_1^2 + (1 - w_1)^2 s_2^2 + 2w_1(1 - w_1) r_{1,2} s_1 s_2$$

Or (equivalently)

$$r_{12} = \frac{s_{12}}{s_1 s_2}$$

$$w_1^2 s_1^2 + (1 - w_1)^2 s_2^2 + 2w_1(1 - w_1) s_{1,2}$$

The New Code

```
simul=function(w1,w2,howmany) {

   newmu = w1*(.1131)+w2*(.04)
   news = sqrt(w1*w1*.2021*.2021 + w2*w2*.07*.07
         +2*w1*w2*(-.2)*.2021*.07)

values = 1:howmany
for(j in 1:howmany) {
   money = 1000
   for(i in 1:30) {
       ret=rnorm(1,newmu,news)
        money = money*(1+ret)
   }
values[j] = money
}
return(values)
}
```

The Output

```
> port1 = simul(1,0,5000)
> port2 = simul(.5,.5,5000)
> port3 = simul(0,1,5000)
➢summary(cbind(port1,port2,port3))

    port1                port2               port3
 Min.    :   424.1   Min.    : 1318   Min.    :  628.5
 1st Qu.:  7245.9   1st Qu.: 5702   1st Qu.: 2347.2
 Median : 15379.9   Median : 8058   Median : 3015.2
 Mean   : 24951.6   Mean   : 9239   Mean   : 3234.4
 3rd Qu.: 31087.1   3rd Qu.:11456   3rd Qu.: 3919.1
 Max.   :461460.2   Max.   :51968   Max.   :11378.5
```

100% stock

50% stock,
50% bonds

100% bonds

Segue……

- Now that we know how to simulate portfolios, we are going to go back and review portfolio ideas from the beginning, and discuss why they are useful.

The word "portfolio" always brings back memories of how my father used it in connection with his clients' accounts after he started his investment counseling firm in 1934. My idea of a portfolio was a fancy leather folder with a sheaf of papers inside. In the world of investing, a portfolio has no physical existence. Rather, it represents the investor's total capital.

Portfolios

- A portfolio is a collection of different securities such as stocks and bonds, that are combined and considered a single asset
- The risk-return characteristics of the portfolio is demonstrably different than the characteristics of the assets that make up that portfolio, especially with regard to risk.
- Combining different securities into portfolios is done to achieve *diversification*.

Asset Classes Make-up Portfolios

- Each modern asset class has an underlying general type of risk, return and correlation to the other asset classes.

Large-cap Passive	
Large-cap Value	Large-cap Growth
Small-cap Value	Small-cap Growth
Hedge Funds	
Real Estate	
Private Equity	
Core Fixed Income	
International Equity	
Emerging Markets Equity	
High Yield Bonds	

By no means a complete list.

New Asset Classes Can be Created

Cramer: High-Growth Stocks In An Asset Class of Their Own

As high-growth names like **Netflix**, **Chipotle** or **Apple** [AAPL 359.90 ▲ 0.72 (+0.2%)] continue to climb, Cramer said Monday investors should evaluate these names differently.

Traditionally, he explained, stocks are judged on a **price-to-earning multiple**. On that basis, however, these momentum stocks are too expensive. High-growth names can't be thought of in that way, he said.

Unlike other stocks, Cramer said high-growth names aren't propelled by news. They don't need a news event, like an analyst's upgrade or a change in management, to send the stock higher. These momentum stocks are in an unique asset class, which is drawing a lot of money into the market.

Weird ETFs

iPath Dow Jones-UBS Livestock Subindex Total Return (COW).

Global X Lithium (LIT).

This fund invests in both lithium mining companies and lithium battery manufacturers. This may be one of the best investment plays available on electric cars, and the future potential of a fund like this is obvious.

Internet HOLDRS (HHH)

iPath Global Carbon (GRN).

This fund is in a class by itself as far as strange goes. It isn't even a play on any type of investment, but on European Union Allowances and Certified Emission Reduction Credits. This is purely a political play, and it's subject to all the complications, re-interpretations and delays that affect just about any idea that's hatched by political bodies.

Introduction

- Harry Markowitz’s “Portfolio Selection” *Journal of Finance* article (1952) set the stage for modern portfolio theory
  - The first major publication indicating the importance of security return correlation in the construction of stock portfolios
  - Markowitz showed that for a given level of expected return and for a given security universe, knowledge of the covariance and correlation matrices are required

Modern Portfolio Theory - MPT

- Prior to the establishment of Modern Portfolio Theory (MPT), most people only focused upon investment returns…they ignored risk.

- With MPT, investors had a tool that they could use to *dramatically reduce the risk* of the portfolio *without a significant reduction* in the expected return of the portfolio.

Diversification

Diversification has two faces:

1. Diversification results in an overall reduction in portfolio risk (return volatility over time) with little sacrifice in returns, and
2. Diversification helps to immunize the portfolio from potentially catastrophic events such as the outright failure of one of the constituent investments.

   (e.g. If only one investment is held, and the issuing firm goes bankrupt, the entire portfolio value and returns are lost. If a portfolio is made up of many different investments, the outright failure of one is more than likely to be offset by gains on others, helping to make the portfolio immune to such events.)

Diversification Reduces Risk

- Assume a hypothetical investor had **$100,000** to invest on January 1, 2000, held his investments through December 31, 2009 and reinvested all distributions. We will also assume the use of Vanguard index mutual funds for our examples.
- Investor 1 puts all of her money in the Vanguard 500 Fund which invests in the stocks making up the Standard & Poor's index in their relative weight in the index.
- How much would it be worth by December 31, 2009? This $100,000 investment would have shrunk to **$90,165** for an average annual loss of 1.03 percent. This was truly a lost decade for this investor.

Diversification Reduces Risk

- Investor 2 added the following funds to his portfolio in addition to the Vanguard 500:
- Vanguard Small Cap Index
- Vanguard Mid Cap Index
- Vanguard Total International Stock Index
- How much would an investment of **$100,000** invested equally in each of these four funds have grown to by December 31, 2009? (We are assuming no taxes or rebalancing in this and all examples. The answer is **$137,511**. This is $47,346 or about 52 percent more than an investment of our investor's cash only in the Vanguard 500 Index.

Diversification Reduces Risk

- Investor 3 added some bonds to her mix. In this case let's add the following funds:
- Pimco Total Return
- T. Rowe Price Short-Term Bond
- American Century Inflation Adjusted Bond
- Templeton Global Bond
- If we now divide the investor's **$100,000** investment equally among the four equity funds from the prior example and among these four bond funds, by the end of 2009, $100,000 investment has grown to **$174,506** or almost double what an investment of $100,000 in the Vanguard 500 Index Fund alone would have yielded.

Diversification is not a new idea

Shakespeare "Merchant of Venice"

My ventures are not in one bottom trusted,
Nor to one place; nor is my whole estate
Upon the fortune of this present year;
Therefore, my merchandise makes me not sad.
Act I, Scene 1

Captain Long John Silver in Treasure Island

I puts it all away, some here, some there,
none too much anywheres, by reason of suspicion.

Chapter 11. Robert Louis Stevenson

Risk Aversion

Portfolio theory assumes that investors are averse to risk

- Given a choice between two assets with equal expected rates of return, risk averse investors will select the asset with the lower level of risk
- It also means that a riskier investment has to offer a higher expected return or else nobody will buy it

Markowitz Portfolio Theory

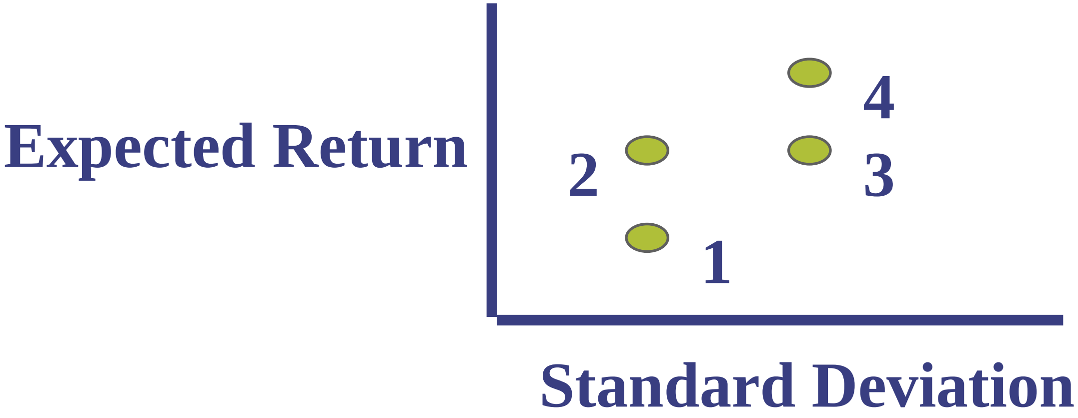
- Derives the expected rate of return for a portfolio of assets and an expected risk measure
- Markowitz demonstrated that the variance of the rate of return is a meaningful measure of portfolio risk under reasonable assumptions
- The portfolio variance formula shows how to effectively diversify a portfolio

Markowitz Portfolio Theory

Assumptions

- Investors base decisions solely on expected return and risk.
- For a given risk level, investors prefer higher returns to lower returns.
- Similarly, for a given level of expected returns, investors prefer less risk to more risk.

Dominance



- 2 dominates 1; has a higher return
- 2 dominates 3; has a lower risk
- 4 dominates 3; has a higher return

How diversification reduces risk

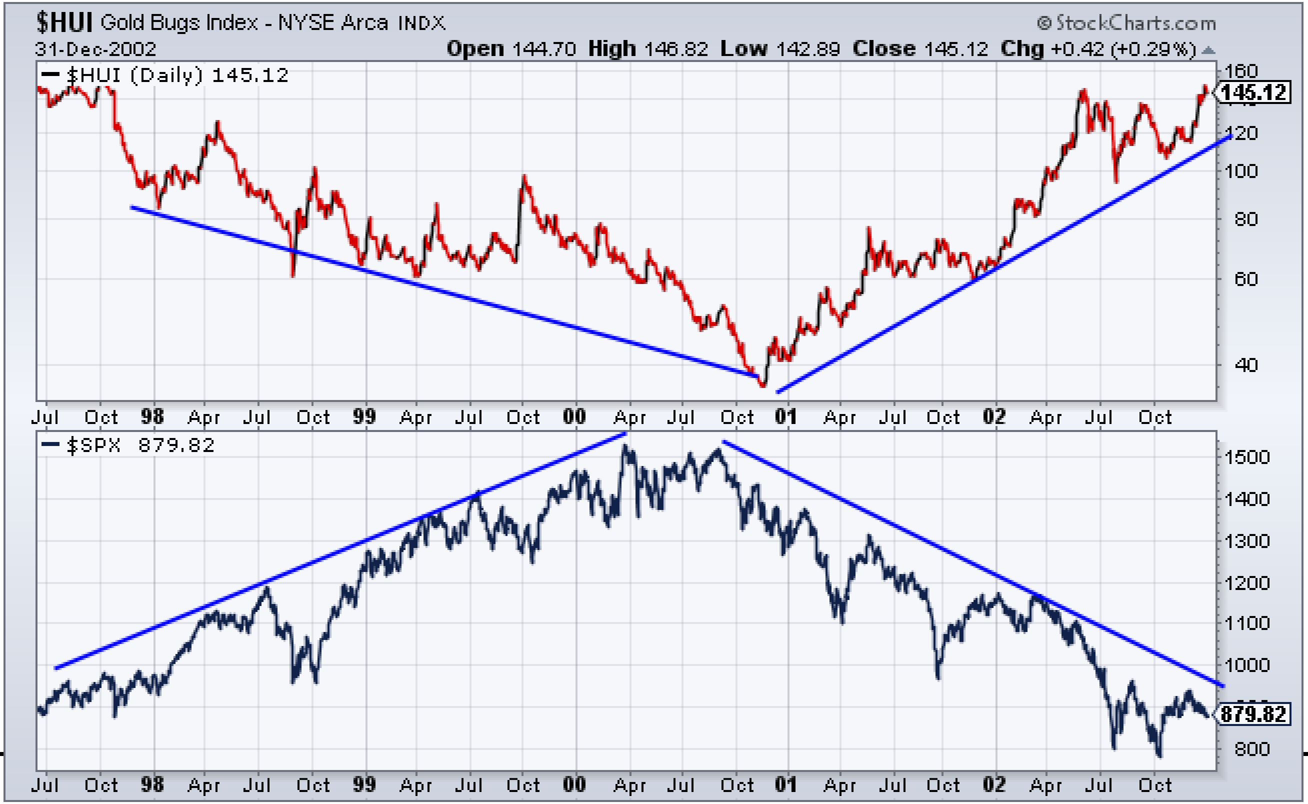
There are two mechanisms by which diversification reduces risk: dilution and interference.

- Dilution is easy enough to understand, if you swap half your shares for cash then you lose half your equity exposure and therefore half your equity risk. If the market crashed tomorrow you'd only lose half as much.

How diversification reduces risk

- “Interference” (a term I pinched from physics where it is used to describe the way waves interact), is where negative movements in some assets are partly cancelled by positive ones in other assets.
- In the old days a good example would be stocks versus gold. Historically, gold goes up when the market goes down and vice versa (but not anymore!).

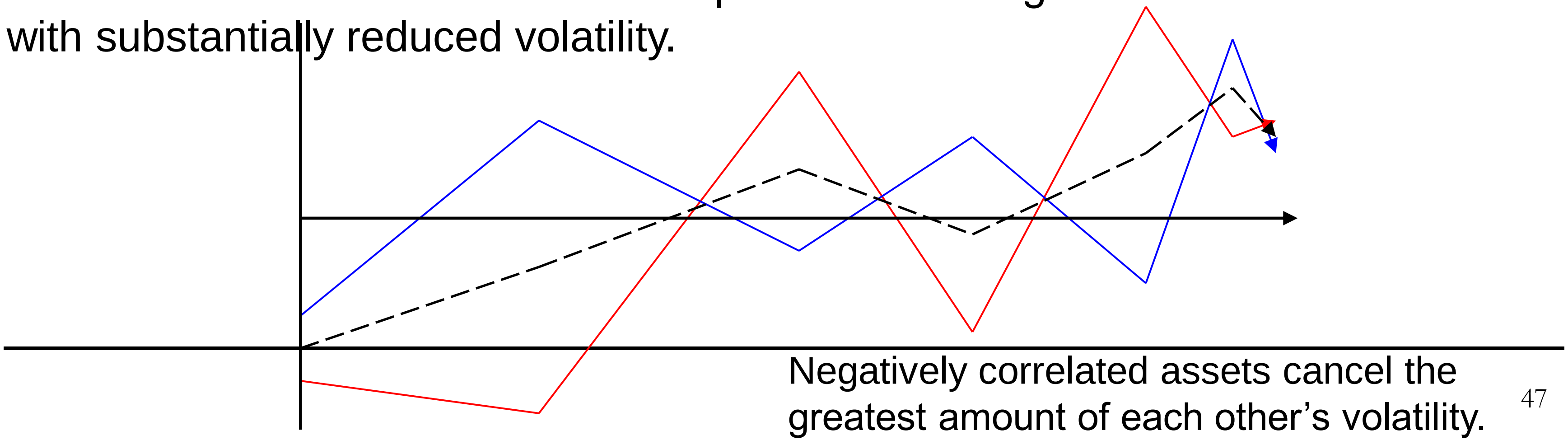
Old Days of Gold and SPY



Interference and correlation

"Correlation" is the word given to the extent to which assets move together, this is measured with statistical formulae. Correlations can range from -1 (perfectly negatively correlated) through to +1 (perfectly positively correlated).

If asset B tends to move in the opposite direction to asset A then these two assets are said to have "negative correlation", and they can be highly effective at cancelling out each other's volatility. If the assets both trend upwards over the longer term a combination of them will have a return equal to the average of the two assets' returns but with substantially reduced volatility.



Negatively correlated assets cancel the greatest amount of each other's volatility.

People Study Correlations

	AAPL	AAL	AA	BBY	C	S	CVS	CVX	HD	
AAPL		-0.17	0.22	0.59	-0.01	0.17	0.18	0.07	0.4	AAPL
AAL	-0.17		0.23	0.04	0.53	0.51	0.2	0.53	0.08	AAL
AA	0.22	0.23		0.34	0.48	0.38	0.42	0.78	0.76	AA
BBY	0.59	0.04	0.34		0.21	0.31	0.17	0.23	0.52	BBY
C	-0.01	0.53	0.48	0.21		0.6	0.29	0.72	0.56	C
S	0.17	0.51	0.38	0.31	0.6		0.23	0.64	0.43	S
CVS	0.18	0.2	0.42	0.17	0.29	0.23		0.38	0.35	CVS
CVX	0.07	0.53	0.78	0.23	0.72	0.64	0.38		0.63	CVX
HD	0.4	0.08	0.76	0.52	0.56	0.43	0.35	0.63		HD

https://www.macroaxis.com/invest/marketCorrelation

Search for Negative Correlation

- https://unicornbay.com/tools/most-less-correlated-assets

Top 1,000 Most and Least correlated assets on the market.

Every day we calculate more than **21,000,000 correlations** (yes, 21 million) among assets all over the world. And from all of these correlations, we pick TOP 1,000 most correlated (or similar) stocks and least correlated (or opposite) stocks. The results you can find on this page. Learn more about asset correlations between each other.
You can also try our Beta Calculator and Asset Correlations free tools.

The Basic Set-Up: Forming Portfolios

Suppose you have \$100 to invest.

Let R_A be the return on asset A.
If $R_A = .1$, and you put all your money into asset A you will have \$110 at the end of the period.

Let R_B be the return on asset B.
If $R_B = .15$, and you put all your money into asset B you will have \$115 at the end of the period.

Suppose you put 1/2 your money into A and 1/2 into B.

How much will you make ?

Portfolio weights

At the end of the period you will have

(100)*.5*(1+.1) + (100)*.5*(1+.15)
=100*(1+.5*.1+.5*.15)

so the return is .5*.1 + .5*.15=.125.

The average of the two returns

To generalize, let w_A be the fraction of your wealth you invest in asset A. Let w_B be the fraction of your wealth you invest in asset B.

The w's are called the <u>portfolio weights</u>, and we usually require that they sum to 1.

Return on a portfolio

Hence the return on a portfolio is given by

$$R_p = w_A R_A + w_B R_B$$

(a weighted sum of the two different returns)

Example

- We have historical country data and suppose that we had put .5 of our money into USA and .5 into Hong Kong.
- What wo

```
. generate portf = .5*honkong + .5*usa

. list honkong usa portf
```

	honkong	usa	portf
1.	.02	.04	.03
2.	.06	-.03	.015
3.	.02	.01	.015
4.	-.03	.01	-.01
5.	.08	.05	.065
6.	.02	0	.01
7.	-.08	-.03	-.055

Example (cont)

```
. summarize honkong usa portf
```

Variable	Obs	Mean	Std. Dev.	Min	Max
honkong	107	.021028	.0722148	-.18	.27
usa	107	.0134579	.0332827	-.09	.11
portf	107	.017243	.0457806	-.1	.14

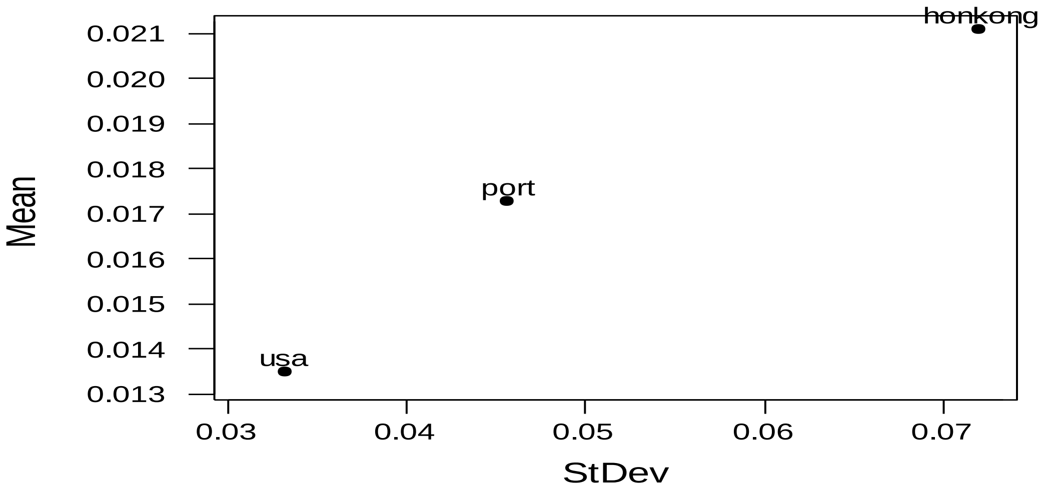
The goal of this section of the notes is to learn what factors control the risk and return of a portfolio.

Comparing returns

How do the returns on this portfolio compare with those of hongkong and usa?

It looks like the mean for my portfolio is right in between the means of usa and hongkong.

What about the sd?

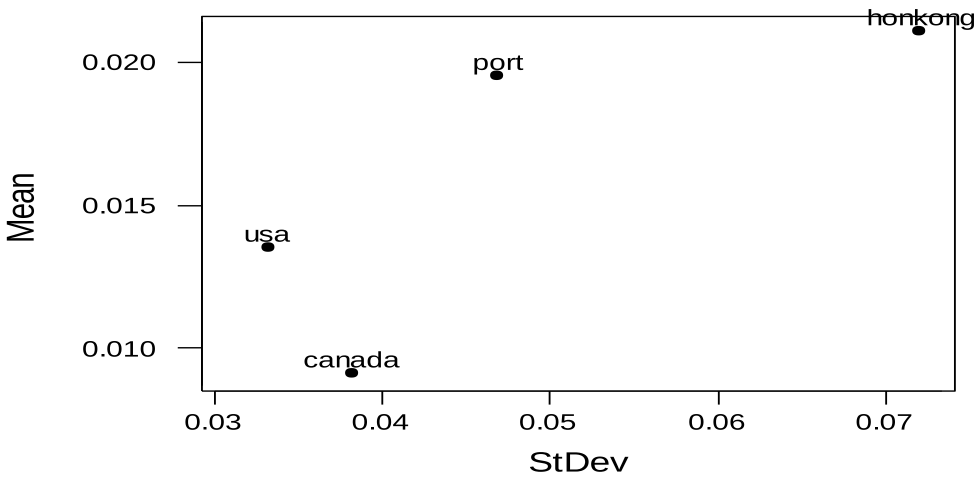


Now try three stocks

Let's try a portfolio with three stocks. The weights must add up to one, but they can be negative, this is called going short in the asset.

```
port = -.5*canada+usa+.5*honkong
```

Clearly, forming portfolios is an interesting thing to do!!



What does “going short” mean ?

Investment Investopedia

Directory > Business > Investment > short sale

Short Sale

A market transaction in which an investor sells borrowed securities in anticipation of a price decline and is required to return an equal amount of shares at some point in the future.

The payoff to selling short is the opposite of a long position. A short seller will make money if the stock goes down in price, while a long position makes money when the stock goes up. The profit that the investor receives is equal to the value of the sold borrowed shares less the cost of repurchasing the borrowed shares.

Investopedia Says: Suppose 1,000 shares are short sold by an investor at $25 apiece and $25,000 is then put into that investor's account. Let's say the shares fall to $20 and the investor closes out the position. To close out the position, the investor will need to purchase 1,000 shares at $20 each ($20,000). The investor captures the difference between the amount that he or she receives from the short sale and the amount that was paid to close the position, or $5,000.

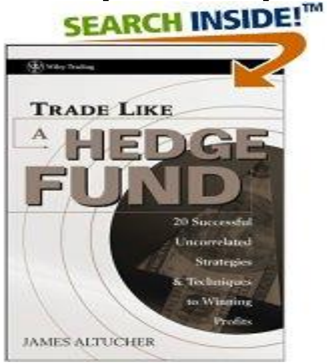
There are also margin rule requirements for a short sale in which 150% of the value of the shares shorted needs to be initially held in the account. Therefore, if the value is $25,000, the initial margin requirement is $37,500 (which includes the $25,000 of proceeds from the short sale). This prevents the proceeds from the sale from being used to purchase other shares before the borrowed shares are returned.

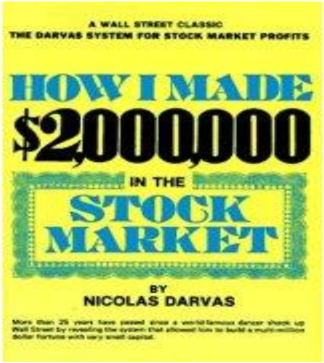
Short selling is an advanced trading strategy with many unique risks and pitfalls. Novice investors are advised to avoid short sales because this strategy includes unlimited losses. A share price can only fall to zero, but there is no limit to the amount it can rise.

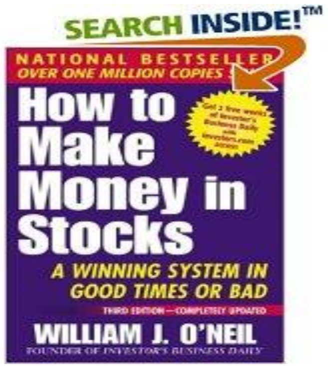
Why form portfolios ?

Maybe the portfolio has a nice mean and variance.

There are some basic formulas that relate the mean and standard deviation of the portfolio returns to the means, variances, and covariances of the returns of the assets that make up the portfolios.







The Stat Setup

- Assume we have n stocks of interest
- Let R_i be a random variable denoting the return of stock i
- Since R_i is random, it has an expected value $E(R_i)$ and a standard deviation σ_i
- We also need to know $Cov(R_i, R_j)$ for all possible combinations.
- Or correlation since

$$Cor(R_i, R_j) = Cov(R_i, R_j) / (\sigma_i \sigma_j)$$

Expected Return of a Portfolio

The Expected Return on a Portfolio is simply the weighted average of the expected returns of the individual assets that make up the portfolio:

$$E(R_p) = \sum_{i=1}^{n} [w_i \times E(R_i)]$$

The portfolio weight of a particular security is the percentage of the portfolio's total value that is invested in that security.

Expected Return of a Portfolio

Suppose *$E(R_A)$= 14%, $E(R_B)$= 6%,*

w_A = *weight of security A* = 28.6%

w_B = *weight of security B* = 71.4%

$$E(R_p) = \sum_{\mathrm{i}=1}^{\mathrm{n}} [w_i \times E(R_i)] = (.286 \times 14\%) + (.714 \times 6\%\ )$$
$$= 4.004\% + 4.284\% = 8.288\%$$

Range of Returns

- In a two asset portfolio, simply by changing the weight of the constituent assets, different portfolio returns can be achieved.
- Because the expected return on the portfolio is a simple weighted average of the individual expected returns of the assets, you can achieve portfolio returns bounded by the highest and the lowest individual asset returns.

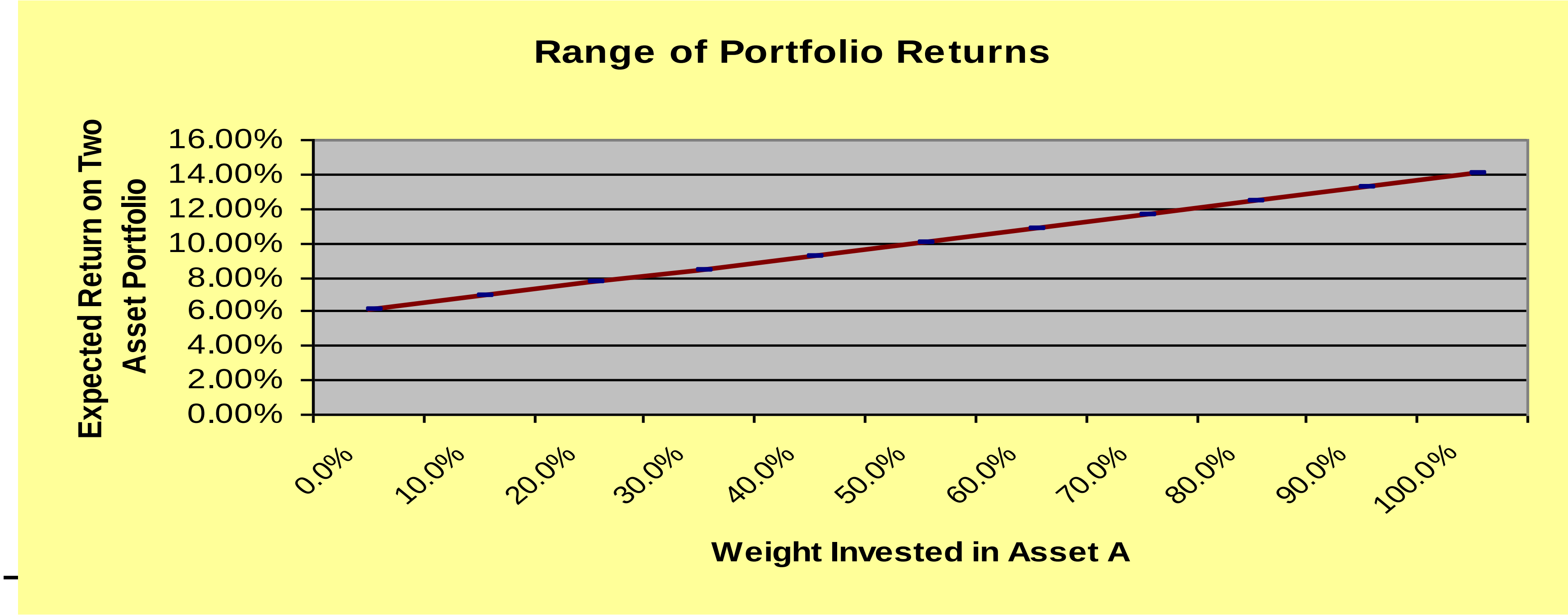
Range of Returns in a Two Asset Portfolio

Expected return on Asset A = 14.0%
Expected return on Asset B = 6.0%

Weight of Asset A	Weight of Asset B	Expected Return on the Portfolio
0.0%	100.0%	6.0%
10.0%	90.0%	6.8%
20.0%	80.0%	7.6%
30.0%	70.0%	8.4%
40.0%	60.0%	9.2%
50.0%	50.0%	10.0%
60.0%	40.0%	10.8%
70.0%	30.0%	11.6%
80.0%	20.0%	12.4%
90.0%	10.0%	13.2%
100.0%	0.0%	14.0%

Range of Returns in a Two Asset Portfolio

$E(r)_A = 14\%$, $E(r)_B = 6\%$



Risk For Portfolios

Standard Deviation of a Two-Asset Portfolio using Covariance

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2(w_A)(w_B)(COV_{A,B})}$$

Risk of Asset A adjusted for weight in the portfolio

Risk of Asset B adjusted for weight in the portfolio

Factor to take into account comovement of returns. This factor can be negative.

Risk For Portfolios

Standard Deviation of a Two-Asset Portfolio using Correlation

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2\underbrace{(w_A)(w_B)(\rho_{A,B})(\sigma_A)(\sigma_B)}}$$

Factor that takes into account the degree of comovement of returns. It can have a negative value if correlation is negative.

	A	B	C	D	E	F
1	**CALCULATING THE MEAN AND STANDARD DEVIATION OF A PORTFOLIO**					
2	Asset returns	**WMT**	**TGT**			
3	Mean return	1.59%	0.46%			
4	Variance	0.93%	0.52%			
5	Standard deviation	9.63%	7.19%			
6	Covariance	0.0038				
7						
8	Proportion of WMT	0.5	<-- In the data table below this is varied from -0.5 to 1.5			
9						
10	Portfolio mean return	1.02%	<-- =B8*B3+(1-B8)*C3			
11	Portfolio return variance	0.0036	<-- =B8^2*B4+(1-B8)^2*C4+2*B8*(1-B8)*B7			
12	Portfolio return standard deviation	6.01%	<-- =SQRT(B11)			

Portfolio expected return

$$E(r_p) = x_{WMT} E(r_{WMT}) + x_{TGT} E(r_{TGT})$$

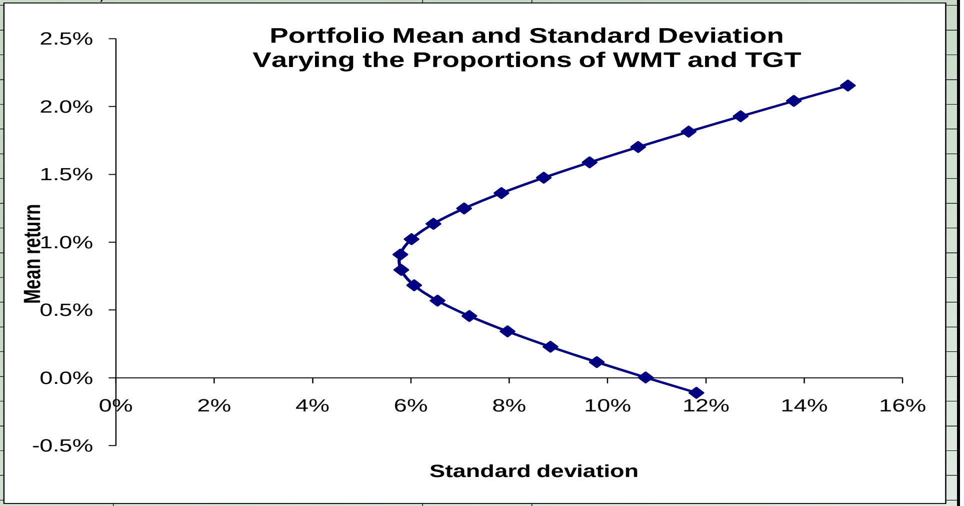
Portfolio variance

$$\sigma_p^2 = w_{WMT}^2 \sigma_{WMT}^2 + w_{TGT}^2 \sigma_{TGT}^2 + 2 * w_{WMT} * w_{TGT} Cov(r_{WMT}, r_{TGT})$$

Note that $x_{TGT} = 1 - x_{WMT}$

In Excel

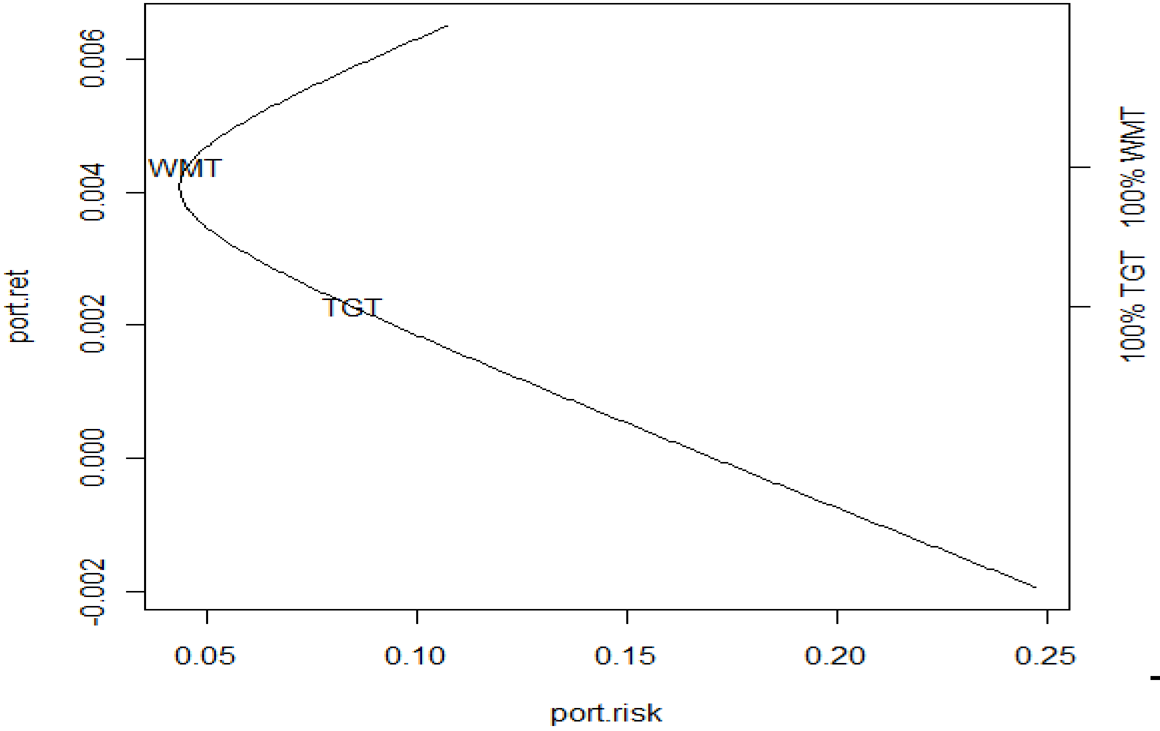
	A	B	C	D
14	**Data table: Varying the proportion of WMT**			
15		Portfolio standard deviation	Porfolio mean return	
16	Proportion of WMT	6.01%	1.02%	<-- =B10, Table header
17	-0.5	11.81%	-0.11%	
18	-0.4	10.77%	0.00%	
19	-0.3	9.78%	0.12%	
20	-0.2	8.84%	0.23%	
21	-0.1	7.96%	0.34%	
22	0	7.19%	0.46%	
23	0.1	6.54%	0.57%	
24	0.2	6.06%	0.68%	
25	0.3	5.80%	0.80%	
26	0.4	5.78%	0.91%	
27	0.5	6.01%	1.02%	
28	0.6	6.46%	1.14%	
29	0.7	7.08%	1.25%	
30	0.8	7.84%	1.36%	
31	0.9	8.70%	1.48%	
32	1	9.63%	1.59%	
33	1.1	10.62%	1.70%	
34	1.2	11.65%	1.82%	
35	1.3	12.71%	1.93%	
36	1.4	13.79%	2.04%	
37	1.5	14.89%	2.16%	



In R (but Excel is fine for this too)

```
myport = function(ticker1,ticker2) {

s1 = getSymbols(ticker1,auto.assign=FALSE)
s2 = getSymbols(ticker2,auto.assign=FALSE)

r1 = monthlyReturn(Ad(s1))
r2 = monthlyReturn(Ad(s2))

w=seq(-1.5,1.5,.01)
port.ret = w*mean(r1)+(1-w)*mean(r2)

port.risk=sqrt(w^2*var(r1)+(1-w)^2*var(r2)+2*w*(1-w)*cov(r1,r2))
plot(port.risk,port.ret,type="l")

text(sd(r1),mean(r1),ticker1)
text(sd(r2),mean(r2),ticker2)
axis(side=4,at=c(mean(r1),mean(r2)),labels=c(paste("100%",ticker1),past
e("100%",ticker2)))

}
```

The Output (more recent data)



Example: The Caffeine Portfolio

- Consider Coke(KO) and Starbucks (SBUX)

```
> getSymbols("SBUX")
[1] "SBUX"
> getSymbols("KO")
[1] "KO"
> r1=monthlyReturn(SBUX)
> r2=monthlyReturn(KO)
> cov(r1,r2)
                monthly.returns
monthly.returns     0.001526567
> cor(r1,r2)
                monthly.returns
monthly.returns       0.2649362
> mean(r1)
[1] 0.004292857
> mean(r2)
[1] 0.006915436
> var(r1)
                monthly.returns
monthly.returns      0.01141654
> var(r2)
                monthly.returns
monthly.returns     0.002908136
>
```

The 50/50 Portfolio by hand (argh!)

Average portfolio return

$$\bar{R}_P = 0.5\bar{R}_{SBUX} + 0.5\bar{R}_{KO} = 0.5(.0043) + (0.5)(0.0069) = 0.056$$

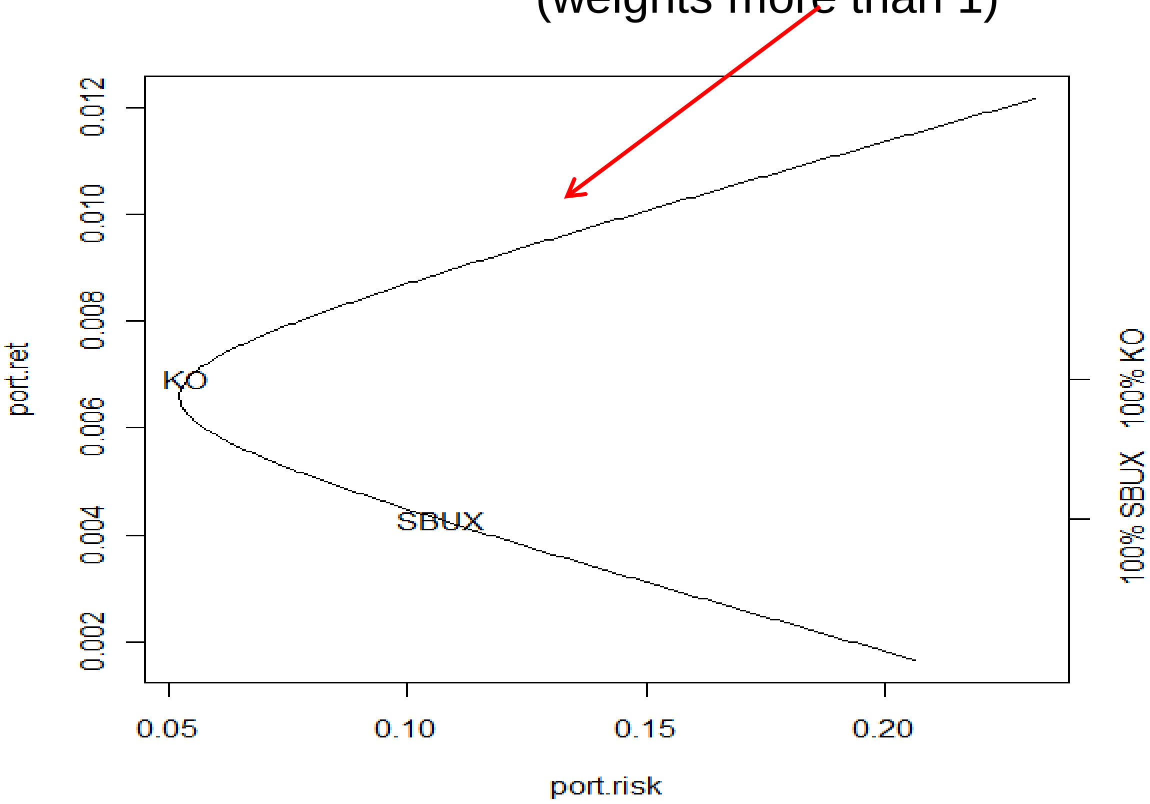
$$s^2_{R_P} = (0.5)^2 s^2_{R_{SBUX}} + (0.5)^2 s^2_{R_{KO}} + 2(0.5)(0.5)s_{R_{SBUX},R_{KO}} = (0.5)^2(.01141)^2 + (0.5)^2(.0029)^2 +$$
$$2(0.5)(0.5)(.0015) = 0.0043$$

$$\mathrm{s}_{\mathrm{R_P}} = \sqrt{s^2_{R_P}} = \sqrt{0.0043} = 0.0655$$

portfolio standard deviation

The Graph

Some leveraged coke portfolios (weights more than 1)



Portfolio Riskiness

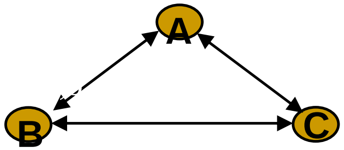
- The riskiness of a portfolio that is made of different risky assets is a function of three different factors:
  - the riskiness of the individual assets that make up the portfolio
  - the relative weights of the assets in the portfolio
  - the degree of comovement of returns of the assets making up the portfolio
- The standard deviation of a two-asset portfolio may be measured using the Markowitz model:

$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

Risk of a Three-Asset Portfolio

- The data requirements for a three-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.

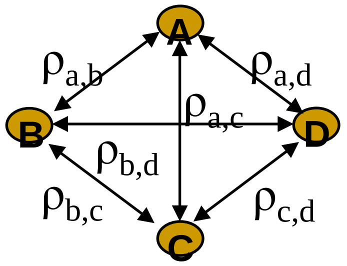
- We need 3 (three) correlation coefficients between A and B; A and C; and B and C.



$$\sigma_p = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + \sigma_C^2 w_C^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B + 2 w_B w_C \rho_{B,C} \sigma_B \sigma_C + 2 w_A w_C \rho_{A,C} \sigma_A \sigma_C}$$

Risk of a Four-asset Portfolio

- The data requirements for a four-asset portfolio grows dramatically if we are using Markowitz Portfolio selection formulae.
- We need 6 correlation coefficients between A and B; A and C; A and D; B and C; C and D; and B and D.



Portfolio Standard Deviation Formula

$$\sigma_{\text{port}} = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{i=1}^{n} w_i w_j \text{Cov}_{ij}}$$

where :

σ_{port} = the standard deviation of the portfolio

W_i = the weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio

σ_i^2 = the variance of rates of return for asset i

Cov_{ij} = the covariance between the rates of return for assets i and j,

where $\text{Cov}_{ij} = r_{ij} \sigma_i \sigma_j$

You probably don't want to hear it, but these formulas look a lot nicer using matrix notation. Note also the curse of dimensionality!

Portfolio Standard Deviation Calculation

- The portfolio standard deviation is a function of:
  - The variances of the individual assets that make up the portfolio
  - The covariances between all of the assets in the portfolio
- The larger the portfolio, the more the impact of covariance and the lower the impact of the individual security variance

Importance of Correlation

- Correlation is important because it affects the degree to which diversification can be achieved using various assets.
- Theoretically, if two assets returns are perfectly positively correlated, it is possible to build a riskless portfolio (sadly though, that portfolio may not have a positive expected return!).

Diversification Potential

- The potential of an asset to diversify a portfolio is dependent upon the degree of co-movement of returns of the asset with those other assets that make up the portfolio.
- In a simple, two-asset case, if the returns of the two assets are perfectly negatively correlated it is possible (depending on the relative weighting) to eliminate all portfolio risk.
- This is demonstrated through the following series of spreadsheets, and then summarized in graph format.

Example of Portfolios and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	1
B	14.0%	40.0%	

Perfect Positive Correlation – no diversification

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	17.5%
80.00%	20.00%	6.80%	20.0%
70.00%	30.00%	7.70%	22.5%
60.00%	40.00%	8.60%	25.0%
50.00%	50.00%	9.50%	27.5%
40.00%	60.00%	10.40%	30.0%
30.00%	70.00%	11.30%	32.5%
20.00%	80.00%	12.20%	35.0%
10.00%	90.00%	13.10%	37.5%
0.00%	100.00%	14.00%	40.0%

Both portfolio returns and risk are bounded by the range set by the constituent assets when $\rho=+1$

Example of Portfolios and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0.5
B	14.0%	40.0%	

Positive Correlation – weak diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	15.9%
80.00%	20.00%	6.80%	17.4%
70.00%	30.00%	7.70%	19.5%
60.00%	40.00%	8.60%	21.9%
50.00%	50.00%	9.50%	24.6%
40.00%	60.00%	10.40%	27.5%
30.00%	70.00%	11.30%	30.5%
20.00%	80.00%	12.20%	33.6%
10.00%	90.00%	13.10%	36.8%
0.00%	100.00%	14.00%	40.0%

When $\rho=+0.5$ these portfolio combinations have lower risk – expected portfolio return is unaffected.

Example of Portfolios and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	0
B	14.0%	40.0%	

No Correlation – some diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	14.1%
80.00%	20.00%	6.80%	14.4%
70.00%	30.00%	7.70%	15.9%
60.00%	40.00%	8.60%	18.4%
50.00%	50.00%	9.50%	21.4%
40.00%	60.00%	10.40%	24.7%
30.00%	70.00%	11.30%	28.4%
20.00%	80.00%	12.20%	32.1%
10.00%	90.00%	13.10%	36.0%
0.00%	100.00%	14.00%	40.0%

Portfolio risk is lower than the risk of either asset A or B.

Example of Portfolios and Correlation

Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-0.5
B	14.0%	40.0%	

Negative Correlation – greater diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	12.0%
80.00%	20.00%	6.80%	10.6%
70.00%	30.00%	7.70%	11.3%
60.00%	40.00%	8.60%	13.9%
50.00%	50.00%	9.50%	17.5%
40.00%	60.00%	10.40%	21.6%
30.00%	70.00%	11.30%	26.0%
20.00%	80.00%	12.20%	30.6%
10.00%	90.00%	13.10%	35.3%
0.00%	100.00%	14.00%	40.0%

Portfolio risk for more combinations is lower than the risk of either asset

Example of Portfolios and Correlation

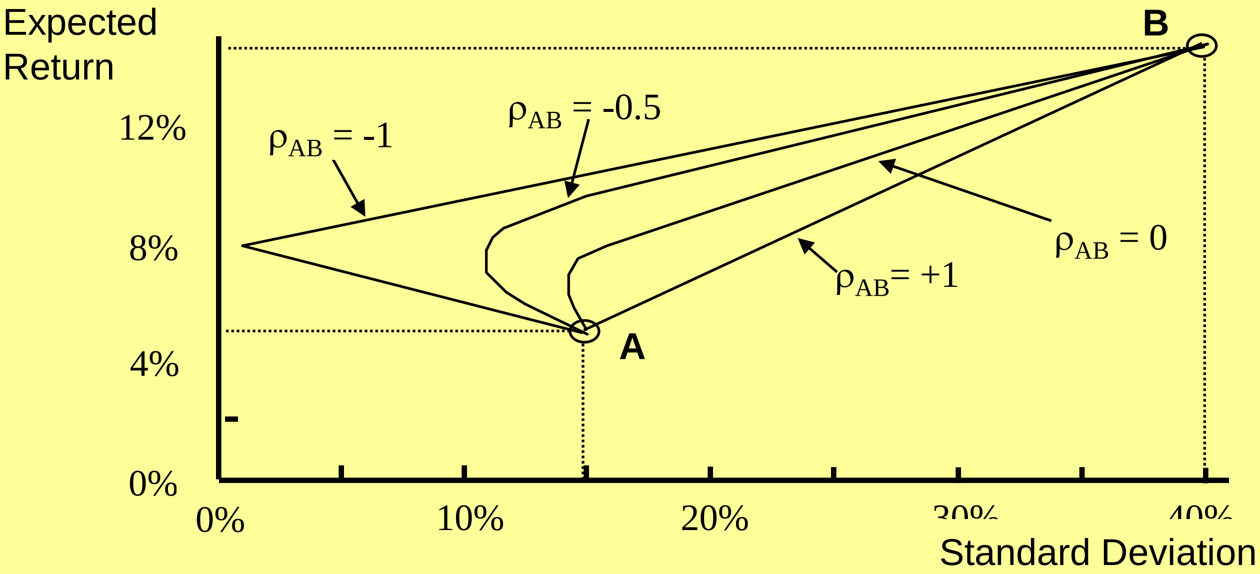
Asset	Expected Return	Standard Deviation	Correlation Coefficient
A	5.0%	15.0%	-1
B	14.0%	40.0%	

Perfect Negative Correlation – greatest diversification potential

Portfolio Components		Portfolio Characteristics	
Weight of A	Weight of B	Expected Return	Standard Deviation
100.00%	0.00%	5.00%	15.0%
90.00%	10.00%	5.90%	9.5%
80.00%	20.00%	6.80%	4.0%
70.00%	30.00%	7.70%	1.5%
60.00%	40.00%	8.60%	7.0%
50.00%	50.00%	9.50%	12.5%
40.00%	60.00%	10.40%	18.0%
30.00%	70.00%	11.30%	23.5%
20.00%	80.00%	12.20%	29.0%
10.00%	90.00%	13.10%	34.5%
0.00%	100.00%	14.00%	40.0%

Risk of the portfolio is almost eliminated at 70% invested in asset A

The Effect of Correlation on Portfolio Risk: The Two-Asset Case



Zero Risk Portfolio

- We can calculate the portfolio that removes all risk.
- When ρ = -1, then

$$\sigma_p = \sqrt{(w_A)^2(\sigma_A)^2 + (w_B)^2(\sigma_B)^2 + 2(w_A)(w_B)(\rho_{A,B})(\sigma_A)(\sigma_B)}$$

- Becomes:

$$\sigma_p = w\sigma_A - (1-w)\sigma_B$$

- Solve this equation for 0.

The Zero Risk Portfolio

- As you can see from the previous slide, if you can find two stocks that have a correlation of -1, you can build a portfolio with 0 risk!
- Mathematically it can be shown that this will happen with

$$w_1 = \frac{\sigma_B}{\sigma_A + \sigma_B}$$

- However, will the portfolio have positive return? Unfortunately, usually no!

Example

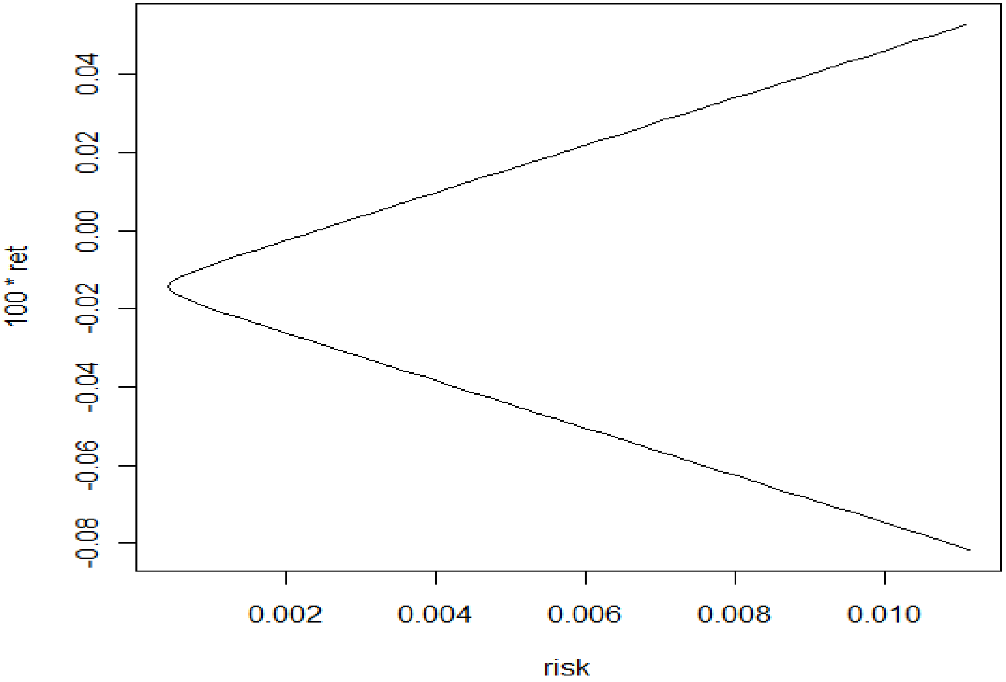
- Consider
  - RYURX (rydex ursa mutual fund)
  - VFINX (Vangaurds S&P 500 mutual fund)

```
cor(vfinx,ryurx)
                RYURX.Adjusted
VFINX.Adjusted      -0.9968647
```

No Free Lunch

- Darn!

As is typical, the zero risk portfolio has a negative expected return!



An Exercise using T-bills, Stocks and Bonds

Base Data:

	Stocks	T-bills	Bonds
Expected Return(%)	12.73383	6.151702	7.0078723
Standard Deviation (%)	0.168	0.042	0.102

Correlation Coefficient Matrix:

Stocks	**1**	-0.216	0.048
T-bills	-0.216	**1**	0.380
Bonds	0.048	0.380	**1**

Portfolio Combinations:

	Weights			Portfolio		
Combination	**Stocks**	**T-bills**	**Bonds**	**Expected Return**	**Variance**	**Standard Deviation**
1	100.0%	0.0%	0.0%	12.7	0.0283	16.8%
2	90.0%	10.0%	0.0%	12.1	0.0226	15.0%
3	80.0%	20.0%	0.0%	11.4	0.0177	13.3%
4	70.0%	30.0%	0.0%	10.8	0.0134	11.6%
5	60.0%	40.0%	0.0%	10.1	0.0097	9.9%
6	50.0%	50.0%	0.0%	9.4	0.0067	8.2%
7	40.0%	60.0%	0.0%	8.8	0.0044	6.6%
8	30.0%	70.0%	0.0%	8.1	0.0028	5.3%
9	20.0%	80.0%	0.0%	7.5	0.0018	4.2%
10	10.0%	90.0%	0.0%	6.8	0.0014	3.8%

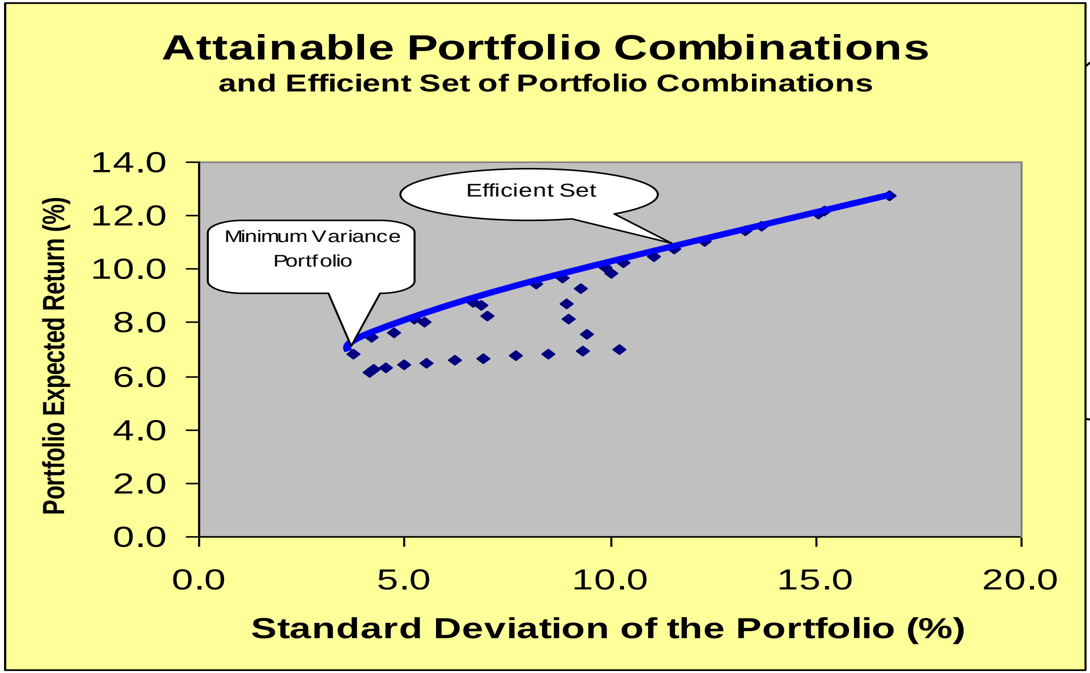
Historical averages for returns and risk for three asset

Historical correlation coefficients between the asset

Each achievable portfolio combination is plotted on expected return, risk (σ) space, found on the following slide.

each combination

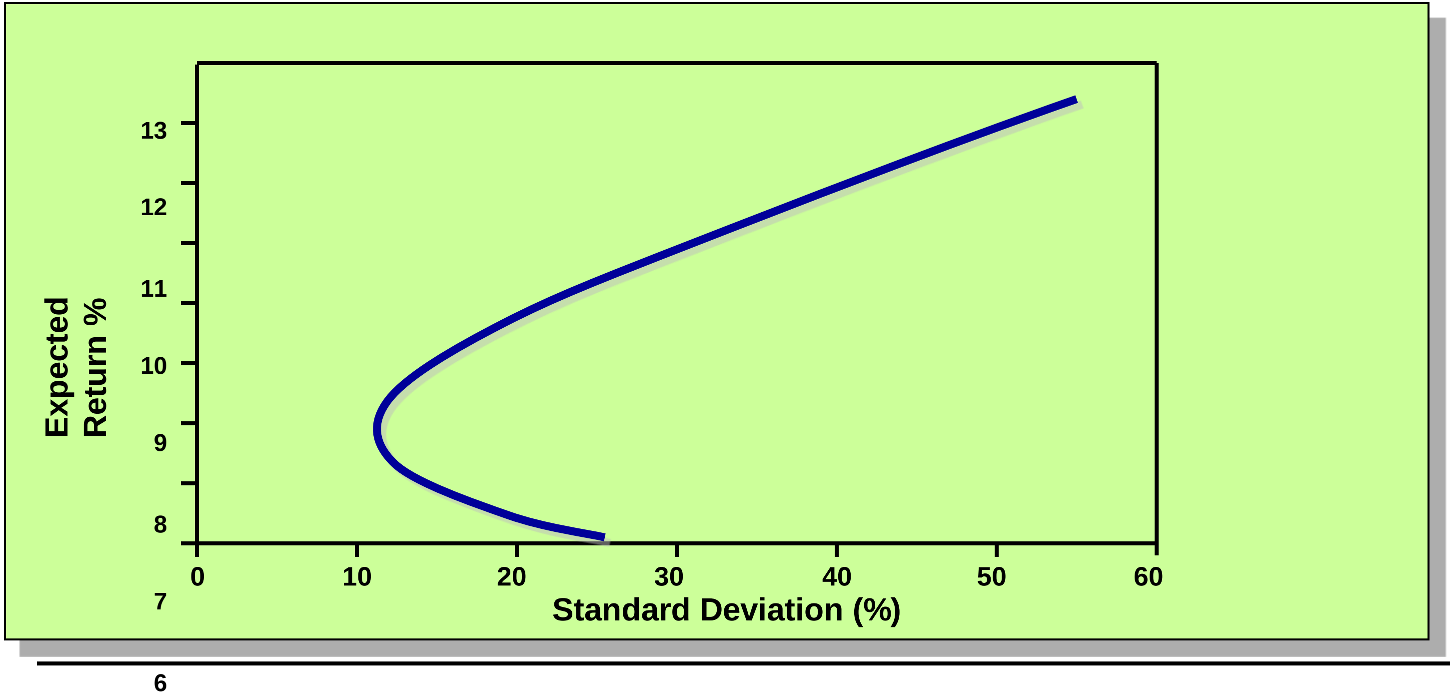
Achievable Portfolios



The efficient set is that set of achievable portfolio combinations that offer the highest rate of return for a given level of risk. The solid blue line indicates the efficient set.

The plotted points are attainable portfolio combinations.

Achievable Two-Security Portfolios

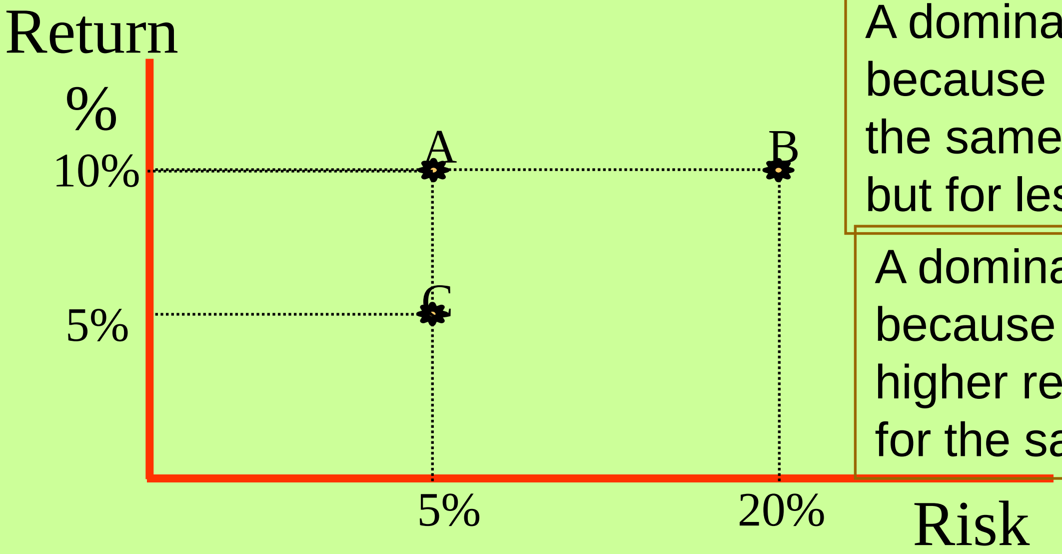


This line represents the set of portfolio combinations that are achievable by varying relative weights and using two non-correlated securities.

Dominance

- It is assumed that investors are rational, wealth-maximizing and risk averse.
- If so, then some investment choices dominate others.

Investment Choices



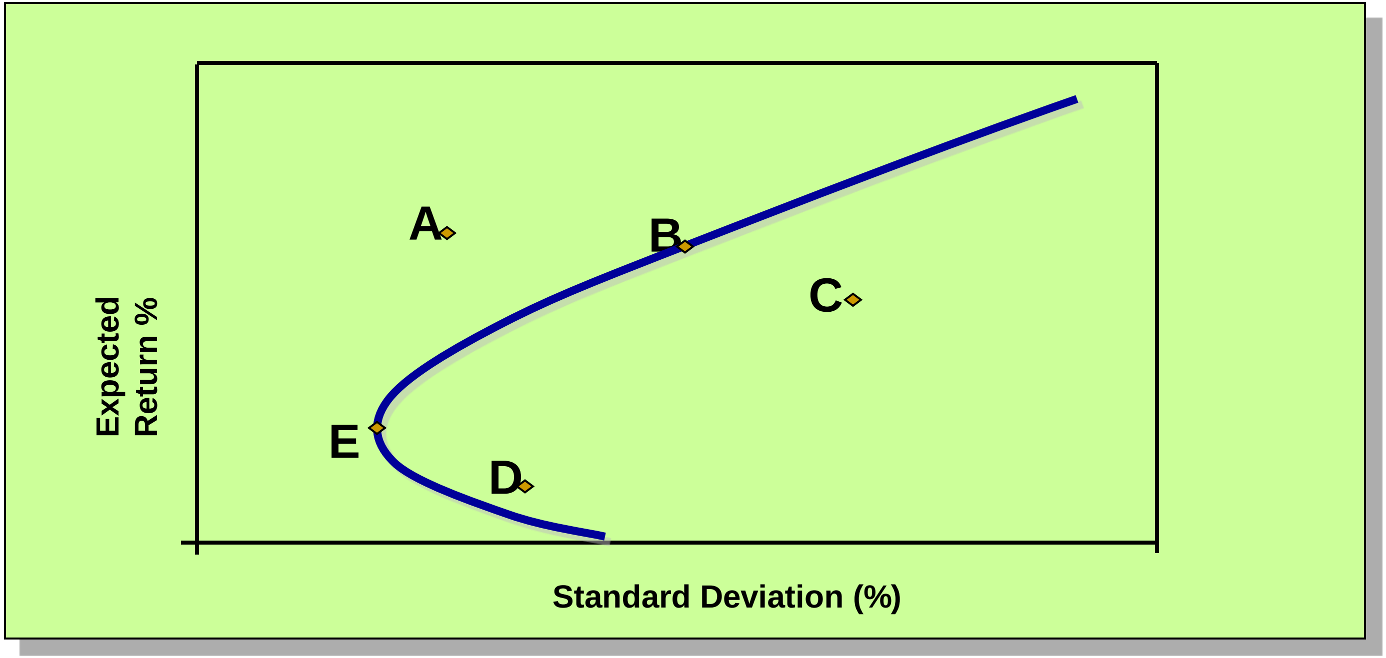
A dominates B because it offers the same return but for less risk.

A dominates C because it offers a higher return but for the same risk.

To the risk-averse wealth maximizer, the choices are clear, A dominates B, A dominates C.

Efficient Frontier

The Two-Asset Portfolio Combinations



A is not attainable

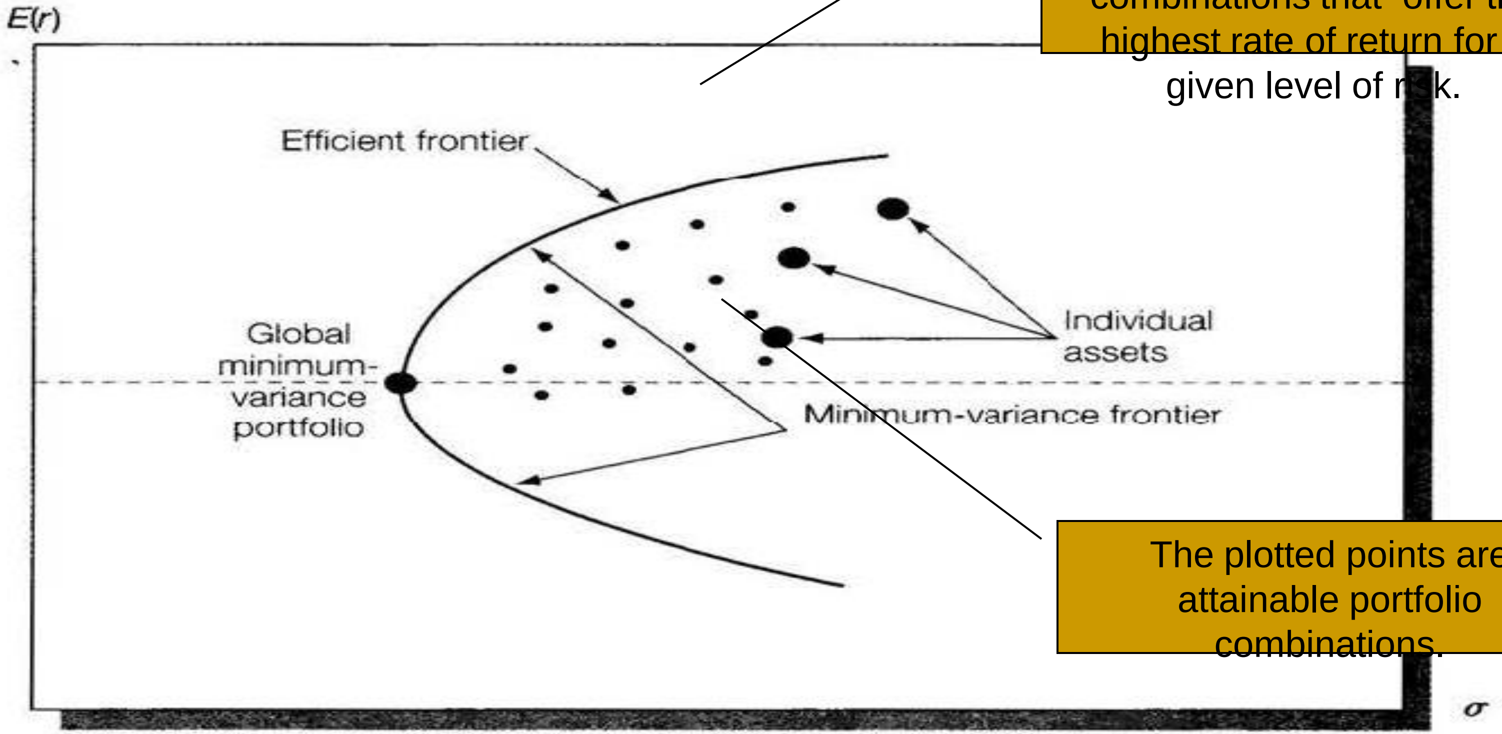
B,E lie on the efficient frontier and are attainable

E is the minimum variance portfolio (lowest risk combination)

C, D are attainable but are dominated by superior portfolios that line on the line above **E**

Achievable Portfolios

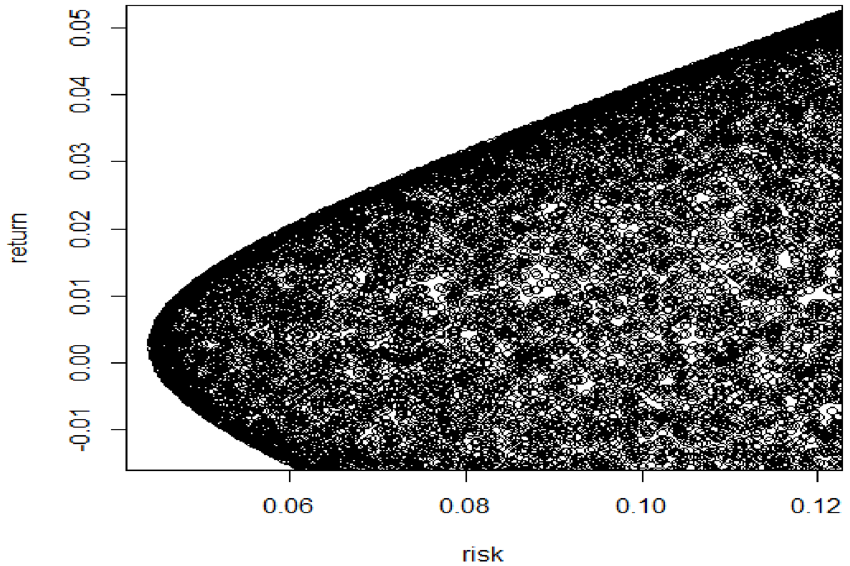
The efficient frontier is that set of achievable portfolio combinations that offer the highest rate of return for a given level of risk.



The plotted points are attainable portfolio combinations.

See port_simu.txt

- Example of portfolios



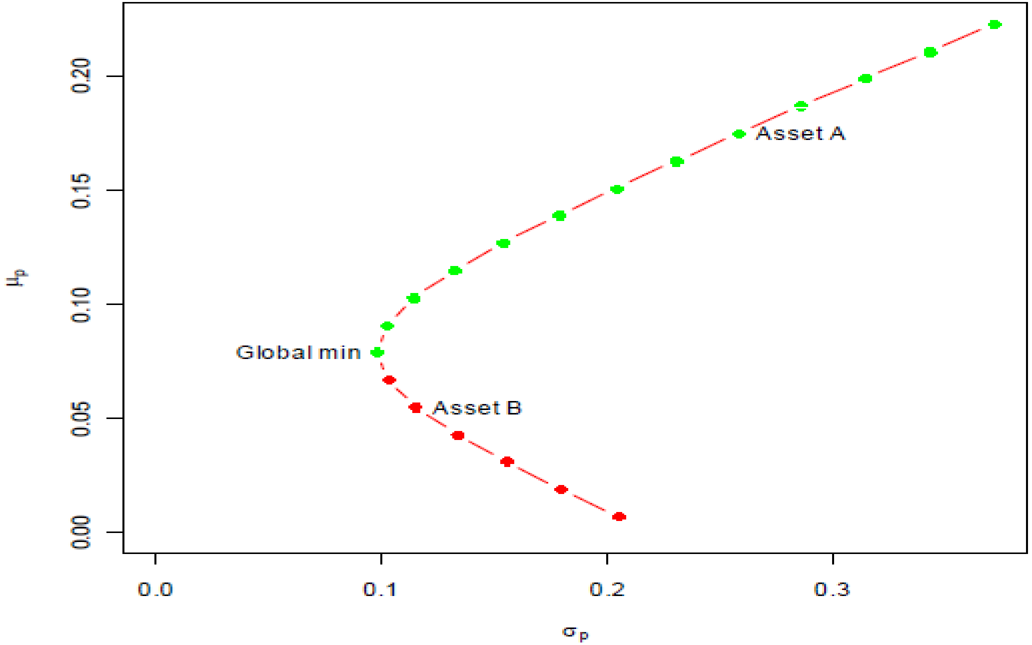
Example in R

- The asset values

μ_A	μ_B	σ_A^2	σ_B^2	σ_A	σ_B	σ_{AB}	ρ_{AB}
0.175	0.055	0.067	0.013	0.258	0.115	-0.004875	-0.164

- Recall efficient portfolios: those portfolios that have the highest expected return for a given level of risk as measured by portfolio variance. These are the portfolios that investors are most interested in holding.

See frag1_lec10.txt



Efficient portfolios are those with the highest expected return for a given level of risk. These portfolios are colored green.

Inefficient portfolios are then portfolios such that there is another feasible portfolio that has the same risk (σ) but a higher expected return (μ). These portfolios are colored red.

Note that the inefficient portfolios are the feasible portfolios that lie below the global minimum variance portfolio, and the efficient portfolios are those that lie above the global minimum variance portfolio.

- The implication for investment decisions is that some portfolios are “inefficient” in the mean variance sense
- Looking at the previous graph we see there are two portfolios which have the same risk, but one has a greater return
- All points below the minimum risk portfolio are said to be inefficient
- All points above the minimum risk portfolio are said to be efficient and make up the efficient frontier.
- The calculation of the efficient frontier is artificially simple when we only have 2 assets in the portfolio
- For the case of 3 or more assets we need to use a constrained optimisation tool like Solver in Excel