Stat 107: Introduction to Business and Financial Statistics
Homework 4: Due Monday, October 3

Xiner Zhou

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(1)

6.1. Seventy-five percent of a portfolio is invested in Honeybell stock and the remaining 25% is invested in MBIB stock. Honeybell stock has an expected return of 6% and an expected standard deviation of returns of 9%. MBIB stock has an expected return of 20% and an expected standard deviation of 30%. The coefficient of correlation between returns of the two securities is expected to be 0.4. Determine the following:

(a) the expected return of the portfolio;
(b) the expected variance of the portfolio;
(c) the expected standard deviation for the portfolio.

(a)

$$E(R_p) = w_H \times E(R_H) + w_M \times E(R_M)$$
$$= 0.75 \times 6\% + 0.25 \times 20\%$$

```
cat('=',0.75*0.06+0.25*0.2)
## = 0.095
```

(b)

$$Var(R_p) = w_H^2 \times Var(R_H) + w_M^2 \times Var(R_M) + 2w_H w_M Cov_{H,M}$$
$$= w_H^2 \times (\sigma_H)^2 + w_M^2 \times (\sigma_M)^2 + 2w_H w_M \sigma_H \sigma_M \rho_{H,M}$$

```
cat('=',0.75^2*0.09^2+0.25^2*0.3^2+2*0.75*0.09*0.25*0.3*0.4)
## = 0.01423125
```

(c)

$$\sigma(R_p) = \sqrt{Var(R_p)}$$

$$= \sqrt{w_H^2 \times (\sigma_H)^2 + w_M^2 \times (\sigma_M)^2 + 2 w_H w_M \sigma_H \sigma_M \rho_{H,M}}$$

```r
cat('=',sqrt(0.75^2*0.09^2+0.25^2*0.3^2+2*0.75*0.09*0.25*0.3*0.4))
## = 0.1192948
```

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(2)

6.4. An equally weighted portfolio will consist of shares from AAB Company stock and ZZY Company stock. The expected return and standard deviation levels associated with the AAB Company stock are 5% and 12%, respectively. The expected return and standard deviation levels for ZZY Company stock are 10% and 20%. Find the expected return and standard deviation levels of this portfolio if returns on the two stocks are:

(a) perfectly correlated;
(b) independent; i.e., a zero correlation coefficient;
(c) perfectly inversely correlated.

.

The expected returns will be the same regardless of correlation:

$$E(R_p) = w_{AAB} \times E(R_{AAB}) + w_{ZZY} \times E(R_{ZZY})$$

$$= 0.5 \times 5\% + 0.5 \times 10\%$$

```r
cat('=',0.5*0.05+0.5*0.1)
## = 0.075
```

The standard deviation is related to the correlation:

$$\sigma(R_p) = \sqrt{Var(R_p)}$$

$$= \sqrt{w_{AAB}^2 \times (\sigma_{AAB})^2 + w_{ZZY}^2 \times (\sigma_{ZZY})^2 + 2 w_{AAB} w_{ZZY} \sigma_{AAB} \sigma_{ZZY} \rho_{AAB,ZZY}}$$

(a)

If perfectly correlated, then

$$\rho_{AAB,ZZY} = 1$$

```r
cat('=',sqrt(0.5^2*0.12^2+0.5^2*0.2^2+2*0.5*0.12*0.5*0.2*1))
## = 0.16
```

(b)

If independent, then

$$\rho_{AAB,ZZY} = 0$$

```
cat('=',sqrt(0.5^2*0.12^2+0.5^2*0.2^2+2*0.5*0.12*0.5*0.2*0))
## = 0.116619
```

(c)

If perfectly inversely correlated, then

$$\rho_{AAB,ZZY} = -1$$

```
cat('=',sqrt(0.5^2*0.12^2+0.5^2*0.2^2+2*0.5*0.12*0.5*0.2*(-1)))
## = 0.04
```

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(3)

6.6. An investor will place one-third of his money into security 1, one-sixth into security 2, and the remainder (one-half) into security 3. Security data is given in the table below:

Security, i	$E[R]$	$\sigma(i)$	$COV(i,1)$	$COV(i,2)$	$COV(i,3)$
1	0.25	0.40	0.16	0.05	0
2	0.15	0.20	0.05	0.04	0
3	0.05	0	0	0	0

Find the expected return and variance of this portfolio.

The expected return:

$$E(R_p) = w_1 \times E(R_1) + w_2 \times E(R_2) + w_3 \times E(R_3)$$

$$= (1/3) \times 0.25 + (1/6) \times 0.15 + (1/2) \times 0.05$$

```
cat('=',(1/3)*0.25 + (1/6)*0.15 + (1/2)*0.05 )
## = 0.1333333
```

The Variance:

$$Var(R_p) = w_1^2 \times (\sigma_1)^2 + w_2^2 \times (\sigma_2)^2 + w_3^2 \times (\sigma_3)^2$$

$$+2w_1w_2Cov_{1,2} + 2w_1w_3Cov_{1,3} + 2w_2w_3Cov_{2,3}$$

```
cat('=',(1/3)^2*0.4^2 + (1/6)^2*0.2^2 + (1/2)*0^2 + 2*(1/3)*(1/6)*0.05+2*(1/3
)*(1/2)*0+2*(1/6)*(1/2)*0 )

## = 0.02444444
```

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(4)

6.8. There exists a market where all securities have a return standard deviation equal to 0.8. All securities are perceived to have independent return outcomes; that is, returns between pairs of securities are uncorrelated.

(a) What would be the return standard deviation of a two-security portfolio in this market?
(b) What would be the return standard deviation of a four-security portfolio in this market?
(c) What would be the return standard deviation of an eight-security portfolio in this market?
(d) What would be the return standard deviation of a 16-security portfolio in this market?
(e) Suppose that all securities in this market have an expected return equal to 0.10. How do the expected returns of the portfolios in parts (a) through (d) differ?

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$$\rho_{i,j} = 0 \forall i \neq j$$

$$\sigma_i = 0.8 \forall i$$

Assume equal weights for all securities, then the standard deviation of portfolio returns is:

$$\sigma(R_{n-security}) = \sqrt{\sum_i^n w_i^2 \sigma_i^2}$$

$$= \sqrt{\sigma^2 \sum_i^n \frac{1}{n}^2}$$

$$= \sigma \sqrt{\frac{1}{n}}$$

$$= \frac{\sigma}{\sqrt{n}}$$

(a)

$$\sigma\left(R_{2-security}\right) = \frac{\sigma}{\sqrt{2}}$$

```
cat('=', 0.8/sqrt(2))
## = 0.5656854
```

(b)

$$\sigma\left(R_{4-security}\right) = \frac{\sigma}{\sqrt{4}}$$

```
cat('=', 0.8/sqrt(4))
## = 0.4
```

(c)

$$\sigma\left(R_{8-security}\right) = \frac{\sigma}{\sqrt{8}}$$

```
cat('=', 0.8/sqrt(8))
## = 0.2828427
```

(d)

$$\sigma\left(R_{16-security}\right) = \frac{\sigma}{\sqrt{16}}$$

```
cat('=', 0.8/sqrt(16))
## = 0.2
```

(e)

If $E(R_i) = 0.10 \forall i$, then the expected returns of portfolio from a) to d) are all the same regardless how many securities and what the portfolio weights are:

$$E\left(R_p\right) = \sum_{i}^{n} w_i \, E(R_i)$$

$$= 0.10 \times \left(\sum_{i}^{n} w_i\right)$$

$$= 0.1 \times 1$$

= 0.1

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(5)

5) One can show mathematically that if two stocks have correlation of -1, then if one puts $\frac{s_2}{s_1 + s_2}$x100% of their money in stock 1, and the rest in stock 2, the resulting portfolio with have 0 risk. But will the portfolio have a positive return? Obtain daily returns for QID and QLD from 2015-01-01 to 2016-01-01.

a) Verify that QLD returns and QID returns have a correlation that is essentially -1
b) What weights are required for QLD and QID to have a 0 risk (standard deviation) portfolio?
c) Compute the mean and standard deviation of the portfolio from part (b). What do you find?

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(a)

```
library(quantmod)
QLD=getSymbols("QLD",from="2015-01-01",to="2016-01-01",auto.assign=FALSE)
QLD.ret=dailyReturn(Ad(QLD))
QID=getSymbols("QID",from="2015-01-01",to="2016-01-01",auto.assign=FALSE)
QID.ret=dailyReturn(Ad(QID))

cor(QLD.ret,QID.ret)

##               daily.returns
## daily.returns    -0.9994757
```

(b)

```
s.QLD<-sd(QLD.ret)
s.QID<-sd(QID.ret)

w.QLD<-s.QID/(s.QLD+s.QID)
w.QID<-1-w.QLD
cat("To have a 0 risk portfolio, the weight for QLD=",w.QLD,'\n',
    "To have a 0 risk portfolio, the weight for QID=",w.QID,'\n')

## To have a 0 risk portfolio, the weight for QLD= 0.5007477
##  To have a 0 risk portfolio, the weight for QID= 0.4992523
```

(c)

```
cat('The mean of the 0 risk portfolio=',mean(w.QLD*QLD.ret+w.QID*QID.ret),'\n
',
```

```
    'The standard deviation of the 0 risk portfolio=',sd(w.QLD*QLD.ret+w.QID*
QID.ret),'\n')

## The mean of the 0 risk portfolio= -4.183764e-05
##  The standard deviation of the 0 risk portfolio= 0.0003614662
```

The expected return of the 0 risk portfolio is negative, so the investors, if using this strategy, are guaranteed to lose money, although the magnitude is small in this case; the standard deviation is very close to 0, which validate the formula about how to achieve a risk-free portfolio, when two assets are perfectly inversely correlated.

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(6)

6) Obtain monthly return data (from adjusted prices) for MRK, KORS, LULU and SPY from 2013-01-01 to 2016-08-01.

   a) What company does each symbol represent? Go to finance.yahoo.com to find out.

   b) What is the average monthly return for each of the stocks ? What is the standard deviation for the returns of the stocks ? What is the correlation between MRK and KORS, MRK and LULU and KORS and LULU ?

   c) Give the expected return and standard deviation of all the possible two stock portfolios (MRK,KORS), (MRK,LULU), (KORS,LULU) with equal amounts invested in each stock (weights of .5 for each stock).

   d) Rank the three portfolios based on their standard deviation. How do they compare with holding one of the individual stocks ?

(a)

```
MRK=getSymbols("MRK",from="2013-01-01",to="2016-08-01",auto.assign=FALSE)
MRK.ret=monthlyReturn(Ad(MRK))
KORS=getSymbols("KORS",from="2013-01-01",to="2016-08-01",auto.assign=FALSE)
KORS.ret=monthlyReturn(Ad(KORS))
LULU=getSymbols("LULU",from="2013-01-01",to="2016-08-01",auto.assign=FALSE)
LULU.ret=monthlyReturn(Ad(LULU))
SPY=getSymbols("SPY",from="2013-01-01",to="2016-08-01",auto.assign=FALSE)
SPY.ret=monthlyReturn(Ad(SPY))
```

MRK is Merck & Co., Inc. KORS is Michael Kors Holdings Limited. LULU is Lululemon Athletica Inc. SPY is SPDR S&P 500 ETF. MRK, KORS, and LULU are individual stocks, while SPY is not.

(b)

```r
cat("The average monthly return for MRK =", mean(MRK.ret),'\n',
    "The standard deviation of monthly return for MRK =", sd(MRK.ret),'\n\n',
    "The average monthly return for KORS =", mean(KORS.ret),'\n',
    "The standard deviation of monthly return for KORS =", sd(KORS.ret),'\n\n
',
    "The average monthly return for LULU =", mean(LULU.ret),'\n',
    "The standard deviation of monthly return for LULU =", sd(LULU.ret),'\n\n
',
    "The average monthly return for SPY =", mean(SPY.ret),'\n',
    "The standard deviation of monthly return for SPY =", sd(SPY.ret),'\n\n')
```

```
## The average monthly return for MRK = 0.01159726
##  The standard deviation of monthly return for MRK = 0.04405822
##
##  The average monthly return for KORS = 0.00576902
##  The standard deviation of monthly return for KORS = 0.1060122
##
##  The average monthly return for LULU = 0.006237978
##  The standard deviation of monthly return for LULU = 0.1001071
##
##  The average monthly return for SPY = 0.01108954
##  The standard deviation of monthly return for SPY = 0.03044601
```

```r
cat("Correlation between MRK and KORS=", cor(MRK.ret, KORS.ret), '\n',
    "Correlation between MRK and LULU=", cor(MRK.ret, LULU.ret), '\n',
    "Correlation between KORS and LULU=", cor(KORS.ret, LULU.ret), '\n')
```

```
## Correlation between MRK and KORS= -0.02751976
##  Correlation between MRK and LULU= 0.08884351
##  Correlation between KORS and LULU= 0.1193581
```

(C)

```r
cat("The expected return of portfolio (MRK,KORS)=", 0.5*mean(MRK.ret)+0.5*mea
n(KORS.ret), '\n',
    "The standard deviation of portfolio (MRK,KORS)=", sqrt(0.5^2*sd(MRK.ret)
^2+0.5^2*sd(KORS.ret)^2+2*0.5*0.5*sd(MRK.ret)*sd(KORS.ret)*cor(MRK.ret,KORS.r
et)), '\n\n',
    "The expected return of portfolio (MRK,LULU)=", 0.5*mean(MRK.ret)+0.5*mea
n(LULU.ret), '\n',
    "The standard deviation of portfolio (MRK,LULU)=", sqrt(0.5^2*sd(MRK.ret)
^2+0.5^2*sd(LULU.ret)^2+2*0.5*0.5*sd(MRK.ret)*sd(LULU.ret)*cor(MRK.ret,LULU.r
et)), '\n\n',
    "The expected return of portfolio (KORS,LULU)=", 0.5*mean(KORS.ret)+0.5*m
ean(LULU.ret), '\n',
    "The standard deviation of portfolio (KORS,LULU)=", sqrt(0.5^2*sd(KORS.re
t)^2+0.5^2*sd(LULU.ret)^2+2*0.5*0.5*sd(KORS.ret)*sd(LULU.ret)*cor(KORS.ret,LU
LU.ret)), '\n\n')
```

```
## The expected return of portfolio (MRK,KORS)= 0.008683138
##  The standard deviation of portfolio (MRK,KORS)= 0.0568389
##
##  The expected return of portfolio (MRK,LULU)= 0.008917617
##  The standard deviation of portfolio (MRK,LULU)= 0.05644964
##
##  The expected return of portfolio (KORS,LULU)= 0.006003499
##  The standard deviation of portfolio (KORS,LULU)= 0.07712557
```

(d)

The best to worst portfolio in terms of risk/standard deviation is: (MRK,LULU), (MRK,KORS), (KORS,LULU). The same rank applies even if we compare based on expected returns. So (MRK,LULU) has the highest expected return with least risk, while (KORS,LULU) has the lowest expected return with highest risk.

All the three portfolios have standard deviation somewhere between individual stocks, but not less than. For (MRK,LULU) and (KORS,LULU), because of their pair-wise correlation are positive, it's not possible to find weights that make the portfolio risk less than the least risky individual stock's. For (MRK, KORS), the correlation is slightly nagative, we could find some weights that make the portfolio less risker than the least risky individual stock, but the designated weights 0.5/0.5 is not one of those weights.