

Stat 107: Introduction to Business and Financial Statistics

Class 18: Project and More Multiple Regression

Important Dates: Final Exam

- December 13th, 2-5pm
- 3 hour take home final similar to midterm

The Class Project-2 to 5 people

■ Important Dates

- ❑ November 16th: project proposal due (ungraded)
- ❑ December 6th: project due (midnight)

■ There are least two directions one can take for the final project:

- ❑ Reproducing an existing study in the literature
- ❑ Simulating or Reporting on a particular stock trading strategy
- ❑ (Your own idea at discretion of Instructor)

Project Section of Class Website

- Many possible ideas are posted here.
- Relatively easy to read articles from
 - The Financial Analysts Journal
 - Journal of Empirical Finance
 - International Review of Financial Studies
- Many different trading ideas
- Examples of projects from previous years

Some Project Ideas

- Sector Switching; see
 - Gummy spreadsheets ,Fidelity Book, Other Papers

Project Ideas: All the Gummy Stuff Spreadsheets

 [The Gummy Stuff Spreadsheets](#)

Project Ideas: Trading

 [A computational implementation of stock charting: abrupt volume increase as signal Composite Index](#)

 [A NOVEL CHART PATTERN RECOGNITION APPROACH: A CASE STUDY ON CUP WITH HAN](#)


 [Applying rough sets to market timing decisions](#)

 [CUSUM TECHNIQUES FOR TECHNICAL TRADING IN FINANCIAL MARKET](#)

 [Dynamic Asset Allocation Using Systematic Sector Rotation](#)

 [Dynamic Asset Allocation Using Systematic Sector Rotation](#)

 [Guide to Fidelity Funds \(introduces the Ulcer Index\)](#)

 [Pairs Trading: Performance of a Relative Value Arbitrage Rule](#)

Test a Sector Switching System

- Use ETFs (exchange traded funds)
- Make a chart as follows:

Table 12–2. Comparison of Diversified and All-Funds Portfolios, combined system, 1981–1988.

| Parameter | 90-Day T-Bills | S&P 500 | Diversified Portfolio | All-Funds Portfolio |
|-------------------------|-------------------|------------|--------------------------|------------------------|
| Weeks tested | 418 | 418 | 418 | 418 |
| Weeks in market | 0 | 418 | 261 | 266 |
| Weeks out of market | 418 | 0 | 157 | 152 |
| Initial investment | \$10,000 | \$10,000 | \$10,000 | \$10,000 |
| Final portfolio value | \$19,896 | \$28,247 | \$120,650 | \$163,080 |
| Bull market return | 7.62% | 40.15% | 60.85% | 69.52% |
| Bear market return | 10.81% | –13.62% | 9.57% | 11.54% |
| Compound annual return | 8.98% | 13.86% | 36.42% | 41.76% |
| Excess return/year | 0.00% | 4.88% | 27.44% | 32.78% |
| Beats market by (/year) | --- | --- | 22.56% | 27.90% |
| Ulcer Index | 0.00% | 9.32% | 2.74% | 2.60% |
| UI Performance | --- | 0.52 | 10.01 | 12.61 |
| SD of weekly return | 0.05% | 2.30% | 1.73% | 1.96% |

Test a Mechanical Pattern

- Similar in spirit to the “Kicker” HW problem

Project Ideas: Trading

 [A computational implementation of stock charting: abrupt volume increase as signal for Composite Index](#)

 [A NOVEL CHART PATTERN RECOGNITION APPROACH: A CASE STUDY ON CUP WITH HANDLE](#)

 [Applying rough sets to market timing decisions](#)

 [CUSUM TECHNIQUES FOR TECHNICAL TRADING IN FINANCIAL MARKET](#)

 [Dynamic Asset Allocation Using Systematic Sector Rotation](#)

 [Guide to Fidelity Funds \(introduces the Ulcer Index\)](#)

 [Market Timing, Charting Heuristic](#)

 [Pairs Trading: Performance of a Relative Value Arbitrage Rule](#)

 [Reverse Engineering RSI](#)

 [Simple Market Timing Model \(faber paper\)](#)

 [Trading Strategies That Work \(or The Big Damn Chapter on Trading Strategies\)](#)

 [Trading With A Day Job: Can Automated Trading Strategies Be Profitable?](#)

I find Chartists a bit weird

Understanding Chart Patterns

Dan's 10 Golden Rules

Charting Programs

Recommended Reading

Useful Stock Resources

Glossary

Understanding Chart Patterns

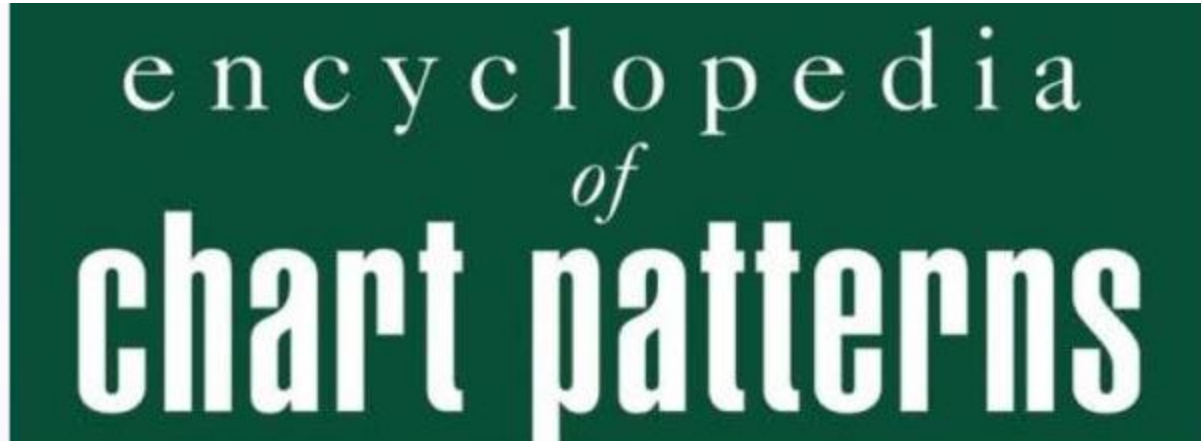
Identifying chart patterns is simply a system for predicting stock market trends and turns! Hundreds of years of price charts have shown that prices tend to move in trends. (I'm sure we've all heard the saying, 'the trend is your friend'.) Well, a trend is merely an indicator of an imbalance in the supply and demand. These changes can usually be seen by market action through changes in price. These price changes often form meaningful chart patterns that can act as signals in trying to determine possible future trend developments. Research has proven that some patterns have high forecasting probabilities. These patterns include: The Cup & Handle, Flat Base, Ascending and Descending Triangles, Parabolic Curves, Symmetrical Triangles, Wedges, Flags and Pennants, Channels and the Head and Shoulders Patterns. In my opinion, these are some of the best patterns to trade.

This section is designed to introduce you to some of these chart patterns, as well as teach you to identify repetitions in the market qualities, to make timely and more accurate decisions when predicting market trends.

11 Most Common Stock Chart Patterns

1. [Cup & Handle](#)
2. [Flat Base](#)
3. [Ascending Triangle](#)
4. [Parabolic Curve](#)
5. [Wedge Formation](#)
6. [Channel Formation](#)
7. [Symmetrical Triangle](#)
8. [Descending Triangle](#)
9. [Flags & Pennants](#)
10. [Head & Shoulders](#)
11. [Inverted Head & Shoulders](#)

Is a pattern repeatable/profitable?



In this revised and expanded second edition of the bestselling **Encyclopedia of Chart Patterns**, Thomas Bulkowski updates the classic with new performance statistics for **both bull and bear markets and 23 new patterns**, including a second section devoted to ten event patterns. Bulkowski tells you how to trade the significant events -- such as quarterly earnings announcements, retail sales, stock upgrades and downgrades -- that shape today's trading and uses statistics to back up his approach. This comprehensive new edition is a must-have reference if you're a technical investor or trader.

There are many websites that will automatically scan for chart patterns:

<http://www.screenulator.com/>

<http://www.nebadawn.com/>

Americanbulls.com

Recommended Shares

BUYS

TRVN (NASDAQ)

MOBQ (OTC PINK)

BULLISH PATTERNS

DTO (NYSE)

RARE (NASDAQ)

GNSZ (OTC PINK)

SELLS

TRUE (NASDAQ)

BEARISH PATTERNS

AAMC (AMEX)

TROX (NYSE)

LNCO (NASDAQ)

AOIFF (OTC PINK)

KMI-WT (WARRANTS)

CanadaBulls.com

BritishBulls.com

AussieBulls.com

Mexicanobulls.com

ChineseBulls.com

IsraeliBulls.com

ItalianBulls.com

Stock Chart

TRVN TREVENA (NASDAQ) Close : 10.1300 [Oversold]



The chart above shows how **TRVN** dropped after the **RED** signal which was issued on 09/25/2015. Since then, the stock price edged down by a hefty 18.54%. This brutal bear move, however, may be over now. Our system verifies this change of mood by issuing a **BUY** signal. The **BULLISH ONE WHITE SOLDIER** pattern that led to this bullish signal indicates that there is an upside potential of 23.16%. The signal has just been issued, so it is not late at all to throw the dice for a possible run-up. The overstretched correction seems to be over, at least for the moment. It may be the right time to join the bulls who started to show a reinstated enthusiasm for this stock.

Thinkorswim.com



Do Chartists Make Money?



Dan Zanger
@DanZanger

World Record Holder Top Swing Trader - Turned \$11,000 to over
\$40,000,000.

Global · <http://chartpattern.com>

14 THE ZANGER PARLAY



FORMER SWIMMING-POOL

contractor Dan Zanger must hold a trading record by having parlayed \$10,775 into \$18 million in the 18 months between June 1998 and December 1999. He bought CMGI in early January 1999 at \$118, rode it to \$305, then sold at \$130*— after which the stock dropped to \$87. The trade netted him more than 210 percent in four days. Eventually, the prop trader's nest egg grew to \$42 million — which shows how career changes can make the best swaps.

*post 2 for 1 stock split

Quantshare.com

- We will spend time later looking at this software (free trial-don't download yet!).

QuantShare is an advanced trading software that offers a unique charting experience, several backtesting features and can handle multiple databases. It includes a lot of plug-ins, like the downloader, the composite, the watch-list, the neural network prediction tool...

This trading software supports US and international markets and can be used by stock, forex, options, futures... traders. We have built a flexible trading software whose possibilities are only limited by your imagination. But because we wanted to provide something unique, we have created a sharing server where you can exchange items or objects with other traders in order to feed your imagination, bring you new ideas, give you the opportunity to use advanced traders tools, and help you improve your trading skills.



Project Idea: Pairs Trading

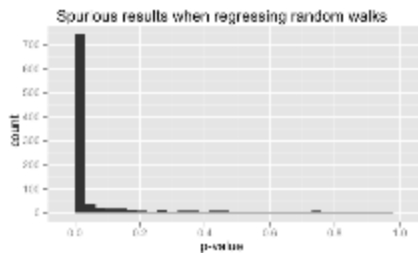
- Pairs trading is also known as statistical arbitrage, and is a very interesting concept.
- In simple terms, one looks for stocks that seem to move together, like GOOG and AAPL, or HD and LOW.
- When their historical relationship gets out of whack, one shorts the overbought one and buys the oversold one.
- It's a market neutral concept, as one invests the same amount of money in each position.

Info about R and pairs trading

Introduction to Cointegration and Pairs Trading

April 15, 2011

By Edwin Chen



Introduction Suppose you see two drunks (i.e., two random walks) wandering around. The drunks know each other (they're independent), so there's no meaningful relationship between their paths. But suppose instead you have a drunk walking with her dog.

This [Continuing reading...](#)

Seasonal pair trading

January 10, 2011

By Dzidorius Martinaitis



quanttrader.info is a good quantitative repository, where I found an idea about seasonal spreads play. The idea of seasonal pair trading differs from pair trading in a way, that it doesn't try to find deviation from the spread's mean read pattern to find...

QuantTrader

.. quantitative trading, programming and more

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[Blog](#) [Pair Trading](#)

Pair-Trading with S&P500 Companies – Part II.

April 10, 2011

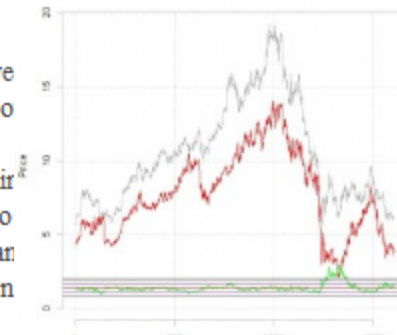
By QuantTrader

Today I'm going to share with you further outcomes of my research in statistical arbitrage trading technique - pair-trading. In the first part of pair-trading with S&P500 Companies I used downloaded data from yahoo to identify co-integrated pairs. ...

Poor man's pairs trading...

April 11, 2010

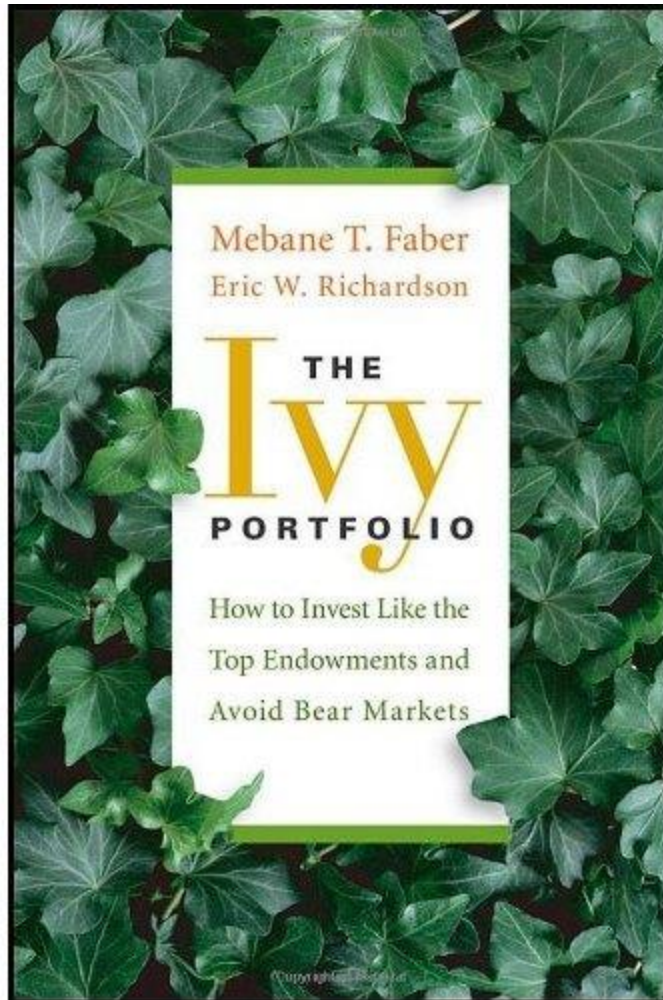
By M. Parzakonis



There is a central notion in Time Series Econometrics, cointegration. Loosely it refers to finding the long run equilibrium of two non-stationary series. As the most known non-stationary series examples come from finance, cointegration is nowadays a tool for traders (not a common one though!). They use it as the theory behind pairs trading...

<http://www.r-bloggers.com/>

Project Idea: The Ivy Portfolio



Well, not exactly, but author Mebane Faber has a nice paper on market timing (available on the course web site) which can be implemented and tested in R or Excel.

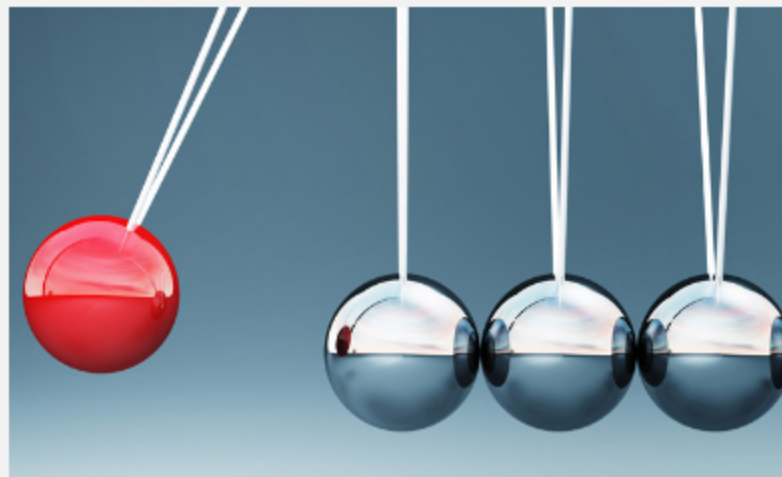
Project Idea: Optimal Momentum

What Is Momentum Investing?

Momentum means persistence in performance. Additional periods of superior performance tend to follow initial periods of superior performance. But it is more than that. Momentum is also a systematic rules-based approach for entering and exiting the markets based on specific, proven criteria.

Why Use It?

Momentum as we define it is still largely undiscovered by mainstream investors. Yet it is now universally accepted by the academic community as an investment approach offering consistently superior risk-adjusted returns. This website will introduce you to the features and benefits of momentum investing.



"A body in motion tends to stay in motion."

-Sir Isaac Newton

Absolute Momentum

Absolute Momentum: a Simple Rule-Based Strategy and Universal Trend-Following Overlay, Our research paper demonstrating the versatility of absolute momentum.

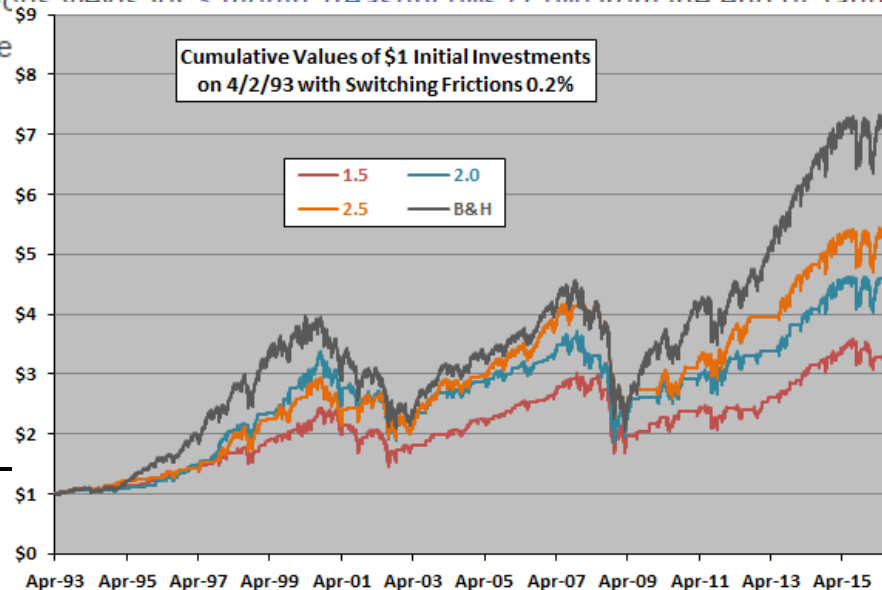
Project Idea: CXO Advisory

🔒 Bollinger Bands: Buy Low and Sell High?

Posted in [Technical Trading](#)

October 26, 2016

Are [Bollinger Bands](#) (BB) useful for specifying when to buy low and when to sell high the overall U.S. stock market? In other words, can an investor beat a buy-and-hold strategy by systematically buying (selling) when the market crosses below (above) the lower (upper) BB? To check, we examine the historical behavior of BBs around the 21-trading day (one month) simple moving average (SMA) of [S&P 500 SPDR \(SPY\)](#) as a tradable proxy for the U.S. stock market. We consider BB settings ranging from 0 to 2.5 standard deviations of daily returns, calculated over the same trailing 21 trading days. Using daily unadjusted closes of [SPY](#) (to calculate BBs), dividend-adjusted closes of SPY (to calculate total returns) and contemporaneous yields for 3 month Treasury bills (T bill) from the end of January 1993 (SPY inception) through e



Timing Ratio Investigation

Bear beta

Bear **beta** is a measure of the sensitivity of a fund's return to negative changes in its benchmark's return.

Beta

Beta represents the slope of the regression line. **Beta** measures a fund's risk relative to the market as a whole (i.e. the "market" can be any index or investment). **Beta** describes the fund's sensitivity to broad market movements. For example, for equities, the stock market is the independent variable and has a **beta** of 1. A fund with a **beta** of 0.5 will participate in broad market moves, but only half as much as the market overall.

Beta timing ratio

Ideally, we would prefer a portfolio manager with a **beta** greater than 1 in rising markets and less than 1 in falling markets. Hence, the **beta timing ratio** is defined as the **ratio** between the **bull beta** and the bear **beta**.

Bull beta

This is a measure of the sensitivity of a fund's return to positive changes in its benchmark's return.

Project Ideas: Replicating Research

- The January Effect (Event Studies)
 - Manager Education on Mutual Fund Performance
 - Both of these papers (and many others in the finance field) are simply regression models.
 - Look in **Financial Analysts Journal** (available online from the Harvard library system or talk to me for ideas).
-

Financial Analysts Journal

Financial Analysts Journal

September/October 2016, Volume 72 Issue 5



[Available Issues](#)

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Information in the Tails of the Distribution of Analysts' Quarterly Earnings Forecasts

[Cameron Truong, Philip B. Shane, and Qihong Zhao](#)

Financial Analysts Journal, September/October 2016, Vol. 72, No. 5: 84–99.

[Preview](#) | [Abstract](#) | [References](#) | [PDF \(373 KB\)](#)



Neither “Normal” nor “Lognormal”: Modeling Interest Rates across All Regimes

[Attilio Meucci, CFA, and Angela Loregian](#)

Financial Analysts Journal, May/June 2016, Vol. 72, No. 3: 68–82.

[Preview](#) | [Abstract](#) | [References](#) | [PDF \(748 KB\)](#)



The Impact of Constraints on Minimum-Variance Portfolios

[Tzee-Man Chow, Engin Kose, and Feifei Li](#)

Financial Analysts Journal, March/April 2016, Vol. 72, No. 2: 52–70.

[Preview](#) | [Abstract](#) | [References](#) | [Summary](#) | [PDF \(599 KB\)](#)

Project Idea: CXO Advisory



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High Prices Mean Good Stocks?

October 25, 2016 • Posted in [Animal Spirits](#), [Equity Premium](#)

Are stocks with high prices or low prices inherently better deals? In their October 2016 paper entitled "[Nominal Stock Price Investing](#)", Ulrich Hammerich, Christian Fieberg and Thorsten Poddig examine the relationship between stock price and future stock performance in the German equity market. Specifically, they each month sort stocks by price and measure the difference in average total returns between the equally weighted tenth (decile) of stocks with the highest prices and the equally weighted decile with the lowest prices. Using monthly prices and total returns for a broad set of German stocks from the end of January 1990 through December 2013, *they find that:*

Manager Education on Mutual Fund Performance

Table 3
Regressions of fund characteristics on manager characteristics

| Independent variables | Dependent variable | | | |
|--|--------------------|-------------|-----------|-------------|
| | BETA | TURNOVER/10 | EXPENSE | LOG(ASSETS) |
| <i>Panel A: regressions using GMAT scores as an independent variable</i> | | | | |
| Intercept | 1.148*** | 18.112** | 3.072*** | 14.819*** |
| SAT/100 | -0.008 | 0.050 | -0.006 | 0.164*** |
| LIBERAL ARTS | 0.033 | 0.789 | 0.054 | 0.178 |
| GMAT/100 | -0.011 | -1.990** | -0.105*** | 0.575*** |
| NO MBA | -0.068 | -12.646* | -0.719*** | 3.730*** |
| CFA | -0.004 | 0.221 | -0.048* | 0.194 |
| OTHERGRAD | 0.043 | 0.474 | 0.071* | -0.068 |
| Ph.D. | -0.089 | -0.290 | -0.085 | 0.833 |
| AGE/10 | -0.006 | 0.918 | 0.040* | -0.436*** |
| MISSINGAGE | -0.020 | 5.840* | 0.197* | -2.102*** |
| TENURE | -0.012*** | -0.385*** | -0.003 | 0.072*** |
| AGGROWTH | 0.237*** | 4.758** | 0.078 | 0.630* |
| EQINCOME | -0.381*** | -5.046*** | -0.147** | 0.219 |
| GRINCOME | -0.178*** | -2.759** | -0.158*** | 0.110 |
| SMALLCOM | -0.026 | 0.112 | 0.139*** | -0.453** |
| LOG(LAG ASSETS) | 0.007 | 0.039 | -0.062*** | |
| No. of observations | 518 | 518 | 518 | 518 |
| Adjusted R^2 | 0.188 | 0.067 | 0.247 | 0.092 |

Running Model in R

```
> fit=lm(rets~tues+wed+thur+fri)
> summary(fit)
```

Call:

```
lm(formula = rets ~ tues + wed + thur + fri)
```

Residuals:

| | Min | 1Q | Median | 3Q | Max |
|--|----------|----------|---------|---------|---------|
| | -0.06520 | -0.00379 | 0.00008 | 0.00390 | 0.04917 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) | |
|-------------|-----------|------------|---------|----------------------|-----|
| (Intercept) | -0.001558 | 0.000218 | -7.13 | 0.00000000000011 | *** |
| tues | 0.001673 | 0.000306 | 5.47 | 0.0000000480860 | *** |
| wed | 0.002607 | 0.000307 | 8.50 | < 0.0000000000000002 | *** |
| thur | 0.002130 | 0.000307 | 6.94 | 0.000000000000042 | *** |
| fri | 0.002640 | 0.000307 | 8.59 | < 0.0000000000000002 | *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.00749 on 6016 degrees of freedom

Multiple R-squared: 0.0163, Adjusted R-squared: 0.0157

F-statistic: 24.9 on 4 and 6016 DF, p-value: <0.0000000000000002

What does the model say?

- Suggests Monday is a downer

| Day of week | Return |
|-------------|--------------|
| Monday | -.0016 |
| Tuesday | -.0016+.0017 |
| Wednesday | -.0016+.0026 |
| Thursday | -.0016+.0021 |
| Friday | -.0016+.0026 |

A Trading Scheme

5. Potential profit from the negative returns for Monday

Even if one were to conclude that the negative returns for Monday are evidence of market inefficiency, the profit to any individual from knowledge of the negative returns is more limited than it may appear. One simple trading strategy based on this information would be for an individual to purchase the Standard and Poor's composite portfolio every Monday afternoon and to sell these investments on Friday afternoon, holding cash over the weekend. Ignoring transactions costs, this trading rule would have generated an average annual return of 13.4 percent from 1953 to 1977, while a buy and hold policy would have yielded a 5.5 percent annual return. However, no investor can ignore transactions costs. If these costs are only 0.25 percent per transaction, the buy and hold policy would have yielded a higher return in each of the 25 years studied.

The January Effect

- Small stocks have tended to produce greater returns than large stocks, and the lion's share of the small-stock premium is earned in the first ten trading days in January.
- The data: All stocks on the NYSE in the CRSP monthly data file from 1926 through 1993.
- Stocks ranked on total market capitalization

The Model

$$r_t = \alpha_0 + \alpha_1 J_t + \varepsilon_t$$

■ Where

r_t = monthly rate of return of stock portfolio in month t

J_t = dummy variable taking a value of 1 if t is a January month and 0 otherwise

What does the model say?

$$r_t = \alpha_0 + \alpha_1 J_t + \varepsilon_t$$

If month is not January

$$r_t = \alpha_0 + \varepsilon_t$$

If month is January

$$r_t = \alpha_0 + \alpha_1 + \varepsilon_t$$

If there is a January effect, what should the value of α_1 be ?

The Output

Table 1. Test of the January Effect: $r_t = a_0 + a_1 J_t + e_t$, 1926–93
(standard errors in parentheses)

| Size Decile | R^2 | a_0 $s(a_0)$ | $a_1 a$ $s(a_1)$ |
|----------------|--------|---------------------|---------------------|
| 1 | 0.0776 | 0.0114* (0.0043) | 0.1244* (0.0148) |
| 2 | 0.0403 | 0.0098* (0.0036) | 0.0729* (0.0123) |
| 3 | 0.0316 | 0.0082* (0.0031) | 0.0562* (0.0108) |
| 4 | 0.0210 | 0.0094* (0.0029) | 0.0432* (0.0102) |
| 5 | 0.0167 | 0.0090* (0.0028) | 0.0359* (0.0095) |
| 6 | 0.0135 | 0.0093* (0.0026) | 0.0310* (0.0092) |
| 7 | 0.0063 | 0.0100* (0.0025) | 0.0202* (0.0088) |
| 8 | 0.0051 | 0.0091* (0.0024) | 0.0173* (0.0083) |
| 9 | 0.0033 | 0.0097* (0.0023) | 0.0133 (0.0080) |
| 10 | 0.0006 | 0.0086* (0.0020) | 0.0049 (0.0071) |
| EW | 0.0227 | 0.0094* (0.0027) | 0.0418* (0.0095) |

^a a_1 measures the magnitude of the January effect over the whole period.

*Significant at the 5 percent level.

Confidence Interval=
0.1244+/- 1.96(.0148)

Event Studies

- The January Effect is called an “Event Study”
- Do markets trade differently around holidays, seasons, elections, earnings announcements, etc... some event of interest.
- These are relatively easy to model as we saw with the January Effect paper.

Other types of events

- Air pollution! ☹️
- Can replicate this paper, “Air pollution and stock returns in the US”
- Paper on web site...abstract below

Health related research documents that air pollution has negative mood effects. Experimental works in psychology relate bad mood to increased risk aversion. Studies in financial economics report an observed link between mood effects and stock market returns. This study therefore investigates whether the mood effects caused by air pollution can have economic implications.

No one said this is rocket science

■ Their first model

estimating the following regression equation:

$$r_t = \beta_0 + \beta_1 D_t + e_t,$$



where D is a dummy variable carrying the value of 1 for Good days and 0 for Unhealthy days,

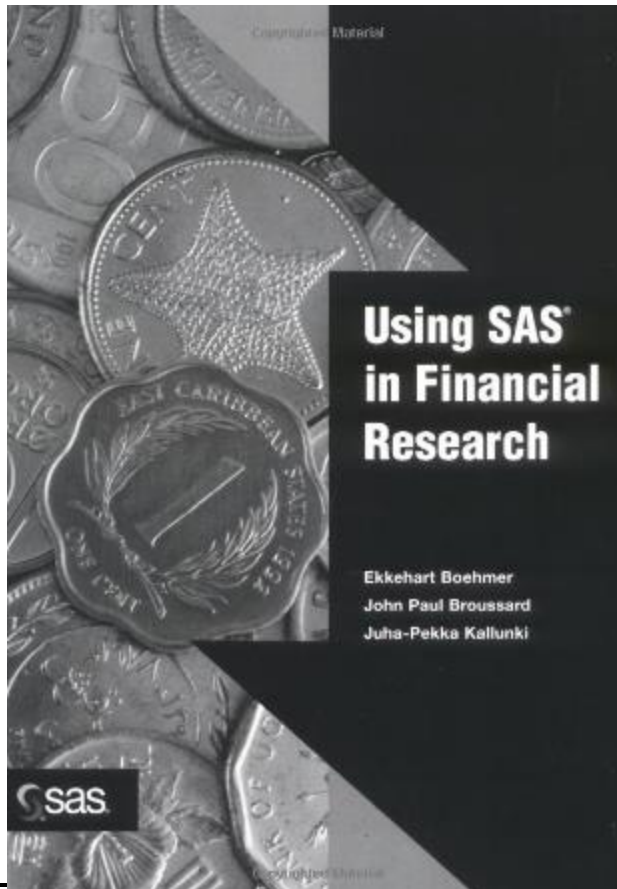
■ Their Second Model

$$r_t = \beta_0 + \beta_1 D_t + \beta_2 \text{Monday} + \beta_3 \text{January} + \beta_4 r_{t-1} + e_t,$$

■ Anyone in this class could replicate this research!

Other Ideas

- The following book is on reserve at Cabot



Don't let the title scare you,
it has some excellent ideas
for a project.

Some Example Chapters

CHAPTER 4 CROSS-SECTIONAL APPROACH TO THE EMPIRICAL TEST OF THE CAPITAL ASSET PRICING MODEL

| | |
|-----------------------|----|
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CHAPTER 5 EVENT STUDIES

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CHAPTER 9 USING ACCOUNTING INFORMATION TO FORECAST MARKET PERFORMANCE

| | |
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| Background for Analyzing Fundamental Accounting Information and Market Performance | 107 |
| The Data | 108 |
| Sample Program to Evaluate Financial Information and Market Performance | 108 |

One more idea

- Systemic Risk (very hot topic)
- See paper for web site:

A Survey of Systemic Risk Analytics

Dimitrios Bisias¹

Mark Flood²

Andrew W. Lo³

Stavros Valavanis⁴

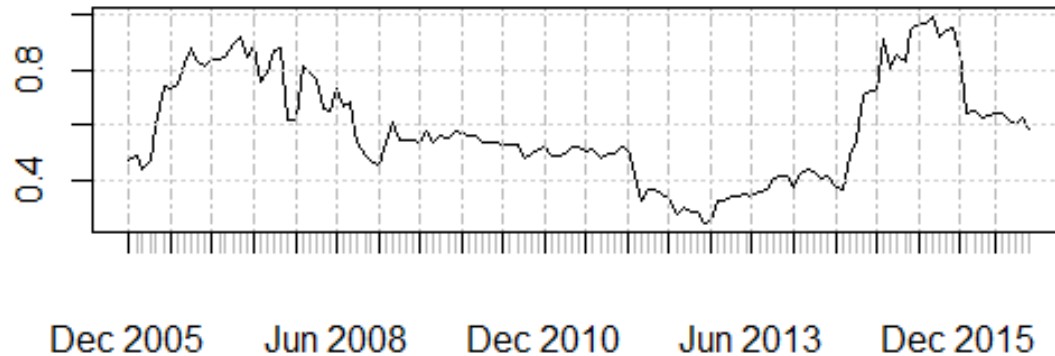
Rolling Betas

- Beta depends on the time period used; there is a package called `roll` that allows one to easily calculate Beta over different time periods.

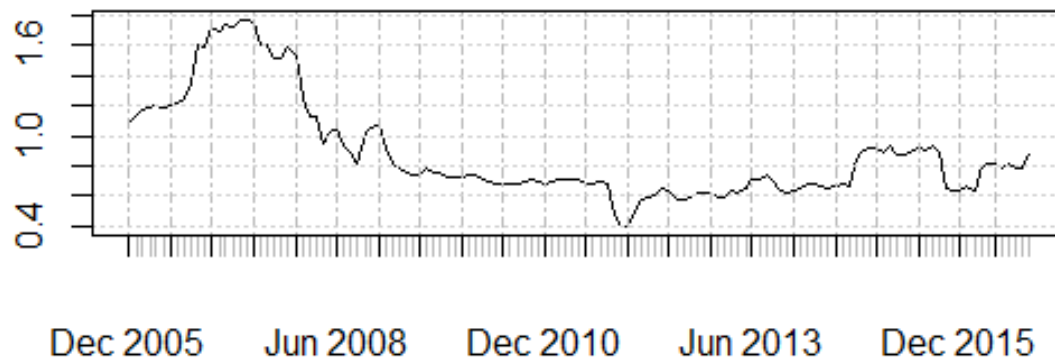
```
library(quantmod)
library(roll)
getSymbols("SPY", from="2000-01-01")
getSymbols("IBM", from="2000-01-01")
getSymbols("PG", from="2000-01-01")
spyret=monthlyReturn(Ad(SPY))
ibmret=monthlyReturn(Ad(IBM))
pgret=monthlyReturn(Ad(PG))
betas=roll_lm(spyret,pgret,width=36)
```

Results

Beta of PG over time



Beta of IBM over time



Systemic Risk

■ *“Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy.”*

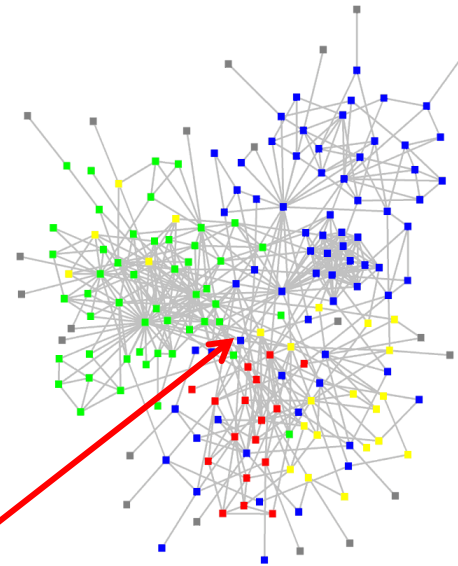
■ Daniel Tarullo

■ Federal Reserve Governor

Model of Systemic Risk

- Stressed firms sell assets (adjust balance sheet)
- Market (Liquidity) failure, firms sell new assets and stress new markets
- Market panic and there are runs on markets

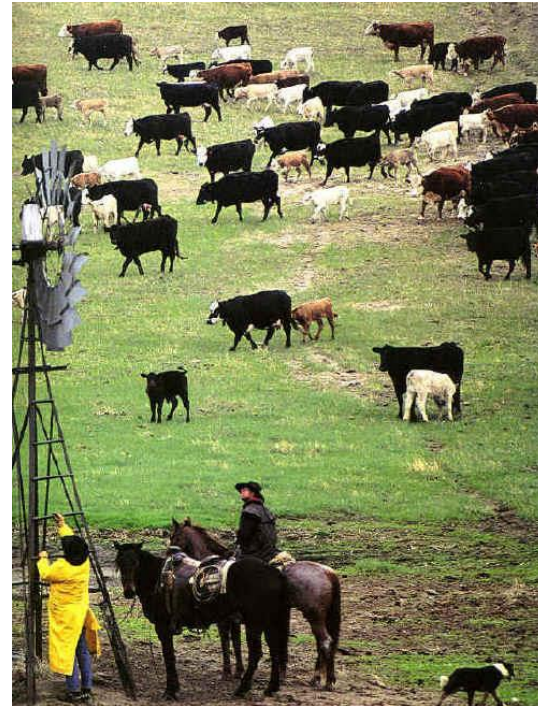
Domino Risk



\$3T CDS Notional outstanding
AIG goes down, who else goes?

\$185b Gov. Loans

Contagion Risk



What is the correlation?

Contagion Risk

- Hunt Brothers 'cornered' Silver market
 - 1980 controlled 1/3 of world silver
 - Family fortune of \$5b
 - Margin requirements were changed
 - Price dropped 50% on March 27, 1980



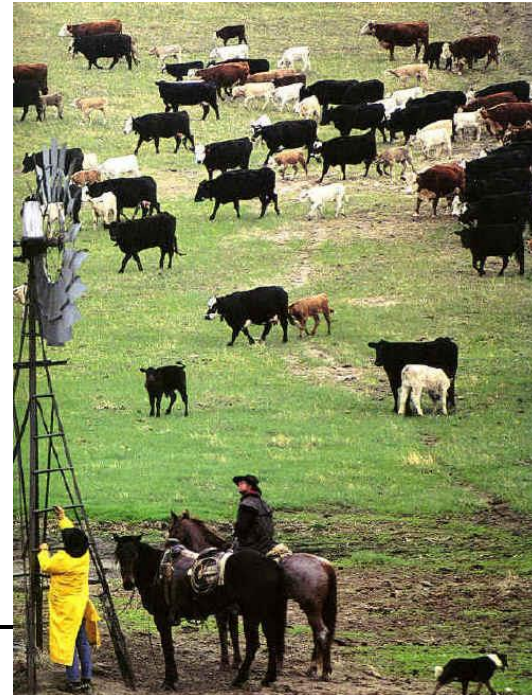
\$21.62



\$10.80

Contagion Risk

- Hunt Brothers 'cornered' Silver market
 - What else did the Hunt Brother's own?
 - Fire sale in silver
 - Liquidity dried up
 - Fire sale in cattle

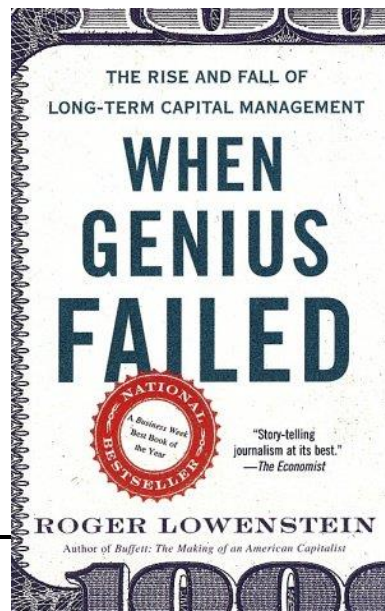


Contagion Risk

■ The Russian Financial Crisis

□ 1998 Long Term Capital Management

- Small exposure to Russian debt
- Large leveraged exposure to Danish debt



Contagion Risk

■ The Russian Financial Crisis

□ 1998 Long Term Capital Management

- Russia defaulted 1998
- Holders of Danish debt hit by default
- Fire sale in Danish debt
- LTCM connected to everyone

Monitoring Systemic Risk

■ Monitoring health of Economy

- Regime shifting models
- Summaries reflecting stress
 - Historical data
 - Derivatives data

■ Looking for Black Swans

- Leverage measures
- Concentrations, Bubbles
- Liquidity capacity
- Linkages and Transparency



The NYU Stern Systemic Risk Rankings

Robert Engle,
NYU Stern School of Business
Macro Prudential Regulation as an Approach to Contain
Systemic Risk

WHICH FINANCIAL FIRMS POSE THE GREATEST RISK TO THE GENERAL ECONOMY?

- HOW CAN FIRMS CAUSE SUCH RISK?
- HOW CAN WE MEASURE THESE RISKS?
- COULD WE HAVE PREDICTED THE FIRMS
THAT WERE THE CASUALTIES OF THE LAST
CRISIS?



■ Over a year ago the Volatility Institute introduced the Volatility Laboratory – an online web site with daily estimates of *volatilities* and *correlations* for a wide range of assets and methods. It has now grown to include hundreds of assets in:

- ❑ Commodities
- ❑ Equity Sectors
- ❑ Equity Names
- ❑ International Equities
- ❑ Exchange Rates
- ❑ Treasuries
- ❑ Corporates
- ❑ Real Estate
- ❑ Volatility Indices

The Stern Risk Page

- Now Stern introduces a page providing estimates of risk for the 102 largest US Financial firms.
 - Risk is estimated both for the firm itself and for its contribution to risk in the system.
 - This is called the *NYU Stern Systemic Risk Ranking*.
 - This is updated weekly/daily to allow regulators, practitioners and academics to see early warnings of system risks.
-

NYU STERN SYSTEMIC RISK RANKINGS

The RISK page of the Volatility Laboratory presents a variety of risk measures for major financial firms, both Globally and in the United States. These measures are updated weekly and reveal several dimensions of risk. Historical estimates of each of these risk measures can be plotted to see the changing performance of individual firms.

Global Systemic Risk Top Ten

| TOP 10 | SRISK% | MES | LVG |
|----------------------------------|--------|-------|--------|
| Deutsche Bank AG | 4.4 | 9.59 | 69.39 |
| BNP Paribas | 3.5 | 9.39 | 41.27 |
| Mitsubishi UFJ Financial Group | 3.4 | 4.62 | 37.84 |
| Barclays PLC | 3.3 | 8.55 | 47.77 |
| Credit Agricole SA | 3.2 | 12.95 | 133.97 |
| Royal Bank of Scotland Group PLC | 3.1 | 8.49 | 46.25 |
| HSBC Holdings PLC | 2.8 | 6.95 | 15.71 |
| Mizuho Financial Group Inc | 2.7 | 5.05 | 51.09 |
| Bank Of America | 2.3 | 5.79 | 22.76 |
| ING Groep NV | 2.3 | 9.88 | 46.02 |

Measuring Systemic Risk

- Acharya, Pedersen, Philippon, and Richardson(2010) propose the use of market data to estimate systemic risk contributions of firms.
- Their central measure is **Marginal Expected Shortfall or MES**.
- This paper develops a new dynamic methodology to estimate MES from equity data.
- MES is combined with leverage and size data to measure systemic risk contributions.

THE APPROACH

- We want to estimate for firm i , the loss in a future crisis:

$$E(Equity\ Loss_i | Crisis)$$

- As we have little data on crises, it is necessary to carefully structure the problem.
- Estimate the expected equity losses for a firm from a modest decline in overall returns.
- Estimate equity loss and capital shortfall for leveraged firms in a crisis.

Mike's Empirical Estimation

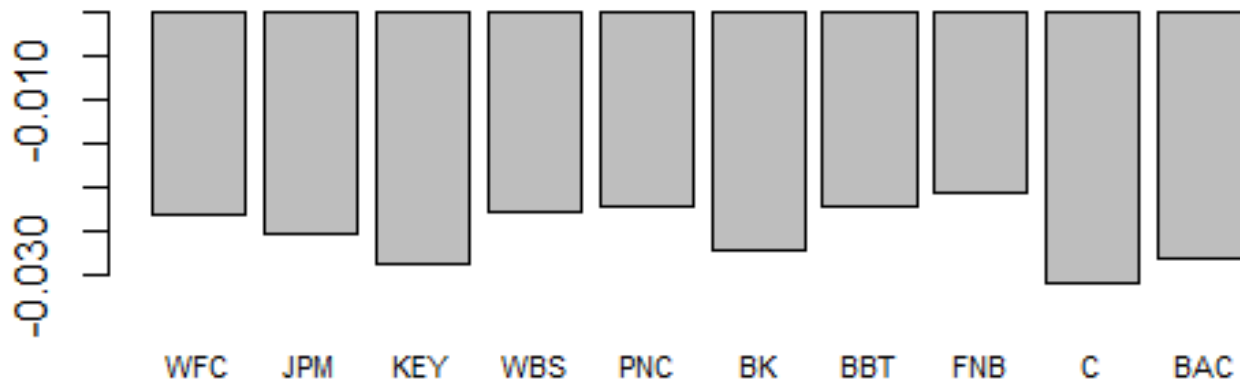
■ MES:

□ Very simple non-parametric estimation:

- find the 5% worst days for the market
- compute each institution's average return on these days

Very Simple Code

```
mymers = function(sname,spyret) {  
  
  x=getSymbols(sname,from="2014-01-01",auto.assign=FALSE)  
  xret=dailyReturn(Ad(x))  
  n=length(spyret)  
  myq = quantile(spyret,.05)  
  which=(1:n)[spyret<myq]  
  return(mean(xret[which]))  
}
```



Our Friend Dan.... in Wired Magazine

Numbers Don't Lie

Dan diBartolomeo head of a Boston financial analysis firm, spotted Bernard Madoff's \$50 billion scam. Here's what he sees coming next. —Daniel Roth

Wired: In 1999, you were hired by a money manager to reverse-engineer Madoff's investment strategy. When did you realize something was amiss?

Dan diBartolomeo: All we had were the monthly returns that Madoff reported to investors. We spent a couple of hours on mathematical analysis, playing around with regressions and spreadsheets, and concluded that the results couldn't have come from the strategy he described.

Wired: Did you immediately think fraud?

diBartolomeo: It was possible that he was using some other strategy he wasn't disclosing. But to get returns like that, he would have needed to be three or four times more skillful than the next-best manager. He also could have been using a strategy that gave him an



Some of Dan's Papers

- Approximating the confidence intervals for Sharpe style weights
- Socially screened portfolios: an attribution analysis of relative performance
- Mutual fund misclassification: Evidence based on style analysis
- Managing Risk Exposures of Socially Screened Accounts

Style Analysis

- Dan likes to talk a little about mutual fund classification.
- In the old days, before a lot of internet watchdog financial sites, mutual funds could self categorize themselves:
 - Growth, Value, Blend, Growth and Income, International, Income, etc.....
- One can also do style analysis on hedge funds

Example: Morningstar

■ Fidelity Magellan

Morningstar Style Box

Large Growth

[\[View Category Definition\]](#)

| | | | |
|-------|-------|--------|---|
| | | | Size Large Medium Small |
| | | | |
| | | | |
| Value | Blend | Growth | Investment Valuation |

[View Top Large Growth Funds](#)

[About the Morningstar Style Box](#)

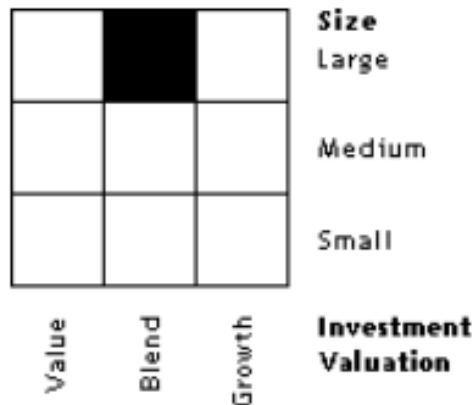
Example: Morningstar

■ Permanent Portfolio (PRPFX)

Morningstar Style Box

Conservative Allocation
[\[View Category Definition\]](#)

[View Top Conservative Allocation Funds](#)
About the Morningstar Style Box



Dan's point on style analysis

- If I am a fund manager compensated for how well I beat my benchmark....
- Maybe I want to be compared to the wrong benchmark....
- If I'm an investor, I absolutely want to make sure I'm investing in what I think I am invested in (I think my fund will win the mile race but its actually entered in the high jump....)

From last week's Crimson

Management Company Reportedly Described as 'Lazy,' 'Stupid' by Employees in 2015 Review

By DAPHNE C. THOMPSON, CRIMSON STAFF WRITER 2 days ago

HMC employees did not mince words in describing their employer to McKinsey, characterizing the culture of HMC as “lazy, fat and stupid” and centered on “stable, rather than smart, capital,” **according to Bloomberg**.

The McKinsey report, which studied fiscal years 2009 through 2014, blamed easily attainable benchmarks—so-called “slow rabbits”—for creating a culture of complacency at HMC, Bloomberg reported.

“This is the only place I’ve seen where people can negotiate the benchmark they get compensated on,” read one “representative quote” from an HMC employee in the McKinsey review.

A true Story

■ Fidelity Asset Manager Mutual Fund

Diversification Made Simple

Fidelity Asset Manager. Now there's an easier way to diversify across a broad range of securities. This fund seeks high total return with reduced risk over the long term by allocating its assets among domestic and foreign stocks, bonds, and short-term/money market instruments.

All It Takes Is One Investment



You simply make one investment, with no sales charge, and join a diversified portfolio which is carefully watched and gradually adjusted by Fidelity professionals who seek to enhance your return in any market environment. Share price and return will vary so you may have a gain or loss when you sell your shares. Call today for a free **Fidelity Asset Manager** Fact Kit. Start with just \$2,500, or \$500 for IRAs.

| |
|---|
| Average Annual Total Returns* as of December 31, 1992 |
| 12.75% |
| 1 Year |
| 14.11% |
| Life of fund 12/28/88-12/31/92 |

■ Convinced girl friend to invest in it late 1993ish

Historically it had done well



■ Historically

| | | |
|------|---|-------|
| 1993 |  | 26.32 |
| 1992 |  | 20.03 |

■ Then 1994

| | | |
|------|---|-------|
| 1994 |  | -7.39 |
|------|---|-------|

■ Compare to Magellan

| | | |
|------|--|-------|
| 1994 |  | -1.81 |
| 1993 |  | 24.66 |
| 1992 |  | 7.01 |

It was supposed to be SAFE

- Manager Robert Beckwitt made a huge, big, bad bet on Mexico.

Causes of the economic crisis of 1994

The crisis is also known in Spanish as *el error de diciembre* — *The December Mistake* — a term coined by outgoing president [Carlos Salinas de Gortari](#) in reference to president Zedillo's sudden reversal of the former administration's policy of tight currency controls. While most critics agree that devaluation was necessary for economic reasons, they argue that the process was mishandled on the political level.

- Love his 1994 interview quote:

Q. BOB, HOW HAS THE FUND
PERFORMED RECENTLY?

A. Better than its peers. That said, the investment climate was such that the absolute performance probably disappointed many investors.

To be fair (and he did recover later)

■ He also said

Q. HAS THE RECENT TURMOIL IN MEXICO CHANGED YOUR THINKING ABOUT THAT COUNTRY'S INVESTMENT POTENTIAL?

A. No. I've had long-standing investments in Mexico - not only stocks but short-term bonds denominated in pesos and longer-term bonds denominated in U.S. dollars. Mexican investments totaled 9% of the fund at the end of March. I increased the fund's stake last summer in Mexican stocks in anticipation of passage of the North American Free Trade Agreement, and the fund benefited as a result. Unfortunately, the January revolt in Chipas and the March assassination of Mexico's leading presidential candidate upset the markets, and the fund gave back about half of its recent gains. I still believe in Mexico's long-term prospects for economic reform. Unless conditions worsen, I plan on staying invested.

My Point

- I don't feel the direction and purpose of the fund was adequately disclosed.
- It was supposed to be a safe, low risk, asset management fund; some equities, some cash, some bonds.
- Who knew it was going to make a big international bet?

Style Analysis

■ From Investopedia

Style Analysis



What Does *Style Analysis* Mean?

The process of determining what type of investment behavior an investor or money manager employs when making investment decisions. Virtually all investors subscribe to a form of investment philosophy, and a prudent analysis of a money manager's style needs to be performed before an investor can determine whether the manager will be good fit for his or her personal investment goals and preferences.



Investopedia explains *Style Analysis*

There is virtually an unlimited number of investment styles; however, some of the most common types of investment styles are categorized as growth investing, value investing, large cap investing, small cap investing and active trading.

Some money managers change their investment styles over time, opting to go with one approach while it is working well and then switching to another when the old approach seems to be losing its luster.

As Dan writes...

- **An inaccurate classification system produces wrong signals, puts investor funds in the hands of those who are not necessarily best qualified to manage them, and ultimately allocates assets into projects that are not optimal.**
 - **On a micro level, misclassification has implications for individual investors. For example, a fund that labels itself "income" but invests a large portion of its assets in small, growth-oriented stocks may have risk and return parameters that are inappropriate for a retired couple.**
 - **The reliability of any classification system that the investing public relies on is of utmost importance.**
 - **Further, Mutual fund data vendors rely heavily on the funds themselves for classification information.**
-

Returns Based Classification Method

- Style analysis is a more recent returns-based approach to measuring the performance of an investment fund.
- Sharpe (1992) pioneered it in the early nineties by developing an 'asset class factor model' to distinguish the performance of different funds with respect to 'style' and 'selection'.
- Style analysis can also be viewed as reverse engineering the asset mix in a portfolio.

Returns Based Classification Method

- The objective of style analysis is to construct a benchmark portfolio, from a set of known indices (for which returns are available), against which to compare the performance of an investment fund's actively-managed portfolio.
- Ideally, the indices should reflect activity in different asset classes, they should be mutually exclusive and exhaustive, and their assets publicly quoted so that they can be tracked 'passively'.
- For example, Sharpe used 12 indices to cover the range of investment options available to US funds. The indices were chosen to have as little overlap as possible.

Style Analysis

- Consider the 6 most widely used categories of equity mutual funds:
 - aggressive growth (f_1)
 - growth (f_2)
 - growth-income (f_3)
 - income (f_4)
 - International (f_5)
 - small capitalization (f_6)

The Model

- We fit the model

$$r_t = b_1 f_{1t} + b_2 f_{2t} + b_3 f_{3t} + b_4 f_{4t} + b_5 f_{5t} + b_6 f_{6t} + e_t$$

- Sharpe suggests finding the weights b_i 's to minimize the tracking error:

$$\sum_t |r_t - b_1 f_{1t} + b_2 f_{2t} + b_3 f_{3t} + b_4 f_{4t} + b_5 f_{5t} + b_6 f_{6t}|$$

- Since they make up a portfolio, the weights are constrained to sum to 100% with the individual weights normally lying between 0% and 100%. (However this latter condition can be modified for funds that are allowed to hold assets short.)

Style Weights

- The weights determined by optimization are called the style weights and, when combined with the indices, form the benchmark portfolio.
- We say that the fund with optimized style weights for the different indices is of the same style as the investment fund.
- The weights can be found using SOLVER in Excel.

Example

- Consider the following indices
 - JKD : Large Core Index Fund
 - JKG : Mid Cap Core Index Fund
 - JKJ : Small Cap Core Index Fund
 - EFA : International Index Fund
 - AGG : Bond Fund

FMAGX Style Analysis

■ What we did in R

```
getSymbols("JKD")
getSymbols("JKG")
getSymbols("JKJ")
getSymbols("EFA")
getSymbols("AGG")
getSymbols("FMAGX")
jkd=monthlyReturn(Ad(JKD))
jkg=monthlyReturn(Ad(JKG))
jkj=monthlyReturn(Ad(JKJ))
efa=monthlyReturn(Ad(EFA))
agg=monthlyReturn(Ad(AGG))
fmagx=monthlyReturn(Ad(FMAGX))

fit=lm(fmagx~-1+jkd+jkg+jkj+efa+agg) (no intercept)
```

The FMAGX Output

■ summary(fit)

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|------------|----------------|----------------|--------------|-------------------|
| jkd | 0.14573 | 0.21408 | 0.681 | 0.49947 |
| jkg | 0.76318 | 0.26326 | 2.899 | 0.00572 ** |
| jkj | -0.13730 | 0.20966 | -0.655 | 0.51580 |
| efa | 0.27465 | 0.12828 | 2.141 | 0.03761 * |
| agg | -0.03724 | 0.23619 | -0.158 | 0.87542 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02168 on 46 degrees of freedom

Multiple R-squared: 0.9124, **Adjusted R-squared: 0.9029**

Rolling the Estimates

- To ensure that the style of the fund doesn't change over time, analysts will roll the data.
- That means they will calculate the style weights over rolling time periods (12-36 month windows), to see if the style weights are drastically changing over time.

Constraining the Weights

- Very easy to read paper on style analysis (on web site)

Returns-Based Style Analysis: An Excel-Based Classroom Exercise

John McDermott ^a

^a Fairfield University , Fairfield, Connecticut, USA

Published online: 07 Aug 2010.

Style Analysis with Solver

- See this week's homework
- Spreadsheet styleanalysis.xls

| Style Weights | |
|----------------------|----------|
| JKD | 0 |
| JKG | 0.027419 |
| JKJ | 0 |
| EFA | 0 |
| AGG | 0.175302 |
| SHY | 0.797279 |
| | |
| constraint | 1 |
| | |
| sum of deviations | 0.096746 |

More than 1 factor?

- The market model is a one factor model: The only determinant of expected returns is the systematic risk of the market. This is the only factor.
- What if there are multiple factors that determine returns?
- Multifactor Models: Allow for multiple sources of risk, that is **multiple risk factors**.

Multifactor Models

- Use other factors in addition to market returns:
 - Examples include industrial production, expected inflation etc.
 - Estimate a **beta** or *factor loading* for each factor using multiple regression

Example: Multifactor Model Equation

$$R_i = \beta_0 + \beta_1 R_M + \beta_2 R_{GDP} + \beta_3 R_{IR} + \varepsilon_i$$

R_i = Return for security i

β_2 = Factor sensitivity for GDP

β_3 = Factor sensitivity for Interest Rate

ε_i = Firm specific events

Multifactor Models

- The Market Model says that a single factor, **Beta**, determines the return between a portfolio and the market as a whole.
- Suppose however there are other factors that are important for determining portfolio returns.
- The inclusion of additional factors would allow the model to improve the model's fit of the data.
- The best known approach is the three factor model developed by Gene Fama and Ken French.

Fama-French 3-Factor Model

- They added these two factors to a standard market model

$$R_{i,t} = \alpha_i + \beta_{i1}(R_{m,t}) + \beta_{i2}SMB_t + \beta_{i3}HML_t + \varepsilon_{i,t}$$

SMB = “small [market capitalization] minus big”

"Size" This is the return of small stocks minus that of large stocks. When small stocks do well relative to large stocks this will be positive, and when they do worse than large stocks, this will be negative.

HML = “high [book/price] minus low”

"Value" This is the return of value stocks minus growth stocks, which can likewise be positive or negative.

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

The Fama-French Three Factor model explains over 90% of stock returns.

Back to Regression-Why Least Squares?

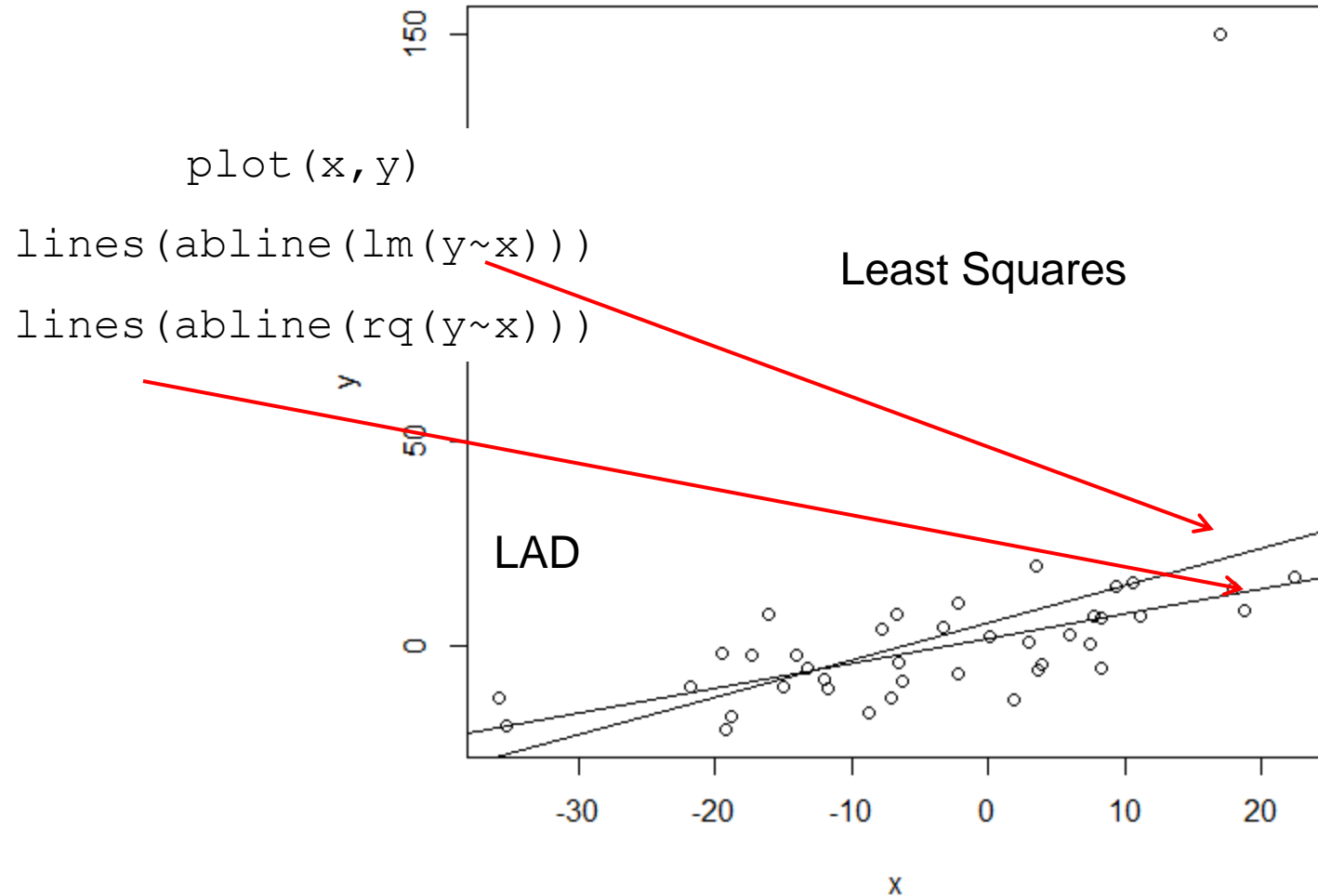
- As a quick aside, (least squares) regression fits a line by solving the equation

$$\min_{b_0, b_1} \sum_{i=1}^n (Y_i - b_0 - b_1 X_i)^2 \quad \text{lm}(y \sim x)$$

- Why not solve the following equation? This is called LAD (least absolute deviation) regression.

$$\min_{b_0, b_1} \sum_{i=1}^n |Y_i - b_0 - b_1 X_i| \quad \begin{array}{l} \text{rq}(y \sim x) \\ \text{[in package} \\ \text{quantreg]} \end{array}$$

Least Squares versus MAD



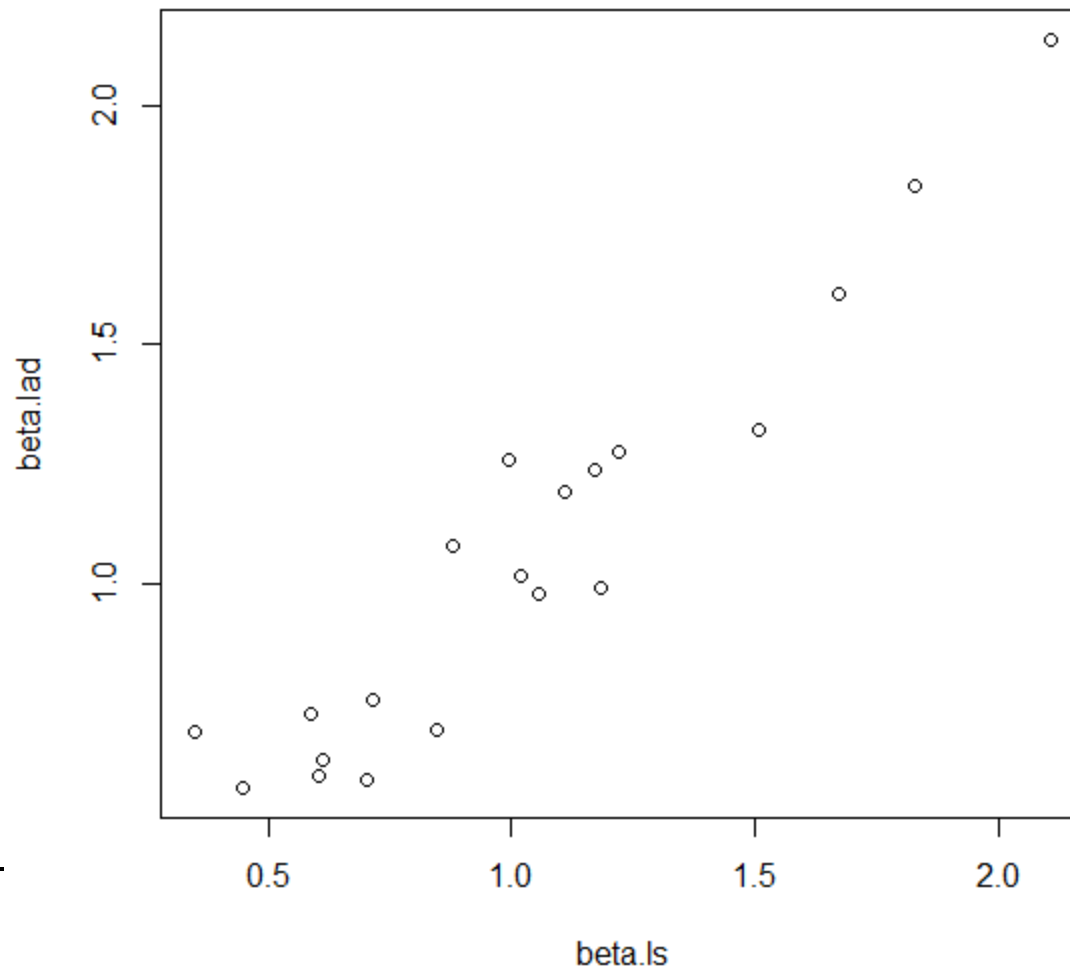
LAD Betas

```
asset.names=c("ATVI", "ADBE", "AKAM", "ALTR", "AMZN",  
"AMGN", "APOL", "AAPL", "AMAT", "ADSK", "ADP", "BIDU",  
"BBBY", "BIIB", "BMC", "BRCM", "CHRW", "CA", "CELG",  
"ANF")
```

```
n=length(asset.names)  
getSymbols("SPY")  
spy.ret=monthlyReturn(Ad(SPY))  
beta.ls=1:n  
beta.lad=1:n  
  
for(i in 1:n){  
x=getSymbols(asset.names[i], auto.assign=FALSE)  
x.ret=monthlyReturn(Ad(x))  
beta.ls[i]=coef(lm(x.ret~spy.ret))[2]  
beta.lad[i]=coef(rq(x.ret~spy.ret))[2]  
}
```

Which ones are correct?

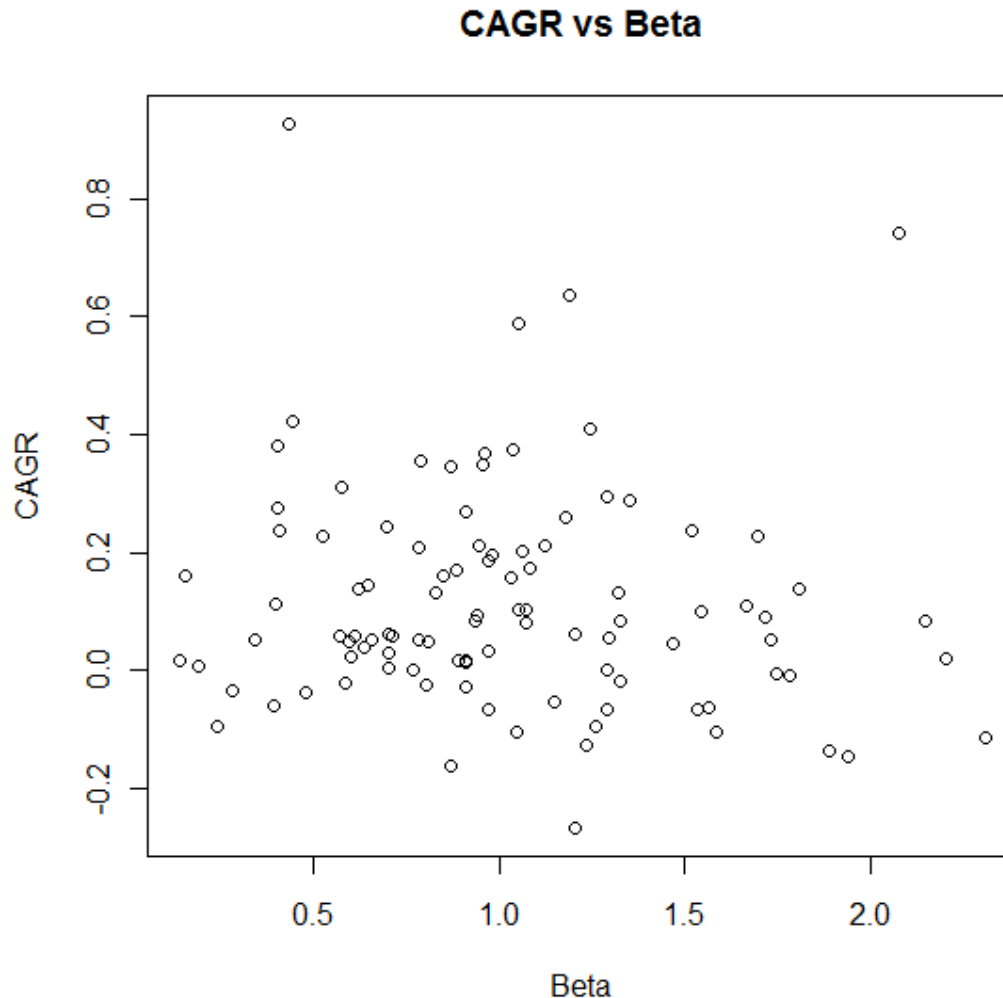
```
> cor(beta.ls,beta.lad)  
[1] 0.9531647
```



CAGR and Beta

- According to the CAPM, a bigger Beta implies a bigger expected stock return.
- So what should a graph of CAGR (compounded annual return) versus Beta look like?
- Probably not what you expect.

Beta vs CAGR for Nasdaq 100 stocks



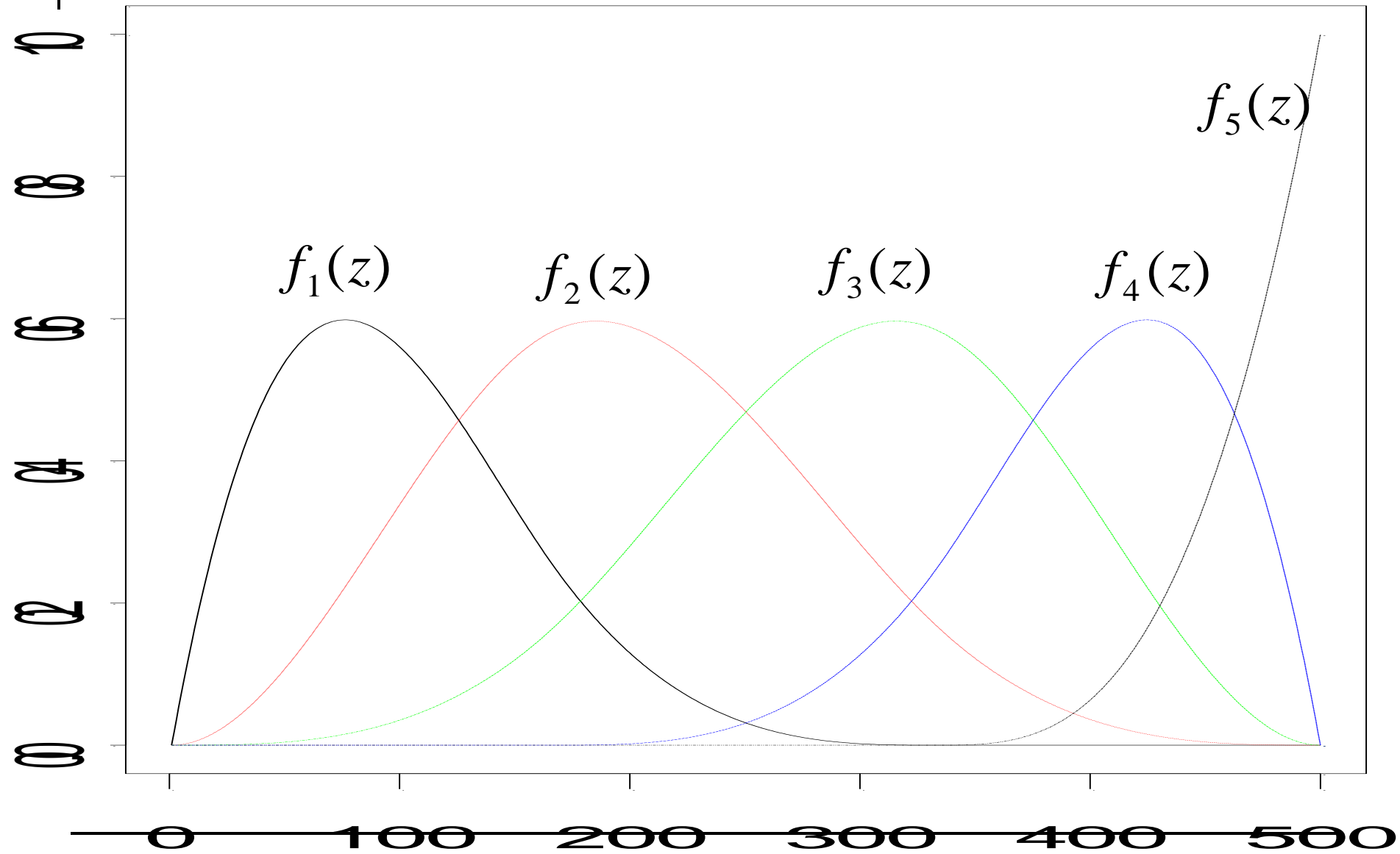
To be fair, we should see what this looks like during a bull market like 1996-2000, but it will look even worse.

Semi-Parametric Modeling

- Before we move to logistic regression, a quick look at some interesting functions built into R.
- We have already seen the idea of splines with density estimation; it can also be used to model non-linear relationships between x and y .

$$Y = f(z) + \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon$$

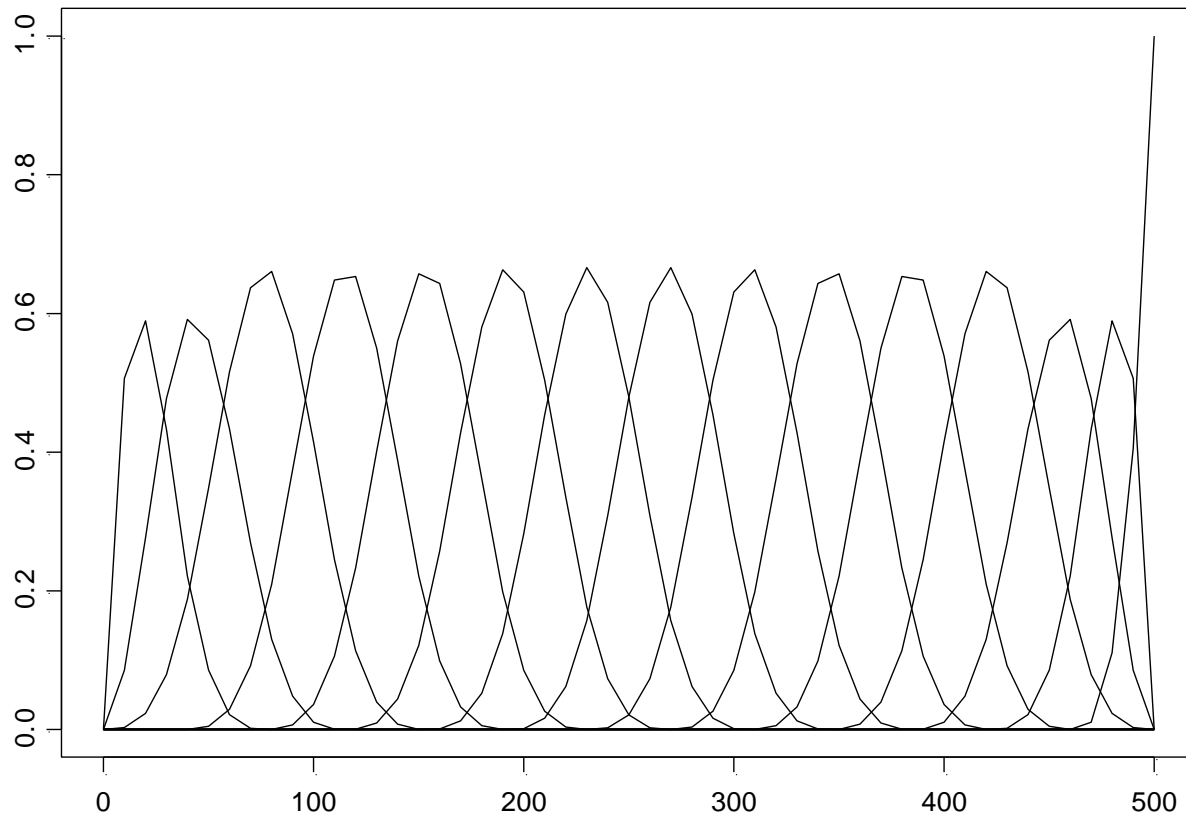
Cubic Spline Basis Functions



The plot on the last slide was made using the following code (library splines was loaded):

```
x=seq(0,500,10)  
foo=bs(x,5)  
matplot(x,foo,type="l")
```

There is almost no limit as to how many basis functions
you can compute:



$$f(z) \approx \sum_{i=1}^5 \alpha_i f_i(z)$$

$$Y = f(z) + \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon$$

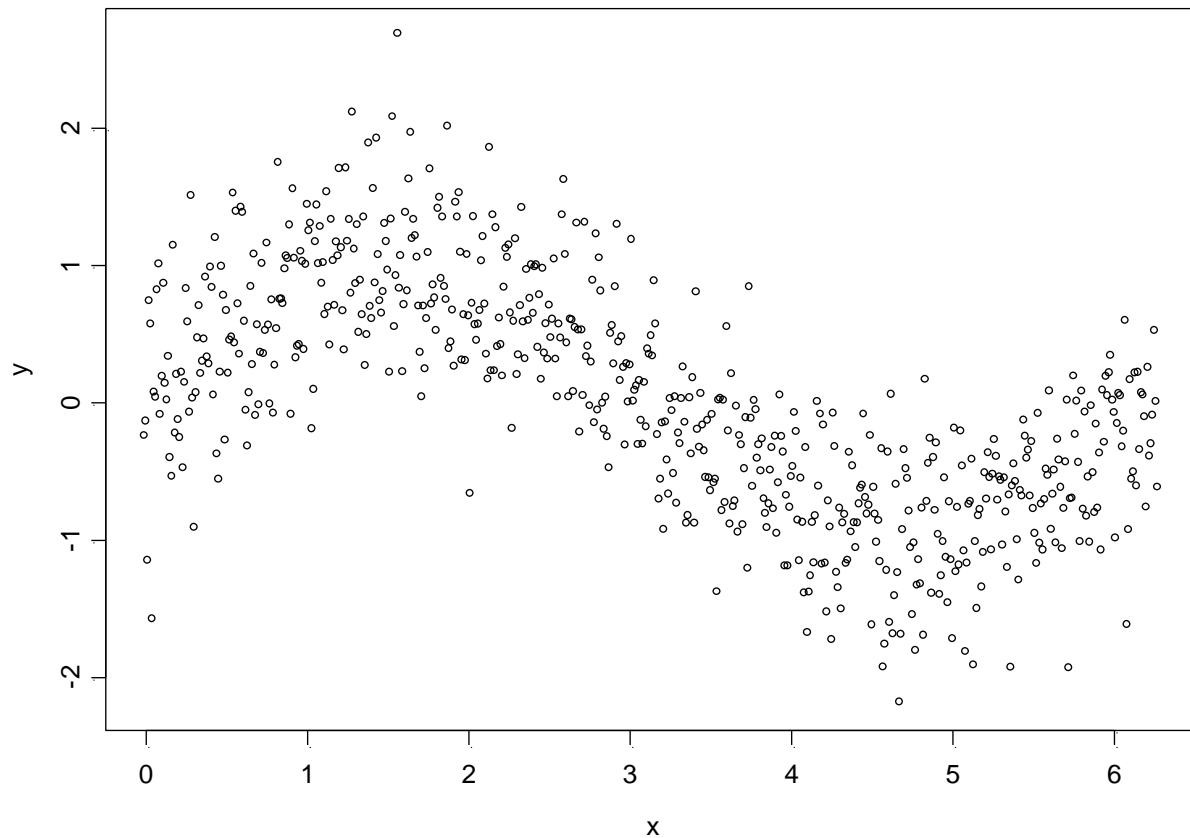
\Rightarrow

$$Y \approx \sum_{i=1}^5 \alpha_i f_i(z) + \beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k + \varepsilon$$

We use ordinary linear regression to maximize over all the unknown parameters.

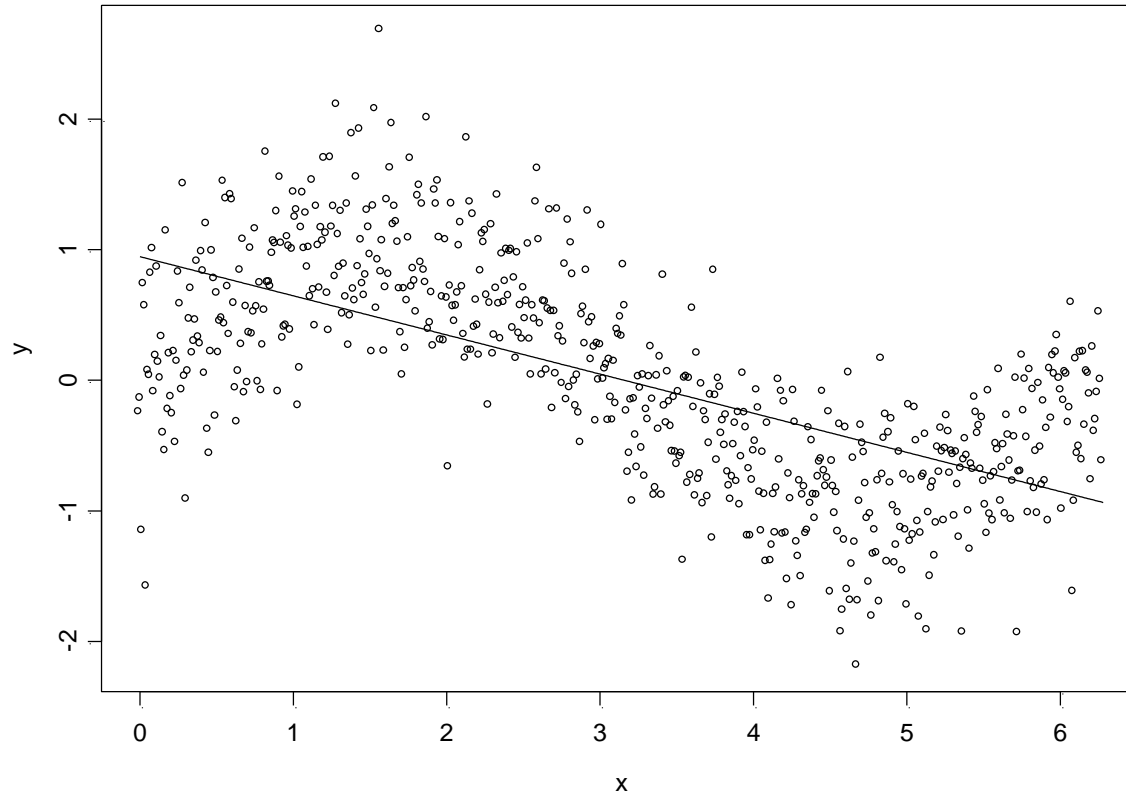
Example

```
> x=seq(0,2*pi,.01)
> length(x)
[1] 629
> y=sin(x)+rnorm(629,s=.5)
```



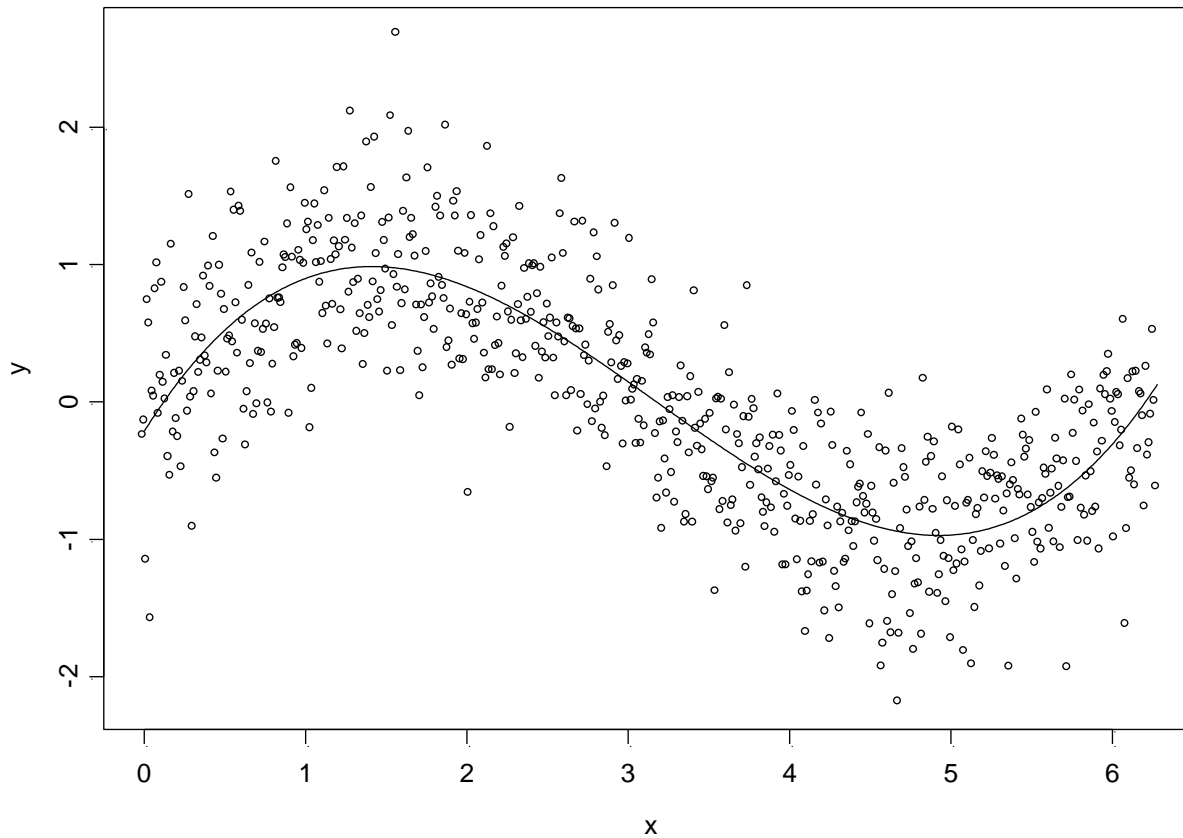
Regression ? (No way)

```
> fit=lm(y~x)
> plot(x,y)
> lines(x,fit$fitted.values,type="l")
```



Spline Fit

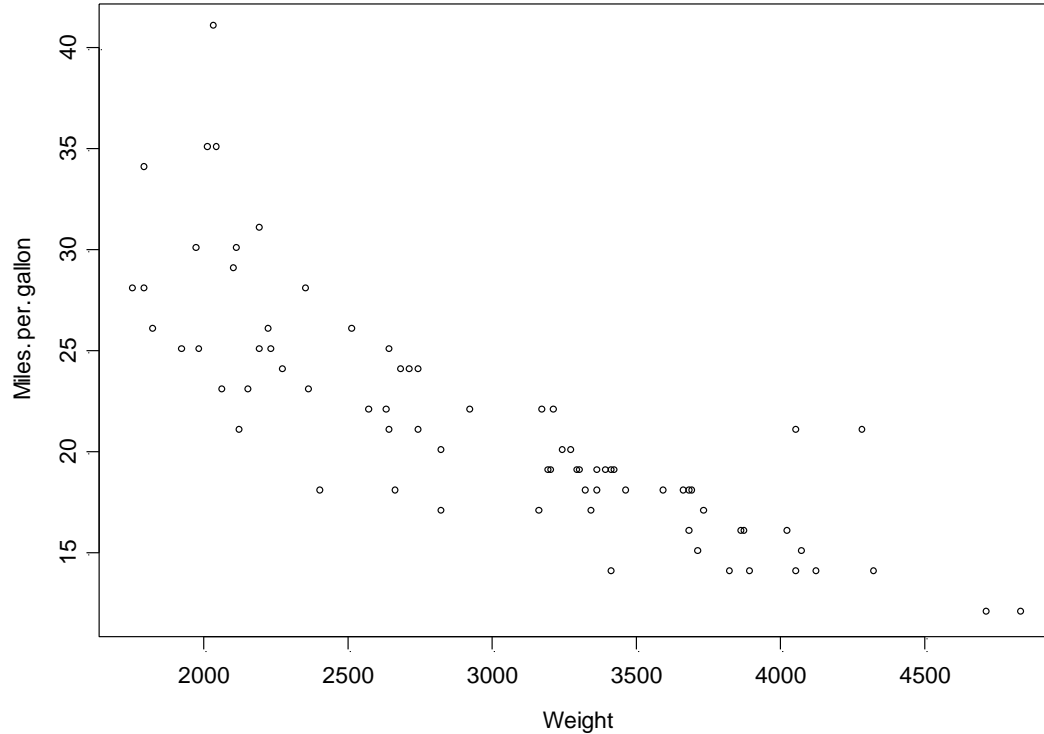
```
> fit=lm(y~bs(x))  
> plot(x,y)  
> lines(x,fit$fitted.values,type="l")
```



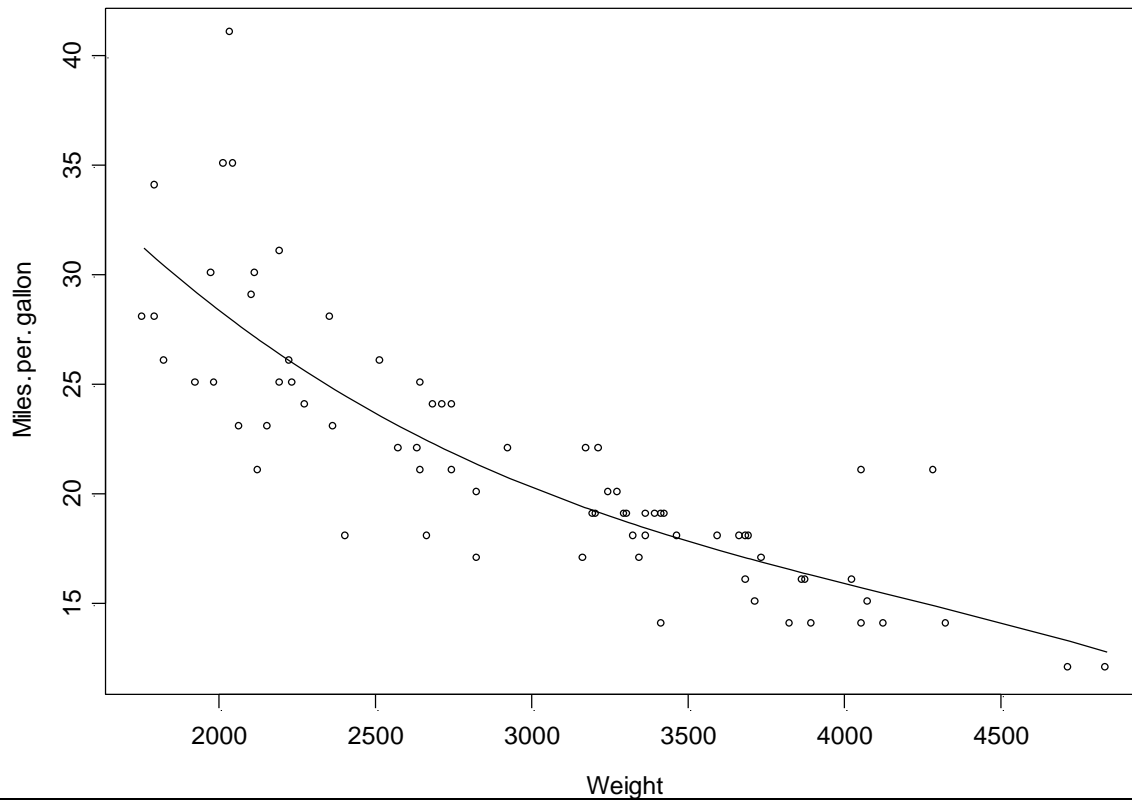
Example:

Car data : model mpg as a function of weight

$$mpg = f(weight) + \varepsilon$$



```
> fit=lm(Miles.per.gallon~bs (Weight,3))  
      > plot(Weight,Miles.per.gallon)  
      > ord=order(Weight)  
> lines(Weight[ord],(fitted(fit))[ord])  
  
>
```



```
> summary(fit)
```

```
Call: lm(formula = Miles.per.gallon ~ bs(Weight, 3))
```

```
Residuals:
```

| Min | 1Q | Median | 3Q | Max |
|--------|--------|---------|-------|-------|
| -6.415 | -1.556 | -0.2815 | 1.265 | 13.06 |

```
Coefficients:
```

| | Value | Std. Error | t value | Pr(> t) |
|----------------|----------|------------|---------|----------|
| (Intercept) | 31.2145 | 1.3047 | 23.9243 | 0.0000 |
| bs(Weight, 3)1 | -13.0441 | 4.1146 | -3.1702 | 0.0023 |
| bs(Weight, 3)2 | -14.0931 | 2.8697 | -4.9110 | 0.0000 |
| bs(Weight, 3)3 | -18.4401 | 2.8696 | -6.4260 | 0.0000 |

```
Residual standard error: 3.209 on 70 degrees of freedom
```

```
Multiple R-Squared: 0.705
```

```
F-statistic: 55.76 on 3 and 70 degrees of freedom, the p-value is 0
```

Notes:

Too few knots give a rough fit, too many knots give too smooth a fit.

One can model each variable in the regression using a spline:

```
Fit=lm(Price~bs(Miles.per.gallon,3)+bs(Weight,3)+bs(Headroom,3))
```

One drawback is that this uses up a lot of degrees of freedom (needs a lot of data).