Stat 107: Introduction to Business and Financial Statistics

Homework 4 Solutions

Note-for the Teall problems the solutions were in the back of the book. We want to make sure work was shown on individual problem sets.

1) Teall book, page 113, problem 6.1

6.1. (a) $\bar{R}_p = (w_M \cdot \bar{R}_M) = (w_H \cdot \bar{R}_H) = (0.25 \cdot 0.20) + (0.75 \cdot 0.06) = 0.09;$

(b) $\sigma_p^2 = w_H^2 \cdot \sigma_H^2 + w_M^2 \cdot \sigma_M^2 + 2 \cdot w_H \cdot w_M \cdot \sigma_H \cdot \sigma_M \cdot \rho_{H,M};$

$\sigma_p^2 = 0.75^2 \cdot 0.09^2 + 0.25^2 \cdot 0.30^2 + 2 \cdot 0.75 \cdot 0.25 \cdot 0.09 \cdot 0.30 \cdot 0.4 = 0.05068;$

(c) $\sigma_p = \sqrt{0.05068} = 0.2251,$

since the standard deviation is the square root of the variance.

2) Teall book, page 113, problem 6.4

6.4. (a) $\bar{R}_p = 0.075, \quad \sigma_p = 0.16;$

(b) $\bar{R}_p = 0.075, \quad \sigma_p = 0.116619;$

(c) $\bar{R}_p = 0.075, \quad \sigma_p = 0.04.$

Show work! Returns for all three correlation situations will be equal since correlation effects risk but not return. The return of a 50/50 portfolio is 5%(.5)+5%(10%)=7.5%

In general the risk is $\sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho}$

For perfect correlation this is

```
> sqrt(.5*.5*.12*.12+.5*.5*.20*.20+2*.5*.5*1*.12*.20)
[1] 0.16
```

For zero correlation this is

```
> sqrt(.5*.5*.12*.12+.5*.5*.20*.20+2*.5*.5*0*.12*.20)
[1] 0.116619
```

For perfectly inversely correlated this is

```
> sqrt(.5*.5*.12*.12+.5*.5*.20*.20+2*.5*.5*(-1)*.12*.20)
[1] 0.04
```

3) Teall book, page 114, problem 6.6

6.6. $E[R_p] = 0.33333 \cdot 0.25 + 0.16667 \cdot 0.15 + 0.5 \cdot 0.05 = 0.13333;$

$$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + w_3^2 \cdot \sigma_3^2 + 2w_1 \cdot w_2 \cdot \sigma_{1,2} + 2w_1 \cdot w_3 \cdot \sigma_{1,3} + 2w_2 \cdot w_3 \cdot \sigma_{2,3};$$

$$0.02444 = 0.33333^2 \cdot 0.40^2 + 0.16667^2 \cdot 0.20^2 + 0.5^2 \cdot 0^2$$
$$+ 2 \cdot 0.16667 \cdot 0.33333 \cdot 0.05 + 2 \cdot 0.16667 \cdot 0.5 \cdot 0 + 2 \cdot 0.33333 \cdot 0.5 \cdot 0;$$

$$\sigma_p^2 = 0.02444, \qquad \sigma_p = 0.15635.$$

4) Teall book, page 114, problem 6.8

6.8. (a) Recall that correlation coefficients (and covariances) equal zero:

$$\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 = 0.5^2 \cdot 0.8^2 + 0.5^2 \cdot 0.8^2 = 0.32, \qquad \sigma_p = 0.565685.$$

(b) $\sigma_p^2 = w_1^2 \cdot \sigma_1^2 + w_2^2 \cdot \sigma_2^2 + w_3^2 \cdot \sigma_3^2 + w_4^2 \cdot \sigma_4^2 = 0.25^2 \cdot 0.8^2 + 0.25^2 \cdot 0.8^2 + 0.25^2 \cdot 0.8^2 + 0.25^2 \cdot 0.8^2 = 0.16, \qquad \sigma_p = 0.4.$

(c) $\sigma_p^2 = 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 + 0.125^2 \cdot 0.8^2 = 0.08, \qquad \sigma_p = 0.282843.$

(d) $\sigma_p^2 = 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.065^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.065^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 + 0.0625^2 \cdot 0.8^2 = 0.04, \qquad \sigma_p = 0.2.$

(e) They don't differ. Expected portfolio returns are always a weighted average of component security expected returns, which is always 0.10 in this example.

5) One can show mathematically that if two stocks have correlation of -1, then if one puts $\frac{s_2}{s_1 + s_2}$ x100% of their money in stock 1, and the rest in stock 2, the resulting portfolio with have 0 risk. But will the portfolio have a positive return? Obtain daily returns for QID and QLD from 2015-01-01 to 2016-01-01.

a) Verify that QLD returns and QID returns have a correlation that is essentially -1

```
> getSymbols("QLD",from="2015-01-01",to="2016-01-01")
[1] "QLD"
> getSymbols("QID",from="2015-01-01",to="2016-01-01")
[1] "QID"
> qldret=dailyReturn(Ad(QLD))
> qidret = dailyReturn(Ad(QID))
> cor(qidret,qldret)
               daily.returns
daily.returns     -0.9994757
```

b) What weights are required for QLD and QID to have a 0 risk (standard deviation) portfolio?

```
> sd(qidret)/(sd(qidret)+sd(qldret))
[1] 0.5007477
```

Essentially 50/50 in each asset

c) Compute the mean and standard deviation of the portfolio from part (b). What do you find?

```
> ret=w1*mean(qidret)+w2*mean(qldret)
> risk = sqrt(w1*w1*var(qidret)+w2*w2*var(qldret)+2*w1*w2*cov(qldret,qidret))
> ret
[1] -4.311394e-05
> risk
             daily.returns
daily.returns  0.0003630051
>
```

You make no money with no risk.

6) Obtain monthly return data (from adjusted prices) for MRK, KORS, LULU and SPY from 2013-01-01 to 2016-08-01.

   a) What company does each symbol represent? Go to finance.yahoo.com to find out.
      MRK = Merk
      KORS = Michael Kors
      LULU = Lulu Lemon
      SPY = S&P500 Index

   b) What is the average monthly return for each of the stocks ? What is the standard deviation for the returns of the stocks ? What is the correlation between MRK and KORS, MRK and LULU and KORS and LULU ?

```
getSymbols("MRK",from="2013-01-01",to="2016-08-01")
getSymbols("KORS",from="2013-01-01",to="2016-08-01")
getSymbols("LULU",from="2013-01-01",to="2016-08-01")
getSymbols("SPY",from="2013-01-01",to="2016-08-01")
mrkret=monthlyReturn(Ad(MRK))
korsret=monthlyReturn(Ad(KORS))
luluret=monthlyReturn(Ad(LULU))
spyret=monthlyReturn(Ad(SPY))

stockrets=cbind(mrkret,korsret,luluret,spyret)
names(stockrets)=c("MRK","KORS","LULU","SPY")
apply(stockrets,2,mean)
apply(stockrets,2,sd)
cor(stockrets)


> apply(stockrets,2,mean)
        MRK        KORS        LULU         SPY
0.011597256 0.005769020 0.006237978 0.011089545
> apply(stockrets,2,sd)
       MRK       KORS       LULU        SPY
0.04405822 0.10601222 0.10010707 0.03044601
> cor(stockrets)
             MRK        KORS       LULU        SPY
MRK   1.00000000 -0.02751976 0.08884351 0.52097750
KORS -0.02751976  1.00000000 0.11935812 0.06530373
LULU  0.08884351  0.11935812 1.00000000 0.02450733
SPY   0.52097750  0.06530373 0.02450733 1.00000000
```

c) Give the expected return and standard deviation of all the possible two stock portfolios (MRK,KORS), (MRK,LULU), (KORS,LULU) with equal amounts invested in each stock (weights of .5 for each stock).

```
retrisk=function(w1,w2,ret1,ret2) {

ret = w1*mean(ret1)+w2*mean(ret2)
risk=sqrt(w1*w1*var(ret1)+w2*w2*var(ret2)+2*w1*w2*cov(ret1,ret2))
cat("Mean = ",ret,"Risk = ",risk,"\n")
}

> retrisk(.5,.5,mrkret,korsret)
Mean =  0.008683138 Risk =  0.0568389
> retrisk(.5,.5,mrkret,luluret)
Mean =  0.008917617 Risk =  0.05644964
> retrisk(.5,.5,luluret,korsret)
Mean =  0.006003499 Risk =  0.07712557
```

d) Rank the three portfolios based on their standard deviation. How do they compare with holding one of the individual stocks ?

The MRK/LULU portfolio has the lowest risk of the stock portfolios but it is still greater risk than just holding MRK by itself. Although the correlation between MRK and LULU is not that high, it is a positive relationship so holding them together doesn't offer diversification.