Stat 107: Introduction to Business and Financial Statistics
Homework 2: Due Monday, Sept 19

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(1)

Suppose monthly returns of AAPL stock are normally distributed with mean 2.48% and standard deviation 12.27%. There is a 1% chance that AAPL monthly returns will be below what value?

```
qnorm(0.01,mean=0.0248,sd=0.1227)
```

```
## [1] -0.2606429
```

---

(2)

Suppose that annual stock returns for a particular company are normally distributed with a mean of 16% and a standard deviation of 10%. You are going to invest in this stock for one year.

(a)

Find that the probability that your one-year return will exceed 30%

```
1-pnorm(0.3, mean=0.16, sd=0.1)
```

```
## [1] 0.08075666
```

(b)

Find that probability that your one-year return will be negative.

```
pnorm(0, mean=0.16, sd=0.1)
```

```
## [1] 0.05479929
```

---

(3)

The variance of Stock A is 0.0016, the variance of the market is 0.0049 and the covariance between the two is 0.0026. What is the correlation coefficient?

```
cat("The correlation coefficient between the stock and the market=",
0.0026/(sqrt(0.0016)*sqrt(0.0049)), "\n")
```

```
## The correlation coefficient between the stock and the market= 0.9285714
```

---

(4)

The distribution of annual returns on common stocks is roughly symmetric, but extreme observations are more frequent than in a normal distribution. Because the distribution is not strongly nonnormal, the mean return over even a moderate number of years is close to normal. Annual real returns on the Standard & Poor's 500-Stock Index over the period 1871 to 2004 have varied with mean 9.2% and standard deviation 20.6%. Andrew plans to retire in 45 years and is considering investing in stocks.

(a)

What is the probability (assuming that the past pattern of variation continues) that the mean annual return on common stocks over the next 45 years will exceed 15%?

```
1-pnorm(0.15, mean=0.092, sd=0.206/sqrt(2004-1871+1))
```

```
## [1] 0.0005586024
```

(b)

What is the probability that the mean return will be less than 5%?

```
pnorm(0.05, mean=0.092, sd=0.206/sqrt(2004-1871+1))
```

```
## [1] 0.009134461
```

---

(5) QIA Workbook: Chapter 4, numbers 14

14. Calculate the covariance of the returns on Bedolf Corporation (R_B) with the returns on Zedock Corporation (R_Z), using the following data.

Probability Function of Bedolf and Zedock Returns

	$R_Z = 15\%$	$R_Z = 10\%$	$R_Z = 5\%$
$R_B = 30\%$	0.25	0	0
$R_B = 15\%$	0	0.50	0
$R_B = 10\%$	0	0	0.25

Note: Entries are joint probabilities.

```
# Expected value of the returns on Bedolf Corporation is
ERb<-0.3*0.25+0.15*0.5+0.1*0.25
# Expected value of the returns on Zedock Corporation is
ERz<-0.15*0.25+0.1*0.5+0.05*0.25

# Covariance is
cat("Covariance=",(0.3-ERb)*(0.15-ERz)*0.25+(0.15-ERb)*(0.1-ERz)*0.50+(0.1-
ERb)*(0.05-ERz)*0.25,"\n")

## Covariance= 0.0025
```

---

(6) QIA Workbook: Chapter 4, numbers 15

15. You have developed a set of criteria for evaluating distressed credits. Companies that do not receive a passing score are classed as likely to go bankrupt within 12 months. You gathered the following information when validating the criteria:

- Forty percent of the companies to which the test is administered will go bankrupt within 12 months: $P(nonsurvivor) = 0.40$.
- Fifty-five percent of the companies to which the test is administered pass it: $P(pass\ test) = 0.55$.
- The probability that a company will pass the test given that it will subsequently survive 12 months, is 0.85: $P(pass\ test \mid survivor) = 0.85$.

A. What is $P(pass\ test \mid nonsurvivor)$?

B. Using Bayes' formula, calculate the probability that a company is a survivor, given that it passes the test; that is, calculate $P(survivor \mid pass\ test)$.

C. What is the probability that a company is a *nonsurvivor*, given that it fails the test?

D. Is the test effective?

A:

By the Law of Total Probability, P(pass test)=P(pass test|nonsurvivor)P(nonsurvivor) + P(pass test|survivor)P(survivor), therefore, P(pass test|nonsurvivor)=(P(pass test)-P(pass test|survivor)P(survivor))/P(nonsurvivor)

```
cat("P(pass test|nonsurvivor)=",(0.55-0.85*(1-0.4))/0.4,"\n")
```

```
## P(pass test|nonsurvivor)= 0.1
```

B:

By the Bayes' Rule, P(survivor|pass test)=P(survivor & pass test)/P(pass test)=P(pass test|survivor)P(survivor)/P(pass test)

```
cat("P(survivor|pass test)=",0.85*(1-0.4)/0.55,"\n")
```

```
## P(survivor|pass test)= 0.9272727
```

C:

By the Bayes' Rule, P(nonsurvivor|not pass test)=P(nonsurvivor & not pass test)/P(not pass test)=P(not pass test|nonsurvivor)P(nonsurvivor)/P(not pass test)=(1-P(pass test|nonsurvivor))P(nonsurvivor)/(1-P(pass test))

```
cat("P(nonsurvivor|not pass test)=",(1-0.1)*0.4/(1-0.55),"\n")
```

```
## P(nonsurvivor|not pass test)= 0.8
```

D:

Answer: The purpose of the test is to detect companies likely to go bankrupcy, the null hypothesis is "The company goes bankrupcy in the next 12 months". The test is effective if the type I error <5%, which is: P(pass test|nonsurvivor)<0.05. However, from A) we know the type I error is 10% whihc is too high, therefore, the test is not effective, it bears too high risk of not detecting true nonsurvivors.

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(7) QIA Workbook; Chapter 5, numbers 4

4. Over the last 10 years, a company's annual earnings increased year over year seven times and decreased year over year three times. You decide to model the number of earnings increases for the next decade as a binomial random variable.

A. What is your estimate of the probability of success, defined as an increase in annual earnings?

For Parts B, C, and D of this problem, assume the estimated probability is the actual probability for the next decade.

B. What is the probability that earnings will increase in exactly 5 of the next 10 years?

C. Calculate the expected number of yearly earnings increases during the next 10 years.

D. Calculate the variance and standard deviation of the number of yearly earnings increases during the next 10 years.

E. The expression for the probability function of a binomial random variable depends on two major assumptions. In the context of this problem, what must you assume about annual earnings increases to apply the binomial distribution in Part B? What reservations might you have about the validity of these assumptions?

A:

The estimate of the probability of success,, defined as an increase in annual earnings, is 0.7

B:

```
dbinom(5, size=10, prob = 0.7)
```

```
## [1] 0.1029193
```

C:

```
cat("Expected number of yearly earnings increases during the next 10
years=",5*0.7,"\n")
```

```
## Expected number of yearly earnings increases during the next 10 years= 3.5
```

D:

```
cat("Variance of the number of yearly earnings increases during the next 10
years=",5*0.7*(1-0.7),"\n")
```

```
## Variance of the number of yearly earnings increases during the next 10
years= 1.05
```

```
cat("Standard deviation of the number of yearly earnings increases during the
next 10 years=",sqrt(5*0.7*(1-0.7)),"\n")
```

```
## Standard deviation of the number of yearly earnings increases during the
next 10 years= 1.024695
```

E:

Answer: The assumptions we make are I.I.D. (Indepedent Identical Distribution), that is, the actual earning increases in each year in the next 10 years is 1) independent 2) identically distributed as Bernoulli distribution with probability of success 0.7. The validity of these assumptions are violated if, the actual earnings have serial correlation, or the probability of increase changes over time.

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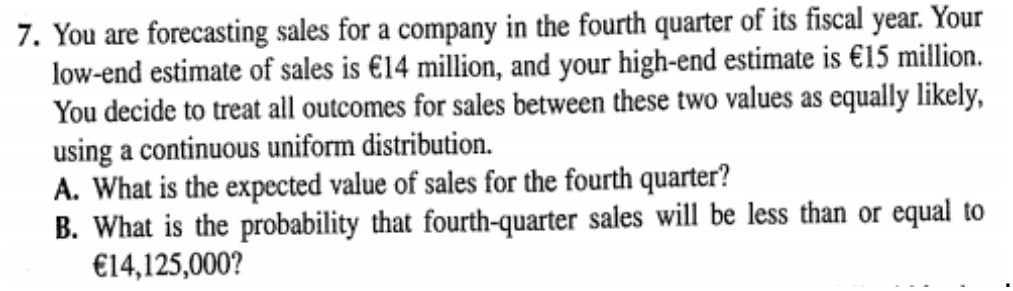
(8) QIA Workbook; Chapter 5, numbers 7



7. You are forecasting sales for a company in the fourth quarter of its fiscal year. Your low-end estimate of sales is €14 million, and your high-end estimate is €15 million. You decide to treat all outcomes for sales between these two values as equally likely, using a continuous uniform distribution.
   A. What is the expected value of sales for the fourth quarter?
   B. What is the probability that fourth-quarter sales will be less than or equal to €14,125,000?

A:

```
cat("The expected value of sales for the fourth quarter=",(14+15)/2,"million
\n")
```

```
## The expected value of sales for the fourth quarter= 14.5 million
```

B:

```
cat("The probability that fourth-quarter sales will be less than or equal to
14,125,000=",(14.125-14)/(15-14),"\n")
```

```
## The probability that fourth-quarter sales will be less than or equal to
14,125,000= 0.125
```

---

(9) QIA Workbook; Chapter 5, numbers 11

11. In futures markets, profits or losses on contracts are settled at the end of each trading day. This procedure is called marking to market or daily resettlement. By preventing a trader's losses from accumulating over many days, marking to market reduces the risk that traders will default on their obligations. A futures markets trader needs a liquidity pool to meet the daily mark to market. If liquidity is exhausted, the trader may be forced to unwind his position at an unfavorable time.

Suppose you are using financial futures contracts to hedge a risk in your portfolio. You have a liquidity pool (cash and cash equivalents) of λ dollars per contract and a time horizon of T trading days. For a given size liquidity pool, λ, Kolb, Gay, and Hunter (1985) developed an expression for the probability stating that you will exhaust your liquidity pool within a T-day horizon as a result of the daily mark to market. Kolb et al. assumed that the expected change in futures price is 0 and that futures price changes are normally distributed. With σ representing the standard deviation of daily futures price changes, the standard deviation of price changes over a time horizon to day T is $\sigma\sqrt{T}$, given continuous compounding. With that background, the Kolb et al. expression is

$$\text{Probability of exhausting liquidity pool} = 2[1 - N(x)]$$

where $x = \lambda/(\sigma\sqrt{T})$. Here x is a standardized value of λ. $N(x)$ is the standard normal cumulative distribution function. For some intuition about $1 - N(x)$ in the expression, note that the liquidity pool is exhausted if losses exceed the size of the liquidity pool at any time up to and including T; the probability of that event happening can be shown to be proportional to an area in the right tail of a standard normal distribution, $1 - N(x)$.

Using the Kolb et al. expression, answer the following questions:

A. Your hedging horizon is five days, and your liquidity pool is $2,000 per contract. You estimate that the standard deviation of daily price changes for the contract is $450. What is the probability that you will exhaust your liquidity pool in the five-day period?

B. Suppose your hedging horizon is 20 days, but all the other facts given in Part A remain the same. What is the probability that you will exhaust your liquidity pool in the 20-day period?

A:

```
cat("Probability of exhausting liquidity pool=",2*(1-
pnorm(2000/(450*sqrt(5)))),'\n')
```

```
## Probability of exhausting liquidity pool= 0.04685418
```

B:

```
cat("Probability of exhausting liquidity pool=",2*(1-
pnorm(2000/(450*sqrt(20)))),'\n')
```

```
## Probability of exhausting liquidity pool= 0.3203164
```

---

(7)

Suppose an investor can choose between two independent assets 1 and 2. The probability distributions for the rates of return for each asset are provided below.

r_1 *outcomes*	5%	8%	12%	15%
r_2 *outcomes*	15%	12%	8%	5%
Probs	0.25	0.25	0.25	0.25

a)

Compute the expected value of each asset

```
Er1<-0.05*0.25+0.08*0.25+0.12*0.25+0.15*0.25
Er2<-0.15*0.25+0.12*0.25+0.08*0.25+0.05*0.25
cat("Expected value of asset 1=", Er1, "\n")
```

```
## Expected value of asset 1= 0.1
```

```
cat("Expected value of asset 2=", Er2, "\n")
```

```
## Expected value of asset 2= 0.1
```

b)

Compute the standard deviation of each asset

```
SDr1<-sqrt((0.05-Er1)^2*0.25+(0.08-Er1)^2*0.25+(0.12-Er1)^2*0.25+(0.15-
Er1)^2*0.25)
SDr2<-sqrt((0.15-Er2)^2*0.25+(0.12-Er2)^2*0.25+(0.08-Er2)^2*0.25+(0.05-
Er2)^2*0.25)
cat("Standard deviation of asset 1=", SDr1, "\n")
```

```
## Standard deviation of asset 1= 0.03807887
```

```
cat("Standard deviation of asset 2=", SDr2, "\n")
```

```
## Standard deviation of asset 2= 0.03807887
```

c)

Based on expected return and the variance of return, which asset do you prefer and why? On what other basis might you prefer one asset to another?

Answer: Since R1 and R2 have the same expected return and variance, I have no prefernece between the two. I might prefer one asset over another if I have other information about the two assets, if I believe one is going to perform better.

d) Assume you hold a portfolio with 50% held in asset 1 and 50% held in asset 2. What is the expected return and variance of your portfolio? Note that the portfolio return is a random variable written as R = 0.5R1+0.5R2.

```
cat("The expected value of the portfolio is:
\n","E(R)=E(0.5R1+0.5R2)=0.5E(R1)+0.5E(R2)=",0.5*Er1+0.5*Er2,'\n')
```

```
## The expected value of the portfolio is:
##  E(R)=E(0.5R1+0.5R2)=0.5E(R1)+0.5E(R2)= 0.1
```

```
cat("The variance of the portfolio is:
\n","Var(R)=Var(0.5R1+0.5R2)=0.25Var(R1)+0.25Var(R2)+2*0.5*0.5Cov(R1,R2)=0.25
Var(R1)+0.25Var(R2)=",0.25*SDr1^2+0.25*SDr2^2,'\n')
```

```
## The variance of the portfolio is:
##
Var(R)=Var(0.5R1+0.5R2)=0.25Var(R1)+0.25Var(R2)+2*0.5*0.5Cov(R1,R2)=0.25Var(R
1)+0.25Var(R2)= 0.000725
```

---

8) In this problem we are going to reproduce the first table from the article article link

Our numbers will be slightly different but close.To grab historical data for the S&P500 do the following

```
library(quantmod)
spydata=getSymbols("^GSPC",from="1950-01-01",auto.assign=FALSE)
spyrets=dailyReturn(Ad(spydata))
spyrets=spyrets[-1]
sd(spyrets)
```

```
## [1] 0.00967957
```

Note we find the historical standard deviation of daily returns to be 0.967% and the quoted paper uses 0.973%. We assume 250 trading days in a year. Simply reproduce the following table using our data from R [so it will be slightly different]. The expected number is 250days*66 years/day*prob of occurrence per year.

Plus / Minus Sigma Level	Probability of occurring on any given day	How often event is expected to occur	Associated S&P 500 percentage move	Actual S&P 500 occurrences (Jan 1950-2016) vs (expected from normal distribution)
>+-1	31.73%	80 trading days per year	+-0.973%	3534 (expected 5276)
>+-2	4.56%	12 trading days per year	+-1.95%	776 (expected 758)
>+-3	0.27%	1 event every 8 months	+-2.92%	229 (expected 44)

```
mu.spyrets<-mean(spyrets)
sd.spyrets<-sd(spyrets)

# Define X=daily return, X is Normally distributed with mean and standard
deviation equal to historical values

# Define empty columns of the final table
  c1<-character(3)
  c2<-character(3)
  c3<-character(3)
  c4<-character(3)
  c5<-character(3)

for(i in 1:3){
  # prob of occuring on any given day
  col2.num<-pnorm(mu.spyrets-i*sd.spyrets, mean=mu.spyrets, sd=sd.spyrets)+1-
pnorm(mu.spyrets+i*sd.spyrets, mean=mu.spyrets, sd=sd.spyrets)
  # how often event is expected to occur
  if (i<3){col3.num<-ceiling(col2.num*250) } else{col3.num<-
ceiling(col2.num*250*8/12) }
  # associated S&P 500 percentage move
  col4.num<-i*sd.spyrets*100
  # actual S&P 500 occurences
  col5.num.actual<-
sum(spyrets>mu.spyrets+i*sd.spyrets)+sum(spyrets<mu.spyrets-i*sd.spyrets)
  # expected S&P 500 occurences
  col5.num.expected<-ceiling(col2.num*250*66)

  # pu into Table cells
  c1[i]<-paste(">+-",i)
```

```
  c2[i]<-paste(round(100*col2.num,digits = 2),"%")
  if (i<3){c3[i]<-paste(col3.num,"trading days per year", sep=" ")}
  else{c3[i]<-paste(col3.num,"event every 8 months", sep=" ")}

  c4[i]<-paste("+-",round(100*i*sd.spyrets,digits = 2),"%")
  c5[i]<-paste(col5.num.actual,"(expected",col5.num.expected,")")
}

# Make Table
myTable<-data.frame(c1,c2,c3,c4,c5)
names(myTable)<-c("Plus/Minus Sigma Level",
                  "Prob of occuring on any given day",
                  "How often event is expected to occur",
                  "Associated percentage move",
                  "Actual occurences vs Expected from normal distribution")

library(knitr)
kable(myTable)
```

Plus/Minus Sigma Level	Prob of occuring on any given day	How often event is expected to occur	Associated percentage move	Actual occurences vs Expected from normal distribution
>+- 1	31.73 %	80 trading days per year	+- 0.97 %	3589 (expected 5236)
>+- 2	4.55 %	12 trading days per year	+- 1.94 %	783 (expected 751)
>+- 3	0.27 %	1 event every 8 months	+- 2.9 %	235 (expected 45)

---

(9)

Test using the S&P 500 data used previously that the daily standard deviation of returns from 1990 to 1995 equals the standard deviation of returns from 2000 to 20005. Use the R routine var.test(x,y).

```
library(quantmod)
spy1=getSymbols("^GSPC",from="1990-01-01",to="1995-12-31",auto.assign=FALSE)
spy2=getSymbols("^GSPC",from="2000-01-01",to="2005-12-31",auto.assign=FALSE)

var.test(dailyReturn(Ad(spy1)), dailyReturn(Ad(spy2)))

##
##  F test to compare two variances
##
```

```
## data:  dailyReturn(Ad(spy1)) and dailyReturn(Ad(spy2))
## F = 0.3638, num df = 1516, denom df = 1507, p-value < 2.2e-16
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
##  0.3288920 0.4024051
## sample estimates:
##               daily.returns
## daily.returns     0.3637992
## attr(,"names")
## [1] "ratio of variances"
```

Answer: Since the p-value is less than 0.05 we reject the null hypothesis. Our conclusion at the 5% level of confidence is, there is sufficient sample evidence to support the claim that the standard deviation of daily adjusted returns from 1990 to 1995 is difference from that from 2000 to 2005.

---

(10)

Again using the historical data, note that the growth of $1 over this period can be obtained using the command prod((1+spyrets)).

a.

What would the growth be if someone had missed the 10 best performing days over this period [this is the argument to not market time but instead buy and hold].

```
spydata=getSymbols("^GSPC",from="1950-01-01",auto.assign=FALSE)
spyrets=as.numeric(dailyReturn(Ad(spydata))) # no time stamp, pure numeric
vecotr
spyrets=spyrets[-1]

# if buy and hold
prod(1+spyrets)
```

```
## [1] 128.401
```

```
# To sort by returns using order()
spyrets.sorted<-spyrets[order(spyrets,decreasing =TRUE)]

# missed the 10 best performing days 1950 until now
spyrets.nobest<-spyrets.sorted[11:length(spyrets.sorted)]
cat("If missed the 10 best performing days,
growth=",prod(1+spyrets.nobest),'\n')
```

```
## If missed the 10 best performing days, growth= 61.9599
```

b.

What would the growth be if someone had missed the 10 worst performing days over this period [this is the argument to market time and instead buy and hold].

```r
# missed the 10 worst performing days 1950 until now
spyrets.noworst<-spyrets.sorted[1:(length(spyrets.sorted)-10)]
cat("If missed the 10 worst performing days,
growth=",prod(1+spyrets.noworst),'\n')

## If missed the 10 worst performing days, growth= 334.0691
```

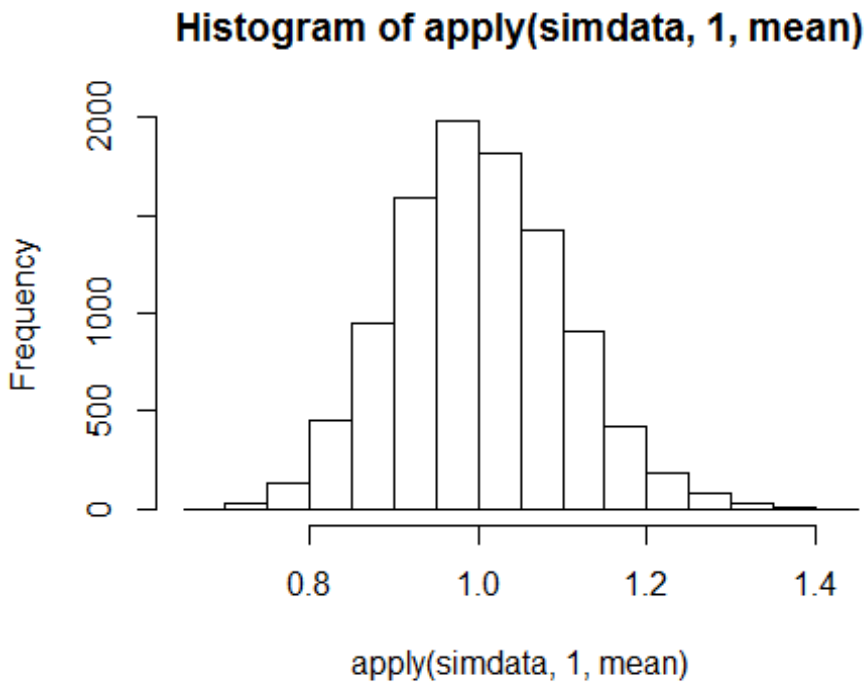
---

(11)

Recall that the R command apply allows us to easily and quickly calculate the mean (or sd or sum or any function) of each row or column of a matrix. Using this command we can quickly write code to demonstrate the Central Limit Theorem.

The following code simulates drawing 10000 samples of size 100 from an exponential distribution with shape parameter=1, and then drawing the resulting histogram.

```r
set.seed(02138)
simdata=matrix(nrow=10000,ncol=100,rexp(100*10000,1))
hist(apply(simdata,1,mean))
```



Explain in easy to understand language what the above code is doing. Using the above code, show how the Central Limit Theorem works for the Gamma distribution with shape=5 and rate=5 parameters [R code is rgamma(n,5,5)]. Use sample sizes of 10, 50 and 100. For each

simulation run, report the mean of the 10000 sample means produced and the standard deviation of the 10000 sample means produced. Comment on what you find.

Answer: The first line of code draws 100*10000 independent random samples from an exponential distribution with shape parameter=1, put them into a matrix, each row of the resulting matrix is a random sample of size 100, therefore, the code essentially generates 10000 samples of size 100. The second line of codes plots a histogram of the means of each sample, by applying mean function to each row.

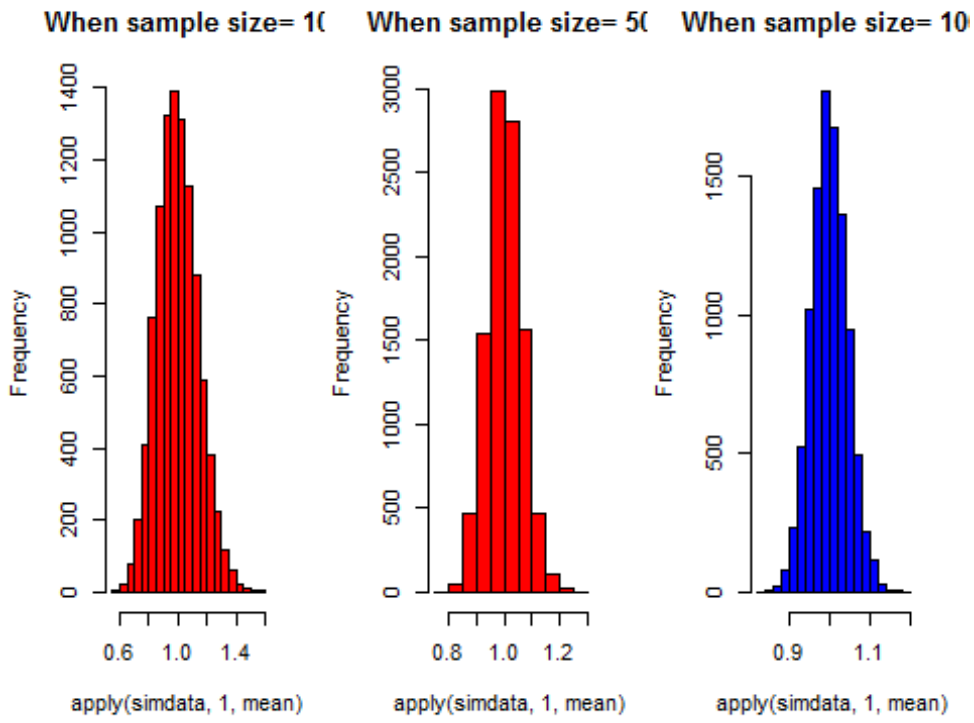
```
set.seed(02138)
n<-c(10,50,100)
par(mfrow=c(1,3))
for(i in 1:3){
  sample.size<-n[i]
  cat("When sample size=",sample.size,'\n')

  simdata<-matrix(nrow=10000, ncol=sample.size,
rgamma(sample.size*10000,5,5))
  cat("The mean of the 10000 sample means=",mean(apply(simdata,1,mean)),'\n')
  cat("The standard deviation of the 10000 sample
means=",sd(apply(simdata,1,mean)),'\n')
  hist(apply(simdata,1,mean),main = paste("When sample size=",sample.size),
col = sample.size)
}
```

```
## When sample size= 10
## The mean of the 10000 sample means= 0.9976744
## The standard deviation of the 10000 sample means= 0.1419841

## When sample size= 50
## The mean of the 10000 sample means= 1.000864
## The standard deviation of the 10000 sample means= 0.06296471

## When sample size= 100
## The mean of the 10000 sample means= 0.9994539
## The standard deviation of the 10000 sample means= 0.04455103
```



Answer: The sample means are all close to the true mean which is 1. However, the standard deviation decreases substantially while increasing the sample size. This tendency is in agreement with the Central Limit Theorem. Compare the histograms, we can easily see that the centrality is all around 1, while the dispersion is smaller and smaller while increasing the sample size.

Statistical Methods Review Problems. This part of this homework is to simply reignite the brain cells involving confidence intervals and hypothesis testing. The R cookbook is the best place to go to see how to compute confidence intervals and hypothesis tests in R. For each problem below, use R to answer the question and clearly state the conclusion of your test, or clearly give the confidence interval.

(12)

In a study of the length of time that students require to earn bachelor's degrees, 80 students are randomly selected and they are found to have a mean of 4.8 years and standard deviation of 2.2 years. Construct a 95% confidence interval estimate of the population mean. Does the resulting confidence interval contradict the fact that 39% of students earn their bachelor's degrees in four years?

```
mu<-4.8
sigma<-2.2
c(mu-1.96*sigma/sqrt(80),mu+1.96*sigma/sqrt(80))
```

```
## [1] 4.317904 5.282096
```

Answer: The 95% confidence interval is (4.32,5.28), it doesn't contradict the fact that 39% of students earn their bachelor's degrees in 4 years. The 95% CI does Not say that, 95% of students earn their degrees between (4.32,5.28). Rather, a confidence interval is an observed interval, in principle different from sample to sample, that frequently includes the value of an unobserved parameter of interest (in this case, the length of time getting a bachelor's degree) if the experiment is repeated. How frequently the observed interval contains the parameter is determined by the confidence level (usually 95%). More specifically, the meaning the CI is that, if CI are constructed across many separate data analyses of replicated experiments, the proportion of such intervals that contain the true value of the parameter will match the given confidence level.

So, what the 95% CI really tells us is that, if replicate the experiment many many times, 95% of times we will see the observed CI include the true average length of time that students requir to earn a bachelor's degree. We can say that, there is a 95% chance that (4.32,5.28) includes the true average length of time that students require to get a bachelor's degree.

---

(13)

Two groups of students are given a problem-solving test, and the results are compared. Find and interpret the 95% confidence interval of the true difference in means.

Mathematics majors	Computer science majors
$\bar{X}_1 = 83.6$	$\bar{X}_2 = 79.2$
$s_1 = 4.3$	$s_2 = 3.8$
$n_1 = 36$	$n_2 = 36$

```
library(BSDA)
tsum.test(83.6,4.3,36,79.2,3.8,36)
```

```
##
##  Welch Modified Two-Sample t-Test
##
## data:  Summarized x and y
## t = 4.6005, df = 68.957, p-value = 1.859e-05
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  2.491991 6.308009
## sample estimates:
## mean of x mean of y
##      83.6      79.2
```

Answer: The 95% CI is (2.491991, 6.308009). If we replicate the experiment many many times, 95% of times we will see the observed CI includes the true difference in means; or, there is a 95% chance that (2.491991, 6.308009) covers the true difference in means.

(14)

A recent Gallup poll consisted of 1012 randomly selected adults who were asked whether "cloning of humans should or should not be allowed." Results showed that 892 of those surveyed indicated that cloning should not be allowed. Construct a 95% confidence interval estimate of the proportion of adults believing that cloning of humans should not be allowed.

```
prop.test(892,1012,correct = FALSE)
```

```
##
##  1-sample proportions test without continuity correction
##
## data:  892 out of 1012, null probability 0.5
## X-squared = 588.92, df = 1, p-value < 2.2e-16
## alternative hypothesis: true p is not equal to 0.5
## 95 percent confidence interval:
##  0.8600478 0.8999133
## sample estimates:
##         p
## 0.8814229
```

Answer: 95% confidence interval is (0.8600478, 0.8999133). There is a 95% chance that (86.00%, 89.99%) includes the true proportion of adults believing that cloning of human should not be allowed.

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(15)

In a sample of 80 Americans, 55% wished that they were rich. In a sample of 90 Europeans, 45% wished that they were rich. Find and interpret the 95% confidence interval of the true difference in proportions.

```
prop.test(c(0.55*80, 0.45*90), c(80, 90), correct = FALSE)
```

```
##
##  2-sample test for equality of proportions without continuity
##  correction
##
## data:  c(0.55 * 80, 0.45 * 90) out of c(80, 90)
## X-squared = 1.6942, df = 1, p-value = 0.1931
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  -0.04982832  0.24982832
## sample estimates:
## prop 1 prop 2
##   0.55   0.45
```

Answer: 95% confidence interval is (-0.04982832, 0.24982832). There is a 95% chance that (-4.98%, 24.98%) includes the true difference (Americans vs Europeans) in proportions of those who wished they were rich.

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(16)

For this problem, download data for the last year (9/01/2014 to 9/01/2015) for AAPL and CAT. We are going to work through some items in the R Cookbook so look there if you get stuck. [use adjusted closing prices for everything].

(a)

Run a hypothesis test to see if AAPL or CAT daily returns are normally distributed. Interpret the output, what is the conclusion of the hypothesis test?

```
AAPL<-getSymbols("AAPL",from="2014-09-01",to="2015-09-01",auto.assign=FALSE)
CAT<-getSymbols("CAT",from="2014-09-01",to="2015-09-01",auto.assign=FALSE)

shapiro.test(as.numeric(dailyReturn(Ad(AAPL))))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(dailyReturn(Ad(AAPL)))
## W = 0.97627, p-value = 0.0003108
```

```
shapiro.test(as.numeric(dailyReturn(Ad(CAT))))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(dailyReturn(Ad(CAT)))
## W = 0.96256, p-value = 3.671e-06
```

Answer: Both p-values are less than 0.05 we reject the null hypothesis. Our conclusion at the 5% level of confidence is that, there is sufficient sample evidence to support the claim that daily returns of AAPL and CAT are not normally distributed.

(b)

Run a hypothesis test to see if AAPL or CAT monthly returns are normally distributed. Interpret the output, what is the conclusion of the hypothesis test?

```
shapiro.test(as.numeric(monthlyReturn(Ad(AAPL))))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(monthlyReturn(Ad(AAPL)))
## W = 0.89676, p-value = 0.1207
```

```
shapiro.test(as.numeric(monthlyReturn(Ad(CAT))))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  as.numeric(monthlyReturn(Ad(CAT)))
## W = 0.96987, p-value = 0.8928
```

Answer: Both p-values are larger than 0.05 we accept the null hypothesis. Our conclusion at the 5% level of confidence is that, there is no sufficient sample evidence to reject the claim that monthly returns of AAPL and CAT are normally distributed.

(c)

Test if AAPL and CAT have the same average daily return (see comparing the means of two samples in the R Cookbook).

```
tsum.test(mean(as.numeric(dailyReturn(Ad(AAPL)))),
          sd(as.numeric(dailyReturn(Ad(AAPL)))),
          length(as.numeric(dailyReturn(Ad(AAPL)))),
          mean(as.numeric(dailyReturn(Ad(CAT)))),
          sd(as.numeric(dailyReturn(Ad(CAT)))),
          length(as.numeric(dailyReturn(Ad(CAT)))))
```

```
##
##  Welch Modified Two-Sample t-Test
##
## data:  Summarized x and y
## t = 1.1644, df = 500.01, p-value = 0.2448
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.001100890  0.004304077
## sample estimates:
##     mean of x     mean of y
##  0.0003627482 -0.0012388453
```

Answer: The p-value is larger than 0.05 we accept the null hypothesis. OUr conclusion at the 5% level of confidence is that, there is no sufficient sample evidence to reject the claim that AAPl and CAT have the same average daily returns.

(d)

Test if AAPL and CAT have the same average monthly return (see comparing the means of two samples in the R Cookbook).

```
tsum.test(mean(as.numeric(monthlyReturn(Ad(AAPL)))),
          sd(as.numeric(monthlyReturn(Ad(AAPL)))),
          length(as.numeric(monthlyReturn(Ad(AAPL)))),
          mean(as.numeric(monthlyReturn(Ad(CAT)))),
          sd(as.numeric(monthlyReturn(Ad(CAT)))),
          length(as.numeric(monthlyReturn(Ad(CAT)))))
```

```
##
##  Welch Modified Two-Sample t-Test
##
## data:  Summarized x and y
## t = 1.2846, df = 23.877, p-value = 0.2112
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.01859108  0.07984341
## sample estimates:
##    mean of x    mean of y
##  0.006328462 -0.024297703
```

Answer: The p-value is larger than 0.05 we accept the null hypothesis. OUr conclusion at the 5% level of confidence is that, there is no sufficient sample evidence to reject the claim that AAPl and CAT have the same average monthly returns.

(e)

Test if AAPL and CAT have the same standard deviation of daily returns. Use the R routine var.test(dailyReturn(Ad(AAPL)),dailyReturn(Ad(CAT)))

```
var.test(dailyReturn(Ad(AAPL)),dailyReturn(Ad(CAT)))
```

```
##
##  F test to compare two variances
##
## data:  dailyReturn(Ad(AAPL)) and dailyReturn(Ad(CAT))
## F = 1.1961, num df = 252, denom df = 252, p-value = 0.1558
## alternative hypothesis: true ratio of variances is not equal to 1
## 95 percent confidence interval:
##  0.9338966 1.5320305
## sample estimates:
##               daily.returns
## daily.returns      1.196143
## attr(,"names")
## [1] "ratio of variances"
```

Answer: The p-value is larger than 0.05 we accept the null hypothesis. OUr conclusion at the 5% level of confidence is that, there is no sufficient sample evidence to reject the claim that AAPl and CAT have the same standard deviation of daily returns.

(f)

Calculate the correlation between AAPL and CAT daily returns, and compute a confidence interval for the correlation. ???

```
cor(dailyReturn(Ad(AAPL)),dailyReturn(Ad(CAT)))
```

```
##               daily.returns
## daily.returns      0.468505
```

```
cor.test(dailyReturn(Ad(AAPL)),dailyReturn(Ad(CAT)))
```

```
##
##  Pearson's product-moment correlation
##
## data:  dailyReturn(Ad(AAPL)) and dailyReturn(Ad(CAT))
## t = 8.4016, df = 251, p-value = 3.297e-15
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.3663443 0.5595049
## sample estimates:
##      cor
## 0.468505
```

Answer: The correlation is 0.468505, the p-value (3.297e-15) is less than 0.05, so the correlation between AAPL and CAT daily returns is significantly different from zero. The confidence interval is (0.3663443, 0.5595049), so there is a 95% chance that (0.3663443, 0.5595049) includes the true correlation.