Stat 107: Introduction to Business and Financial Statistics

Homework 5 Solutions

Solutions to the Teall problems are in the back of the book. Work must be shown to receive credit.

1) Teall book, page 137, problem 7.1

7.1. (a) $\begin{bmatrix} 7 \\ 0 \\ 10 \end{bmatrix}$;

(b) $\begin{bmatrix} 7 & 0 \\ 10 & 2.5 \end{bmatrix}$.

2) Teall book, page 138, problem 7.4

(a) $\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -4+\frac{12}{2} & 2-\frac{4}{2} \\ -6+\frac{12}{2} & 3-\frac{4}{2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$;

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$;

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$;

(d) $[4 \quad 5 \quad 6]\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 77$;

(e) $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}[4 \quad 5 \quad 6] = \begin{bmatrix} 16 & 20 & 24 \\ 20 & 25 & 30 \\ 24 & 30 & 36 \end{bmatrix}$.

3) Teall book, page 138, problem 7.6

7.6. (a) Form a returns vector as follows:

$$\mathbf{r} = \begin{bmatrix} 0.07 \\ 0.09 \\ 0.13 \end{bmatrix}.$$

(b) The covariance matrix is as follows:

$$\mathbf{V} = \begin{bmatrix} 0.04 & 0.01 & 0.02 \\ 0.01 & 0.16 & 0.08 \\ 0.02 & 0.08 & 0.36 \end{bmatrix}.$$

(c) The weights vector is as follows:

$$\mathbf{w} = \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix}.$$

(d) (a) 3×1; (b) 3×3; (c) 3×1.

(e) The expected portfolio return is given as follows:

$$\mathrm{E}[R_p] = [0.30 \quad 0.50 \quad 0.20] \begin{bmatrix} 0.07 \\ 0.09 \\ 0.13 \end{bmatrix} = 0.092,$$

$$\mathrm{E}[R_p] = \qquad \mathbf{w}' \qquad \mathbf{r}.$$

(f) The portfolio variance is found as follows:

$$\sigma_p^2 = [0.30 \quad 0.50 \quad 0.20] \begin{bmatrix} 0.04 & 0.01 & 0.02 \\ 0.01 & 0.16 & 0.08 \\ 0.02 & 0.08 & 0.36 \end{bmatrix} \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix},$$

$$\sigma_p^2 = \qquad \mathbf{w}' \qquad \mathbf{V} \qquad \mathbf{w};$$

$$\sigma_p^2 = [0.021 \quad 0.099 \quad 0.118] \begin{bmatrix} 0.30 \\ 0.50 \\ 0.20 \end{bmatrix} = 0.0794,$$

$$\sigma_p^2 = \qquad \mathbf{w}'\,\mathbf{V} \qquad \mathbf{w} \quad = \sigma_p^2.$$

4) Teall book, page 139, problem 7.7

7.7. (a) $1/8 = 0.125$.

(b) The inverse of the identity matrix is the identity matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(c) The inverse of a diagonal matrix is found by inverting each of the principle diagonal elements:

$$\begin{bmatrix} 0.25 & 0 \\ 0 & 2 \end{bmatrix}.$$

Part (d)

Thus, the inverse matrix is

$$\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}.$$

Part (e)

The inverse matrix is

$$\begin{bmatrix} -100 & 50 \\ 75 & -25 \end{bmatrix}.$$

(f) The inverse matrix is

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

(g) The inverse matrix is

$$\begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.16 \end{bmatrix}.$$

Part (h)

The inverse matrix is

$$\begin{bmatrix} 0.5 & 0 & 0 \\ -0.25 & 0.25 & 0 \\ 0 & -0.1 & 0.05 \end{bmatrix}.$$

$$\mathbf{C}^{-1} = \begin{bmatrix} 0.04 & 0.04 \\ 0.04 & 0.16 \end{bmatrix};$$

5) Teall book, page 139, problem 7.9

Our original system of equations is represented as follows:

$$\begin{bmatrix} 0.08 & 0.08 & 0.1 & 1 \\ 0.08 & 0.32 & 0.2 & 1 \\ 0.1 & 0.2 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$\mathbf{C} \cdot \mathbf{x} = \mathbf{s}.$$

The solution is given by $x = C^{-1}s$

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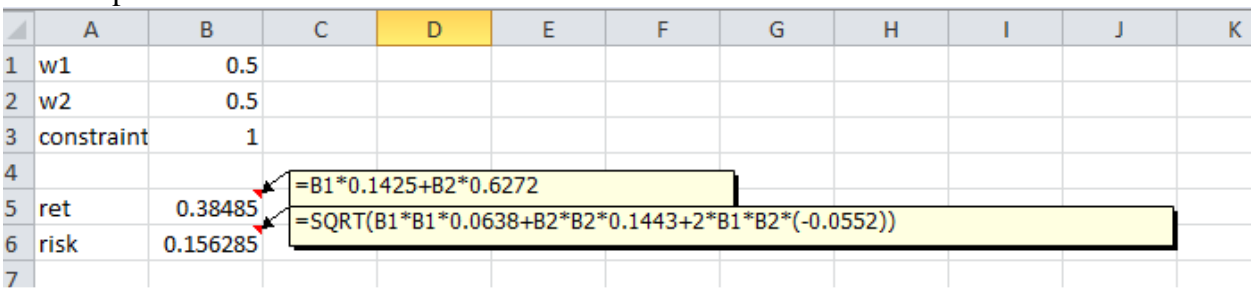
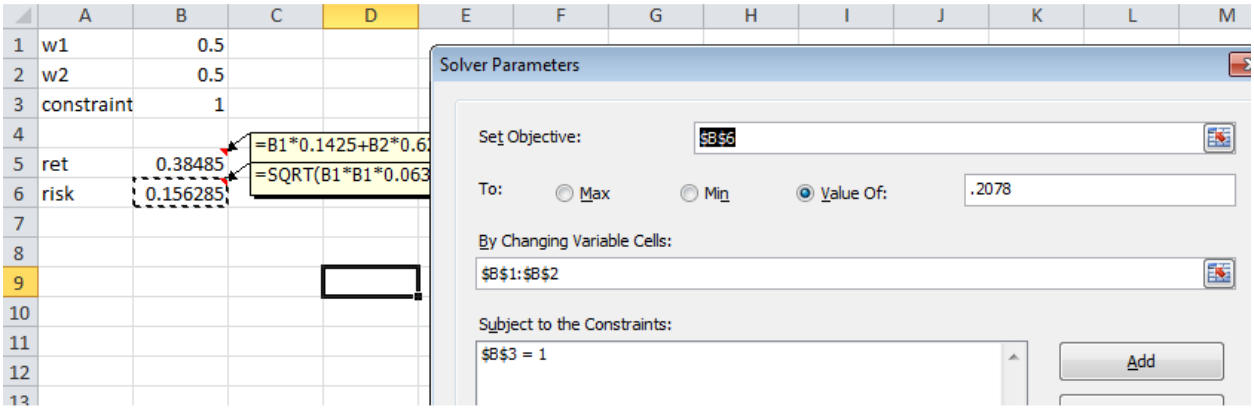
> row1=c(.08,.08,.1,1)
> row2=c(.08,.32,.2,1)
> row3=c(.1,.2,0,0)
> row4=c(1,1,0,0)
> C=rbind(row1,row2,row3,row4)
> s=c(.1,.1,.1,.1)
> x=solve(C)%*%s
>
> x
       [,1]
[1,] -0.800
[2,]  0.900
[3,] -2.160
[4,]  0.308
```

6) Consider the data for stock A and Stock B below. A portfolio composed of 90% of Stock A and 10% Stock B stock has expected return of 19.1% and standard deviation of 20.78%. Find another portfolio with the same standard deviation and a higher return. (You can do this by trial and error, but you can also use Solver.)

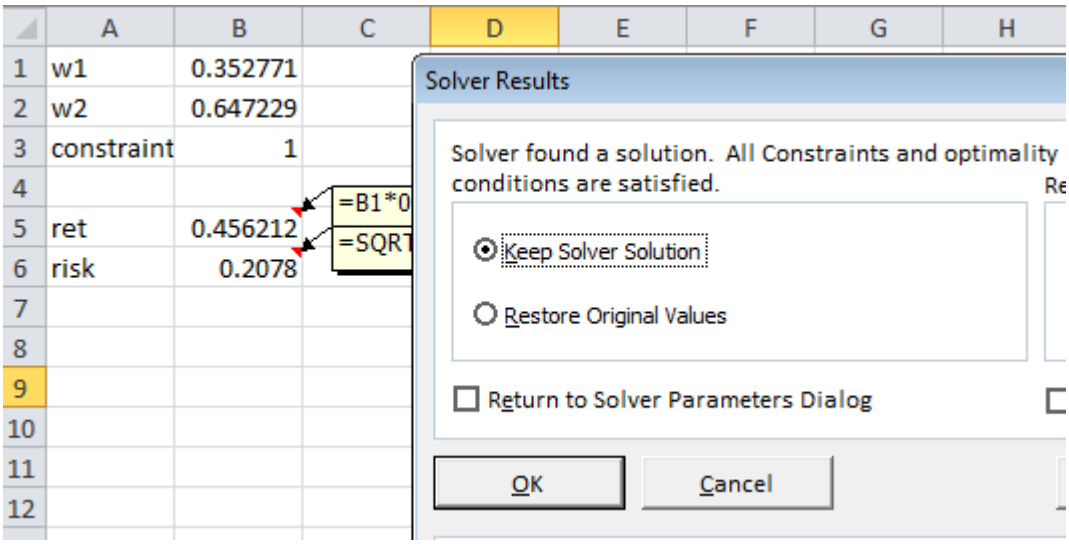
	A	B	C
1		Stock A	Stock B
2	Expected return	14.25%	62.72%
3	Variance	6.38%	14.43%
4	Sigma	25.25%	37.99%
5	Covariance of returns	-5.52%	

We set up an Excel worksheet and then use Solver





The solution is weights of 35.2% and 64.8% as seen below



7) For this question, you will need to use the Excel spreadsheet **hw5sheet1.xls**..

The spreadsheet contains statistical summaries of stock indices from seven countries, with their average returns, standard deviations, and correlations over the period 1980-1993. We assume that these data provide good approximations for future expected returns, volatilities and correlations. [Note: this may not necessarily be the case in reality; past realizations are not necessarily a good indicator of future performance]. The spreadsheet also contains the covariance matrix of these seven indices.

a) Calculate the mean and standard deviation for the Global Minimum Variance portfolio (i.e., the portfolio that has the lowest variance of all possible portfolios that can be created using the assets provided). Assume short sales are permitted. Show also the portfolio weights.

The minimum variance portfolio has expected return 17.12 and standard deviation of 17.20. The weights are as follows:

US	0.3759
Germany	0.1976
UK	0.0725
Japan	0.2073
Australia	0.1143
Canada	0.0345
France	-0.0021

b) Suppose the riskless rate is 5.5% (hahahaha) for both borrowing and lending. What is the expected return and standard deviation of the Tangent Portfolio? What are the portfolio weights in this case? Assume that short sales are permitted.

The tangent portfolio has expected return of 22.88 and standard deviation of 21.03. The weights are as follows:

US	0.6463
Germany	0.5403
UK	0.2630
Japan	0.1980
Australia	0.1494
Canada	-0.6363
France	-0.1605

c) Redo part (b) assuming that no short sales are permitted. Do the weights change greatly?

The expected value is now 19.06 with standard deviation 18.44. The weights do change quite a bit as now Canada and France have 0 weight.

US	0.2245
Germany	0.4417
UK	0.1769
Japan	0.1546
Australia	0.0023
Canada	0.0000
France	0.0000

For the following, suppose you are a fund manager whose available assets are limited to the above seven country indices and a riskless investment rate of 5.5% (for example, you could assume that this is the annual rate on bank Certificates of Deposit). You have been asked to advise two clients on their optimal portfolio mix based on their risk-return preferences and income/liquidity needs. Assume that there are no taxes, no inflation, and no transactions costs (i.e., ignore your fee for advising and investing on behalf of your clients).

d) Consider a client who is a relatively conservative middle-aged man with a reasonable level of income and a family to take care of. He wants to earn a better return than the 5.5% CD rate at the bank, but indicates that the maximum annual standard deviation he could tolerate is 12%. He also has a strong aversion to short selling. He asks you to invest $250,000 for him.

i. Can you create a portfolio solely with the seven country indices that has a portfolio standard deviation of 12% or less? Explain.

No it is not possible. Each individual standard deviation is more than 12% and all the covariances are positive.

ii. Consider a portfolio consisting of the tangency portfolio with no short sales and a bank Certificate of Deposit. What percentage of the client's wealth should be placed in each asset (the tangency portfolio and the Certificate of Deposit) to achieve a portfolio standard deviation of 12%?

The no short sale tangency portfolio has std dev 18.44 We want to solve $sqrt(w^2 18.44^2) = 12$ where w is the weight in the tangency portfolio. Solving this gives 65.1% in the tangency portfolio and 34.9% in the cd.

iii. What is the expected return of the portfolio found in part (ii)?

.651*19.06 + (1-.651)*5.5=14.33