CS 134: Networks	Lecture 3 **outline**, Monday, 1/30
Prof. Yaron Singer	Spring 2017

1 Overview

Question: *What are the properties of social networks that explain the six degrees of separation?*

2 Candidates for "Small World" Models

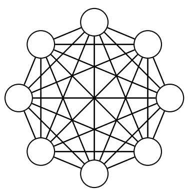

Figure 1: A clique.

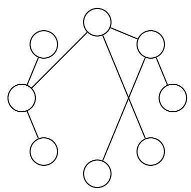

Figure 2: A tree.

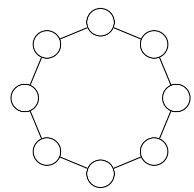

Figure 3: A cycle.

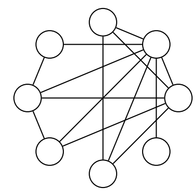

Figure 4: $G(8, 1/2)$.

Question: *What are the properties that we make graphs represent social networks?*

Definition. *Given a graph $G = (V, E)$, the **clustering coefficient of a vertex** v, denoted $C(v)$, is the fraction, over all pairs of neighbors of v, of those pairs who are neighbors of each other. Formally,*

$$C(v) := \frac{|\{(u, w) \in E : u, w \in N(v)\}|}{\binom{d(v)}{2}}.$$

*The **clustering coefficient of a graph** is the average clustering coefficient of its nodes.*

3 Square Lattices, Expansion, and Regular Random Graphs

Definition. *In a **square lattice** (see Figure 4) on $n = r^2$ nodes, the nodes are positioned in an $r \times r$ lattice and are connected to nodes at a lattice distance at most k away on the lattice for some given parameter $k \in \mathbb{N}$. The **lattice distance** between two nodes on a lattice is the minimum number of horizontal and vertical steps between the two nodes.*

Question: *What is the longest distance between two nodes in a square lattice?*

Definition. *A d-regular random graph is a graph where all nodes have degree d and every two nodes have the same likelihood to be connected. That is, fix some number of vertices n, consider all graphs on n vertices that are d-regular, and pick one of these uniformly at random.*

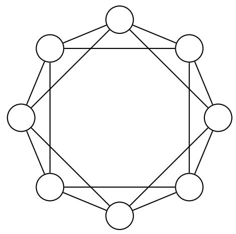

Figure 5: A ring lattice with $n = 8$ nodes and $k = 2$

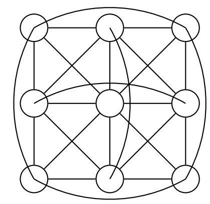

Figure 6: A square lattice with $r \times r = 3 \times 3 = 9$ nodes and $k = 2$.

Question: *Do d-regular random graphs have short distances?*

Definition. *The* expansion *of a graph $G = (V, E)$ is the minimum, over all cuts we can make (dividing the graph in two pieces), of the number of edges crossing the cut divided by the number of vertices in the smaller half of the cut. Formally, it is*

$$\alpha = \min_{S \subseteq V, 1 \leq |S| \leq \frac{|V|}{2}} \frac{|e(S)|}{|S|}$$

where $e(S)$ is the number of edges leaving the set S.

Theorem. *Suppose a graph G is d-regular, for some constant $d \geq 3$, and has constant expansion α. Then the diameter of G is $O\left(\frac{d}{\alpha} \log n\right)$.*

Question: *What is the clustering coefficient of d-regular random graphs?*

4 Constructing a Small World Model

	diameter	clustering coefficient	maximum degree
Lattice	bad (too large)	good (constant)	good (constant)
d-regular random	good (logarithmic)	bad (too small)	good (constant)
Watts-Strogatz	good	good	good

References

[1] Duncan J. Watts and Steven H. Strogatz. *Collective dynamics of small-world networks.* Nature, 393:440442, 1998.