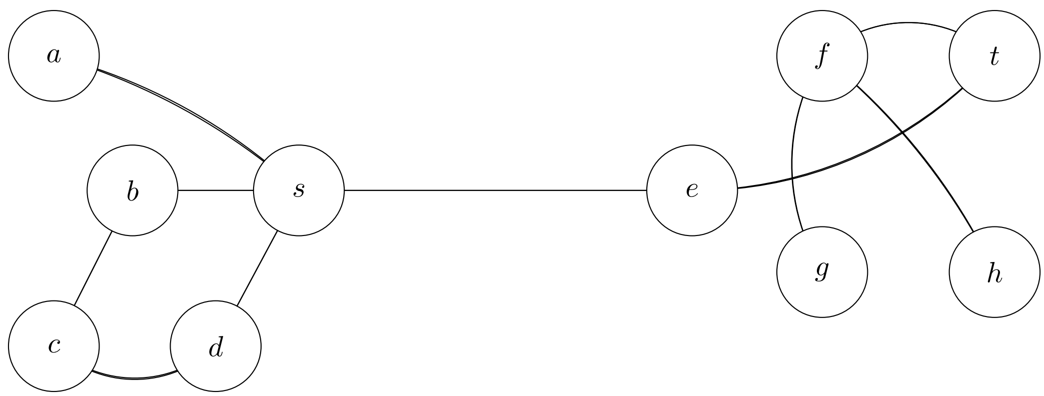
CS 134: Networks	**Practice Midterm**
Prof. Yaron Singer	**Friday 2/10/2017**

Instructions. This exam consists of five questions. You have to answer the first two questions, and *any two of the last three questions* (in case you answer all three, please indicate clearly which two you want us to grade).

The following hold of all graphs **unless otherwise stated**: they are undirected; there are never multiple edges between the same pair of nodes $\{i, j\}$; there are no self-edges (ones starting and ending at the same node).

1. Graph Terms. (25 points)



a. (5 points) How many connected components are there in this graph?

b. (5 points) Write down the neighborhood of $\{s, t\}$.

c. (5 points) Compute the expected distance from t to a node v in the graph chosen uniformly at random.

d. (10 points) What is the size of the smallest cut in this graph?

2. True or False. (25 points) For each statement, indicate whether it is true or false. If it is true, nothing else is required. If it is false, correct the statement or give a counterexample.

a. (10 points) Consider an Erdős-Rényi random graph $G(n, p)$, with $n \geq 3$ nodes and probability $p > 0$ of any link forming. The probability that the graph is connected is at least p^M, where $M = n(n-1)/2$ is the number of potential edges.

b. (5 points) In the BFS algorithm, the first node that enters the queue (excluding the source itself) is at distance 2 from the source.

c. (5 points) If n is even and G is a clique of n nodes, the expansion of G is n.

d. (5 points) Let G be a two-dimensional square lattice of dimensions $r \times r$. Let d be a natural number smaller than r, and let v be a fixed node in G. The number of nodes at distance at most d from v in G is $\Theta(d^2)$.

3. Power laws. (25 points) Let X be a discrete random variable such that

$$\mathbb{P}[X = d] = \begin{cases} c \cdot d^{-2} & \text{if } d \in \{1, 2, \ldots, \Delta\} \\ 0 & \text{otherwise.} \end{cases}$$

Here Δ is an integer and $c > 0$ is a constant that depends on Δ. This constant is set to a value that makes this a probability distribution (i.e., makes the probabilities sum up to exactly 1).

a. (10 points) Indicate which of the following describes the behavior of c as a function of Δ:

(i) As $\Delta \to \infty$ we have that $c \to +\infty$.

(ii) As $\Delta \to \infty$ we have that $c \to 0$.

(iii) As $\Delta \to \infty$ we have that c converges to a finite, nonzero constant.

(iiii) None of the above.

b. (15 points) Let m_Δ denote the expected value of X for a given value of the constant Δ. Write a formula for m_Δ and explain briefly why $m_\Delta = O(\Delta)$ as $\Delta \to \infty$. You may freely (without explanation) use the approximations we have learned for summations.

4. Configuration model. (25 points) Consider a configuration model defined on 3 nodes and degree distribution $\{2, 1, 1\}$.

a. (10 points) What is the exact probability that there is a self-edge in the graph?

b. (15 points) What is the exact probability that the graph is connected?

5. Diameter. Let G be an $r \times r$ square lattice.

a. (10 points) Write its diameter as a function of r.

b. (15 points) Let v be the node at a corner. How many nodes (in terms of r) are there at distance r from v? (Here, distance refers to geodesic distance in the graph G, which is the same thing as lattice distance.)