CS 134: Networks Lecture 6 **outline**, Wednesday, 2/8
Prof. Yaron Singer Spring 2017

1 Overview

Definition. *A non-negative random variable X has a* ***power law distribution*** *if there exist constants $c, \alpha > 0$ s.t.:*

$$Pr(X \geq x) = c \cdot x^{-\alpha}.$$

In the case where X is a discrete random variable, then there exist constants $c', \alpha' > 0$ s.t.:

$$Pr(X = x) = c' \cdot x^{-\alpha'}.$$

2 Power-Laws in Graphs

- The **configuration model**: generating random graphs with arbitrarily degree distribution:
  - We are given a degree distribution $(d_1, \cdots, d_n)$, indicating that node i has degree d_i.
  - For each $i \in \{1, \ldots, n\}$, create d_i "half-edges" exiting v_i.
  - Connect the half edges by choosing a matching uniformly at random, i.e., create disjoint pairs of half-edges uniformly at random.[1]
- Generating a **power-law like** degree distribution on a *finite* graph:
  - The largest degree in a graph is Δ.
  - There is exactly one node with degree Δ.
  - Thus:
    $$\Pr(X = \Delta) = c \cdot \Delta^{-\alpha} = \frac{1}{n}$$
    and we get that $\Delta = (c \cdot n)^{1/\alpha}$.

3 The Rich Get Richer Model

- Nodes arrive in order $1, \ldots, n$;
- When node j arrives it generates a link as follows:
  - With probability p, j connects to $i \in \{1, \cdots, j-1\}$ uniformly at random;
  - With probability $1-p$, j chooses $i \in \{1, \cdots, j-1\}$ uniformly at random and connects to the node i connects to.

[1] It is possible that we end up with self-loops though this is rare.