CS 134: Networks

Prof. Yaron Singer | Spring 2017

1 Overview

Last week we learned some game theory to allow us to access some more sophisticated network models. In this lecture, we will cover cascades. We wish to use a model to answer the following questions:

- When does the network adopt a new behavior?
- When does something go viral?

2 Network Coordination Model

Let us begin by defining our model formally.

Definition. *In our network coordination model, nodes in a network have two possible stategies, A and B. For each pair of connected nodes u and v:*

- *u and v get payoff $a > 0$ when both adopt A;*
- *u and v get payoff $b > 0$ when both adopt B;*
- *otherwise their payoff is* 0

In matrix form, this game is:

	A	B
A	a, a	$0, 0$
B	$0, 0$	b, b

Question: Suppose that a node u currently uses B. When will the node u adopt A?
Answer: Suppose that u has d neighbors and p fraction of them adopt A. Then the utility to u from adopting A is dpa, and utility from keeping B is $d(1-p)b$. We have that u will adopt A iff $dpa \geq d(1-p)b$ (assuming ties are broken lexicographically). Simplifying the algebra, we get that u will adopt A iff $p \geq \frac{b}{a+b}$.

We have that nodes will follow a **threshold rule** for adopting behavior A. A node adopts behavior A when at least a fraction q of its neighbors use A.

There are two obvious equilibria in this model, namely all A and all B. We are often interested to see how easy it is to change between these two equilibria, and if it is possible for other equilibria to arise.

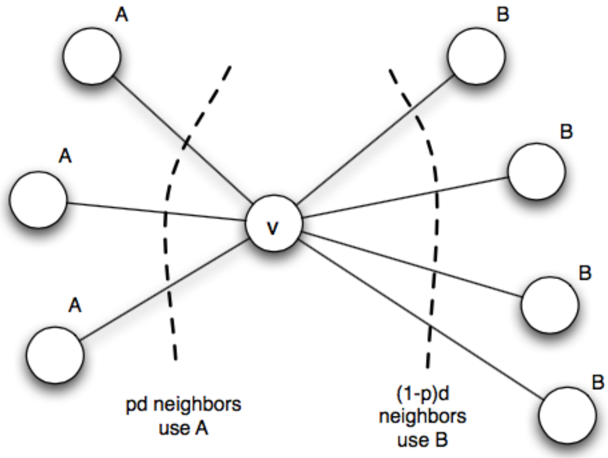


Figure 1: The node v has $d = 7$ and $p = \frac{3}{7}$.

3 Cascading Behavior

Let us begin with some definitions, followed by some examples.

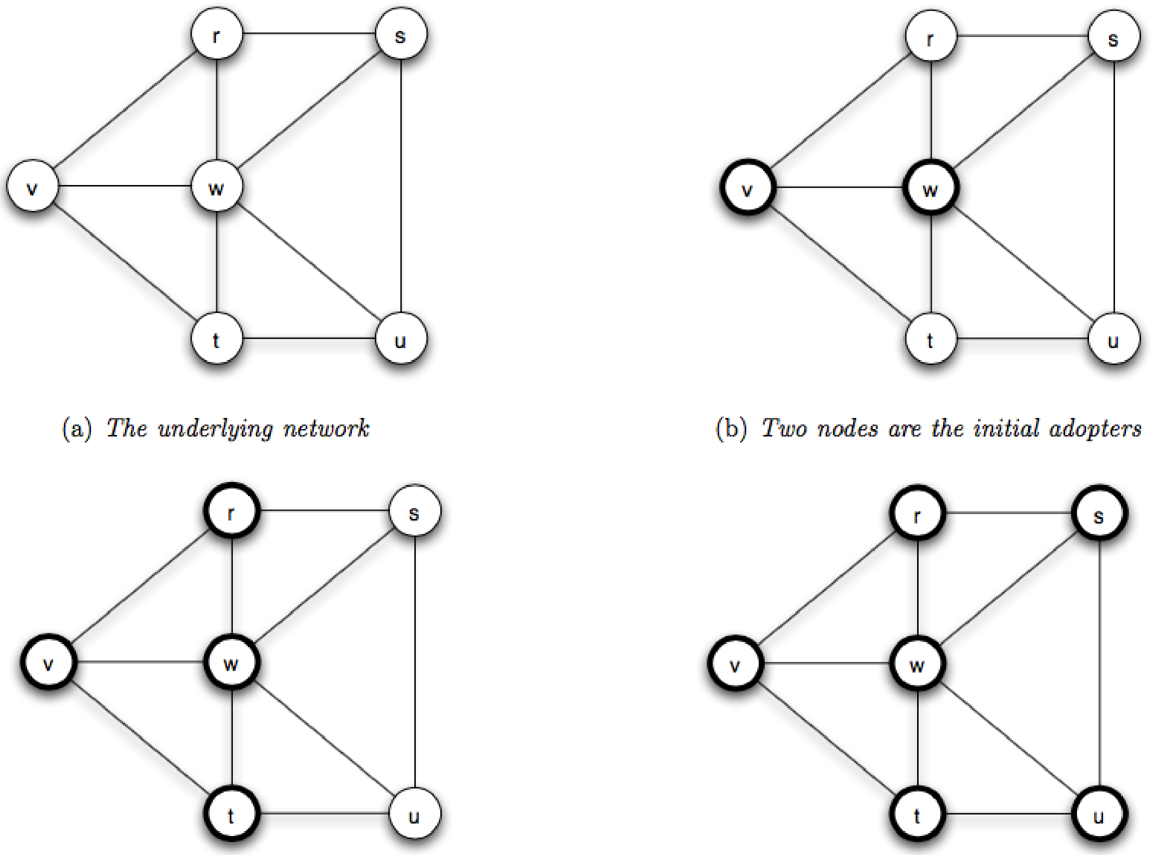
Definition. *In the cascading behavior model, nodes in a network play the network coordination game defined above. Initally, all nodes adopt strategy B.*

- *at time step $t = 0$, initialize a small set of initial adopters to adopt behavior A*
- *at every time step $t = 1, 2, \ldots$ each node that has not yet adopted A revises its decision and adopts A if a greater than $q = \frac{b}{a+b}$ fraction of its neighbors at time step t are using strategy A*

Definition. *Complete cascade. Consider a set of initial adopters of behavior A while others adopt B. Nodes re-evaluate decision to adopt A based on a threshold q. If all nodes end up switching to A, then then the initial set is said to create a* **complete cascade** *with threshold q.*

In order to increase the number of nodes that adopt A and the possibility of a complete cascade, we can try increasing the size of the initial set of adopters, or decrease the threshold q e.g. by increasing the payout a.

Note that in Figure 3, behavior A could not have a complete cascade was because it could not penetrate the cluster on the right.



(a) *The underlying network*

(b) *Two nodes are the initial adopters*

(c) *After one step, two more nodes have adopted*

(d) *After a second step, everyone has adopted*

Figure 19.3: Starting with v and w as the initial adopters, and payoffs $a = 3$ and $b = 2$, the new behavior A spreads to all nodes in two steps. Nodes adopting A in a given step are drawn with dark borders; nodes adopting B are drawn with light borders.

Figure 2: An example from the text of cascading behavior leading to a complete cascade.

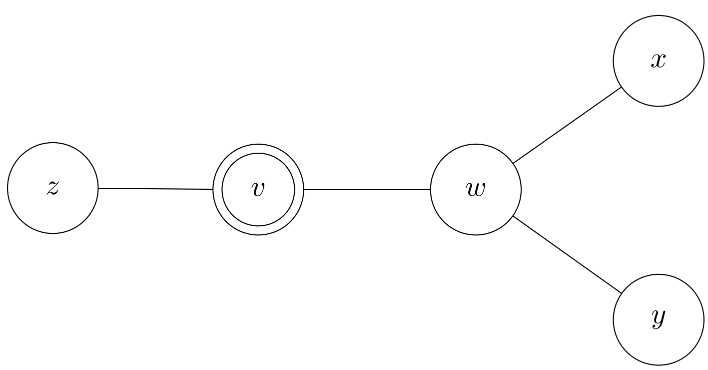


Figure 3: An example of an incomplete cascade when $a = 3, b = 2$. If the set of intial adopters of A is just $\{v\}$, this cascade only reaches z and terminates after one step.

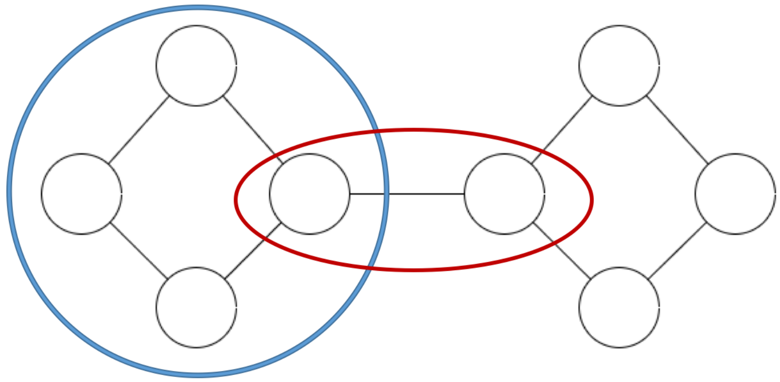


Figure 4: The four nodes circled in blue form a cluster of density $\frac{2}{3}$, but the two nodes circled in red do not.

Definition. *A* **cluster of density** *p is a set of nodes C such that every node in C has at least p fraction of its neighbors also in C.*

Theorem. *In a cascading behavior network, consider a set of initial adopters of behavior A with threshold q.*

1. *If the remaining network has a cluster of density greater than $1 - q$, then there is not a complete cascade.*
2. *If there is not a complete cascade, then there is a cluster of density greater than $1 - q$ in the remaining network.*

Proof. ($\Rightarrow$) Assume for the sake of contradiction that there exists a cluster S of density greater than $1 - q$ in the remaining network, but we also have a complete cascade. Let v be the first node in S that adopts A. But by the definition of a cluster with density greater than $1-q$, v has at least $1-q$ fraction of neighbors in S that have not adopted A (since v was the first node in S to do so), which is a contradiction to v adopting A since the threshold is not met. Hence, there is no complete cascade.

($\Leftarrow$) Assume that there is no complete cascade. Let S be the nodes in the network that did not adopt A. We have that $\forall w \in S$, w did not adopt A because at least $1 - q$ fraction of its neighbors did not adopt A. Hence, at least $1 - q$ fraction of its neighbors are in S, so S is a cluster of density at least $1 - q$. $\square$