HW2_PCA_xz2735

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Problem 1

1.

The picture is shown on the next page.

2.

(a)

There are 10304 principle components in total.

(b)

As $X = UDV^T$, D is a 10304×10304 diagonal matrix where the diagonal element $d_1 \geq d_2 \geq ... \geq d_{10304} \geq 0$, U is an 100×10304 orthogonal matrix which $UU^T = I$, V is an 10304×10304 diagonal matrix. $u_i d_i$ is ith principle component under new system. Let

$$\xi_i = u_{i1} d_1$$

is the elements for the first principle component, so

$$x_i V = [u_{11}d_1, u_{21}d_1, ..., u_{10304,1}d_1] = [\xi_1, \xi_2, ..., \xi_{10304}]$$

, and we choose to use first 48 principle component to represent x, so

$$\hat{x} = \bar{x} + \xi_1 v_1 \ldots + \xi_2 v_2 + ... + \xi_{48} v_{48}$$

Problem 2

1.

```r
library(quantmod)
library(factoextra)
symbols <- read.csv("dow.csv", header = F, stringsAsFactors = F)
nrStocks = length(symbols[,1])
z =matrix(data=NA,nrow=251,ncol=30)
colnames(z)=symbols[1:30,1]
for(i in 1:nrStocks){
  x<-getSymbols(symbols[i,1], auto.assign = F, from = "2018-01-01",
                to = "2019-01-01")[,4]
  row.names(z) <- format(index(getSymbols(symbols[i,1],
                   auto.assign = F, from = "2018-01-01",
                   to = "2019-01-01")[,4]),"%y/%m/%d")
  z[,i]=x
}
```

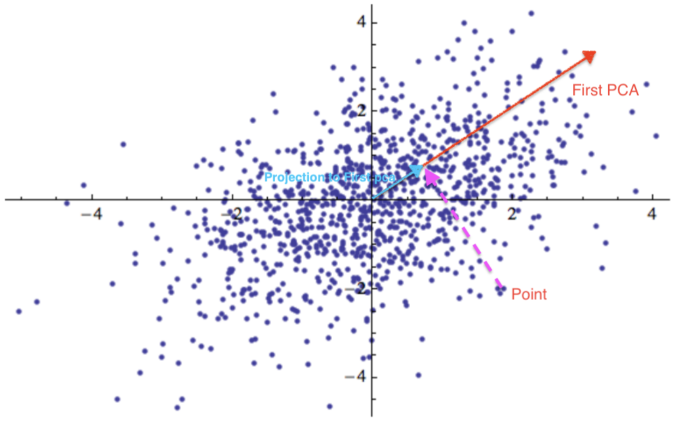
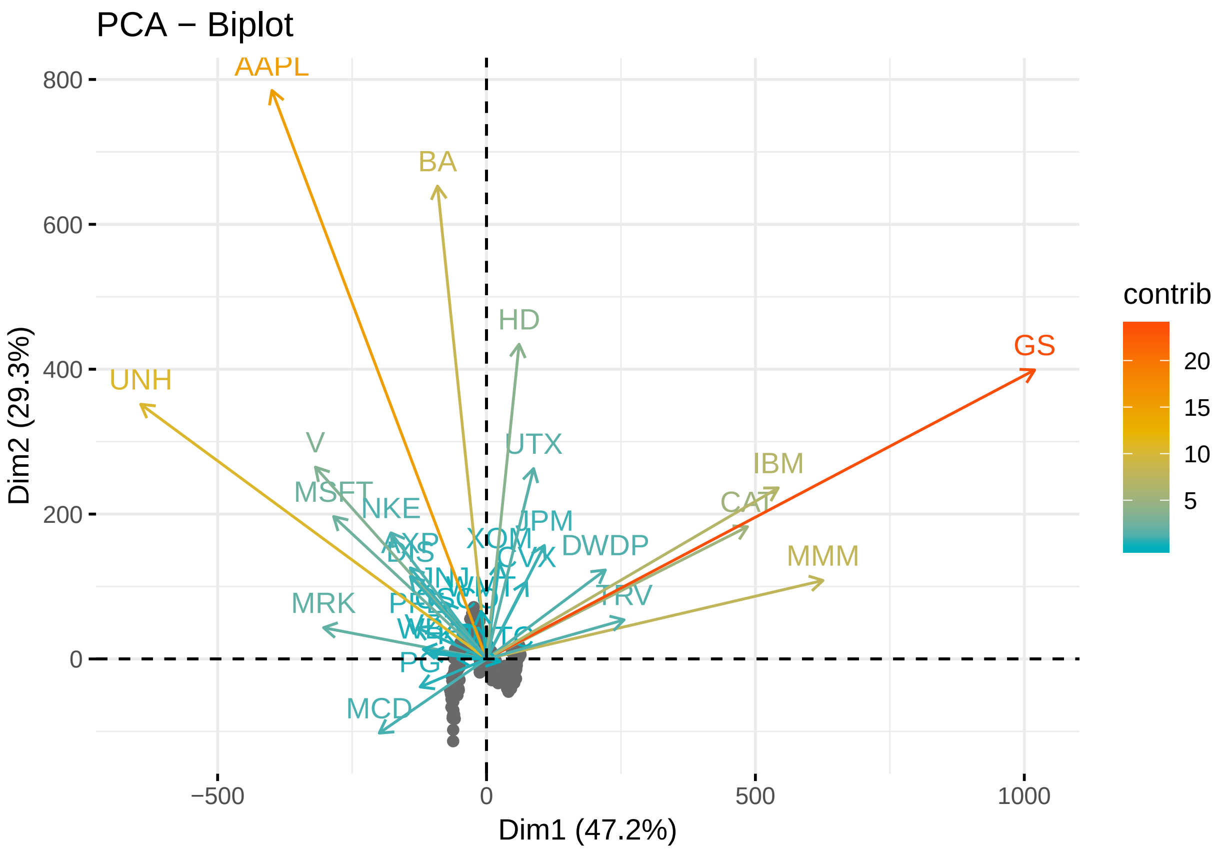


Figure 1: Caption for the picture.

2

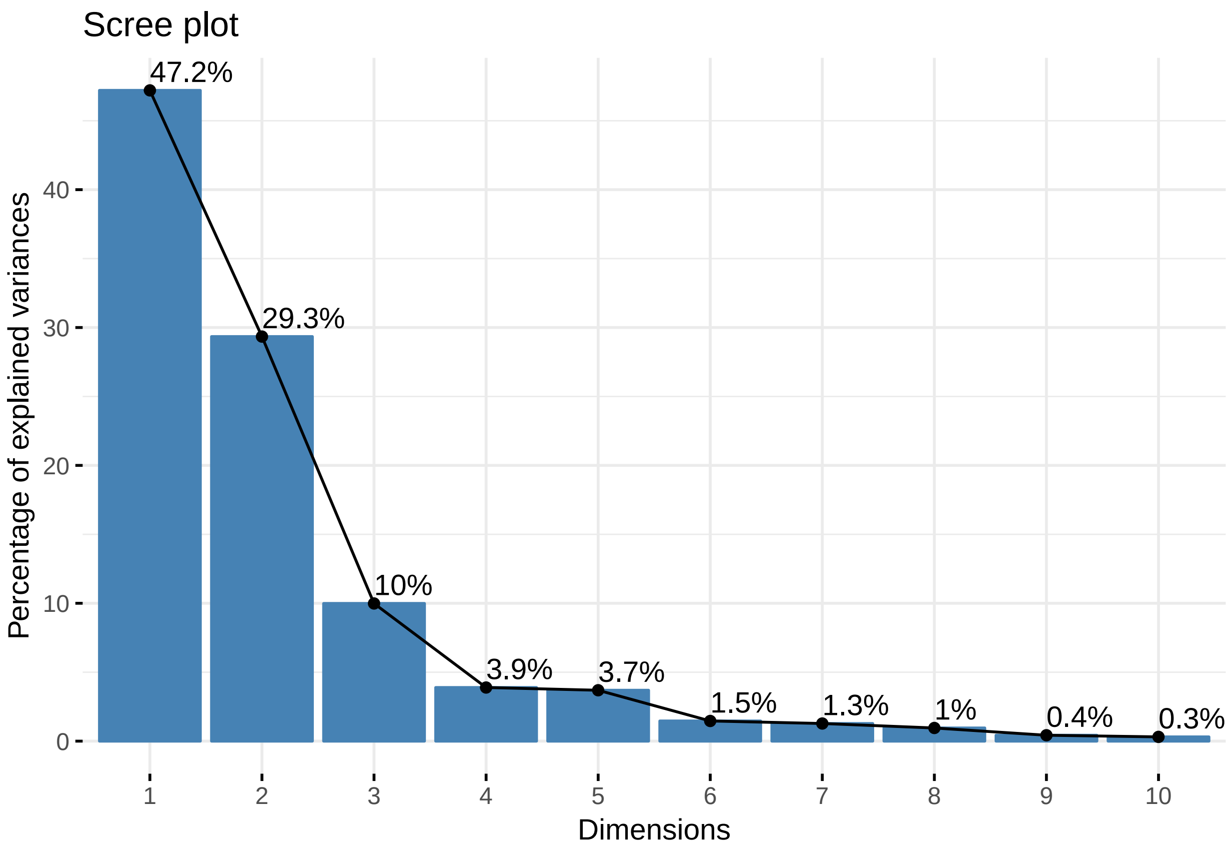
```
pca = princomp(z,cor=F)
```

```
fviz_pca_biplot(pca, repel = F,
                col.var="contrib",
                gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
                col.ind = "#696969" , # Individuals color
                label="var"
                )
```



In the biplot, there are some structures: Most companies are on the upside of the plot. 1. Companies in chemical, natural resource and construction are located in the first quadrant. 2. Other entertainment,food, pharmacy companies are in the second quadrant.

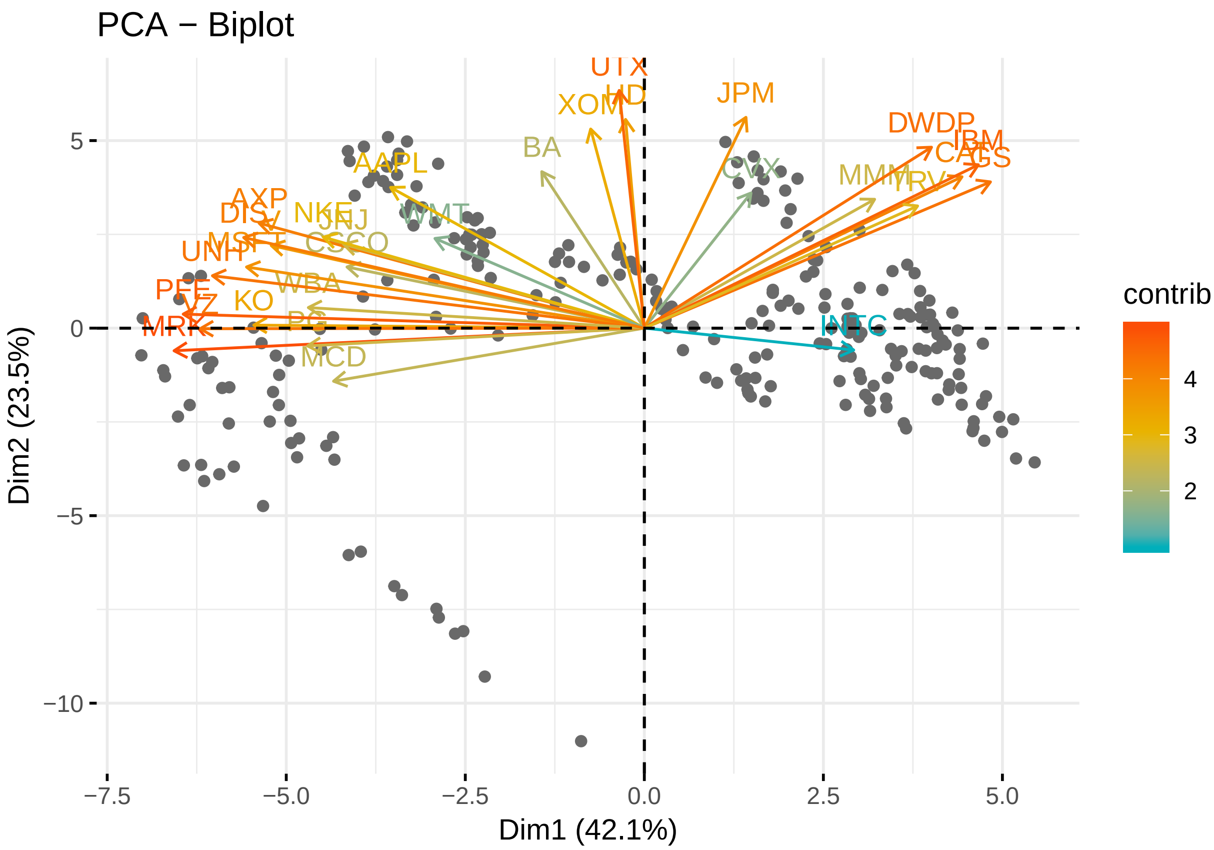
```
fviz_screeplot(pca, addlabels = TRUE)
```



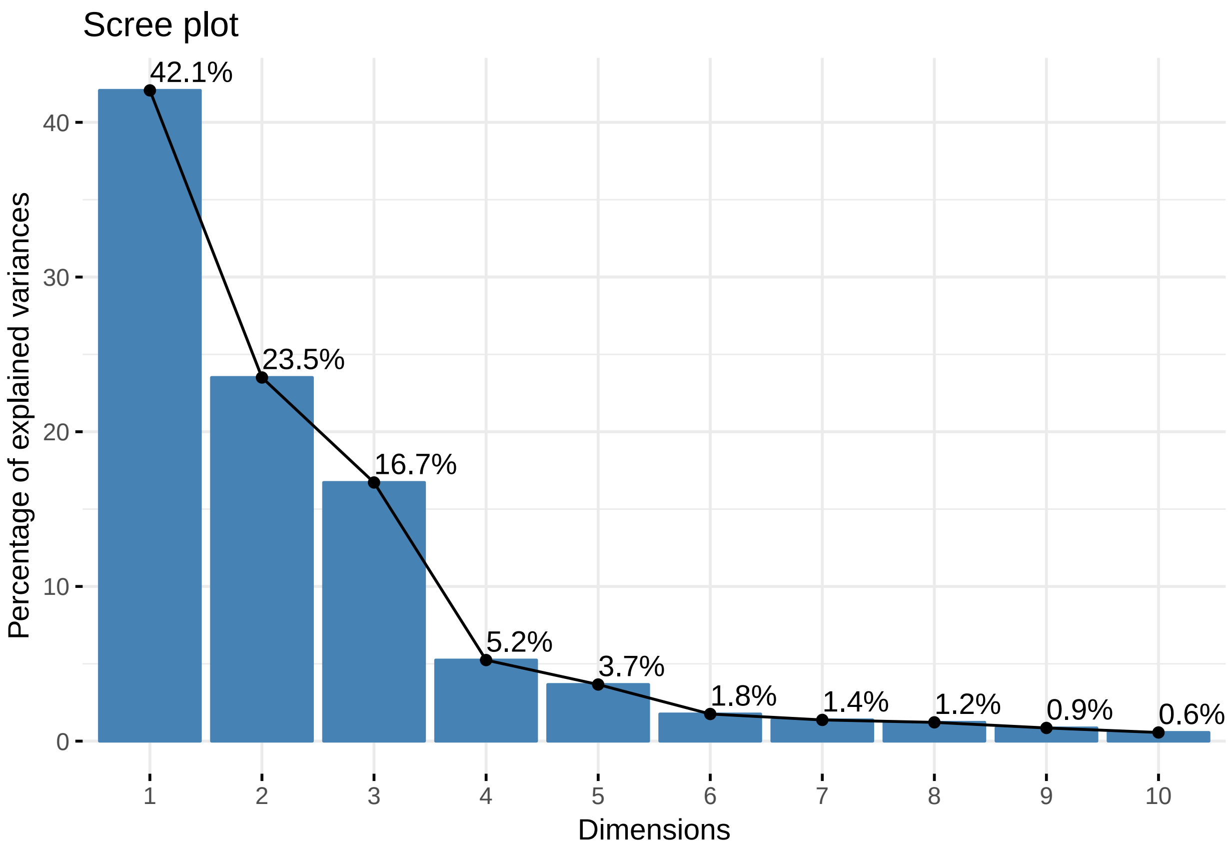
There are mainly 8 dimension that are important and it contains about 95% information of the data.

3

```
pca1 = princomp(z,cor=T)
fviz_pca_biplot(pca1, repel = F,
                col.var="contrib",
                gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
                col.ind = "#696969" , # Individuals color
                label="var"
                )
```



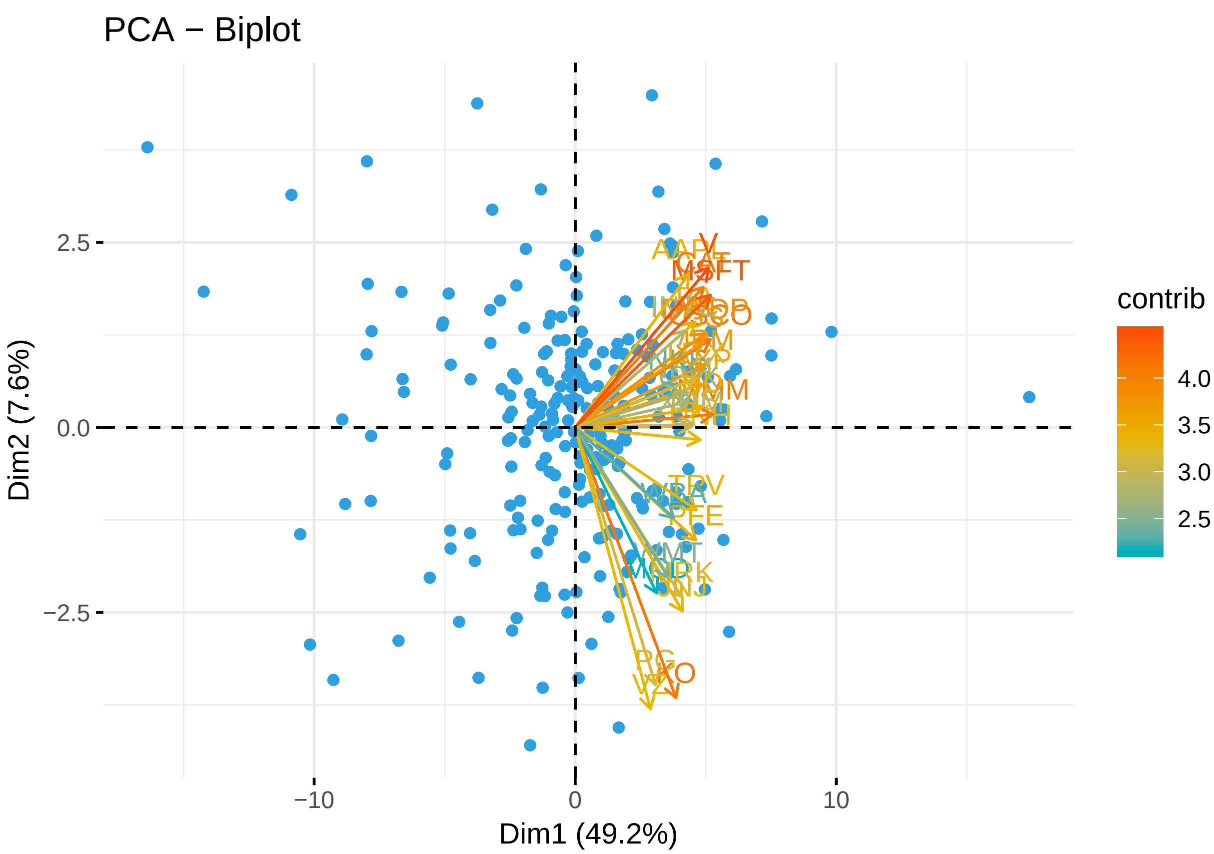

```r
fviz_screeplot(pca1, addlabels = TRUE)
```



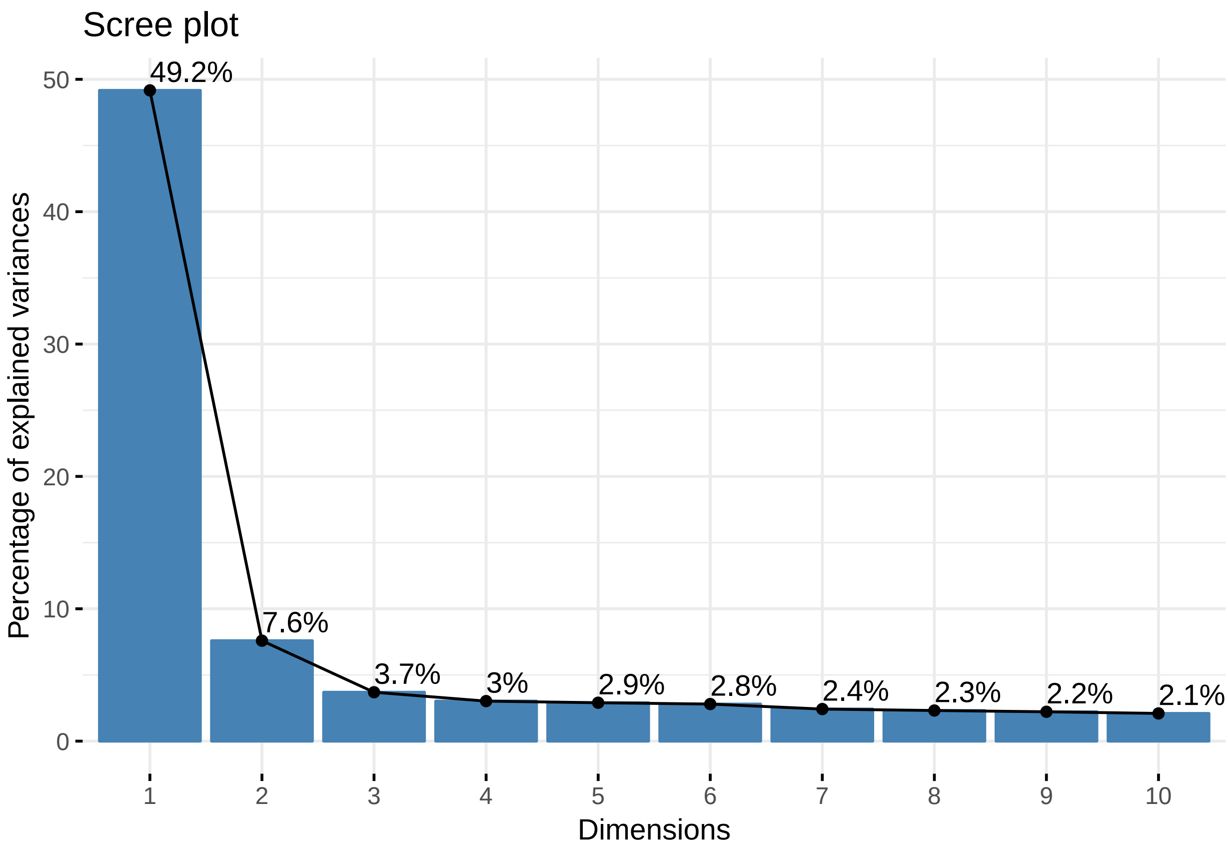
In this case, the data is scaled.

4

```
data.now <- z[1:dim(z)[1]-1,]
data.next <- z[2:dim(z)[1],]
return <- (data.next-data.now)/data.now
pca.return = princomp(return,cor=T)
fviz_pca_biplot(pca.return, repel = F,
                col.var="contrib",
                gradient.cols = c("#00AFBB", "#E7B800", "#FC4E07"),
                col.ind = "#2E9FDF" , # Individuals color
                label="var"
                )
```



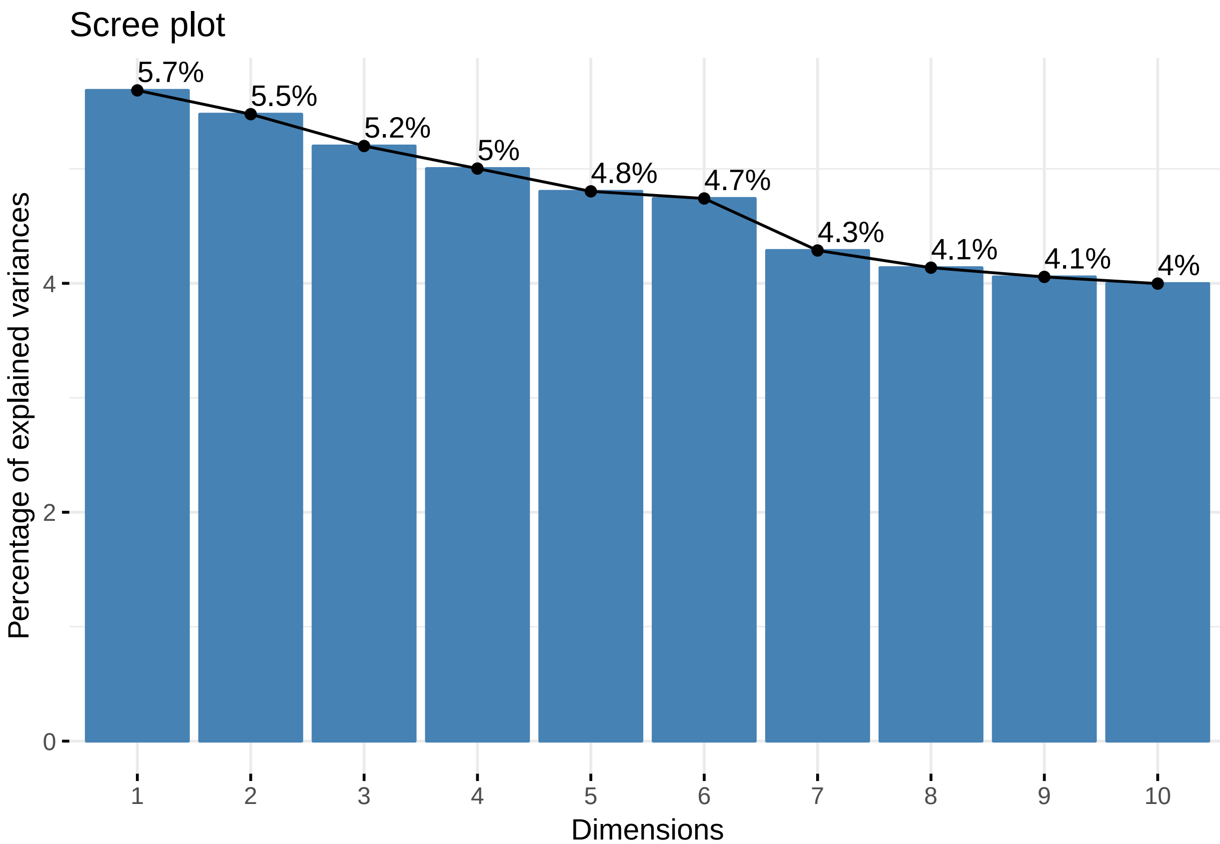

```r
fviz_screeplot(pca.return, addlabels = TRUE)
```



With the scree plot, we can see the first component contains almost half information of the data already. The components on the shallow slope(after component 2) contribute little to the solution. We can drop them. But is we want to contain 95% information of the data, we should keep at least the first 10 PCA.

If each stock fluctuating up and down randomly and independently for each stock,the variance perception of each compnent would be very similar. In that case, the scree plot will be quite flat and the perception of each component will be very small. So for this situation, there is no need to do PCA. It can be shown as below.

```
a = runif(30*250,-1,1)
data = matrix(a,nrow=250,ncol=30)
pca.rand = princomp(data)
fviz_screeplot(pca.rand, addlabels = TRUE)
```



Problem 3

(1)

Suppose $s_{ij} =< x_i - \bar{x}, x_j - \bar{x} >$, $t_{ij} =< z_i - \bar{z}, z_j - \bar{z} >$, $a_{ij} = s_{ij} - t_{ij}$ then

$$(S-T)^2 = \begin{pmatrix} \sum_i a_{1i} & & \\ & \ddots & \\ & & \sum_i a_{ni} \end{pmatrix}$$

So

$$tr[(S-T)^2] = \sum_i \sum_j a_{ij}^2 = \sum_i \sum_j (< x_i - \bar{x}, x_j - \bar{x} > - < z_i - \bar{z}, z_j - \bar{z} >)^2$$

(2)

By property of trace, such that $tr[A+B] = tr[A] + tr[B]$,$tr[AB] = tr[BA]$, we know

$$tr[UD^4U^T] = tr[U^TUD^4] = tr[D^4]$$

. And

$$tr[(UD^2U^T - \tilde{U}\tilde{D}^4\tilde{U}^T)] = tr[D^4] + tr[\tilde{D}^4] - tr[\tilde{U}\tilde{D}^4\tilde{U}^TUD^2U^T) - tr[UD^2U^T\tilde{U}\tilde{D}^4\tilde{U}^T]$$

$$= tr[D^4] + tr[\tilde{D}^4] - 2tr[D^2U^T\tilde{U}\tilde{D}^4\tilde{U}^TU] =$$
$$tr[D^4 + \tilde{D}^4 - 2D^2U^T\tilde{U}\tilde{D}^4\tilde{U}^TU]$$

(3)

$A = U^T\tilde{U}$, so

$$S_c = tr[D^4 + \tilde{D}^4 - 2D^2A\tilde{D}^2A^T] = tr[D^4] + tr[\tilde{D}^4] - tr[2A^TD^2A\tilde{D}^2] = \sum_j d_j^4 + \sum_j \tilde{d}_j^4 - 2\sum_i\sum_j a_{ij}^2 d_i^2 \tilde{d}_j^2$$

, so

$$\frac{\partial S_c}{\partial \tilde{d}_j^2} = 2\tilde{d}_j^2 - 2\sum_j d_i^2 a_{ij}^2$$

for $j = 1, 2...k$

(4)

Let

$$\frac{\partial S_c}{\partial \tilde{d}_j^2} = 0$$

, so

$$\tilde{d}_j^2 = \sum_j d_i^2 a_{ij}^2$$

Now, to minimize (2), we should maximize $\sum_j^k(\sum_i d_i^2 a_{ij}^2)^2 = \sum_j^k(\tilde{u}_j^T U D^2 U \tilde{u}_j)^2$, as we know U is the martix with each columns are one of first k eigenvectors of matrix S.