Quantum Computing Project

Introduction to Quantum Computation

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1 Introduction

This project is designed to culminate the foundational knowledge of single-qubit systems, quantum gates, multiple-qubit systems, quantum circuits, and quantum measurements.

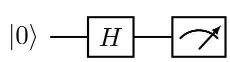
We will design a single-qubit circuit demonstrating superposition, a two-qubit circuit demonstrating entanglement, and a complex circuit involving at least three qubits and a combination of gates in the following sections accociate with the corresponding circuit analysis and the discussion.

2 Single-qubit

The H gate and rotation gate can help us to generate a qubit in a superposition.

2.1 Example 1

2.1.1 Design



2.1.2 Analysis

After applying the H gate the qubit state is

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

with the probability of measuring $|0\rangle$:$|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$ and the probability of measuring $|1\rangle$:$|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$

In other word, when we measure, we obtain 0 or 1, each with 50% probability which in a state of a superposition.

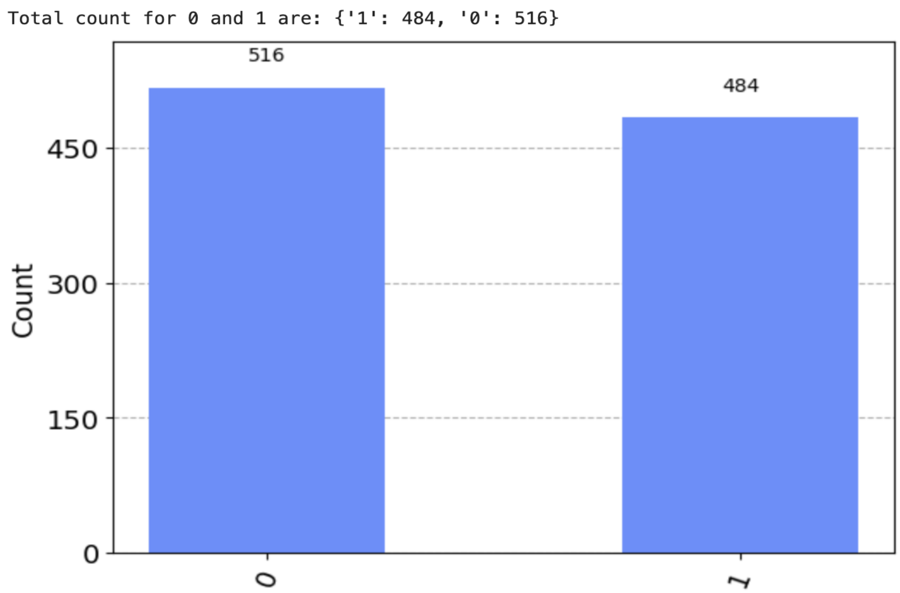


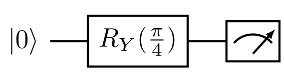
Figure 1: Simulation result of 2.1 Example.

2.1.3 Simulation

Figure 1 shows the simulation result of example 1 in 2.1, after 1000 simulation, the proportion of getting $|0\rangle$ is 51.6% and getting $|1\rangle$ is 48.4% which are very close to the theoretical results shown in 2.1.2.

2.2 Example 2

2.2.1 Design



2.2.2 Analysis

In this example, we consider a rotation gate $R_Y(\theta) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{bmatrix}$.

The output state will be

$$R_Y(\frac{\pi}{4})\,|0\rangle = \begin{bmatrix} \cos(\frac{\pi}{8}) & -\sin(\frac{\pi}{8}) \\ \sin(\frac{\pi}{8}) & \cos(\frac{\pi}{8}) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cos(\frac{\pi}{8}) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \sin(\frac{\pi}{8}) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \cos(\frac{\pi}{8})\,|0\rangle + \sin(\frac{\pi}{8})\,|1\rangle$$

with the probability of measuring $|0\rangle$:$|\cos(\frac{\pi}{8})|^2 \approx 0.85$ and the probability of measuring $|1\rangle$:$|\sin(\frac{\pi}{8})|^2 \approx 0.15$.

So we will obtain $|0\rangle$ with 85% probability and $|1\rangle$ with 15% probability which in a state of a superposition.

2.2.3 Simulation

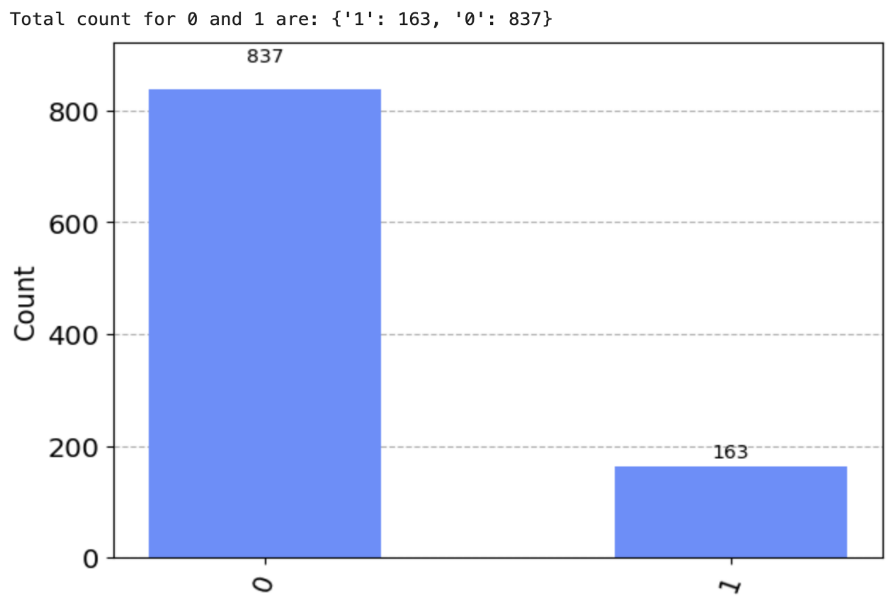
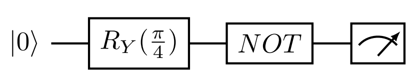


Figure 2: Simulation result of 2.2 Example.

Figure 2 shows the simulation result of example 2 in 2.2, after 1000 simulation, the proportion of getting $|0\rangle$ is 83.7% and getting $|1\rangle$ is 16.3%. The empirical results are close to the theoretical results shown in 2.2.2 .

2.3 Example 3

2.3.1 Design



2.3.2 Analysis

In additon to example 2, we add an extra NOT gate after the rotation gate.

The output state will be

$$NOT(R_Y(\frac{\pi}{4})\,|0\rangle) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\frac{\pi}{8}) & -\sin(\frac{\pi}{8}) \\ \sin(\frac{\pi}{8}) & \cos(\frac{\pi}{8}) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \sin(\frac{\pi}{8})\,|0\rangle + \cos(\frac{\pi}{8})\,|1\rangle$$

with the probability of measuring $|0\rangle$:$|\sin(\frac{\pi}{8})|^2 \approx 0.15$ and the probability of measuring $|1\rangle$:$|\cos(\frac{\pi}{8})|^2 \approx 0.85$.

The output will obtain $|0\rangle$ with 15% probability and $|1\rangle$ with 85% probability which in a state of a superposition.

2.3.3 Simulation

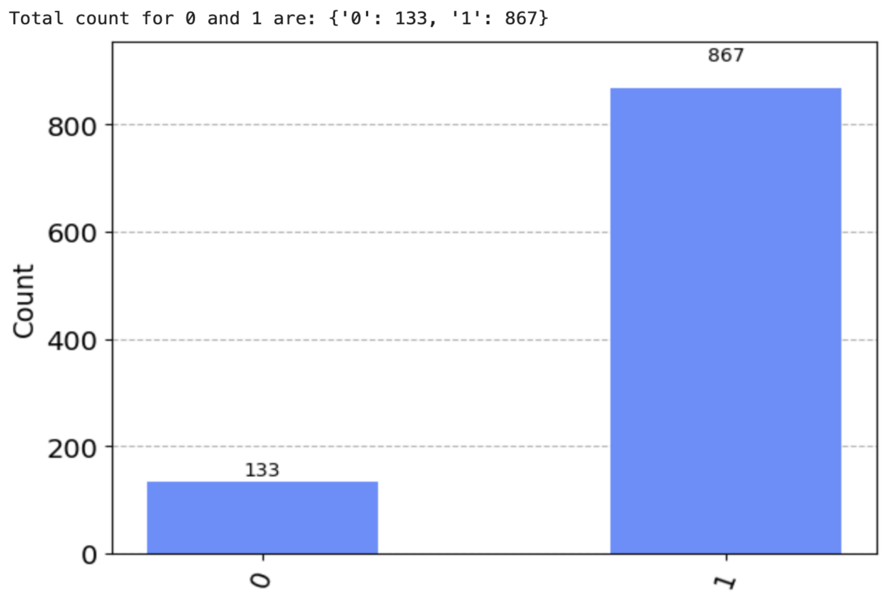


Figure 3: Simulation result of 2.3 Example.

Figure 3 shows the simulation result of example 3 in 2.3, after 1000 simulation, the proportion of getting $|0\rangle$ is 13.3% and getting $|1\rangle$ is 86.7%. The empirical results are close to the theoretical results shown in 2.2.3.

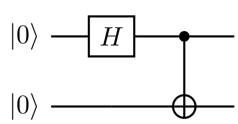
2.4 Discussion

The three examples are all in a state of superposition and can distinguish with each other since they have the different probability of collapsing to 0 and 1 after the measurements. The corresponding simulation results are all consistant with the theoretical results.

3 Two-qubits circuit

3.1 Example 1($|\Phi^+\rangle$)

3.1.1 Design



3.1.2 Analysis

$$
\begin{aligned}
CNOT((H|0\rangle)\otimes|0\rangle) &= CNOT\left(\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)\otimes\begin{bmatrix}1\\0\end{bmatrix}\right)\\
&= CNOT\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}\right)\\
&= CNOT\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\\
&= \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}\\
&= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle
\end{aligned}
$$

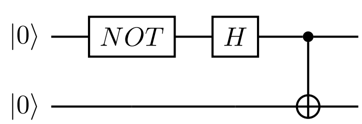
The output state will collapse to $|00\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$, and collapse to $|11\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$.

3.1.3 Simulation

Figure 4 shows the simulation result of example 1 in 3.1, after 1000 simulation, the proportion of getting $|00\rangle$ is 50.8% and getting $|11\rangle$ is 49.2% which are close to the theoretical results shown in 3.1.2 .

3.2 Example 2($|\Phi^-\rangle$)

3.2.1 Design



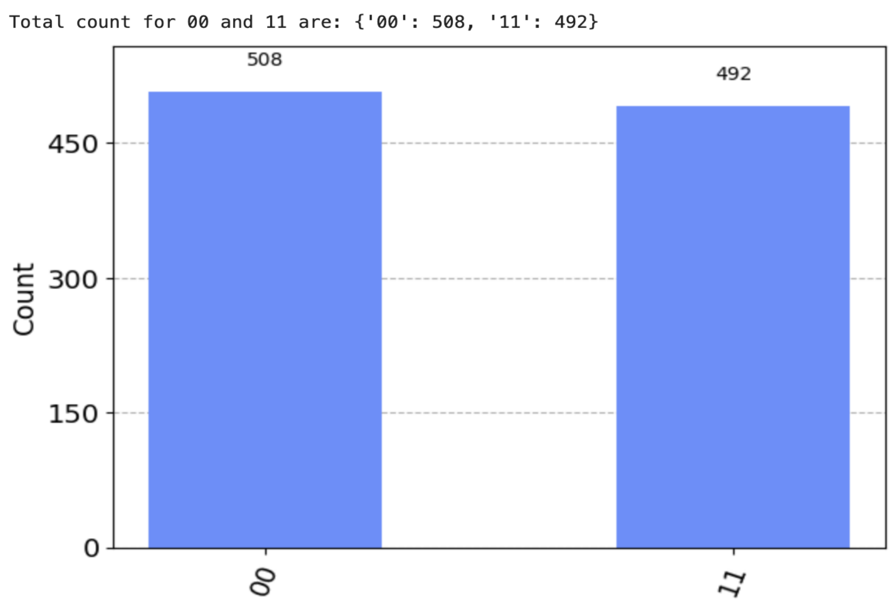


Figure 4: Simulation result of 3.1 Example.

3.2.2 Analysis

$$
\begin{aligned}
CNOT((H(NOT\,|0\rangle)) \otimes |0\rangle) &= CNOT\left(\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\right)\right) \otimes \begin{bmatrix}1\\ 0\end{bmatrix}\right)\\
&= CNOT\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ -1\end{bmatrix} \otimes \begin{bmatrix}1\\ 0\end{bmatrix}\right)\\
&= CNOT\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\\ -1\\ 0\end{bmatrix}\\
&= \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\\ -1\\ 0\end{bmatrix}\\
&= \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 0\\ 0\\ -1\end{bmatrix} = \frac{1}{\sqrt{2}}\,|00\rangle - \frac{1}{\sqrt{2}}\,|11\rangle
\end{aligned}
$$

The output state will collapse to $|00\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$, and collapse to $|11\rangle$ with probability $|-\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$.

3.2.3 Simulation

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Total count for 00 and 11 are: {'00': 506, '11': 494}
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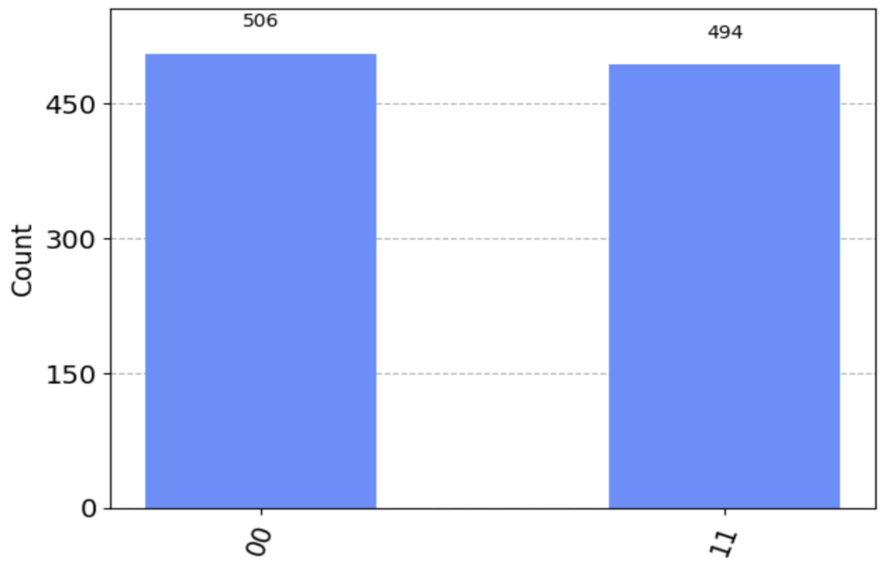
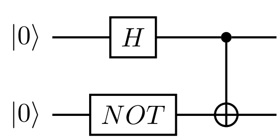


Figure 5: Simulation result of 3.1 Example.

Figure 5 shows the simulation result of example 2 in 3.2, after 1000 simulation, the proportion of getting $|00\rangle$ is 50.6% and getting $|11\rangle$ is 49.4% which are close to the theoretical results shown in 3.2.2 .

3.3 Example 3($|\Psi^+\rangle$)

3.3.1 Design



3.3.2 Analysis

$$
\begin{aligned}
CNOT((H\left|0\right\rangle) \otimes (NOT\left|0\right\rangle))) &= CNOT\left(\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)\otimes\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)\\
&= CNOT\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}\right)\\
&= CNOT\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}\\
&= \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}\\
&= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\left|01\right\rangle + \frac{1}{\sqrt{2}}\left|10\right\rangle
\end{aligned}
$$

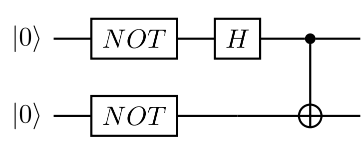
The output state will collapse to $\left|01\right\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$, and collapse to $\left|10\right\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$.

3.3.3 Simulation

Figure 6 shows the simulation result of example 3 in 3.3, after 1000 simulation, the proportion of getting $\left|01\right\rangle$ is 49.8% and getting $\left|10\right\rangle$ is 50.2% which are close to the theoretical results shown in 3.3.2 .

3.4 Example 4($\left|\Psi^-\right\rangle$)

3.4.1 Design



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Total count for 01 and 10 are: {'01': 498, '10': 502}
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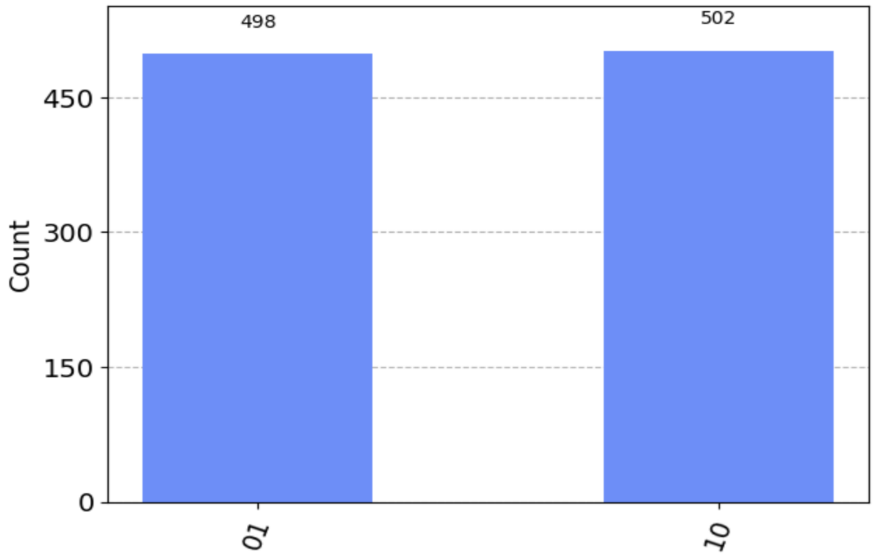


Figure 6: Simulation result of 3.3 Example.

3.4.2 Analysis

$$
\begin{aligned}
CNOT(NOT(H|0\rangle) \otimes NOT|0\rangle)) &= CNOT\left(\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)\otimes\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\right)\\
&= CNOT\left(\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}\right)\\
&= CNOT\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}\\
&= \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix}\\
&= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle
\end{aligned}
$$

The output state will collapse to $|01\rangle$ with probability $|\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$, and collapse to $|10\rangle$ with probability $|-\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$.

3.4.3 Simulation

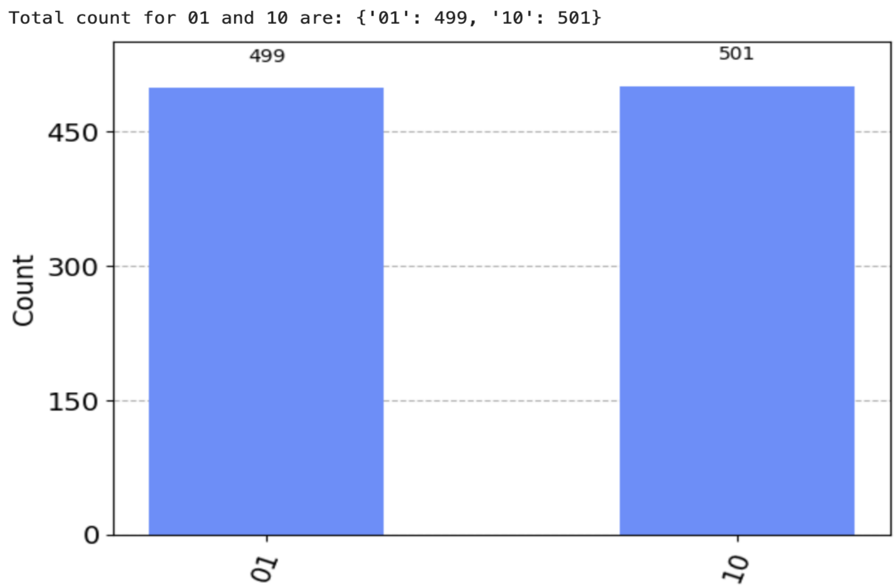
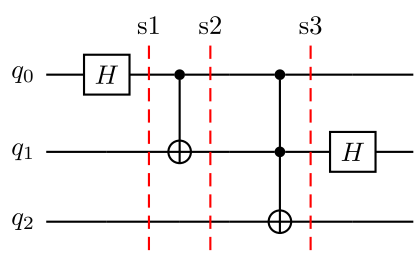


Figure 7: Simulation result of 3.4 Example.

Figure 7 shows the simulation result of example 4 in 3.4, after 1000 simulation, the proportion of getting $|01\rangle$ is 50.6% and getting $|10\rangle$ is 50.1% which are close to the theoretical results shown in 3.4.2 .

4 Three-qubits circuit

4.1 Design



In this design, we have three qubits q_0, q_1 and q_2.We apply a Hadamard gate (H) to the first qubit q_0 and use a CNOT gate (X) to entangle q_0 and q_1. We also apply the Toffoli gate (T) to q_0, q_1, and q_2. Finally, we have a Hadamard gate applied to q_1.

4.2 Analysis

At s1,

$$|\psi\rangle = H|0\rangle \otimes |0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}(|000\rangle + |100\rangle).$$

At s2,

$$\begin{aligned}(CNOT \otimes I)|\psi\rangle &= \left(\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix} \otimes \begin{bmatrix}1&0\\0&1\end{bmatrix}\right)|\psi\rangle \\ &= \begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\1\\0\\0\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\0\\1\\0\end{bmatrix} \\ &= \frac{1}{\sqrt{2}}(|000\rangle + |110\rangle)\end{aligned}$$

At s3,

$$T(\frac{1}{\sqrt{2}}(|000\rangle+|110\rangle)) = \begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\0\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)$$

Finally, the output will be

$$\begin{aligned}(I \otimes H \otimes I)\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) &= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1&0\\0&1\end{bmatrix} \otimes \begin{bmatrix}1&1\\1&-1\end{bmatrix} \otimes \begin{bmatrix}1&0\\0&1\end{bmatrix}\right)\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\ &= \frac{1}{\sqrt{2}}\begin{bmatrix}1&0&1&0&0&0&0&0\\0&1&0&1&0&0&0&0\\1&0&-1&0&0&0&0&0\\0&1&0&-1&0&0&0&0\\0&0&0&0&1&0&1&0\\0&0&0&0&0&1&0&1\\0&0&0&0&1&0&-1&0\\0&0&0&0&0&1&0&-1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\0\\1\\0\\0\\1\\0\\-1\end{bmatrix} \\ &= \frac{1}{2}|000\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|101\rangle - \frac{1}{2}|111\rangle\end{aligned}$$

The outcome will obtain $|000\rangle$ with probability $|\frac{1}{2}|^2 = 0.25$,$|010\rangle$ with probability 0.25, $|101\rangle$ with probability 0.25 and $|111\rangle$ with probability $|-\frac{1}{2}|^2 = 0.25$.

4.3 Simulation

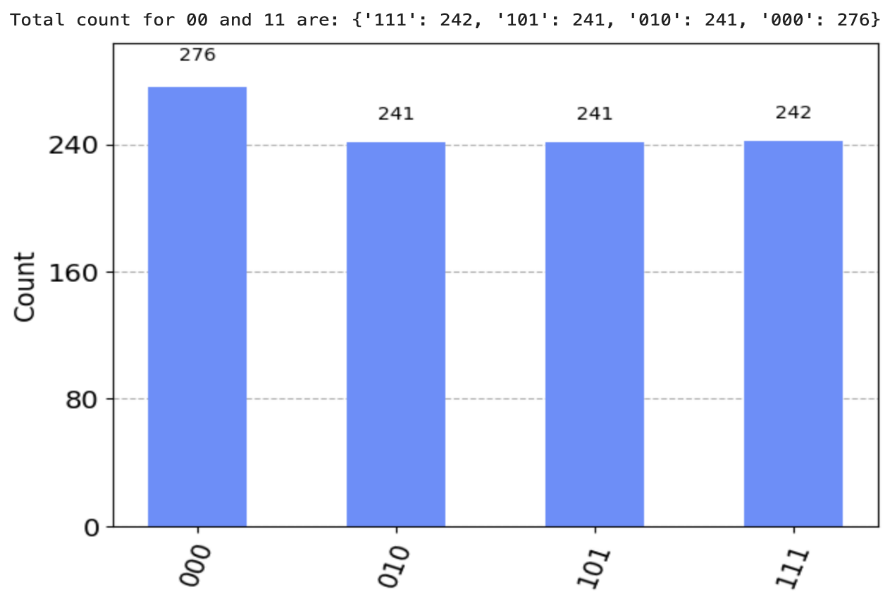


Figure 8: Simulation result of 4.1 Example.

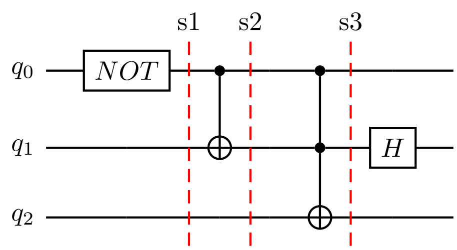
Figure 8 shows the simulation result of example 4 in 3.4, after 1000 simulation, the proportion of getting $|000\rangle$ is 27.6%, getting $|010\rangle$ is 24.1%, getting $|101\rangle$ is 24.1% and getting $|111\rangle$ is 24.1%. These results are close to the theoretical results shown in 4.2 .

4.4 Decoherence

Suppose we encountered an unexpected issue with decoherence at s1 or s3 in 4.1.

4.4.1 Decoherence occures at point s1

If decoherence occurs after the Hadamard gate, the information about q_0 superposition may be lost. The qubit q_0 might end up in a mixed state, and the quantum advantage gained from superposition could be diminished. If the output of q_0 at s1 comes to $|0\rangle$ due to the decoherence, then we can treat it as NOT gate.



The output at s1 would be

$$|\psi\rangle = |1\rangle \otimes |0\rangle \otimes |0\rangle = |100\rangle .$$

sequentially, at s2, we have

$$\begin{aligned}
(CNOT \otimes I)\,|100\rangle &= \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) |100\rangle \\
&= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \\
&= |110\rangle)
\end{aligned}$$

At s3,

$$T(|100\rangle) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |111\rangle$$

Finally,

$$
\begin{aligned}
(I \otimes H \otimes I)\,|111\rangle &= \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) |111\rangle \\
&= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \\
&= \frac{1}{\sqrt{2}} (|101\rangle - |111\rangle)
\end{aligned}
$$

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{'111': 517, '101': 483}
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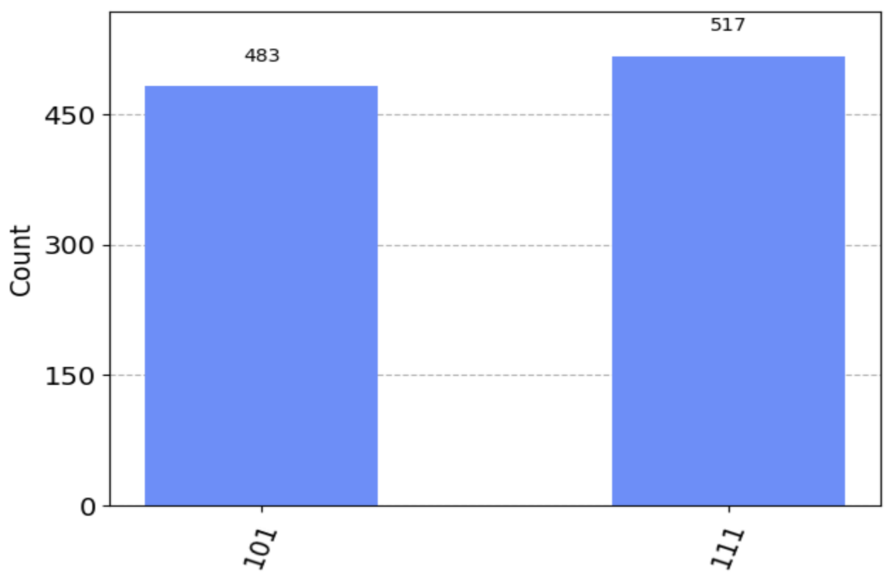
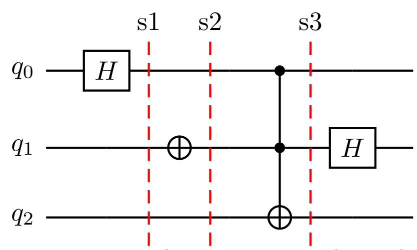


Figure 9: Simulation result of 4.4.1.

The result only have two combinations with 48.3% of $|101\rangle$ and 51.7% of $|111\rangle$ in this situation while the output shown in Figure 8 has four combinations.

4.4.2 Decoherence occures at point s2

If decoherence occurs after the CNOT gate, the correlations between q_0 and q_1 may be disrupted. Assume that the decoherence occurs on the control gate , the 4.1 will change to



Under this case, there is no engagement between q_0 and q_1, the output at s2 will be

$$(H \otimes NOT \otimes I)\left|000\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\otimes\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\otimes\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\left|000\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\0\\0\\0\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}(\left|010\right\rangle + \left|110\right\rangle)$$

At s3,

$$T(\frac{1}{\sqrt{2}}(\left|010\right\rangle + \left|110\right\rangle)) = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}(\left|010\right\rangle + \left|111\right\rangle)$$

Finally,

$$\begin{aligned}(I \otimes H \otimes I)\left|111\right\rangle &= \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\otimes\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\otimes\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\right)\frac{1}{\sqrt{2}}(\left|010\right\rangle + \left|111\right\rangle)\\
&= \frac{1}{2}\begin{bmatrix}1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 1 & 0 & 0 & 0 & 0\\1 & 0 & -1 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & -1 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 1 & 0 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 1 & 0 & 1\\0 & 0 & 0 & 0 & 1 & 0 & -1 & 0\\0 & 0 & 0 & 0 & 0 & 1 & 0 & -1\end{bmatrix}\begin{bmatrix}0\\0\\1\\0\\0\\0\\0\\1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1\\0\\-1\\0\\0\\1\\0\\-1\end{bmatrix}\\
&= \frac{1}{2}(\left|000\right\rangle - \left|010\right\rangle + \left|101\right\rangle - \left|111\right\rangle)\end{aligned}$$

Even though there is a decoherence occured and the theoretical results are not the same from 4.2.1. But the final simulation results are the same because the theoretical probability prediction cannot distinguish with each other.

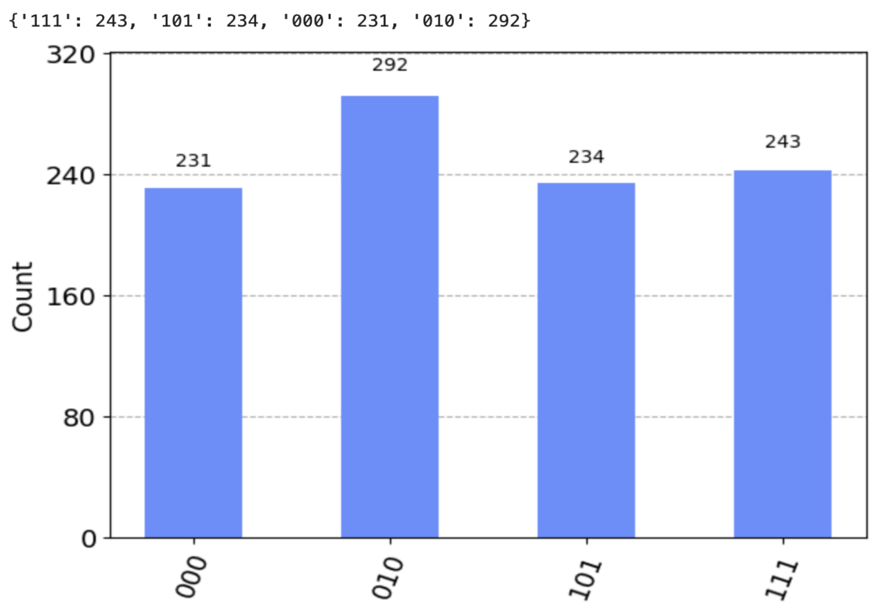


Figure 10: Simulation result of 4.4.2.

4.5 Discussion

We redomly choose two points, and result from 4.4.1 is changed while result from 4.4.2 keep the same. The situation we considered is not only just assume the state $|1\rangle$. In the situation 4.4.1, the decoherence will effect the engagement between q_0 and q_1,the result will change sequentially after the decoherence occured.