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A Partitioning Method for Distributed Capacitor Control of Electric Power Distribution Systems

Michael R. Kleinberg, *Student Member, IEEE*, Karen Miu, *Member, IEEE*, Nicole Segal, *Student Member, IEEE*, Horst Lehmann, and Timothy R. Figura

***Abstract*—In order to implement distributed control of an electric power distribution system, typically a feeder is first partitioned into control areas. In this work, an analytical partitioning method based on capacitor reactive power domains is presented. Once partitioned, a distributed control algorithm is employed to support distribution operation applications with a focus on voltage spread reduction. The partitioning method is also applied to the commonly studied capacitor control objective of loss minimization. Simulation results applying the method to a real-world distribution feeder are presented and compared with an alternate geographically-based partitioning method. Results show the analytical partitioning and distributed control methodology is capable of producing high quality solutions to the capacitor control problem.**

***Index Terms*—Capacitor control, distributed control, distribution system control.**

I. Introduction

HISTORICALLY, distribution system planners and operators use offline studies to compute capacitor controller set points based on local feeder conditions such as: bus voltage, VAR flow, temperature, and/or time of day [1]. However, with renewed investment in advanced capacitor control systems, the potential for online distributed control is increasing. Switched feeder capacitors are being embedded with intelligent controllers capable of measurement, communication, and actuation functions. Allowing this emerging distributed intelligence to resolve and coordinate control actions will enable advanced distribution automation applications and reduce costs associated with constructing centralized distribution control centers.

Thus, a partitioning method for distributed capacitor control of electric power distribution feeders is presented. In a distributed environment, the issues of controller coordination, communication, and feeder partitioning are of critical importance. Multi-agent systems, which address coordination and communication of distributed controllers, have been shown viable [2] and applied in distribution networks for service restoration [3] and distributed generation control [4]. However, the issue of feeder partitioning has received less attention.

Previous works on partitioning have focused solely on improving computation speed of steady-state and/or transient analysis. These methods have primarily been system structure based approaches, such as partitioning a system: by feeders [5], at the locations of power electronic converters [6], or, in shipboard systems, along tie-lines using a diakoptics approach [7]. Recent work has proposed partitioning feeders in balanced clusters to handle dispatch of variable generation [8]. In this work, reactive power domains are employed to partition a distribution feeder for distributed control.

The power domain concept was first introduced as a loss allocation method for transmission systems [9], [10] and later extended to multi-phase distribution systems [11], [12]. Power domains allow for a means to explicitly assign load and loss in a contiguous portion of a feeder to a specific source or sources of real or reactive power. Here, the boundaries of these contiguous zones, or domains, are then used as natural decoupling points for distributed analysis and control, thus relaxing the need for structure based partitioning methods.

Once partitioned, a distributed control algorithm can be employed to support distribution operation applications. Here, capacitor control for voltage spread reduction (VSR) is investigated. A reference signal must be identified for each control area to implement VSR in a distributed environment [13]. Distributed analysis methods, which provide an estimate of the operating state within each control area [5]–[7], [14], [15], can provide these references as will be demonstrated in this work.

A preliminary study of distributed capacitor control was presented in [16]. This work is extended here to include a formalized problem statement, distributed control algorithm, and simulation results on real-world, large-scale distribution feeders. Specifically, the paper addresses:

- centralized and distributed voltage spread reduction problem formulations;
- a control area partitioning methodology based on reactive power domains for multi-phase distribution systems;
- a distributed control solution algorithm;
- simulation results for a 948-bus distribution system.

II. Problem Formulation

The partitioning method presented here will be applied to a distributed implementation of capacitor control for VSR. Both centralized and distributed VSR problem formulations are presented below. In the general case, VSR minimizes the voltage differential between any two nodes on the same phase in a network. However, often times in practice a reference voltage, such as the substation voltage magnitude, is selected.

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M. R. Kleinberg, K. Miu, and N. Segal are with the Department of Electrical and Computer Engineering, Drexel University, Philadelphia, PA 19104 USA (e-mail: mrk26@drexel.edu; karen@coe.drexel.edu; nus23@drexel.edu).

H. Lehmann and T. R. Figura are with PPL Electric Utilities, Allentown, PA 18101 USA.

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Selection of this reference can be viewed as application specific. For example, the general case may be required for feeders which experience voltage rise and can be employed to ensure safe operating conditions in the presence of automated switching and/or distributed energy resources. Alternatively, by minimizing the voltage spread with respect to the substation voltage magnitude, the problem is useful for conservation voltage reduction (CVR) applications. Impacts of the selected voltage reference will be illustrated with simulation results in Section V.

A. Centralized Problem Formulation

The centralized VSR problem is formulated as a mixed integer, nonlinear constrained optimization problem. Specifically, the objective will minimize the per-phase voltage magnitude differential between any node in the system and a specified reference.

In summary

$$\min_{u \in U} \max_{\substack{k \in N \\ p \in a,b,c}} \left| \left| V^p_{ref,0} \right| - |V^p_k| \right| \tag{1}$$

subject to

$$f(V, u) = 0 \tag{2}$$
$$|I^p_k| \leq I^{\max}_k \; \forall \, k \in N \tag{3}$$
$$P^2_l + Q^2_l \leq (S^{\max}_l)^2 \; \forall \, l \in F \tag{4}$$
$$V^{\min}_k \leq |V^p_k| \leq V^{\max}_k \; \forall \, k \in N, p \in a, b, c \tag{5}$$

where

$f(V, u)$	multi-phase power flow equations;
F	set of system branches;
I^p_k	current flow entering bus k, phase p;
$I^{\max}_k$	maximum current flow or upstream protection setting of branch entering bus k, whichever is least;
N	set of all buses;
$S_l = \sqrt{P^2_l + Q^2_l}$	total apparent power entering feeder l;
$S^{\max}_l$	maximum capacity of feeder l or its supplying transformer, whichever is least;
u	capacitor control scheme;
U	set of capacitor controls;
V	vector of node voltages;
V^p_k	voltage at bus k, phase p;
$\lvert V^p_{ref,0} \rvert$	centralized reference voltage;
$V^{\min}_k$	minimum operating voltage, bus k;
$V^{\max}_k$	maximum operating voltage, bus k.

B. Distributed Problem Formulation

The distributed formulation requires that control areas in a system are identified. The formulation is similar to the centralized problem but focuses on each control area independently. The control reference here may be obtained from within or outside of each capacitor's respective control area. For each control area

$$\min_{u_i \in U_i} \max_{\substack{j \in N_i \\ p \in a,b,c}} \left| \left| V^p_{ref,i} \right| - \left| V^p_j \right| \right| \tag{6}$$

subject to

$$f(V_i, u_i) = 0 \tag{7}$$
$$|I^p_k| \leq I^{\max}_k \; \forall \, k \in N_i \tag{8}$$
$$P^2_l + Q^2_l \leq (S^{\max}_l)^2 \; \forall \, l \in F_i \tag{9}$$
$$V^{\min}_k \leq |V^p_k| \leq V^{\max}_k \; \forall \, k \in N_i, p \in a, b, c \tag{10}$$

where

F_i	set of all feeders, control area i;
N_i	set of all buses, control area i;
u_i	capacitor control scheme, control area i;
U_i	set of all capacitor controls, control area i;
V_i	vector of node voltages, control area i;
$\lvert V^p_{ref,i} \rvert$	distributed reference voltage, control area i.

For the distributed case, reference voltage selection for each control area is again application specific. However, now it may also take into account the level of communication and coordination available among controllers. When communication channels between control areas are intact, references may be coordinated and shared between partitions. Lacking coordination, each control area may operate in stand-alone operation drawing its reference voltage magnitude from within the partition based on local results of a distributed analysis, assuming communication within the area is available. Lastly, when all communication fails, the controller can revert back to a purely local control mode.

III. Control Area Partitioning Methodology

In this work, each capacitor bank on the feeder is assigned a corresponding control area. Here, the reactive power domains of shunt capacitors are used to identify these areas.

Capacitor reactive power domains are a function of the capacitor location, capacitor size, system component parameters, and load distribution. Therefore, the domains change with the system parameters and are defined for a specific load distribution and configuration. Domains are computed by post-processing a power flow solution or state estimate. Determination of reactive power domains will be explained below and illustrated via a small example system.

A. Reactive Power Domains

For each capacitor and the substation, and a given reactive power output, Q_{Gi}

$$Q_{Gi} = Q^{load}_{Gi} + Q^{loss}_{Gi} \quad i = 0, 1, 2, \ldots, n_{caps} \tag{11}$$

with

$$Q^{load}_{Gi} = Q^{load,a}_{Gi} + Q^{load,b}_{Gi} + Q^{load,c}_{Gi} \tag{12}$$
$$Q^{loss}_{Gi} = Q^{loss,a}_{Gi} + Q^{loss,b}_{Gi} + Q^{loss,c}_{Gi} \tag{13}$$

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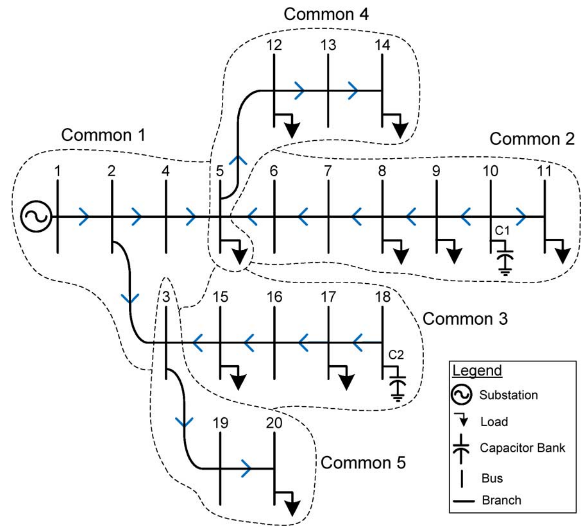

Fig. 1. Example 20-bus distribution system—arrows indicate positive reactive power flow direction on phase a, reactive power commons highlighted.

where

n_{caps}	number of capacitor bus locations;
$Q_{Gi}^{load}, Q_{Gi}^{loss}$	total reactive power load and loss associated with the substation $(i = 0)$ and each capacitor $(i = 1, 2, \ldots, n_{caps})$, respectively;
$Q_{Gi}^{load,p}, Q_{Gi}^{loss,p}$	reactive power load and loss associated with the substation and each capacitor, phase p, respectively.

The three-phase domain of a capacitor is defined as the contiguous set of nodes and branches by phase, whose reactive power is supplied by the respective capacitor. For each bus, the reactive power domains vary across each phase and are assigned based on positive power flow direction. Positive power flow direction is used to trace power back to a generator, capacitor, or substation and to allocate loads to several sources for common areas. For two directly connected buses, bus i and bus j:

- if $\mathrm{Im}(V_i^p I_{ij}^{p^*}) - \mathrm{Im}(V_j^p I_{ij}^{p^*}) > 0$, reactive power flows from bus i to bus j over phase p.

The loss on a branch or load at a node may be supplied by multiple sources. The domains of these sources then intersect at these common branches or nodes. A contiguous set of branches and nodes by phase supplied by the same capacitor is defined as a capacitor common. The domain of each capacitor is then the set of all commons which attribute a portion or all of its load and loss to the respective capacitor. A proportionality assumption is applied to separate the contributions from each capacitor to the load and loss in the common [9]. These concepts will be illustrated on an example system.

B. 20-Bus Example

A 20-bus multi-phase distribution system, shown in Fig. 1, is used to illustrate the proposed partitioning process. System parameters include:

- peak load of 2881 kW and 1863 kVAR;
- two 900-kVAR capacitor banks (bus 10 and bus 18), denoted C1 and C2, respectively.

After running a power flow on the system, a directed graph of positive reactive power flow direction is overlaid on the system, as seen in Fig. 1. The portions of the system served by the same source or sources of reactive power were then grouped into commons. This resulted in 5 commons as shown in Fig. 1. The domain of each reactive power source can then be expressed as follows:

- Substation domain: Common 1, Common 4, and Common 5
- Capacitor C1 domain: Common 2 and Common 4
- Capacitor C2 domain: Common 3 and Common 5

The contribution of each reactive power source to the load served in a common is needed. Thus, the proportionality assumption is applied, for example to phase a, as follows:

$$Q_{G0}^{load,a} = Q_{com1}^{load,a} + \alpha Q_{com4}^{load,a} + \beta Q_{com5}^{load,a} \tag{14}$$
$$Q_{G1}^{load,a} = Q_{com2}^{load,a} + \gamma Q_{com4}^{load,a} \tag{15}$$
$$Q_{G2}^{load,a} = Q_{com3}^{load,a} + \zeta Q_{com5}^{load,a} \tag{16}$$

with

$$\alpha = \frac{Q_{4-5}^a}{Q_{4-5}^a + Q_{6-5}^a}, \quad \beta = \frac{Q_{2-3}^a}{Q_{2-3}^a + Q_{15-3}^a}$$
$$\gamma = \frac{Q_{6-5}^a}{Q_{4-5}^a + Q_{6-5}^a}, \quad \zeta = \frac{Q_{15-3}^a}{Q_{2-3}^a + Q_{15-3}^a}$$

where is the reactive power flow from bus i to bus j along phase a. A similar treatment can be applied for phase b and c and to the reactive losses per phase in each common.

With the reactive power domains identified, the feeder is then partitioned into control areas. Here, shared commons are assigned to one control area and the feeder is partitioned only at three-phase buses. Thus, the following control area identification process is employed:

- Assign each shared common, per phase, to the source supplying it with the largest proportion of reactive power.
  - Start with capacitors having no further downstream capacitors. Identify the first upstream three-phase bus in the path between the capacitor and the substation such that for each phase, the entire per-phase common is located downstream.
  - Repeat this process for each capacitor, progressing towards the substation.
- The sub-system bounded between the three-phases buses identified above is the control area of each respective capacitor.

The reactive power domains will change each time the load or capacitor settings change. In this work, control areas are defined based on the reactive power domains at peak loading conditions with all capacitors at their maximum tap setting. This approach is taken to resolve the tradeoffs between model flexibility and computational burden.

Applying this process to the system shown in Fig. 1 results in the distributed system represented in Fig. 2. Common 5, for example, has a larger proportion of load and loss served by capacitor C2 as opposed to the substation. Therefore, it was assigned to the capacitor C2 control area as shown in Fig. 2. Further analysis of this system can be found in [16]. In the next section, a distributed solution algorithm is presented and used to compute capacitor control decisions in each control area.

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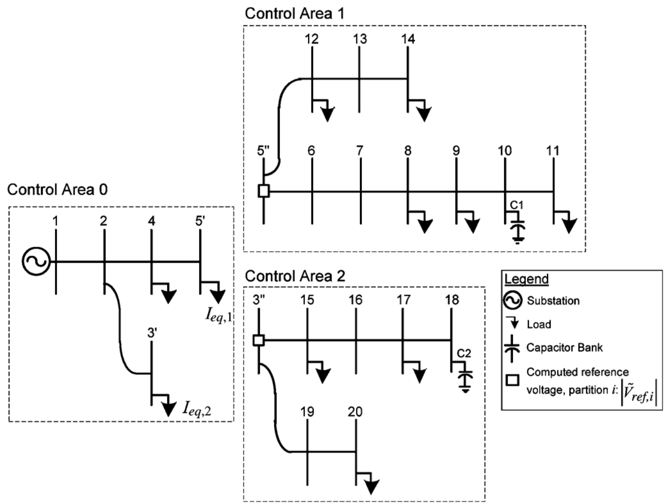

Fig. 2. Example system partitioned based on reactive power domains.

IV. Distributed Control Algorithm

Changes to capacitor settings in one control area may affect the control reference in all other control areas. Lacking additional coordination, this may lead to controller hunting. Localcontroller methods typically enforce a time-delay between successive control actions to prevent this situation [1]. In a distributed environment, a distributed control algorithm should compute and coordinate control actions. Here, communication between control areas will be used to coordinate control actions and share voltage reference signals.

The algorithm is intended for online control, enabled with AMI systems such as those implemented at PPL Electric Utilities [17]. For systems with limited online load information available, a measurement based implementation, as presented in [16], may be more appropriate.

The following outlines the proposed control algorithm. Given a distribution system partitioned into control areas:

Step 1) Specify the desired control reference for each control area

Step 2) Order control areas in increasing order of minimum voltage magnitude

Step 3) Select the next control area on list. If no more exist, then go to Step 7

Step 4) Compute and implement capacitor control action for the selected control area
— if no change to capacitor settings in the control area are found, then go to Step 3

Step 5) Communicate resulting network equivalents [13] to adjacent partitions, compute a new system solution, and re-compute references

Step 6) Evaluate constraints for each control area
— if a control area has violations, then undo and disallow selected control option and go to Step 4;
— else, accept the control action and go to Step 3

Step 7) If the list of all control areas has been evaluated with no changes to capacitor settings, then exit. Else, go to Step 2

An optimization technique is required to compute the optimal control settings in Step 4. For the simulation results presented here, an exhaustive search is used. However, any appropriate existing optimization method may be employed [18]. If a feasible solution is not found for a new operating state, then no control action will be taken and the system would remain at the same feasible operating point. Simulation results are presented below to demonstrate the performance of the proposed distributed control.

V. Simulation Results

Case studies on a real-world, large-scale distribution system are presented next. These results are used to demonstrate the proposed partitioning methodology for distributed control and the impacts of voltage reference selection on solution quality. Specifically, this section will present the following:

- details of a 948-bus, 1224-node test system
- a set of centralized and distributed controller voltage references
- centralized and distributed VSR control results employing each voltage reference
- an alternative, geographically based partitioning method and resulting VSR distributed control results
- results obtained by changing the controller objective from VSR to another common application, loss minimization

A. Test Circuit

A system diagram of the 948 bus, 1224 node, multi-phase test distribution system, can be seen in Fig. 3. The peak load of this system is 8361 kW and 2746 kVar. The base voltage of the system is 12.9 kV. A count of system components can be found in Table I. All loads are modeled as constant power. In these studies, the system was modeled with capacitors placed as follows:

- C1: Bus 1104
- C2: Bus 1292
- C3: Bus 1333
- C4: Bus 1937

All capacitors in the system are gang-operated across all phases. In this study, the capacitors are switched in two banks of 300 kVar each. Each capacitor then has three available settings:

- Bank Setting 0: 0 kVar
- Bank Setting 1: 300 kVar
- Bank Setting 2: 600 kVar

Three load levels were available for testing corresponding to light, average, and peak loading conditions experienced in the system. The load levels include:

- Load Level 1: 26% (light load)
- Load Level 2: 70% (average load)
- Load Level 3: 100% (peak load)

The test circuit was partitioned based on the reactive power domains at peak load with all capacitors set at 600 kVar. The proposed partitioning procedure was applied and the resulting control areas are shown in Fig. 4. Here, five control areas were identified, each corresponding to one of the four capacitor banks and the substation, respectively.

The distributed capacitor control program was coded in MATLAB and all power flow calculations were performed using an unbalanced multi-phase distribution power flow solver [14], [19]. The simulations results aim to assess the solution quality obtainable by the partitioning method and distributed controller. Therefore, to compute the distributed control solution, Step 4 of the distributed control algorithm employs an

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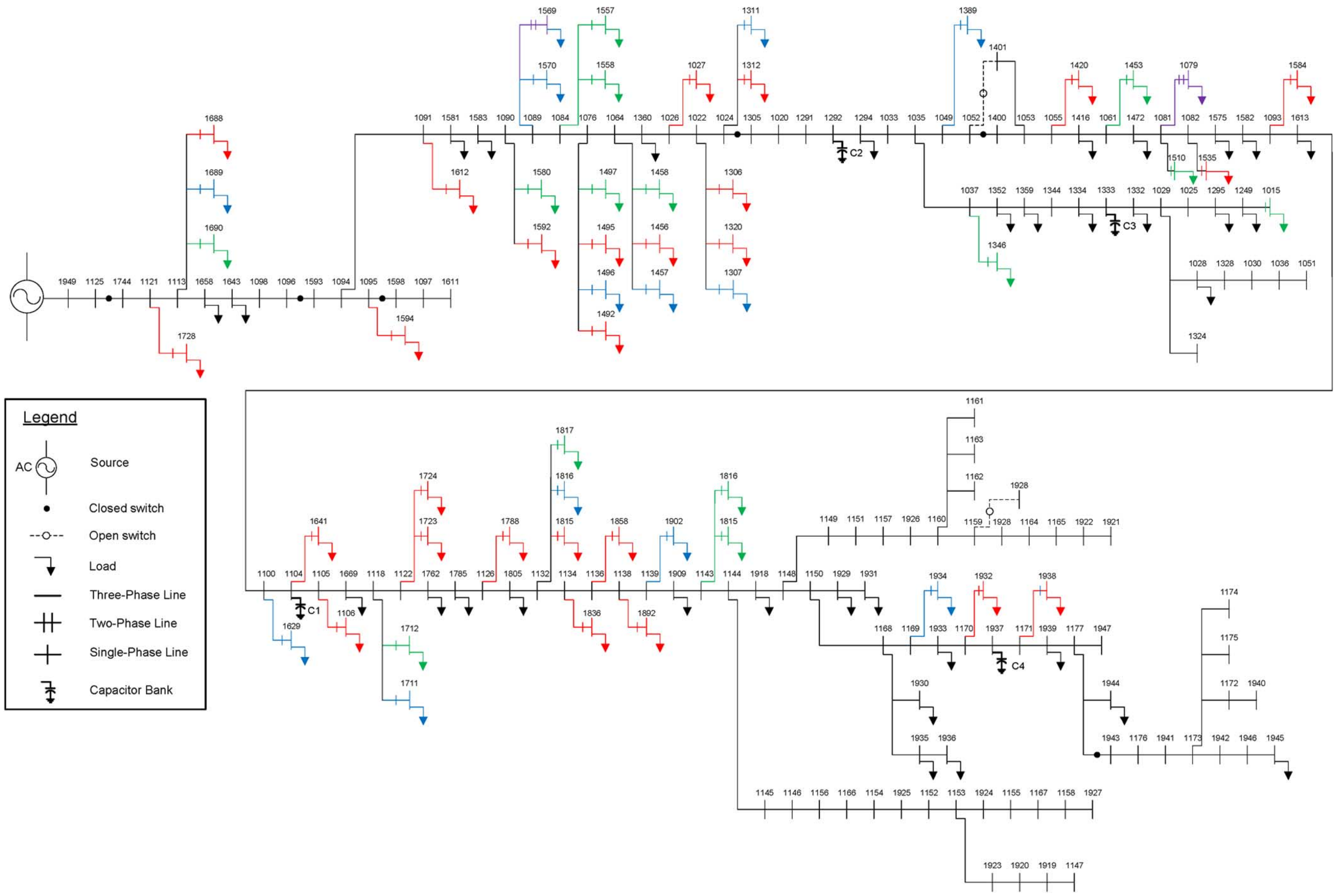

Fig. 3. 948-bus, 1224-node multi-phase unbalanced distribution system.

TABLE I
SYSTEM COMPONENT LIST

Component type	# of each
Distribution lines	941
Loads	282
Capacitor banks	4
Sectionalizing switches	6
Tie switches	17

exhaustive search over all bank settings in the respective control area. For comparison, centralized solutions are computed using an exhaustive search over all available bank settings to produce a global optima.

B. Reference Signal Selection

Various reference voltages are investigated for both the centralized and distributed formulations. Selection of the reference will be application specific. For the centralized problem, two reference voltages are investigated: (17), the substation voltage, and (18), the maximum voltage magnitude in the system:

$$\left|V^p_{ref,0}\right| = \left|\overline{V}^p_{sub}\right| \tag{17}$$

$$\left|V^p_{ref,0}\right| = \max_{j\in N}\left|V^p_j\right| \tag{18}$$

where $|\overline{V}^p_{sub}|$ is the specified substation phase voltage magnitude.

Similarly, various reference voltages may be employed in each control area of the distributed formulation. The three investigated in this work can be stated as

$$\left|V^p_{ref,i}\right| = \left|\overline{V}^p_{sub}\right| \tag{19}$$

$$\left|V^p_{ref,i}\right| = \max_{k\in N_i}\left|V^p_k\right| \tag{20}$$

$$\left|V^p_{ref,i}\right| = \left|\tilde{V}^p_{eq,i}\right| \tag{21}$$

where $|\tilde{V}^p_{eq,i}|$ is the control area equivalent source bus voltage [14] computed value from distributed analysis. E.g., the furthest upstream bus identified in the partitioning procedure, in Fig. 4, buses 1033, 1089, 1132, and 1333.

Each of these distributed references represents varying levels of measurement, communication, and coordination requirements between distributed controllers and within each control area. With intact communication channels, a globally shared reference, such as (19), may be employed. However, in the event of communication failures, distributed controllers may continue to optimize based solely on conditions within their respective control areas, e.g., using (20) or (21).

Lastly, the references are formulated to allow for a different reference voltages on each phase. If voltage imbalance is a concern for a given circuit, the proposed approach may be used to minimize imbalance.

C. VSR Results

Using the proposed distributed control algorithm, the resulting centralized and distributed VSR control results were

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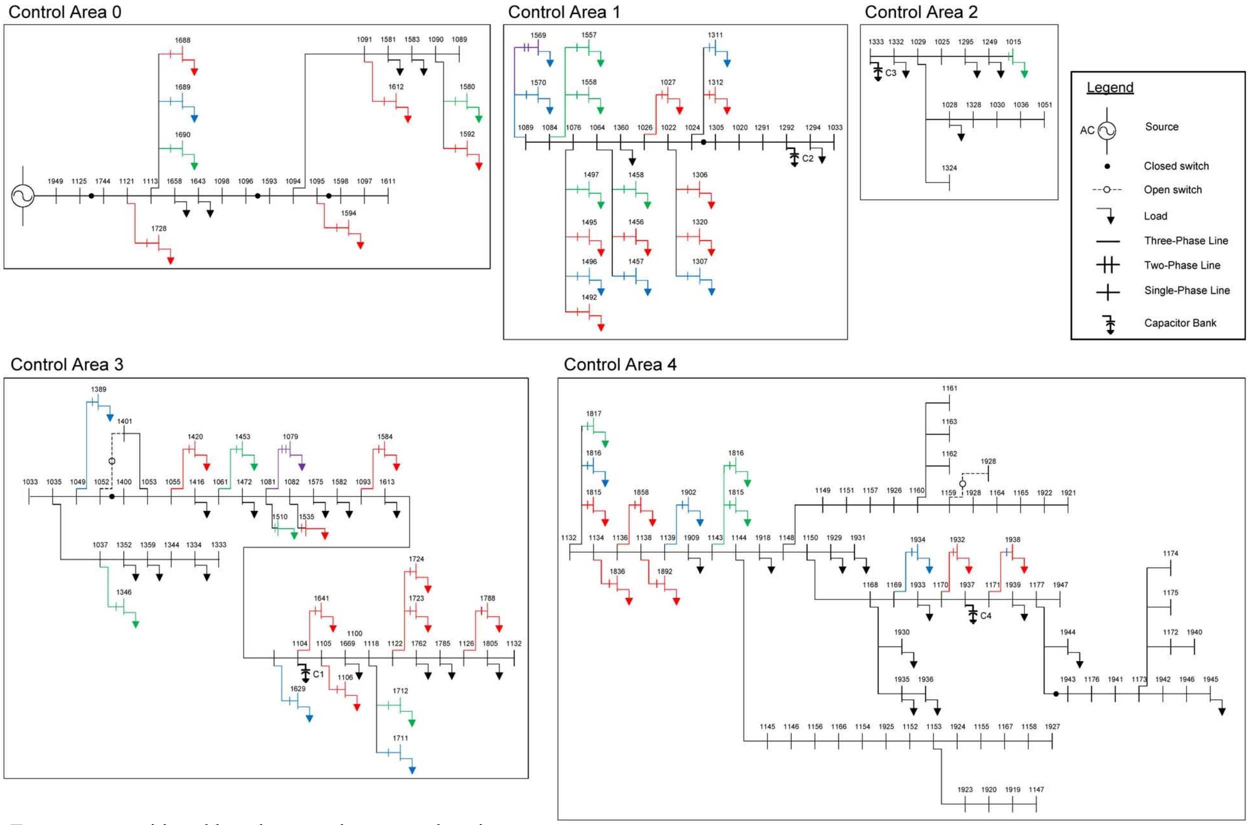

Fig. 4. Test system partitioned based on reactive power domains.

TABLE II
REACTIVE POWER DOMAIN PARTITIONING—CAPACITOR BANK SETTING (C1-C2-C3-C4) FOR EACH LOAD LEVEL AND REFERENCE VOLTAGE

Load Level	Centralized Controllers		Distributed Controllers		
	$\left\|V^p_{ref,0}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,0}\right\| = \max\limits_{j\in N}\left\|V^p_j\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \max\limits_{k\in N_i}\left\|V^p_k\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\tilde{V}^p_{eq,i}\right\|$
Light	2-1-2-0	2-1-1-1	0-2-1-1	1-0-2-1	0-1-2-1
Average	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2
Peak	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2

TABLE III
REACTIVE POWER DOMAIN PARTITIONING—RESULTING VOLTAGE SPREAD IN p.u. AT EACH LOAD LEVEL WITH $|V^p_{ref,0}| = |\overline{V}^p_{sub}|$

Load Level	Centralized Controller	Distributed Controllers		
	$\left\|V^p_{ref,0}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \max\limits_{k\in N_i}\left\|V^p_k\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\tilde{V}^p_{eq,i}\right\|$
Light	0.00182032	0.00224203	0.00224428	0.00224330
Average	0.01795475	0.01795475	0.01795475	0.01795475
Peak	0.02321949	0.02321949	0.02321949	0.02321949

computed for each reference. The resulting capacitor bank settings at each load level for all references can be seen in Table II. The corresponding voltage may be seen in Table III.

Remarks: For the test system, the following observations are made:

- At peak load, it is typical that all capacitors would be switched to their maximum tap setting. This is observed for both centralized and distributed controllers and all references considered.
- At average load, the distributed controller matched the centralized solutions for all references considered.
- At peak and average load, the distributed controller finds the globally optimal solution for the test system across varying levels of coordination.
- At light load, the distributed control resulted in sub-optimal solutions compared to the centralized solution.
- While suboptimal, light load typically represents non-critical operating periods, with worst case voltage spreads here an order of magnitude less than the minimum achieved at average and peak loading.

D. Geographical Partitioning

In order to study the impact of partitioning methods, the distributed controller was also applied to an alternative, geographically-based partitioning scheme. This partitioning scheme identifies control areas based on the communication range of each

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TABLE IV
GEOGRAPHICAL PARTITIONING—CAPACITOR BANK SETTING (C1-C2-C3-C4) FOR EACH LOAD LEVEL AND REFERENCE VOLTAGE

Load Level	Centralized Controllers		Distributed Controllers		
	$\left\|V^p_{ref,0}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,0}\right\| = \max_{j\in N}\left\|V^p_j\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \max_{k\in N_i}\left\|V^p_k\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\tilde{V}^p_{eq,i}\right\|$
Light	2-1-2-0	2-1-1-1	2-1-0-1	1-1-0-2	0-1-2-1
Average	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2
Peak	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2	2-2-2-2

TABLE V
GEOGRAPHICAL PARTITIONING—RESULTING VOLTAGE SPREAD IN p.u. AT EACH LOAD LEVEL WITH $|V^p_{ref,0}| = |\overline{V}^p_{sub}|$

Load Level	Centralized Controller	Distributed Controllers		
	$\left\|V^p_{ref,0}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\overline{V}^p_{sub}\right\|$	$\left\|V^p_{ref,i}\right\| = \max_{k\in N_i}\left\|V^p_k\right\|$	$\left\|V^p_{ref,i}\right\| = \left\|\tilde{V}^p_{eq,i}\right\|$
Light	0.00182032	0.00263882	0.00354705	0.00224330
Average	0.01795475	0.01795475	0.01795475	0.01795475
Peak	0.02321949	0.02321949	0.02321949	0.02321949

TABLE VI
CAPACITOR BANK SETTING (C1-C2-C3-C4) AND SYSTEM REAL POWER LOSS FOR EACH LOAD LEVEL AND CONTROL SCHEME—LOSS MINIMIZATION

Load Level	Centralized Controllers		Distributed Controller (Reactive Power Domains)		Distributed Controller (Geographical Partitioning)	
	Capacitor bank settings	P_{loss} (kW)	Capacitor bank settings	P_{loss} (kW)	Capacitor bank settings	P_{loss} (kW)
Light	0-1-1-0	8.38079	1-1-0-0	8.40419	0-1-0-1	8.57990
Average	2-2-2-2	55.17571	2-1-2-1	57.02187	2-1-1-2	57.17159
Peak	2-2-2-2	126.96227	2-2-2-1	127.52827	2-1-2-2	127.56687

capacitor controller. Each control area then encompasses a contiguous electrical network located within a coverage area centered at the location of each capacitor, e.g., the wireless base station communication range.

Portions of the system which are overlapped between multiple communication areas are assigned to the geographically closest capacitor controller. After applying this process to each capacitor, the remaining contiguously connected system is assigned to the control area associated with the substation. Portions of the system not in a coverage area are assigned to the closest contiguously connected control area. Here, the geographically based partitions were identified using a communication area of approximately one square mile centered at the location of each capacitor.

VSR capacitor bank control settings obtained using the geographical partitioning scheme are shown in Table IV. The resulting voltage spread may be seen in Table V.

Remarks: For the test system, the following observations are made:

- At peak and average load, the geographical partitioning scheme produced the same results as the reactive power domain approach for all references for the test system.
- At light load, geographical partitioning produced the same or sub-optimal results compared to reactive power domain partitioning, depending on the reference.

Thus, to investigate solution quality with respect to partitioning techniques, other applications can be investigated. A study of loss minimization follows.

E. Loss Minimization Objective

From a partitioning aspect, it is useful to assess the applicability of the method for other distribution applications. As such, the effect of changing the controller objective to loss minimization is investigated next. For the centralized formulation, the VSR objective shown in (1) is replaced by the following, again subject to (2)–(5):

$$\min_{u\in U} P_{loss}(V,u) \tag{22}$$

where $P_{loss}(V,u)$ is the system real power losses. The distributed loss formulation replaces the objective in (6), with the following, subject to (7)-(10):

$$\min_{u_i\in U_i} P_{loss,i}(V_i,u_i) \tag{23}$$

where $P_{loss,i}(V_i,u_i)$ is the system real power losses in partition i.

Using the loss reduction objective, the capacitor control solutions were again computed using the distributed control algorithm presented in Section IV for each of the three load levels. Step 2 of the distributed control algorithm was changed to order control areas in decreasing order of partition losses. The loss minimization results may be seen in Table VI. Of note, loss minimization objective does not require a reference.

Remarks: For the test system, the following observations are made:

- At all load levels, distributed control resulted in sub-optimal solutions as compared to centralized control.
- At all load levels, distributed control using reactive power domain partitions provided higher quality solutions than geographical partitioning.

F. Comments

For the given test system, distributed control produced promising results for both VSR and loss minimization applications. The proposed analytically based partitioning scheme is adaptive to network configuration and load distribution and requires low computational overhead to determine control areas. This enables distributed controllers to adapt to changing system conditions or operational objectives.

The proposed distributed control method was also shown to be robust to varying levels of coordination, represented here by various reference selections. Distributed control schemes can then be employed to strike a balance between the operational requirements on communication, computation, and solution quality.

VI. Conclusions

In this work, a partitioning method for distributed capacitor control of electric power distribution systems has been presented. The partitioning method employs reactive power domains to analytically partition a distribution feeder into control areas. A distributed control algorithm was proposed to then coordinate capacitor controllers in each control area. The method was applied to the distribution application functions of voltage spread reduction and loss minimization.

For the VSR objective, results showed that the distributed control technique was able to compute the same solution as a centralized controller at average and peak loading levels. For loss minimization, while sub-optimal when compared with a centralized solution, the reactive power domain based distributed controller produced higher quality solutions than a proposed alternative geographical partitioning scheme.

As the push towards advanced distribution automation continues, distributed online control will both enable advanced distribution automation applications and help ease costs associated with constructing centralized distribution control centers. In addition, for systems with existing centralized control, distributed controllers can provide back-up support in the event of communication and/or controller failures.

References

[1] T. A. Short, *Electric Power Distribution Handbook*. Boca Raton, FL, USA: CRC, 2003.

[2] S. D. J. McArthur, E. M. Davidson, V. M. Catterson, A. L. Dimeas, N. D. Hatziargyriou, F. Ponci, and T. Funabashi, "Multi-agent systems for power engineering applications—part I: Concepts, approaches, and technical challenges," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1743–1751, Nov. 2007.

[3] J. M. Solanki, S. Khushalani, and N. N. Schulz, "A multi-agent solution to distribution systems restoration," *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1026–1034, Aug. 2007.

[4] B. A. Robbinsy, A. D. Domínguez-Garcíay, and C. N. Hadjicostisz, "Control of distributed energy resources for reactive power support," in *Proc. IEEE North American Power Symp.*, Boston, MA, USA, Aug. 2011.

[5] Y. Fukuyama, Y. Nakanishi, and H.-D. Chiang, "Fast distribution power flow using multiple processors," *Elect. Power Energy Syst.*, vol. 18, no. 5, pp. 331–337, Sep. 1996.

[6] G. D'Antona, A. Monti, F. Ponci, and L. Rocca, "A distributed state estimator for electric power systems in avionic and naval applications," in *Proc. Instrumentation and Measurement Technology Conf.*, Sorrento, Italy, Apr. 24–27, 2006.

[7] F. M. Uriarte and K. L. Butler-Purry, "Diakoptics in shipboard power system simulation," in *Proc. IEEE North American Power Symp.*, Sep. 2006, pp. 201–210.

[8] D. Bakken, A. Bose, K. M. Chandy, P. P. Khargonekar, A. Kuh, S. Low, A. von Meier, K. Poolla, P. P. Varaiya, and F. Wu, "GRIP-Grids with intelligent periphery: Control architectures for Grid2050," in *Proc. IEEE Int. Conf. Smart Grid Commun.*, Brussels, Belgium, 2011.

[9] G. Strbac, D. Kirschen, and S. Ahmed, "Allocating transmission system usage on the basis of traceable contributions of generators and loads to flows," *IEEE Trans. Power Syst.*, vol. 13, no. 2, pp. 527–532, May 1998.

[10] D. Kirschen and G. Strbac, "Tracing active and reactive power between generators and loads using real and imaginary currents," *IEEE Trans. Power Syst.*, vol. 14, no. 4, pp. 1212–1319, Nov. 1999.

[11] S. Tong and K. Miu, "A network-based distributed slack bus model for DGs in unbalanced power flow studies," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 835–842, May 2005.

[12] S. Tong, M. Kleinberg, and K. Miu, "A distributed slack bus model and its impact on distribution system application techniques," in *Proc. oIEEE International Symp. Circuits and Systems*, Kobe, Japan, May 2005.

[13] T. Senjyu, S. Toma, K. Tanaka, A. Yona, N. Uraski, and T. Funabashi, "Optimal control of voltage in distribution systems by voltage reference management," *IEEJ Trans. Elect. Electron. Eng.*, pp. 1239–1244, Dec. 2008.

[14] M. Kleinberg, K. Miu, and C. Nwankpa, "Distributed multi-phase distribution power flow: Modeling, solution algorithm, and simulation results," *Trans. Soc. Modeling and Simulation Int.*, vol. 4, no. 8–9, pp. 403–412, Aug.–Sep. 2008.

[15] M. M. Nordman and M. Lehtonen, "Distributed agent-based state estimation for electrical distribution networks," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 652–658, May 2005.

[16] M. Kleinberg and K. Miu, "A study of distributed capacitor control for electric power distribution systems," in *Proc. IEEE North American Power Symp.*, Boston, MA, USA, Aug. 2011.

[17] B. Bujnowski, "Lessons from a successful AMR implementation," *Transm. Distrib. World*, 2005.

[18] H. N. Ng, M. M. A. Salama, and A. Y. Chikhani, "Classification of capacitor allocation techniques," *IEEE Trans. Power Del.*, vol. 15, no. 1, pp. 387–392, Jan. 2000.

[19] R. D. Zimmerman, "Comprehensive distribution power flow: Modeling, formulation, solution algorithms and analysis," Ph.D. dissertation, Cornell Univ., Ithaca, NY, USA, Jan. 1995.

Michael R. Kleinberg (S'05) received the B.S. and M.S. degrees in electrical engineering from Drexel University, Philadelphia, PA, USA. He is currently pursuing the Ph.D. degree in the Electrical and Computer Engineering Department, Drexel University.

His research interests include analysis and optimization of electric power distribution systems.

Karen Miu (M'98) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Cornell University, Ithaca, NY, USA.

She is currently an Associate Professor in the Electrical and Computer Engineering Department, Drexel University, Philadelphia, PA, USA. Her research interests include distribution system analysis, distribution automation, and optimization techniques applied to power systems.

Dr. Miu is a recipient of the 2000 National Science Foundation Career award, the 2001 Office of Naval Research Young Investigator Award, and the 2005 Eta Kappa Nu (HKN) Outstanding Young Electrical and Computer Engineer Award.

Nicole Segal (S'05) received the B.S. and M.S. degrees in electrical engineering from Drexel University, Philadelphia, PA, USA. She is currently pursuing the Ph.D. degree in the Electrical and Computer Engineering Department, Drexel University.

Her research interests include electric power distribution optimization and policy.

Horst Lehmann is with PPL Electric Utilities in Allentown, PA, USA.

Timothy R. Figura is with PPL Electric Utilities in Allentown, PA, USA.