Polarization Game over Social Networks

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joint work with Emrah Akyol and Zeynep Ertem

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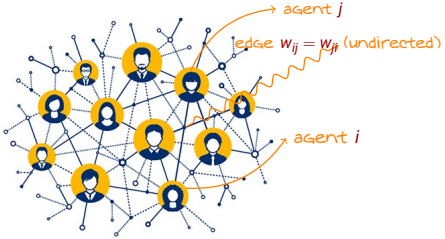
Problem Setting



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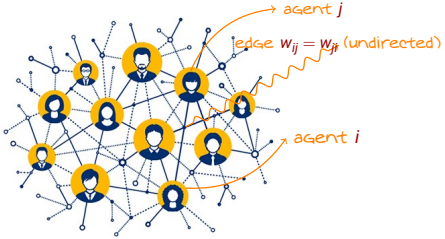

Opinion dynamics: Friedkin-Johnson

expressed opinion

innate opinion

$$z_i(t) = \alpha_i \, s_i + (1-\alpha_i) \sum_{j \in V} w_{ij} z_j(t-1)$$

$\alpha_i \in [0, 1]$

network influence

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$\alpha_i \in [0, 1]$

network influence

$\mathbf{s} = (s_i, s_j, s_{-(i,j)})$
original

Maximizer$[i^A, s_{i^A}] \rightarrow [i, s'_i]$ Minimizer$[i^B, s_{i^B}] \rightarrow [j, s'_j]$

$\mathbf{s}' = (s'_i, s'_j, s_{-(i,j)})$
modified

Opinion dynamics

$\mathbf{z}' = \lim_{t \to \infty} \mathbf{z}(t)$
steady-state opinion

$P(\mathbf{z}') = (\mathbf{z}' - \bar{\mathbf{z}})^T (\mathbf{z}' - \bar{\mathbf{z}})$
polarization

Nash and Stackelberg Equilibria

- There are two notions of equilibrium of interest: (Generalized) Nash and Stackelberg.

Generalized Nash Equilibrium

A tuple (i_A, s_I^A) and (i_B, s_I^B) is a GNE if these three are simultaneously satisfied:

$$i_A, s_I^A = \underset{i_A \in \mathcal{S}_A, s_I^A \in [0,1]}{\arg\max} \; P$$

$$i_B, s_I^B = \underset{i_B \in \mathcal{S}_B, s_I^B \in [0,1]}{\arg\min} \; P$$

$$i_A \neq i_B$$

- Fictitious play is a method to find Nash Equilibrium (Brown, 1949)
- In a two-player zero-sum game, it is guaranteed that fictitious play will converge to an NE
- We need a modified version of FP to find the GNE.

Prior Work

- Limited prior work on polarization dynamics over social networks.
  - Minimizing polarization (Chen and Racz, 2021)
  - Maximizing polarization (Musco et al., 2018)
  - Change network parameters (Zhu et al., 2021)
- Some focuses on minimization or maximization of polarization but not the game setting where both players (maximizer and minimizer) exist.
  - New models of opinion dynamics (Perra and Rocha, 2019)

Equilibrium Details

- Fictitious play (FP) is commonly used to find NE in clasical zerosum games.
- Here, we modify fictitious play so that it can used to determine GNE. We first need the following:

Theorem

For a given i_A, i_B, pair, $s_i^A \in \{0, 1\}$ and

$$s_i^B = \frac{-\sum_{j \neq i_B} s_j (a_j - \frac{1}{n})^T (a_i - \frac{1}{n})}{(a_i - \frac{1}{n})^T (a_i - \frac{1}{n})}$$

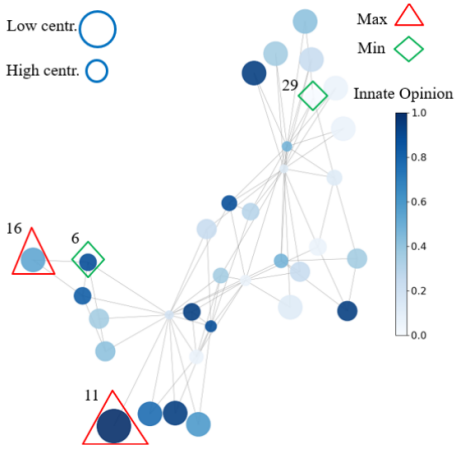
where a_i is the i'th column of $A = (I + L)^{-1}$ and L is the Laplacian of the network.

- Proof Sketch: Note that $\mathbf{z}' = (I + L)^{-1}\mathbf{s}$ and $P(\mathbf{z}')$ is convex in $\mathbf{z}'$. Since P is linear in $\mathbf{s}$, we are maximizing/minimizing a convex functional. Maximizer is on the boundary and the minimizer can be found by KKT conditions.
- Exhaustive search for i_A, i_B.

Main Takeaway

- The novel setting requires new tools to analyze generalized NE
- Heuristics inspiration of determining GNE to avoid computationally demanding method
  - maximizer ⟶ less connected(lonely) agent
  - minimizer ⟶ extreme opinion

Simulation - Karate Network



Data	Nodes	Edges	Network	Source
Karate(Karate club)	34	78	undirected	Zachary (1977)

Player	**Max**		**Min**	
Probability	75%	25%	90%	10%
Nodes	11	16	6	29
Innate opinion	0.77	0.51	0.72	0.21
New opinion	1	1	0	1

Table Karate Network NE result.

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	Max	**Min**
Nodes	16	6
Innate opinion	0.51	0.72
New opinion	1	0
Polarization	0.17	

Table Karate Network Maxmin result.

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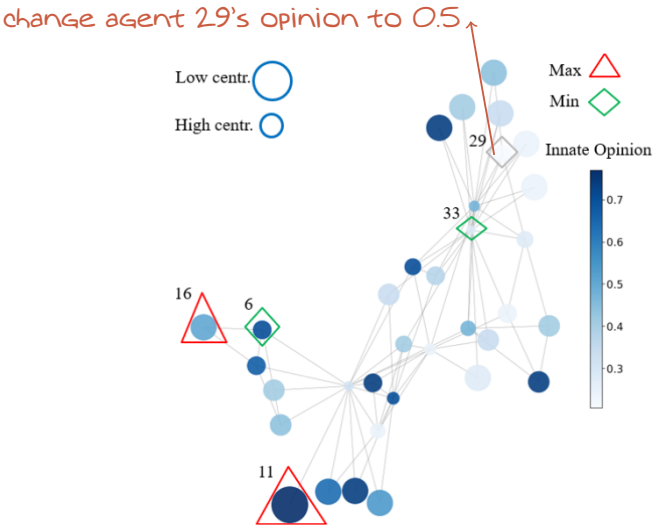
Simulation - Change innate opinion



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Probability	75%	25%	90%	10%
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New opinion	1	1	0	1

Table Karate Network NE result.

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	Max	Min
Nodes	11	33
Innate opinion	0.77	0.29
New opinion	1	1
Polarization	0.16	

Table Karate Network Maxmin result.

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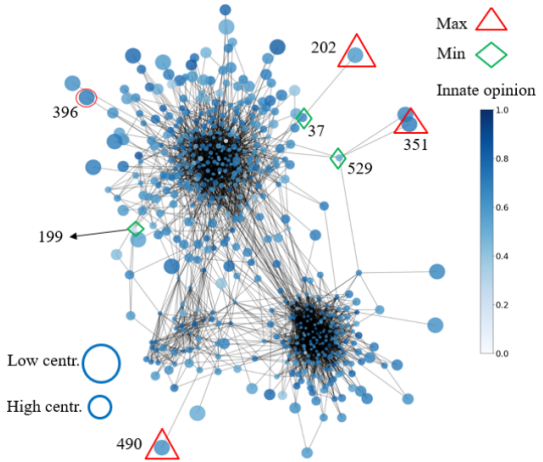
Simulation - Twitter Network



Data	Nodes	Edges	Network	Source
Twitter(elections)	548	5,271	undirected	De et al. (2014)

Player	**Max**			**Min**		
Probability	45%	45%	10%	40%	40%	20%
Nodes ID	202	351	490	529	37	199
Innate opinion	0.56	0.64	0.57	0.49	0.60	0.23
New opinion	0	0	0	0.76	0.8	0.7

Table Twitter Network NE result.

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	Max	**Min**
Nodes	396	199
Innate opinion	0.65	0.23
New opinion	0	0.71
Polarization	0.26	

Table Twitter Network Maxmin result.

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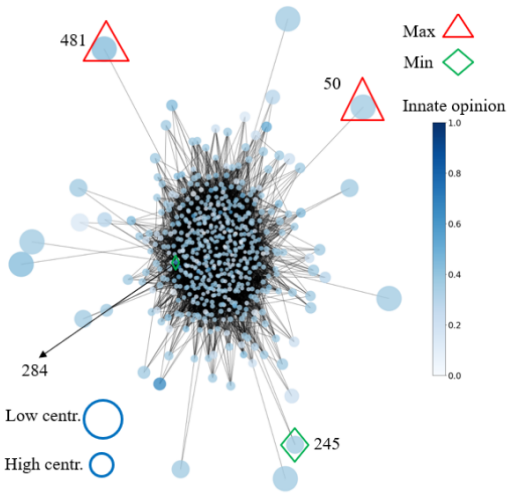
Simulation - Reddit Network



Data	Nodes	Edges	Network	Source
Reddit(politics network)	553	94,312	undirected	De et al. (2014)

Player	**Max**		**Min**	
Probability	29%	71%	84%	16%
Node ID	50	481	245	284
Innate opinion	0.5	0.5	0.409	0.625
New opinion	1	1	0.5	0.29

Table Reddit Network NE result.

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	Max	**Min**
Nodes	481	284
Innate opinion	0.5	0.63
New opinion	1	0
Polarization	0.07	

Table Reddit Network Maxmin result.

Summary

- Maximizer tend to choose the node with low centrality and neutral opinion.
- Minimizer tends to choose the node with min/max opinion among all agents.
- Any questions/Comments? ⇒ xzhan176@binghamton.edu

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