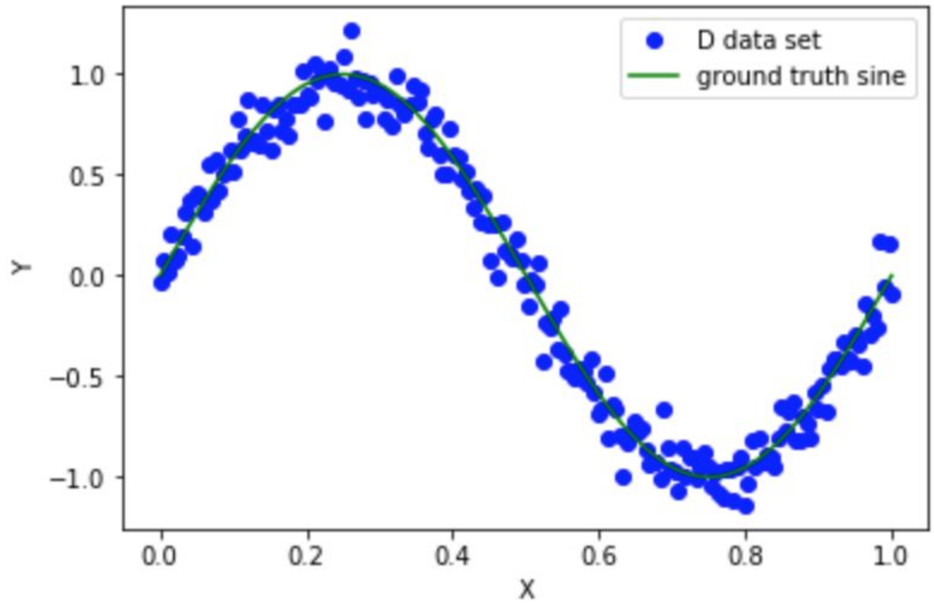
Machine Learning HW2
Xiaoyang Zhang
B00708854

Question 1:
Q1.1.

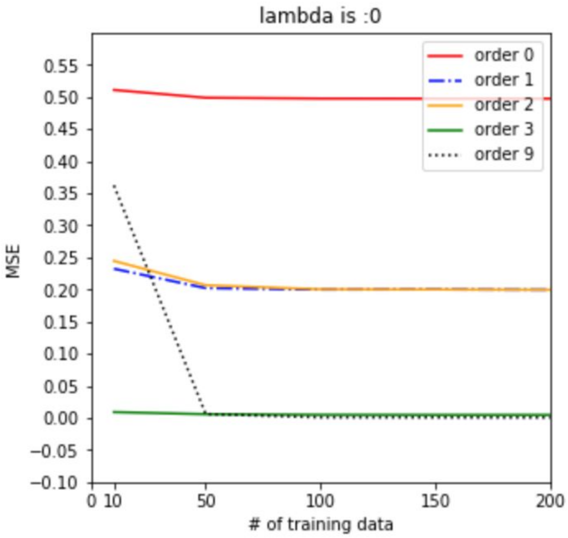


Q1.2. **Note : in this question, I use the whole Dtrain set (Ntrain == 200) as the test set, it is convenient to compare the results from next question.**
Parameters in the best case:

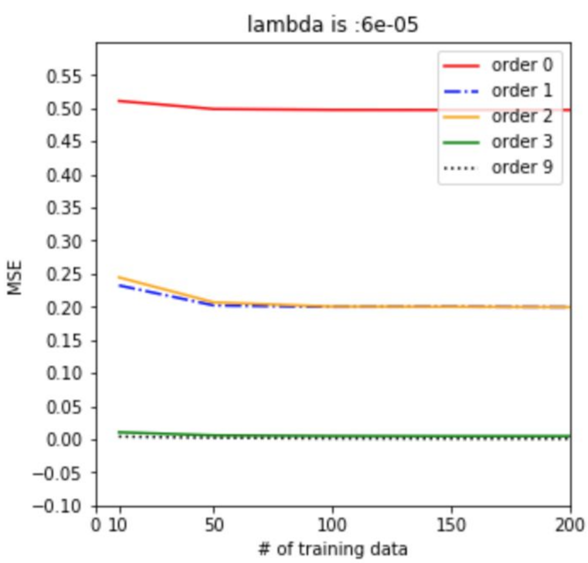
```
lambda0 = 0
lambda1 = 0.00006
lambda2 = 0.00007
lambda3 = 0.000075
lambda4 = 0.00008
lambda_array = [lambda0, lambda1, lambda2, lambda3, lambda4]
```

```
order0 = 0
order1 = 1
order2 = 2
order3 = 3
order9 = 9
order_array = [order0, order1, order2, order3, order9]
```

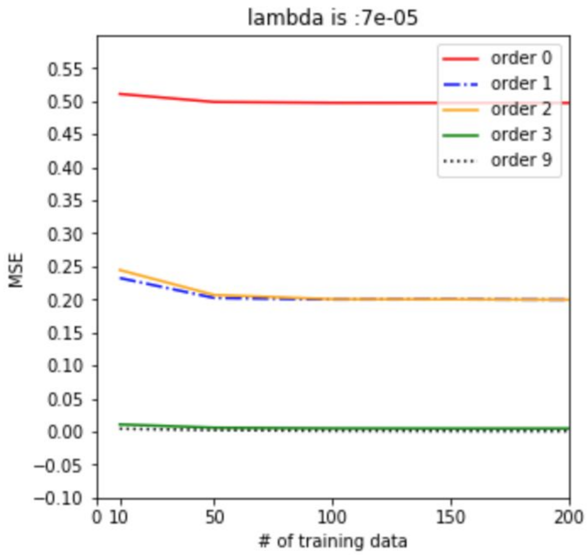
According to the parameters, the related plots and theta values are below:



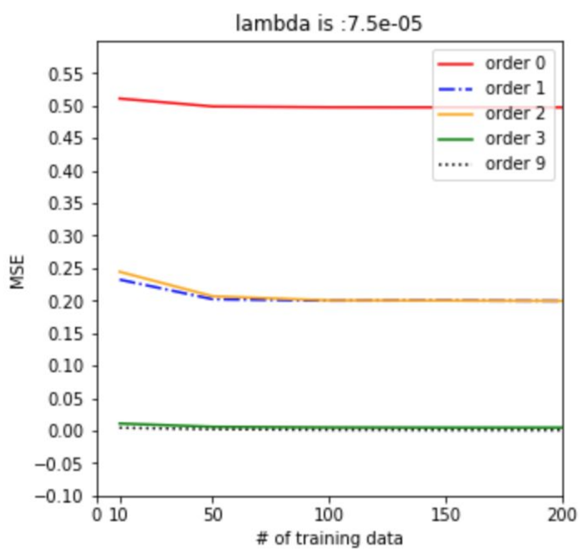
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.95	0.94	-0.18	-0.06
w1	NaN	-1.89	-1.86	11.75	12.73
w2	NaN	NaN	-0.02	-34.14	-127.14
w3	NaN	NaN	NaN	22.74	1023.72
w4	NaN	NaN	NaN	NaN	-4762.72
w5	NaN	NaN	NaN	NaN	12534.92
w6	NaN	NaN	NaN	NaN	-19594.25
w7	NaN	NaN	NaN	NaN	18129.45
w8	NaN	NaN	NaN	NaN	-9161.00
w9	NaN	NaN	NaN	NaN	1944.29



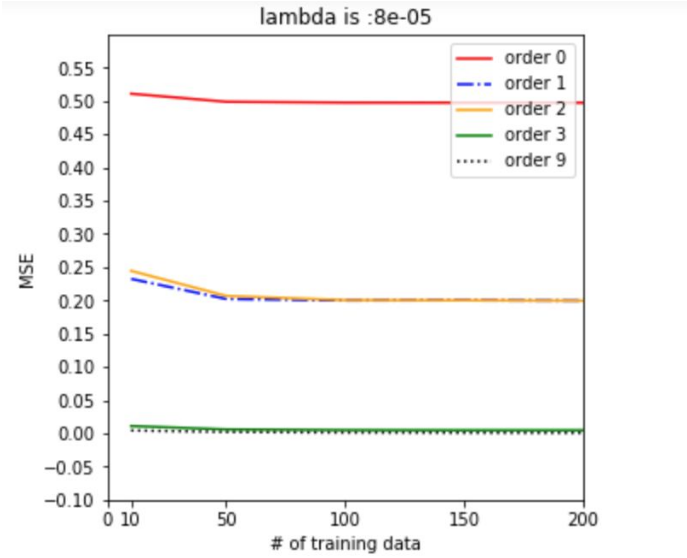
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.95	0.94	-0.17	-0.03
w1	NaN	-1.89	-1.86	11.71	7.85
w2	NaN	NaN	-0.02	-34.04	-12.82
w3	NaN	NaN	NaN	22.68	-12.72
w4	NaN	NaN	NaN	NaN	7.13
w5	NaN	NaN	NaN	NaN	13.29
w6	NaN	NaN	NaN	NaN	7.10
w7	NaN	NaN	NaN	NaN	-1.59
w8	NaN	NaN	NaN	NaN	-5.79
w9	NaN	NaN	NaN	NaN	-2.52



	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.95	0.94	-0.17	-0.03
w1	NaN	-1.89	-1.86	11.70	7.89
w2	NaN	NaN	-0.02	-34.02	-13.14
w3	NaN	NaN	NaN	22.67	-11.96
w4	NaN	NaN	NaN	NaN	6.81
w5	NaN	NaN	NaN	NaN	12.71
w6	NaN	NaN	NaN	NaN	7.08
w7	NaN	NaN	NaN	NaN	-1.09
w8	NaN	NaN	NaN	NaN	-5.38
w9	NaN	NaN	NaN	NaN	-3.00



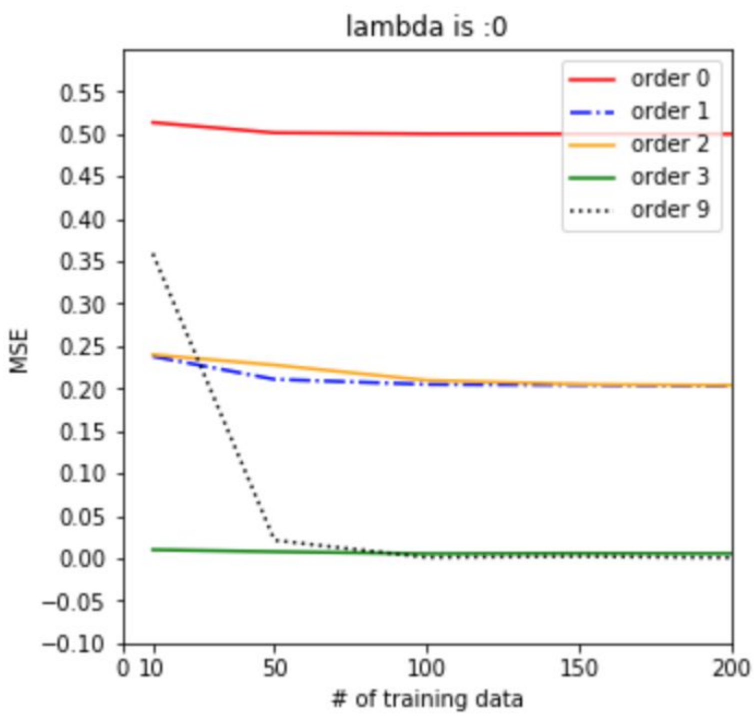
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.95	0.94	-0.17	-0.03
w1	NaN	-1.89	-1.86	11.70	7.91
w2	NaN	NaN	-0.02	-34.01	-13.28
w3	NaN	NaN	NaN	22.66	-11.64
w4	NaN	NaN	NaN	NaN	6.68
w5	NaN	NaN	NaN	NaN	12.47
w6	NaN	NaN	NaN	NaN	7.07
w7	NaN	NaN	NaN	NaN	-0.87
w8	NaN	NaN	NaN	NaN	-5.20
w9	NaN	NaN	NaN	NaN	-3.21

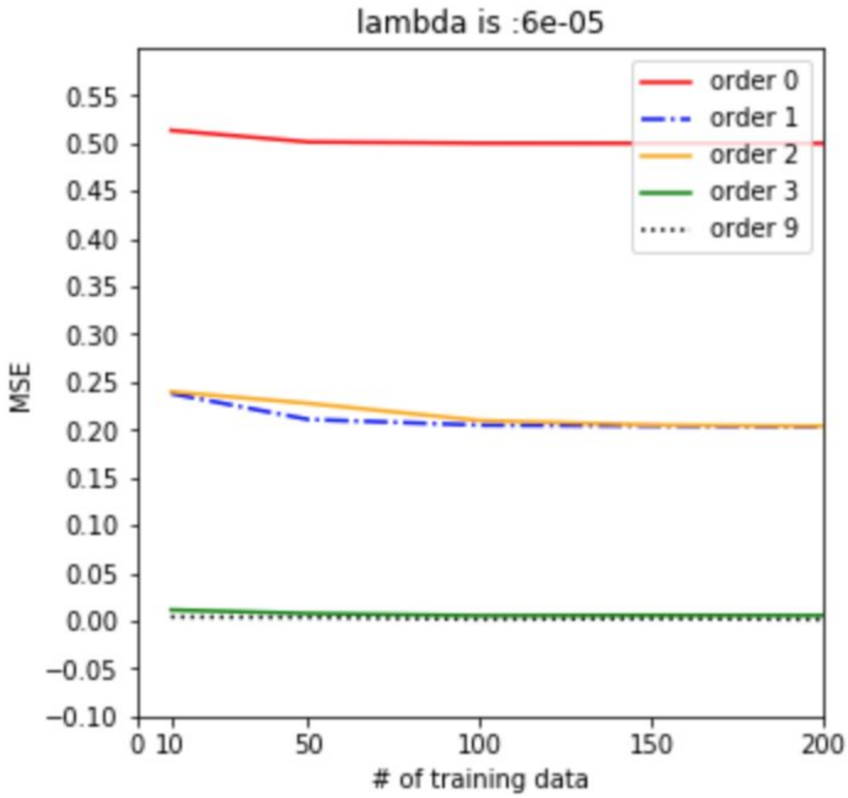


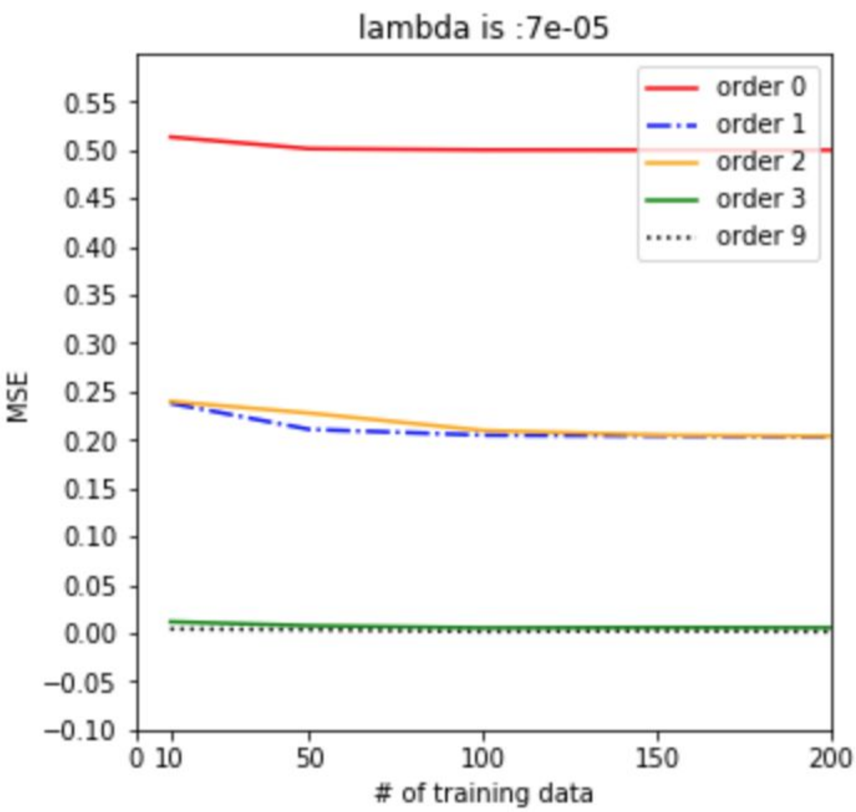
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.95	0.94	-0.17	-0.03
w1	NaN	-1.89	-1.86	11.69	7.93
w2	NaN	NaN	-0.02	-34.01	-13.40
w3	NaN	NaN	NaN	22.66	-11.36
w4	NaN	NaN	NaN	NaN	6.56
w5	NaN	NaN	NaN	NaN	12.25
w6	NaN	NaN	NaN	NaN	7.06
w7	NaN	NaN	NaN	NaN	-0.68
w8	NaN	NaN	NaN	NaN	-5.04
w9	NaN	NaN	NaN	NaN	-3.39

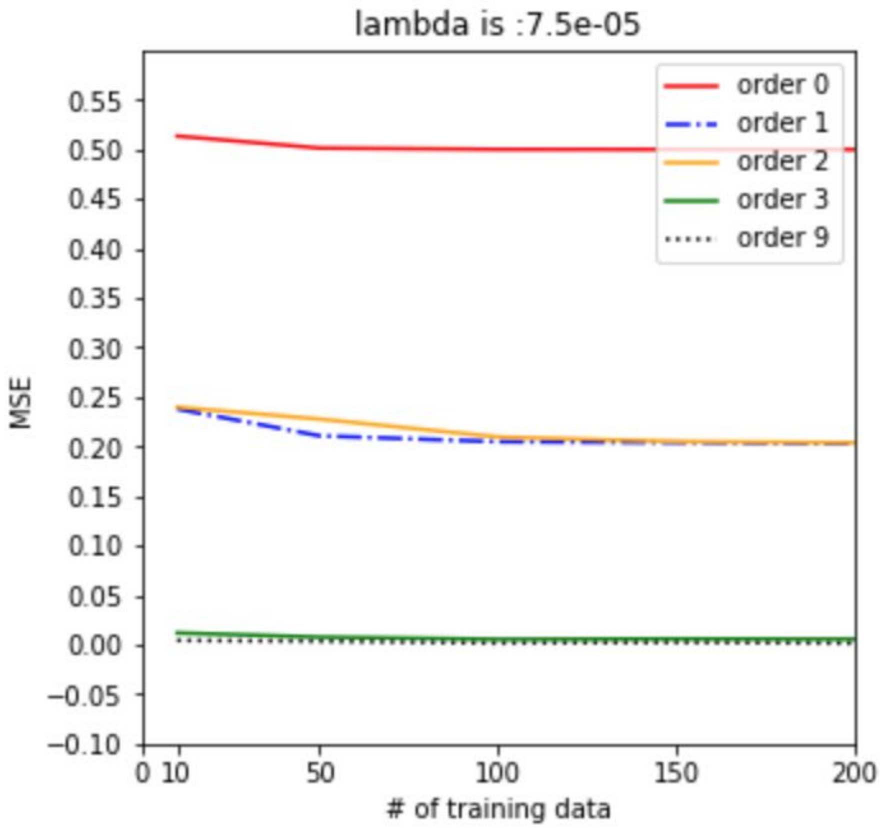
Q1.3. Test part, the test case same as question 2. The only difference is using Dtest set (Ntest == 199) as the test set. It is easy to compare the results from question 2 case by case.

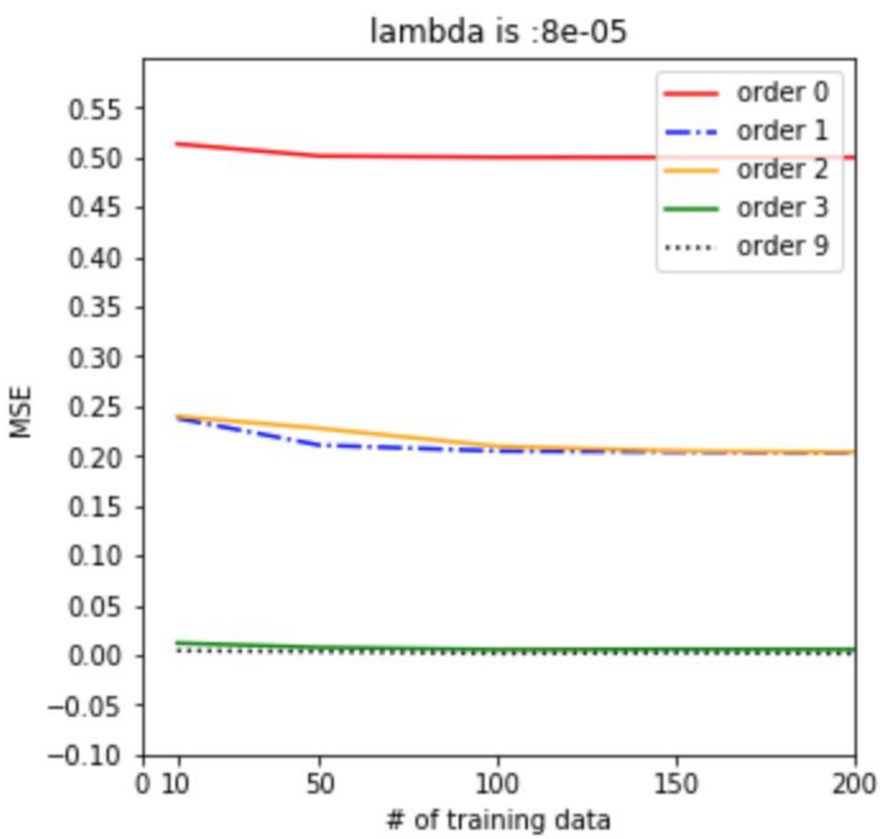
In order to get the better comparison, I modify the requirement to create (Xtest).
Like this: Xtest = Xtrain + 10/(2*(Ntrain-1)), i = 1 ~ (Ntrain -1)




lambda is :6e-05
order 0
order 1
order 2
order 3
order 9
MSE
# of training data


lambda is :7e-05
order 0
order 1
order 2
order 3
order 9
MSE
# of training data


lambda is :7.5e-05
order 0
order 1
order 2
order 3
order 9
MSE
# of training data

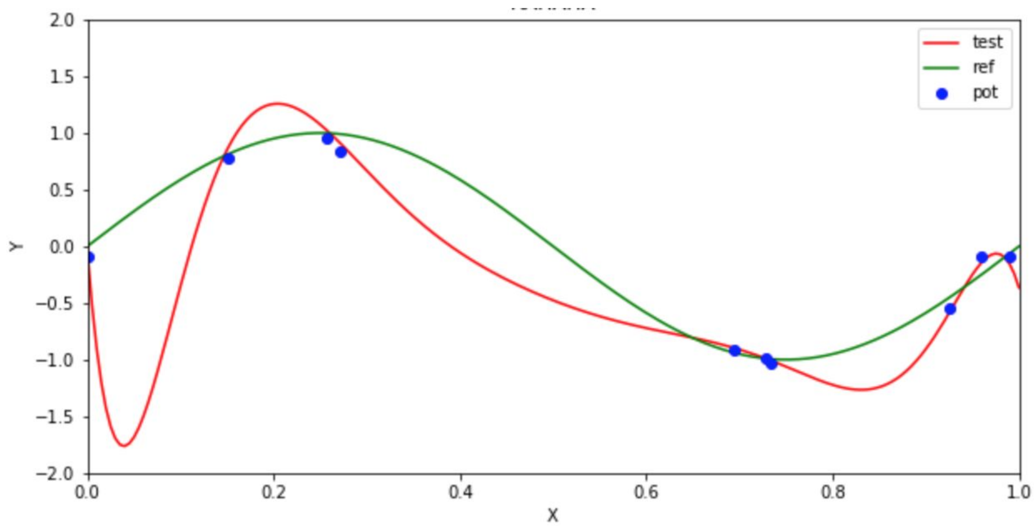

lambda is :8e-05
order 0
order 1
order 2
order 3
order 9
MSE
# of training data

Q1.4.

Q1: which one has the best performance and why?

A1:

In my case, the model with higher order(order 3 and 9) and enough training data set(larger than 100) has the best performance. Higher order are more flexible, it can make the model fit the original data samples well. But complex Models is always with High Variance. If the training data set was small, it is very easy to get over fitting, just like below. When I increase the training data set up to 200 samples, the performance is better.



What's more, appropriate lambda is good for performance. In my case, the model has the best performance when lambda is between 0.00007 to 0.00008. The reason is lambda can modify the weight of regularization, which can penalize coefficient values. It is one way to control the over fitting, especially in the case when there is no enough training data. (for example: 10~50 training data in this question)

Q2: Which one has the worst performance and why?

A2:

The model with less training data and high order, no regularization(lambda == 0) has the worst performance. The reason just like the explanation in above question.

Q3: How about the balance of variance and bias?

A3:

Variance and bias are trade-off performance, it is impossible to get best performance on both of them. In order to get better bias performance, some variance performance should be sacrificed:

a. Smaller lambda should be used.
b. Using high order model.

In order to get better variance performance, some bias performance should be sacrificed:

a. Larger lambda should be used.
b. Using lower order model.
c. Using large training data set.

Question 2:

Q2.1:

Implement Sigmoid():

```
def sigmoid(z):
    return 1.0 / (1 + np.exp(-z))
```

Implement LR_GD():

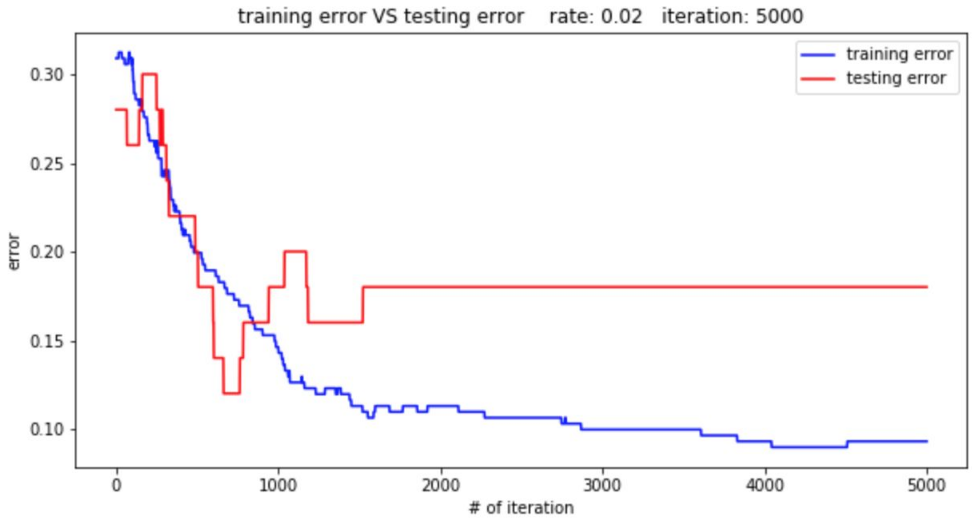
```
def LR_GD(train_x, train_y, test_x, test_y, alpha, iteration): # train_x: m * n; train_y: m * 1
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of test points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)
    for i in range(iteration):
        temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n * m] -> [1 * m]
        temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [1 * m]
        temp2 = np.multiply(temp1, train_x.T) # [1 * m]
        theta_vector = theta_vector - new_alpha * np.sum(temp2, axis=1)

        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x, train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, test_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

Implement LR_Test():

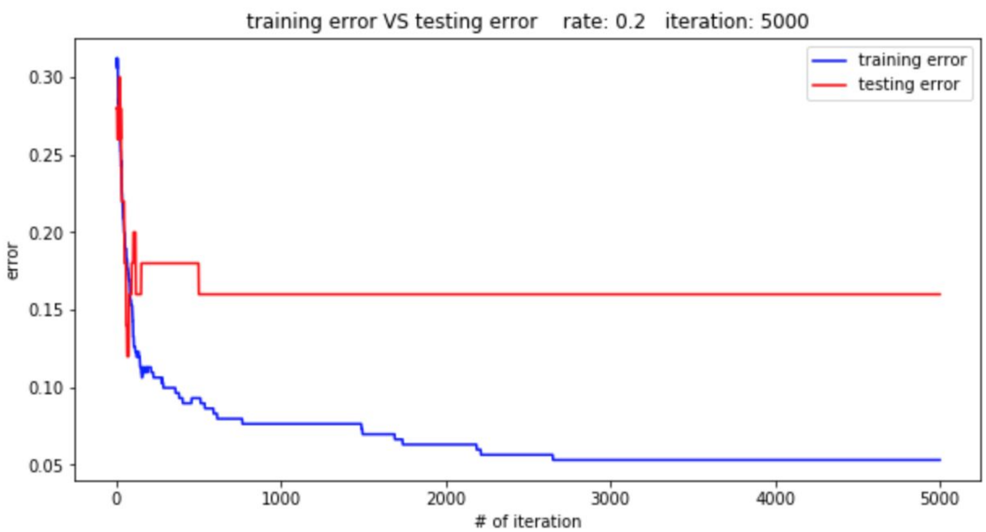
```
def LR_Test(theta_set, test_x, test_y):
    m, n = np.shape(test_x) # m is # of points, n is # of features
    matchCount = 0
    for i in range(m):
        test = np.array(np.dot(theta_set, test_x[i].T).tolist())
        predict = test[0] > 0
        if predict == bool(test_y[i]):
            matchCount += 1
        accuracy = float(matchCount) / m
    return accuracy
```

Try 3 different learning rate:

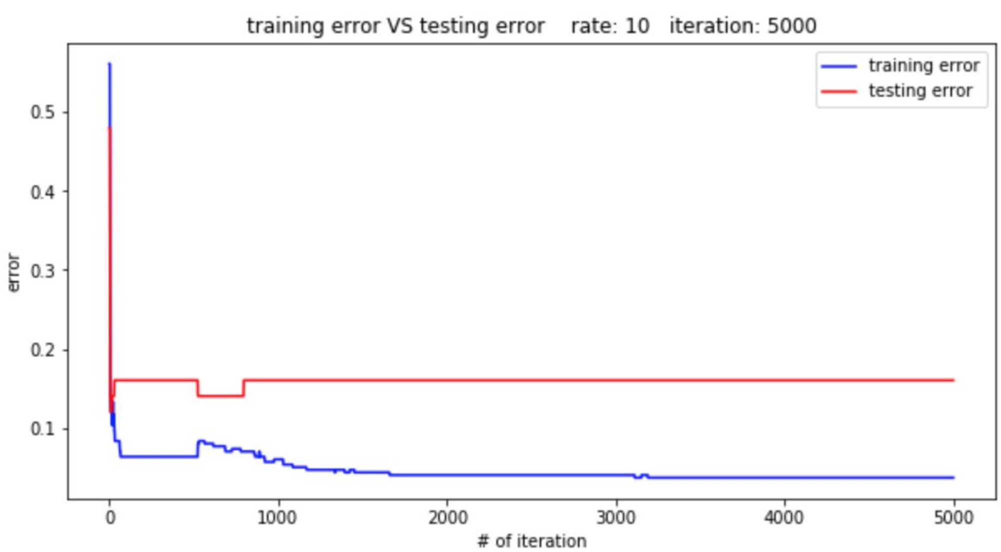
Case 1:



Case 2:



Case 3:



Q2.2:

Implement LR_SGD():

```
def LR_SGD(train_x, train_y, test_x, test_y, alpha, iteration):
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)
    for i in range(iteration):
        for j in range(0,n):
            temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n * m] -> [1 * m]
            temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [1 * m]
            temp2 = np.multiply(temp1, train_x[:,j]) # [1 * m]
            theta_vector[0][j] = theta_vector[0][j] - new_alpha * np.sum(temp2) # 1

        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x, train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, test_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

Try 3 different learning rate:

Case 1:

Case 2:

Case 3:

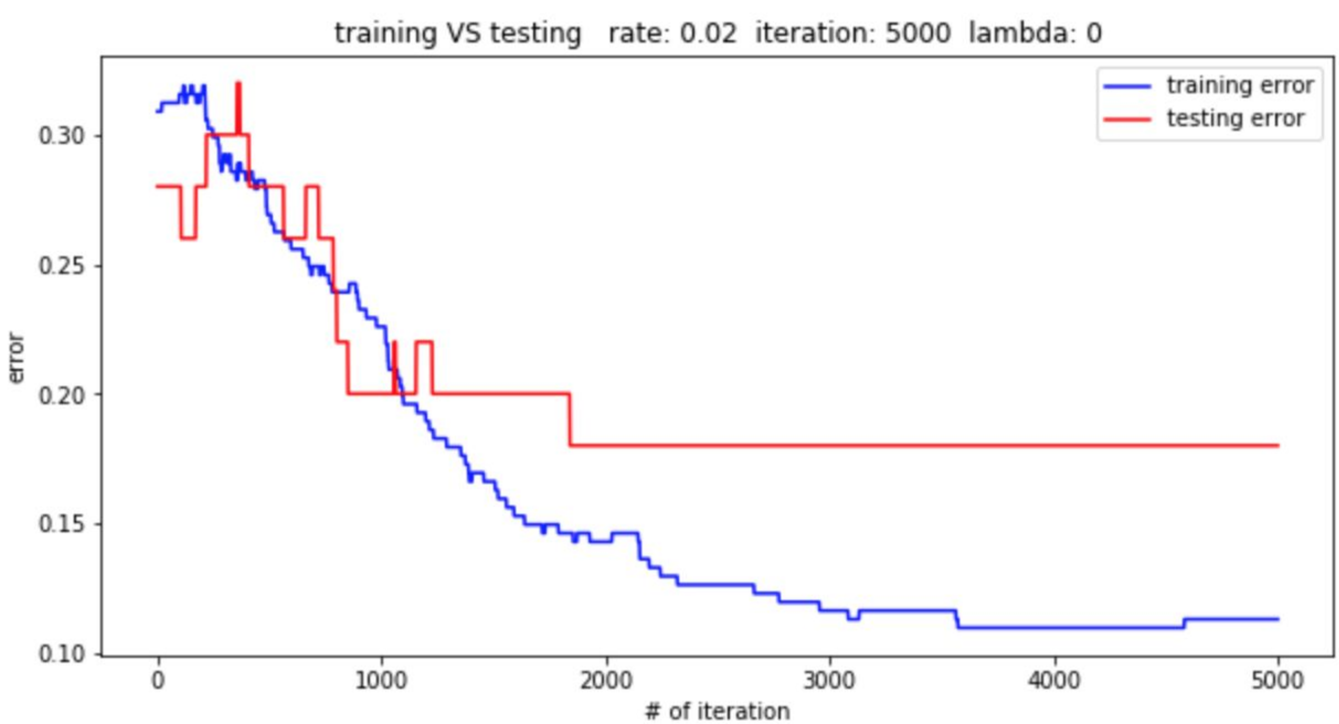
Q2.3:

Implement LR_GD_REG():

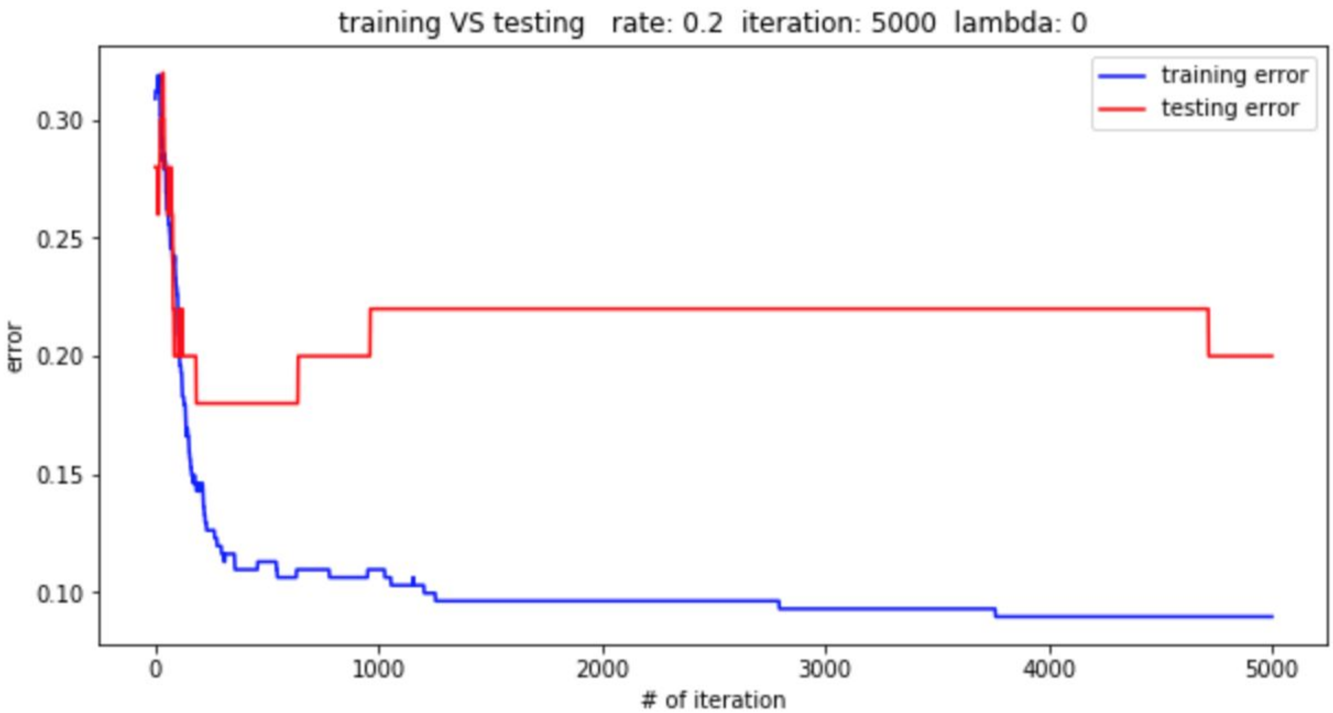
```
def LR_GD_REG(train_x, train_y, test_x, test_y, alpha, iteration, lambd): # train_x: m * n; train_y: m * 1
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of test points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)

    train_x_one = train_x[:,0]
    train_x_rest = train_x[:,1:n]
    for i in range(iteration):
        theta_vector_one = theta_vector[0][0:1]
        theta_vector_rest = theta_vector[0][1:n]
        ## only this part is different
        temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n * m] -> [1 * m]
        temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [1 * m]
        temp2 = np.multiply(temp1, train_x_rest.T) # [n * m]
        theta_vector_rest = theta_vector_rest - new_alpha * (np.sum(temp2, axis=1) - lambd*theta_vector_rest)
        ## in np.sum(X, axis=?), no matter axis is 1 or 0, the result should be one row.
        ##
        theta_vector = np.concatenate((np.array([theta_vector_one]), np.array([theta_vector_rest])),axis=1)
        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x, train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, test_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

Try 3 different learning rate:

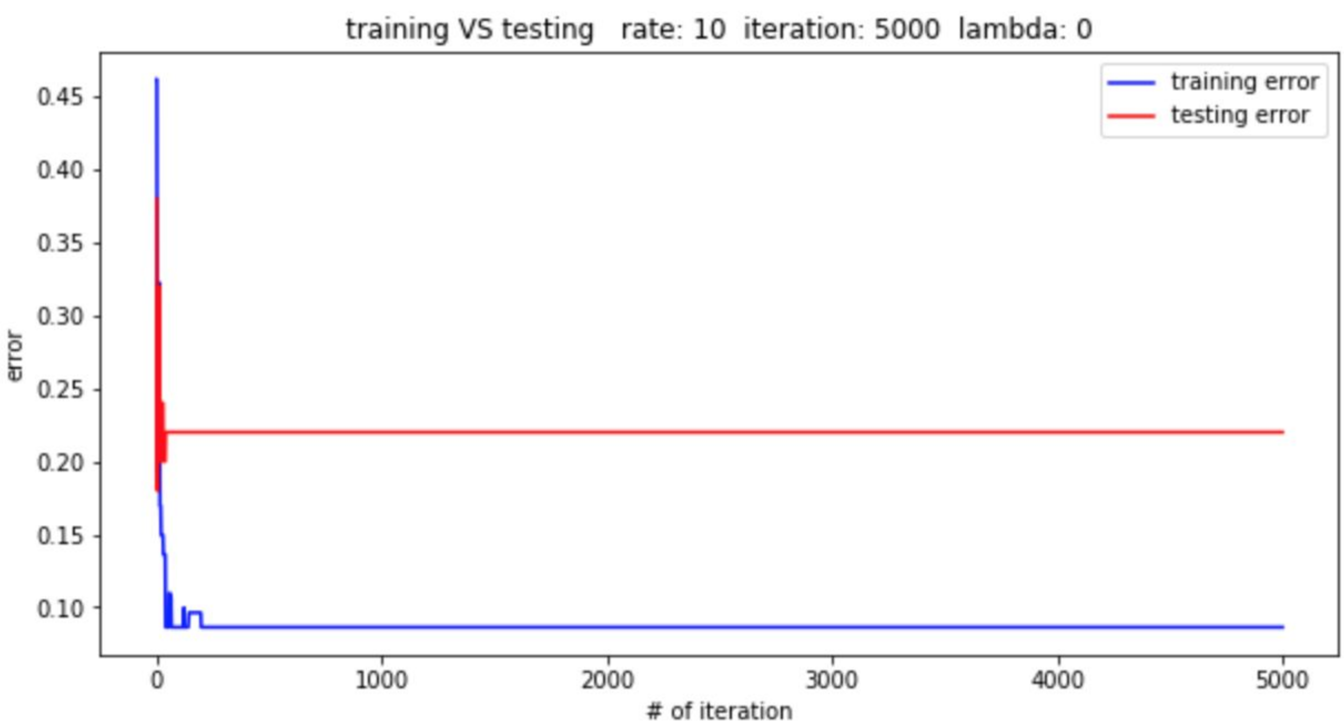
Case 1:



Case 2:

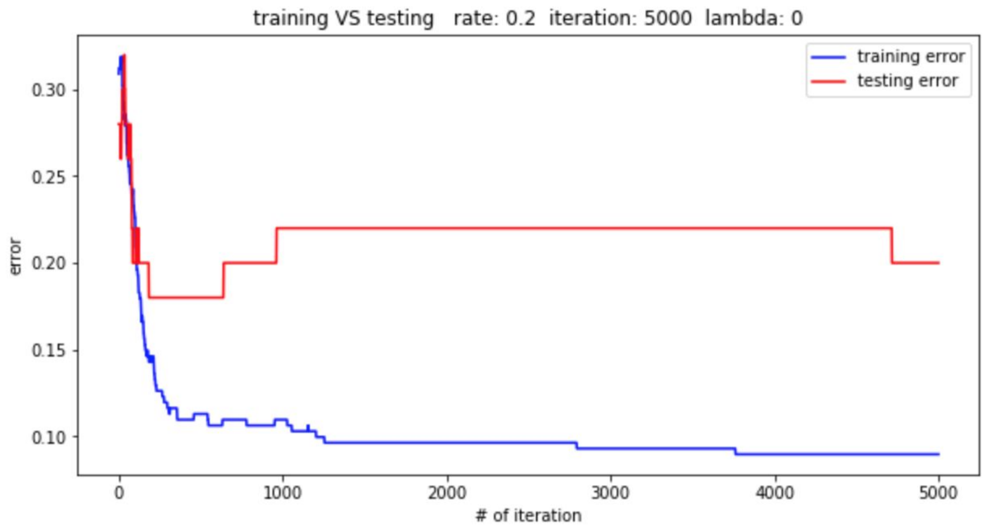

training VS testing rate: 0.2 iteration: 5000 lambda: 0
training error
testing error
error
# of iteration
0.30
0.25
0.20
0.15
0.10
0
1000
2000
3000
4000
5000

Case 3:

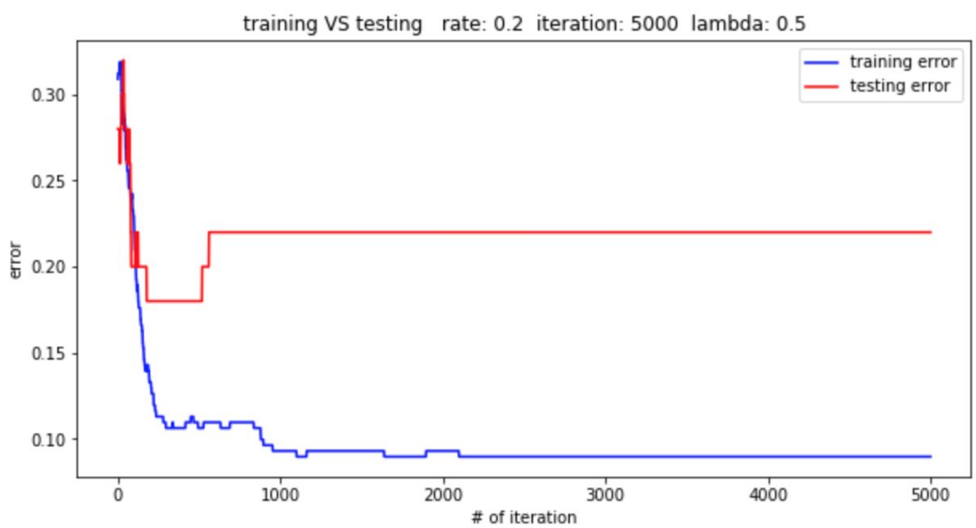

training VS testing rate: 10 iteration: 5000 lambda: 0
training error
testing error
error
# of iteration
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0
1000
2000
3000
4000
5000

Try 5 different lambda:

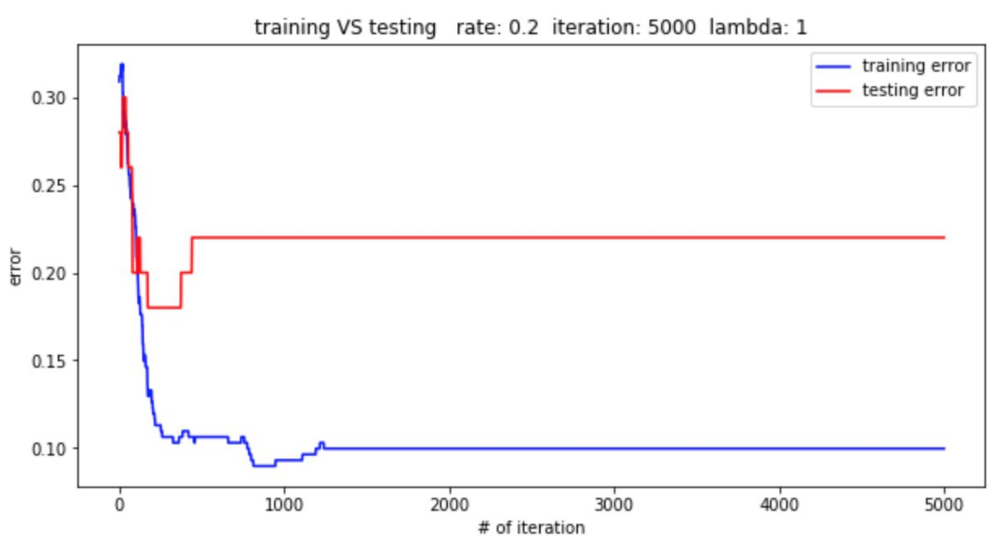
Case 1:



Case 2:



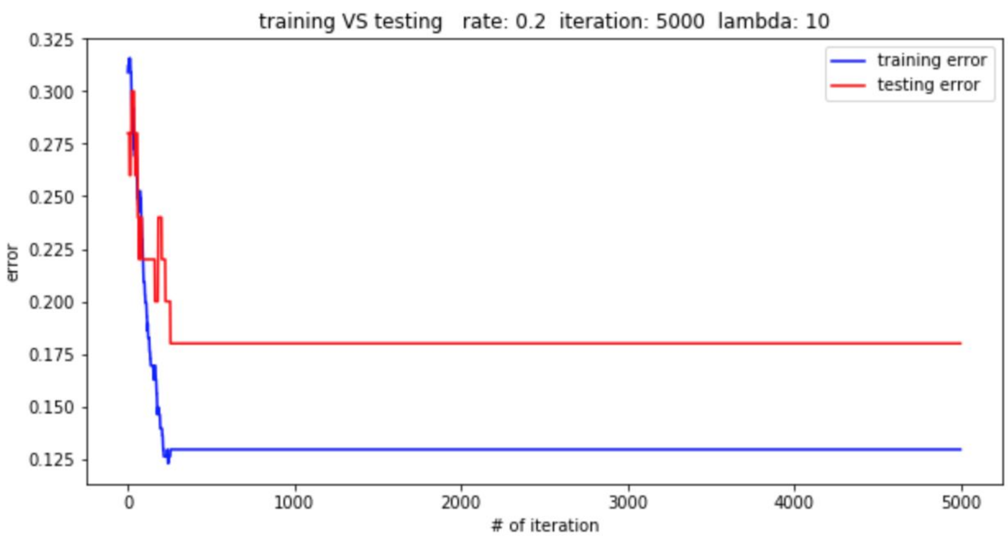
Case 3:



Case 4:

training VS testing rate: 0.2 iteration: 5000 lambda: 4
training error
testing error
error
0.30
0.25
0.20
0.15
0.10
0
1000
2000
3000
4000
5000
# of iteration

Case 5:



training VS testing rate: 0.2 iteration: 5000 lambda: 10
training error
testing error
error
0.325
0.300
0.275
0.250
0.225
0.200
0.175
0.150
0.125
0
1000
2000
3000
4000
5000
# of iteration

Q: What do you observe by using different learning rate?
A:

1. When learning rate is larger, the error curve is more steep. If no divergence happened, the curve can go to the lower error point more quickly.
2. But larger learning rate is easy to pass the minimum J(theta) point. In that case, the value of cost function will get bigger and bigger, just like rebound.

Q: What is the best learning rate?
A:

1. Could not be too small. Otherwise it will spend a long time to find the appropriate parameter(theta) set which relates to the minimum J(theta), slow convergence.
2. Could not be too large. Otherwise it is easy to miss the minimum J(theta) point. J(theta) may not decrease on every iteration, may not converge.

Q: Explain how to find it?
A:

1. Try different learning rate, find the one with better performance(fast, converge), for example: 0.001 -> 0.003 -> 0.01 -> 0.03 -> 0.3 -> 1
2. From experience. Different models in different occasion(data set size, number of features...) have different empirical value.

Q: What do you observe by using different weights?
A:

1. convergence speed will be affected by weights. When increasing weight, the convergence speed is faster.
2. When increasing weight, model's variance becomes smaller. But it does this at the expense of adding bias.

Q: What is the best weight?
A:

1. Could not be too large. Otherwise, that will make the model less flexible to match/trace data set. In other word, it would be hard to train the model to get the appropriate parameters. In this case, it is easy to meet under fitting.
2. Could not be too small. Especially when we haven't got enough training data and the model is complex with high order. In this case, it is easy to meet over fitting.
3. How much bias are you willing to tolerate in your estimate? Depend on different problem.

Q: Explain how to find it?

1. Try different weight, find the one with better performance(converge fast, acceptable bias, no over fitting….), for example: 0 -> 0.001 -> -> 0.01 -> 0.1 -> 1 -> 10

Appendix: Below part is source Code

Appendix: Below part is source Code

Machine Learning HW2

Xiaoyang Zhang B00708854

In [3181]:
```
import pandas as pd
import numpy as np
import random
import math
from sklearn.metrics import mean_squared_error
import matplotlib.pyplot as plt
%matplotlib inline
```

Question 1.

1.

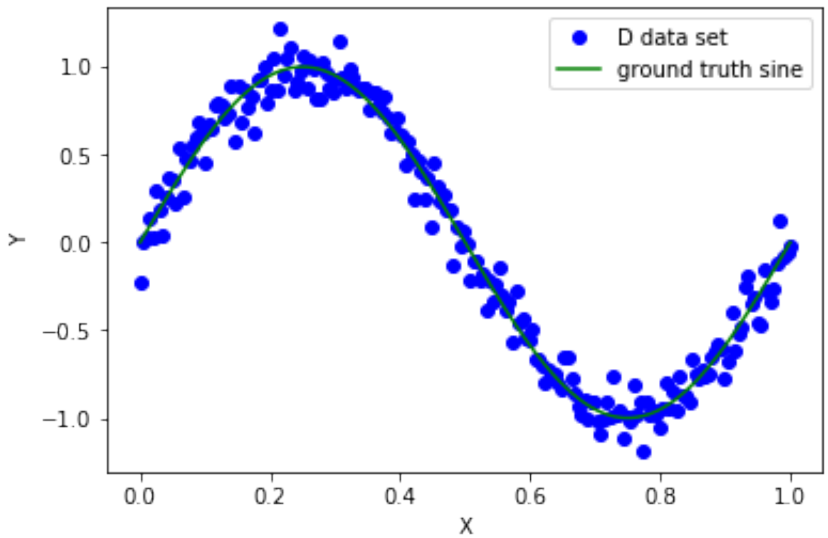
In [3182]:
```
#someone has generated data by sampling the sinusoidal function f(x)
= sin(2pix)  x is between [0,1]
N_train = 200
i_train = np.arange(N_train) # [0...199]
```

In [3183]:
```
X_train = i_train / (N_train - 1) #[0...1, 0.005 alternative]
```

In [3184]:
```
Y_ground_true = np.sin(2*math.pi*X_train)
```

In [3185]:
```
Y_noise = Y_ground_true + np.random.normal(0, 0.1, N_train) # white G
aussian noise
```

In [3186]:
```
plt.plot(X_train, Y_noise, 'ro',color='blue', label='D data set')
plt.plot(X_train, Y_ground_true, color='green', label='ground truth s
ine')
plt.legend()
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



In [3187]:
```
# Generate a dataset D_set consisting of N = 200
X_buf = np.array([X_train])
Y_buf = np.array([Y_noise])
D_set = np.concatenate((X_buf,Y_buf),axis=0).T
D = D_set.tolist()
```

2.

Parameter set : sub test set, lambda, order

In [3188]:
```
#D is [X_buf[i], Y_buf[i]]
subD_10 = random.sample(D, 10)
subD_50 = random.sample(D, 50)
subD_100 = random.sample(D, 100)
subD_150 = random.sample(D, 150)
subD_200 = random.sample(D, 200)
subD_200_test = random.sample(D, 200)
subD_array = [subD_10, subD_50, subD_100, subD_150, subD_200, subD_20
0_test]
```

In [3189]:
```
lambda0 = 0
lambda1 = 0.00006
lambda2 = 0.00007
lambda3 = 0.000075
lambda4 = 0.00008
lambda_array = [lambda0, lambda1, lambda2, lambda3, lambda4]
```

In [3190]:
```
order0 = 0
order1 = 1
order2 = 2
order3 = 3
order9 = 9
order_array = [order0, order1, order2, order3, order9]
```

Below part is version 1, only for debugging

single training case

In [3191]:
```
subD = subD_10 #list, manual modify
test_qty = 200
qty = len(subD)
order = 9 #manual modify
lambda_v = lambda0 #manual modify
#above are input parameters
subD.sort()
subD_temp = np.array(subD)
subD_X = subD_temp.T[0]
subD_Y = subD_temp.T[1]
```

In [3192]:
```
#def create_X(X_input):
#    X_temp = X_input.tolist()
#    sub = np.ones([1,order])
#    for x in X_temp:
#        temp = np.array([[x, pow(x,2), pow(x,3)]]) # order == 3, sho
uld modify, follow order, hard code
#        sub = np.concatenate((sub,temp),axis=0)
#    sub = np.delete(sub,0,axis=0)
#    return sub

#X_sub = create_X(subD_X)
```

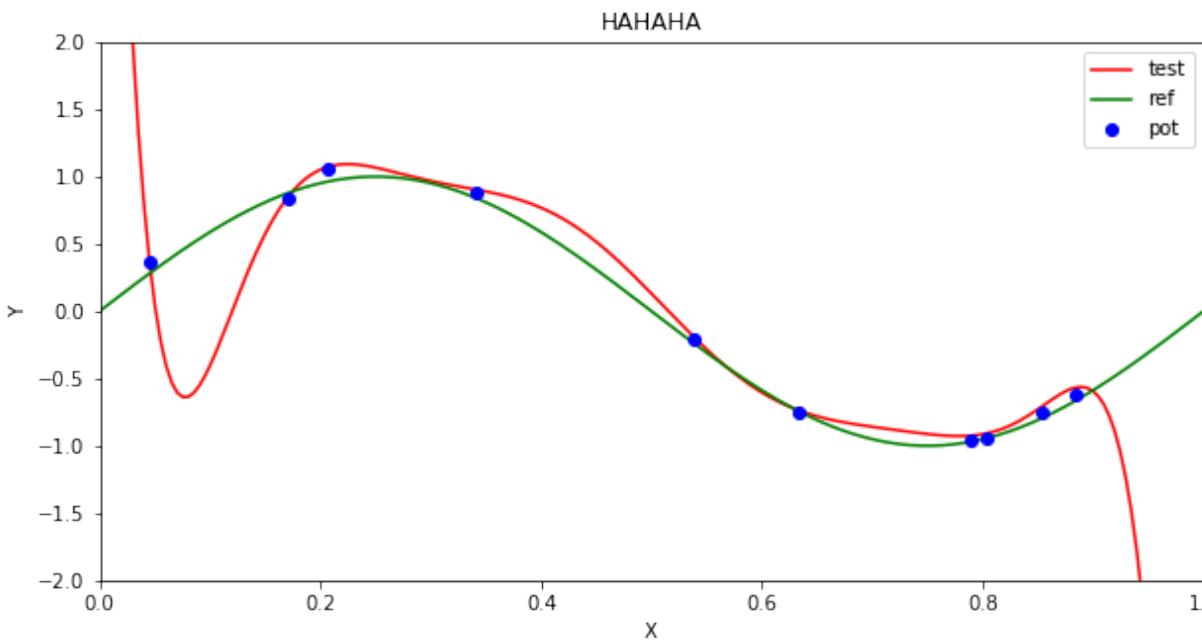
In [3193]:
```
X = np.array([np.ones(qty,int)]).T # first col is 1, all 1
for i in range(1, order+1):
    temp = np.array([np.power(subD_X,i)])
    X = np.concatenate((X,temp.T),axis=1)

Y = np.array([subD_Y]).T
E = np.eye(order+1)
E[0][0] = 0
```

In [3194]:
```
theta = np.dot(np.dot(np.linalg.inv((np.dot(X.T,X) + lambda_v*E)), X.
T),Y)
```

In [3195]:
```
#theta.T[0].tolist()
```

In [3196]:
```
#this block is for testing
#what I want to do is to create ground truth y and test y, than calcu
late the MSE
test_X_ground = create_subD_X(subD_200_test)
test_Y_ground = np.sin(2*math.pi*test_X_ground)

test_X_array = np.array([np.ones(test_qty,int)]).T # first col is 1,
all 1
for i in range(1, order+1):
    temp = np.array([np.power(test_X_ground,i)])
    test_X_array = np.concatenate((test_X_array,temp.T),axis=1)

test_Y_result = np.dot(test_X_array, theta)
```

In [3197]:
```
plt.figure(num=0, figsize=(10, 5))
plt.title('HAHAHA')
plt.plot(test_X_ground, test_Y_result,color='red', label='test')
plt.plot(test_X_ground, test_Y_ground,color='green', label='ref')
plt.plot(subD_X, subD_Y, 'ro', color='blue', label='pot')
plt.xlim((0, 1))
plt.ylim((-2, 2))
plt.legend()
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



In [3198]:
```
mean_squared_error(test_Y_result, test_Y_ground)
```

Out[3198]: 4.7842102546473715

Above is first version, for debugging

Below is second version, for submitting

Using function:

In [3199]:
```
# no change
def create_subD_X(subDataSet):
    subDataSet.sort()
    temp = np.array(subDataSet)
    return temp.T[0]
def create_subD_Y(subDataSet):
    subDataSet.sort()
    temp = np.array(subDataSet)
    return temp.T[1]
```

In [3200]:
```
def create_MSE_set(subD_set, lambda_set, order_set):
    Mse_set = []
    theta_set = []
    len_subD_set = len(subD_set)-1 # the last one is test set, so ign
ore here
    len_lambda_set = len(lambda_set)
    len_order_set = len(order_set)
    sub_50_test = subD_set[len_subD_set] # the last one in subD_set,
only for testing
    qty_test = len(sub_50_test)

    for l in range(0,len_lambda_set):
        lamb = lambda_set[l]
        for o in range(0,len_order_set):
            order = order_set[o]
            E = np.eye(order+1)
            E[0][0] = 0
            for s in range(0,len_subD_set):
                sub = subD_set[s]
                quantity = len(sub)
                sub_X = create_subD_X(sub)
                sub_Y = create_subD_Y(sub)

                Xc = np.array([np.ones(quantity,int)]).T # first col
is 1, all 1
                for i in range(1, order+1):
                    temp = np.array([np.power(sub_X,i)])
                    Xc = np.concatenate((Xc,temp.T),axis=1)

                Yc = np.array([sub_Y]).T

                #get theta
                theta = np.dot(np.dot(np.linalg.inv((np.dot(Xc.T,Xc)
+ lamb*E)), Xc.T) ,Yc)
                #create theta set
                theta_set.append(theta.T[0].tolist())

                #get MSE
                t_X_ground = create_subD_X(sub_50_test)
```

```
                t_Y_ground = np.sin(2*math.pi*t_X_ground)

                t_X_array = np.array([np.ones(qty_test,int)]).T
                for i in range(1, order+1):
                    temp = np.array([np.power(t_X_ground,i)])
                    t_X_array = np.concatenate((t_X_array,temp.T),axi
s=1)

                t_Y_result = np.dot(t_X_array, theta)

                t_MSE = mean_squared_error(t_Y_result, t_Y_ground)


                #create MSE set
                Mse_set.append(t_MSE)

    return theta_set, Mse_set
```

In [3201]:

```
theta_all, mse_all = create_MSE_set(subD_array, lambda_array, order_a
rray)
print (len(theta_all))
print (len(mse_all))
```

```
125
125
```

In [3202]:

```
# theta set, contain 125 different models
theta_lam0 = theta_all[0:25]
theta_lam1 = theta_all[25:50]
theta_lam2 = theta_all[50:75]
theta_lam3 = theta_all[75:100]
theta_lam4 = theta_all[100:125]
```

In [3203]:

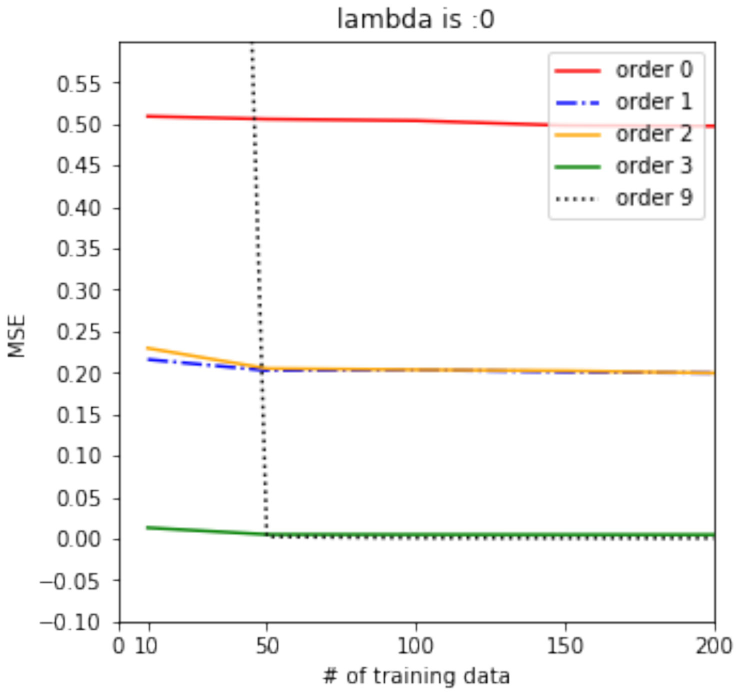
```
mse0 = mse_all[0:25] # lam0
mse1 = mse_all[25:50] # lam1
mse2 = mse_all[50:75] # lam2
mse3 = mse_all[75:100] # lam3
mse4 = mse_all[100:125] # lam4
```

Below part is for graph & theta set

In [3204]:
```
def display(lambda_value, mse_set):
    X_array = np.array([10,50,100,150,200])
    name = 'lambda is :' + str(lambda_value)
    #create figure
    plt.figure(figsize=(5, 5))
    #title
    plt.title(name)
    #plot 1
    plt.plot(X_array, mse_set[0:5], color='red', label='order 0')
    #plot 2
    plt.plot(X_array, mse_set[5:10],'-.' ,color='blue', label='order
1')
    #plot 3
    plt.plot(X_array, mse_set[10:15], color='orange', label='order 2'
)
    #plot 4
    plt.plot(X_array, mse_set[15:20], color='green', label='order 3')
    #plot 5
    plt.plot(X_array, mse_set[20:25], ':', color='black', label='orde
r 9')
    #set axis range
    plt.xlim((0, 200))
    plt.ylim((0, 0.6))
    #set aixs name
    plt.xlabel('# of training data')
    plt.ylabel('MSE')
    #set aixs mark
    my_x_ticks = np.array([0,10,50,100,150,200])
    my_y_ticks = np.arange(-0.1, 0.6, 0.05)
    plt.xticks(my_x_ticks)
    plt.yticks(my_y_ticks)
    #legned
    plt.legend()
    #display figure
    plt.show()
```

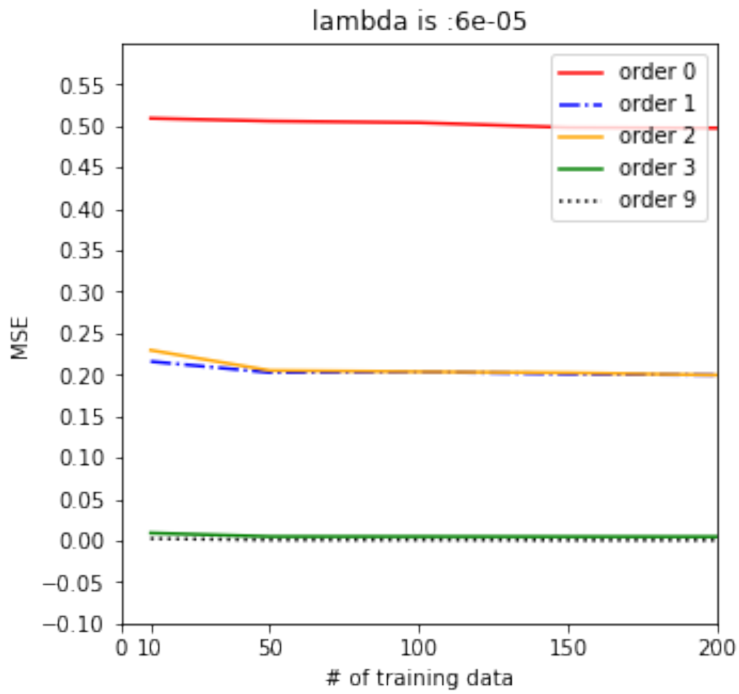
In [3205]:
```
import pandas as pd
def display_theta(theta_set):
    buffer = theta_set[slice(4,25,5)]
    d = {'order 0' : pd.Series(np.around(buffer[0],2), index=['w0']),
         'order 1' : pd.Series(np.around(buffer[1],2), index=['w0', '
w1']),
         'order 2' : pd.Series(np.around(buffer[2],2), index=['w0', '
w1', 'w2']),
         'order 3' : pd.Series(np.around(buffer[3],2), index=['w0', '
w1', 'w2', 'w3']),
         'order 9' : pd.Series(np.around(buffer[4],2), index=['w0', '
w1', 'w2', 'w3', 'w4', 'w5', 'w6', 'w7', 'w8', 'w9']),
        }
    df = pd.DataFrame(d)
    print(df)
```

In [3206]:
```
#figure 0, lam0
display(lambda0, mse0)
display_theta(theta_lam0)
```



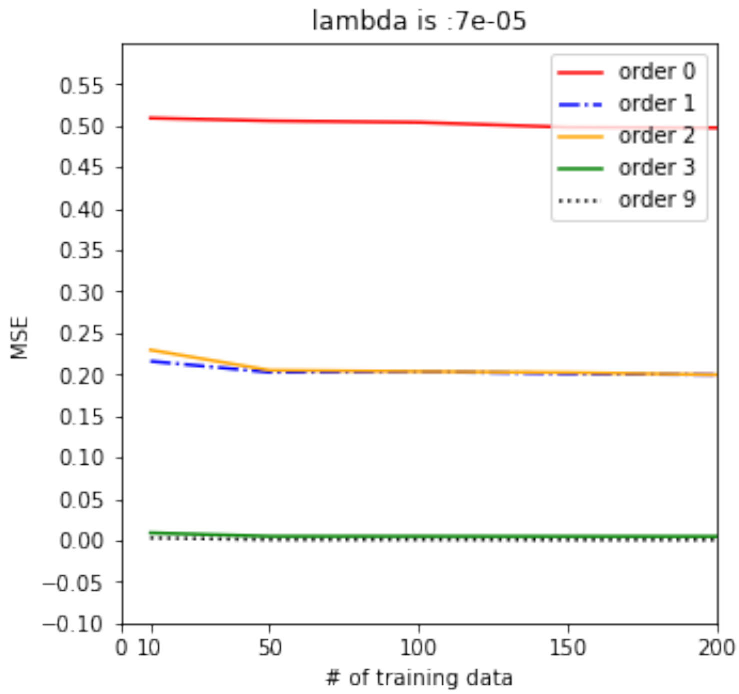
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.93	0.93	-0.20	-0.14
w1	NaN	-1.87	-1.83	11.83	14.86
w2	NaN	NaN	-0.04	-34.28	-140.64
w3	NaN	NaN	NaN	22.82	997.13
w4	NaN	NaN	NaN	NaN	-4175.15
w5	NaN	NaN	NaN	NaN	10016.53
w6	NaN	NaN	NaN	NaN	-14386.42
w7	NaN	NaN	NaN	NaN	12363.38
w8	NaN	NaN	NaN	NaN	-5880.99
w9	NaN	NaN	NaN	NaN	1191.45

In [3207]:
```
#figure 1, lam1
display(lambda1, mse1)
display_theta(theta_lam1)
```



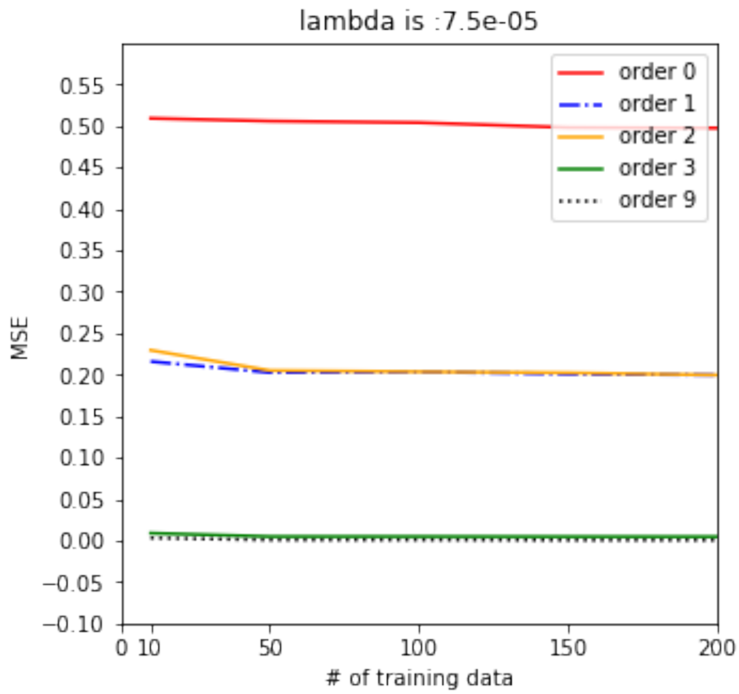
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.93	0.93	-0.19	-0.08
w1	NaN	-1.87	-1.83	11.79	8.58
w2	NaN	NaN	-0.04	-34.18	-15.03
w3	NaN	NaN	NaN	22.76	-12.63
w4	NaN	NaN	NaN	NaN	10.34
w5	NaN	NaN	NaN	NaN	15.72
w6	NaN	NaN	NaN	NaN	5.35
w7	NaN	NaN	NaN	NaN	-6.50
w8	NaN	NaN	NaN	NaN	-8.90
w9	NaN	NaN	NaN	NaN	3.10

In [3208]:
```
#figure 2, lam2
display(lambda2, mse2)
display_theta(theta_lam2)
```



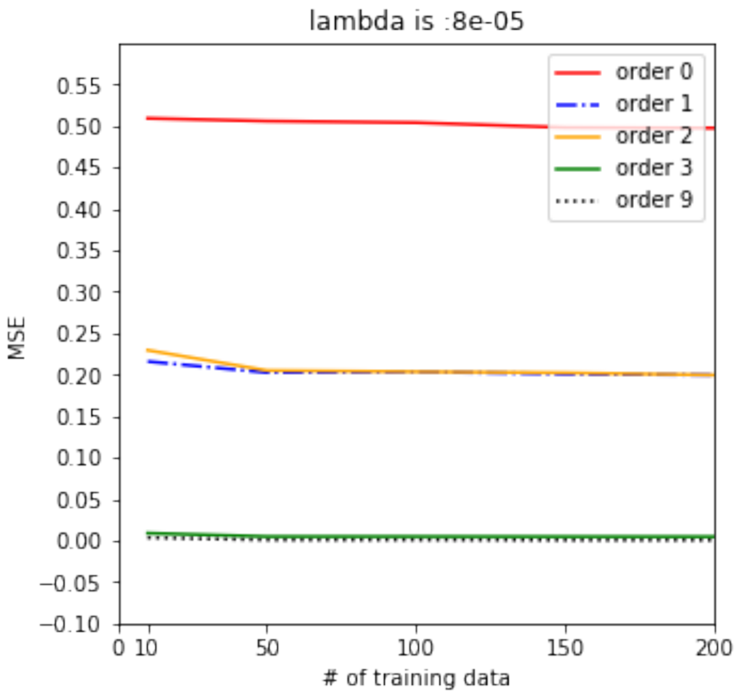
	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.93	0.93	-0.19	-0.08
w1	NaN	-1.87	-1.83	11.78	8.64
w2	NaN	NaN	-0.04	-34.16	-15.47
w3	NaN	NaN	NaN	22.75	-11.62
w4	NaN	NaN	NaN	NaN	9.97
w5	NaN	NaN	NaN	NaN	14.85
w6	NaN	NaN	NaN	NaN	5.25
w7	NaN	NaN	NaN	NaN	-5.72
w8	NaN	NaN	NaN	NaN	-8.19
w9	NaN	NaN	NaN	NaN	2.32

In [3209]:
```
#figure 3, lam3
display(lambda3, mse3)
display_theta(theta_lam3)
```



	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.93	0.93	-0.19	-0.08
w1	NaN	-1.87	-1.83	11.78	8.66
w2	NaN	NaN	-0.04	-34.15	-15.65
w3	NaN	NaN	NaN	22.74	-11.19
w4	NaN	NaN	NaN	NaN	9.81
w5	NaN	NaN	NaN	NaN	14.48
w6	NaN	NaN	NaN	NaN	5.21
w7	NaN	NaN	NaN	NaN	-5.39
w8	NaN	NaN	NaN	NaN	-7.89
w9	NaN	NaN	NaN	NaN	1.99

In [3210]:
```
#figure 4, lam4
display(lambda4, mse4)
display_theta(theta_lam4)
```



	order 0	order 1	order 2	order 3	order 9
w0	0.0	0.93	0.93	-0.19	-0.08
w1	NaN	-1.87	-1.83	11.78	8.68
w2	NaN	NaN	-0.04	-34.14	-15.81
w3	NaN	NaN	NaN	22.74	-10.80
w4	NaN	NaN	NaN	NaN	9.66
w5	NaN	NaN	NaN	NaN	14.15
w6	NaN	NaN	NaN	NaN	5.18
w7	NaN	NaN	NaN	NaN	-5.09
w8	NaN	NaN	NaN	NaN	-7.62
w9	NaN	NaN	NaN	NaN	1.68

3.

In [3211]:
```
N_test = 199
X_test = X_train + 10/(2*(N_train-1))
X_test = np.delete(X_test,len(X_test)-1,axis=0)  #[0...198]
Y_test_ref = np.sin(2*math.pi*X_test)
```

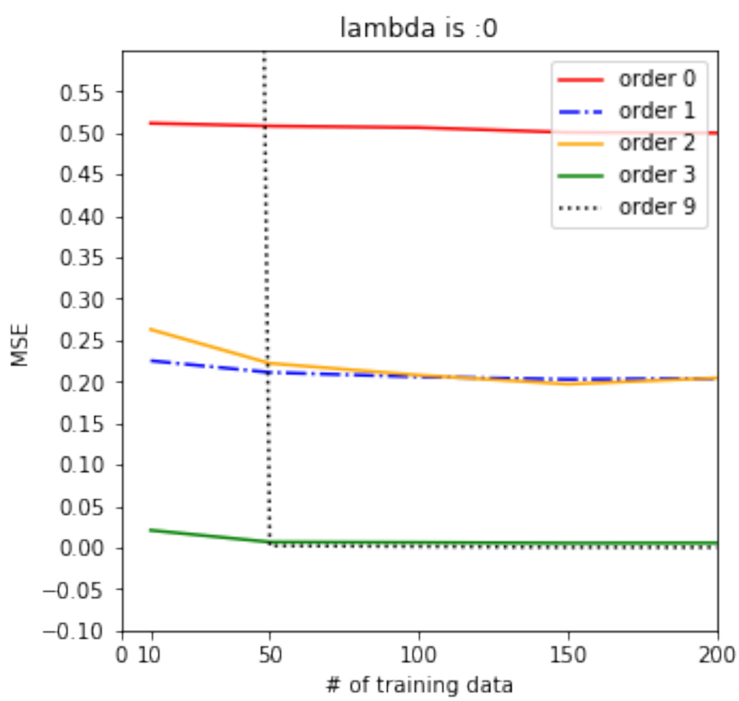
In [3212]:
```
# Generate a dataset D_set consisting of N = 200
X_test_buf = np.array([X_test])
Y_test_buf = np.array([Y_test_ref])
D_test_set_ref_buf = np.concatenate((X_test_buf,Y_test_buf),axis=0).T
D_test_ref = D_test_set_ref_buf.tolist()
```

In [3213]:
```
subD_test_comp_array = [subD_10, subD_50, subD_100, subD_150, subD_20
0, D_test_ref]
```

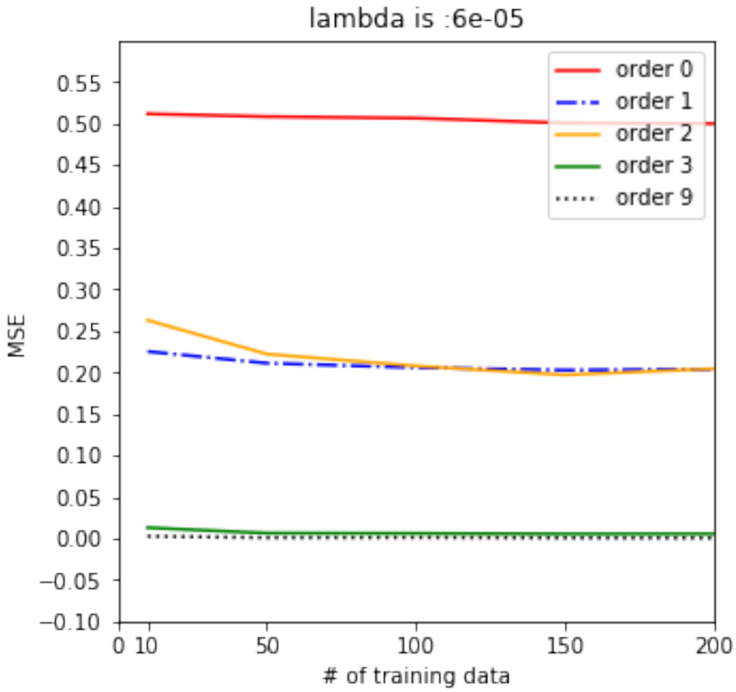
In [3214]:
```
theta_all_useless, mse_all_test = create_MSE_set(subD_test_comp_array
, lambda_array, order_array)
```

In [3215]:
```
mse0_test = mse_all_test[0:25] # lam0
mse1_test = mse_all_test[25:50] # lam1
mse2_test = mse_all_test[50:75] # lam2
mse3_test = mse_all_test[75:100] # lam3
mse4_test = mse_all_test[100:125] # lam4
```

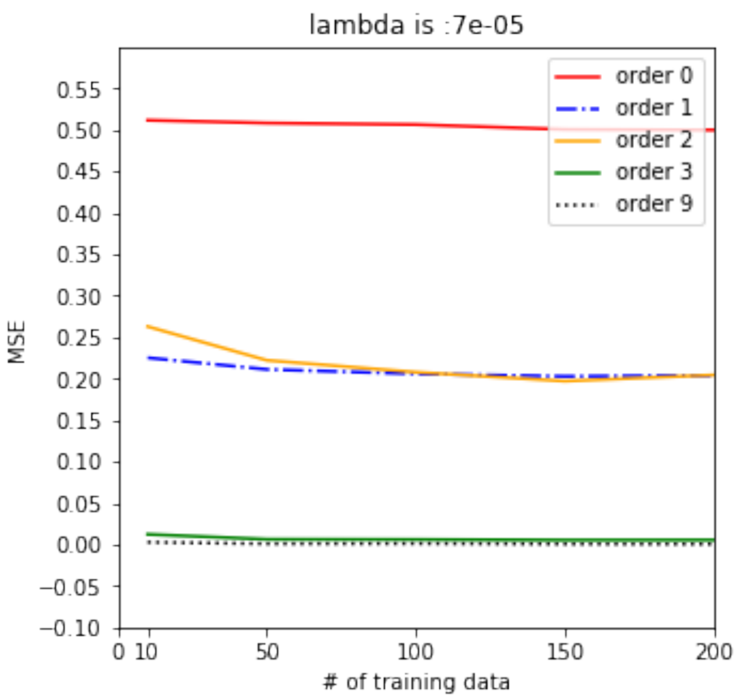
In [3216]:
```
display(lambda0, mse0_test)
```



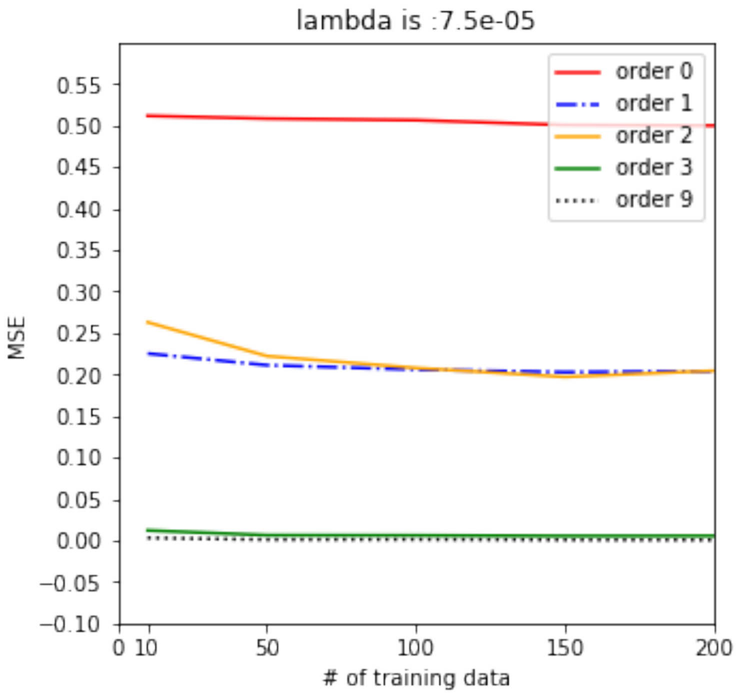
In [3217]:
```
display(lambda1, mse1_test)
```



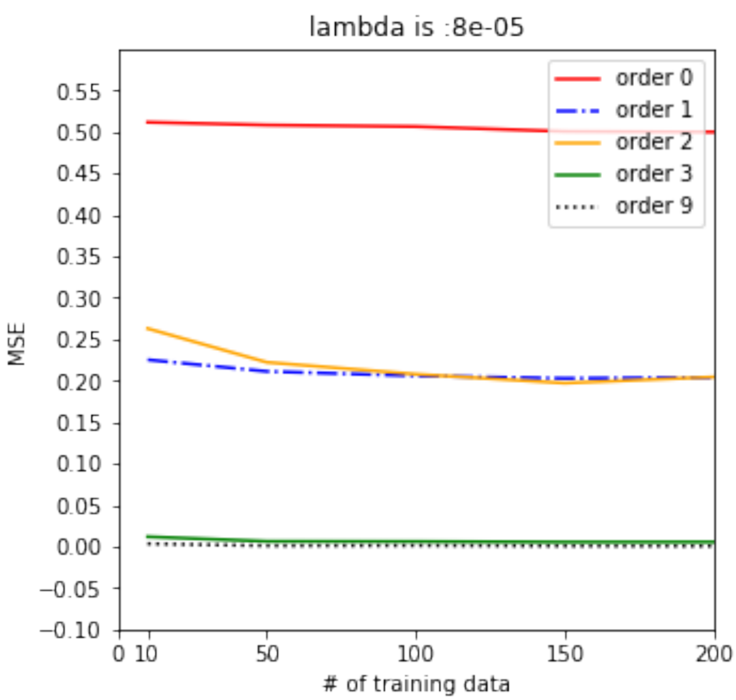
In [3218]:
```
display(lambda2, mse2_test)
```



In [3219]:
```
display(lambda3, mse3_test)
```



In [3220]:
```
display(lambda4, mse4_test)
```



4. conclustion

which one has the best performance and why?

A: In my case, the model with higher order(order 3 and 9) and enough training data set(larger than 100) has the best performance. Higher order are more flexible, it can make the model fit the original data samples well. But complex Models is always with High Variance. If the training data set was small, it is very easy to get over fitting, just like below. When I increase the training data set up to 200 samples, the performance is better.

What's more, appropriate lambda is good for performance. In my case, the model has the best performance when lambda is between 0.00007 to 0.00008. Because the lambda can modify the weight of regularization, which can penalize coefficient values. It is one way to control the over fitting, especially in the case when there is no enough training data. (for example: 10~50 training data in this question)

Which one has the worst performance and why?

A: The one with less training case and high order, no regularization(lambda == 0) has the worst performance. The reason just like the explaination in above question.

How about the balance of variance and bias?

A: variance and bias are trade-off performance, it is impossible to get best performance on both of them. In order to get better bias performance, some variance performance should be sacrificed: Smaller lambda should be used. Using high order model. In order to get better variance performance, some bias performance should be sacrificed: Larger lambda should be used. Using lower order model. Using large training data set.

Question 2.

preprocessing

In [3221]:
```
import pandas as pd
import time
df = pd.read_csv('ionosphere.data.txt',sep=',')
df.describe()
```

Out[3221]:

	1	1.1	1.2	1.3	1.4	1.5	1.6
count	351.000000	351.0	351.000000	351.000000	351.000000	351.000000	351.000000
mean	0.891738	0.0	0.641342	0.044372	0.601068	0.115889	0.550095
std	0.311155	0.0	0.497708	0.441435	0.519862	0.460810	0.492654
min	0.000000	0.0	-1.000000	-1.000000	-1.000000	-1.000000	-1.000000
25%	1.000000	0.0	0.472135	-0.064735	0.412660	-0.024795	0.211310
50%	1.000000	0.0	0.871110	0.016310	0.809200	0.022800	0.728730
75%	1.000000	0.0	1.000000	0.194185	1.000000	0.334655	0.969240
max	1.000000	0.0	1.000000	1.000000	1.000000	1.000000	1.000000

8 rows × 34 columns

In [3222]:
```
def shuffle_data(df_in):
    arr = np.array(df_in)
    np.place(arr, arr=='g', 1)
    np.place(arr, arr=='b', 0)
    np.random.shuffle(arr)
    train = arr[0:301]
    test = arr[301:351]

    train_X = np.ones([1,301]).T
    train_X = np.concatenate((train_X,train[:,0:34]),axis=1) # 0~34 f
eature (0 + 1~34 features)
    #train_X = train[:,0:34]
    train_Y = train[:,34] # no. 34 feature is output

    test_X = np.ones([1,50]).T
    test_X = np.concatenate((test_X,test[:,0:34]),axis=1) # 0~34 feat
ure (0 + 1~34 features)
    #test_X = test[:,0:34]
    test_Y = test[:,34]
    return train_X, train_Y, test_X, test_Y
```

choose logistic regression as the classification model

In [3223]:
```
def sigmoid(z):
    return 1.0 / (1 + np.exp(-z))
```

below block is only for debugging

In [3224]:
```
alpha = 0.1
iteration = 1
m, n = np.shape(test_set_X) # m is # of test points, n is # of featur
es
new_alpha = alpha/m
theta_vector = np.ones([1,n]) # [1 * n]
theta_vector_temp = np.zeros([1,n])
for i in range(iteration):
    temp0 = np.dot(theta_vector, test_set_X.T) # [1 * n] * [n * m] ->
[1 * m]
    temp_buf = np.array(temp0.tolist())
    #print(temp_buf)
    temp1 = sigmoid(temp_buf) # [1 * m]
    #print(temp1)
    temp2 = temp1 - test_set_Y.T # [1 * m]
    #print(temp2)
    for j in range(0,n):
        temp3 = np.multiply(temp2, test_set_X[:,j]) # [1 * m]
        theta_vector_temp[0][j] = theta_vector[0][j] - new_alpha * np
.sum(temp3) # 1
        #print(theta_vector_temp)
    theta_vector = theta_vector_temp
    #print(i)
    #print(theta_vector)
```

above block is only for debugging

In [3225]:
```
def checkJ(train_x, train_y, theta_vector):
    m, n = np.shape(train_x)
    temp0 = np.dot(theta_vector, train_x.T) # [1*n] * [n*m]
    temp_buf = np.array(temp0.tolist()) # [1*m]
    temp1 = sigmoid(temp_buf) # [1*m]
    temp2 = np.multiply(train_y, np.log(temp1+0.0000000000000000000000
000001))
    temp3 = np.multiply((1-train_y), np.log(1-temp1+0.000000000000000
000000000001))
    J_value = -1/m * np.sum(temp2 + temp3)
    return J_value
```

In [3226]:
```
def LR_Test(theta_set, test_x, test_y):
    m, n = np.shape(test_x) # m is # of points, n is # of features
    matchCount = 0
    for i in range(m):
        test = np.array(np.dot(theta_set, test_x[i].T).tolist())
        predict = test[0] > 0
        if predict == bool(test_y[i]):
            matchCount += 1
        accuracy = float(matchCount) / m
    return accuracy
```

In [3227]:

```
def LR_GD(train_x, train_y, test_x, test_y, alpha, iteration): # train_x: m * n; train_y: m * 1
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of test points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)
    for i in range(iteration):
        temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n * m] -> [1 * m]
        temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [1 * m]
        temp2 = np.multiply(temp1, train_x.T) # [1 * m]
        theta_vector = theta_vector - new_alpha * np.sum(temp2, axis=1)

        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x, train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, test_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

In [3228]:
```
def LR_SGD(train_x, train_y, test_x, test_y, alpha, iteration):
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)
    for i in range(iteration):
        for j in range(0,n):
            temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n *
m] -> [1 * m]
            temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [
1 * m]
            temp2 = np.multiply(temp1, train_x[:,j]) # [1 * m]
            theta_vector[0][j] = theta_vector[0][j] - new_alpha * np.
sum(temp2) # 1

        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x,
train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, t
est_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

In [3229]:
```
def display_cost(cost_set, cost_min):
    plt.figure(num=0, figsize=(10, 5))
    plt.title('J(theta) vs Iteration')
    plt.plot(cost_set,color='green')
    plt.xlabel('# of Iteration')
    plt.ylabel('J(theta)')
    plt.show()
    print(cost_min)
```

In [3230]:
```
def LR_GD_REG(train_x, train_y, test_x, test_y, alpha, iteration, lambd): # train_x: m * n; train_y: m * 1
    startTime = time.time()
    train_accuracy_set = []
    test_accuracy_set = []
    J_min = 50000 # default max cost value
    m, n = np.shape(train_x) # m is # of test points, n is # of features
    new_alpha = alpha/m
    theta_vector = np.full((1,n),1) # [1 * n]
    best_theta_vector = np.full((1,n),1)

    train_x_one = train_x[:,0]
    train_x_rest = train_x[:,1:n]
    for i in range(iteration):
        theta_vector_one = theta_vector[0][0:1]
        theta_vector_rest = theta_vector[0][1:n]
        ## only this part is different
        temp0 = np.dot(theta_vector, train_x.T) # [1 * n] * [n * m] -> [1 * m]
        temp1 = sigmoid(np.array(temp0.tolist())) - train_y.T # [1 * m]
        temp2 = np.multiply(temp1, train_x_rest.T) # [n * m]
        theta_vector_rest = theta_vector_rest - new_alpha * (np.sum(temp2, axis=1) - lambd*theta_vector_rest)
        ## in np.sum(X, axis=?), no matter axis is 1 or 0, the result should be one row.
        ##
        theta_vector = np.concatenate((np.array([theta_vector_one]), np.array([theta_vector_rest])),axis=1)
        J = checkJ(train_x, train_y, theta_vector)
        if(J < J_min):
            best_theta_vector = theta_vector
            J_min = J
        train_accuracy_set.append(LR_Test(best_theta_vector, train_x, train_y))
        test_accuracy_set.append(LR_Test(best_theta_vector, test_x, test_y))

    print('time >> %fs' %(time.time() - startTime))
    return train_accuracy_set, test_accuracy_set
```

note about below steps:

Line 1: shuffle data, split data into training and testing part

Line 2: LR_GD with runing time

Line 3: plot and show best theta set

Line 4: using the best thera set to test

In [3231]:
```
#train_set_X, train_set_Y, test_set_X, test_set_Y = shuffle_data(df)
#cost_min, cost_set, best_theta = LR_GD(test_set_X, test_set_Y, 3, 20
)
#display_cost(cost_set, cost_min)
#print(LR_Test(best_theta, test_set_X, test_set_Y))

#print('==========================')

#cost_min, cost_set, best_theta = LR_GD_REG(test_set_X, test_set_Y, 3
, 20, 0)
#display_cost(cost_set, cost_min)
#print(LR_Test(best_theta, test_set_X, test_set_Y))
```

In [3232]:
```
#train_set_X, train_set_Y, test_set_X, test_set_Y = shuffle_data(df)
#cost_min, cost_set, best_theta = LR_SGD(test_set_X, test_set_Y, 0.01
, 5)
#display_cost(cost_set, cost_min)
#LR_Test(best_theta, test_set_X, test_set_Y)
```

Below part is tring different learning rates

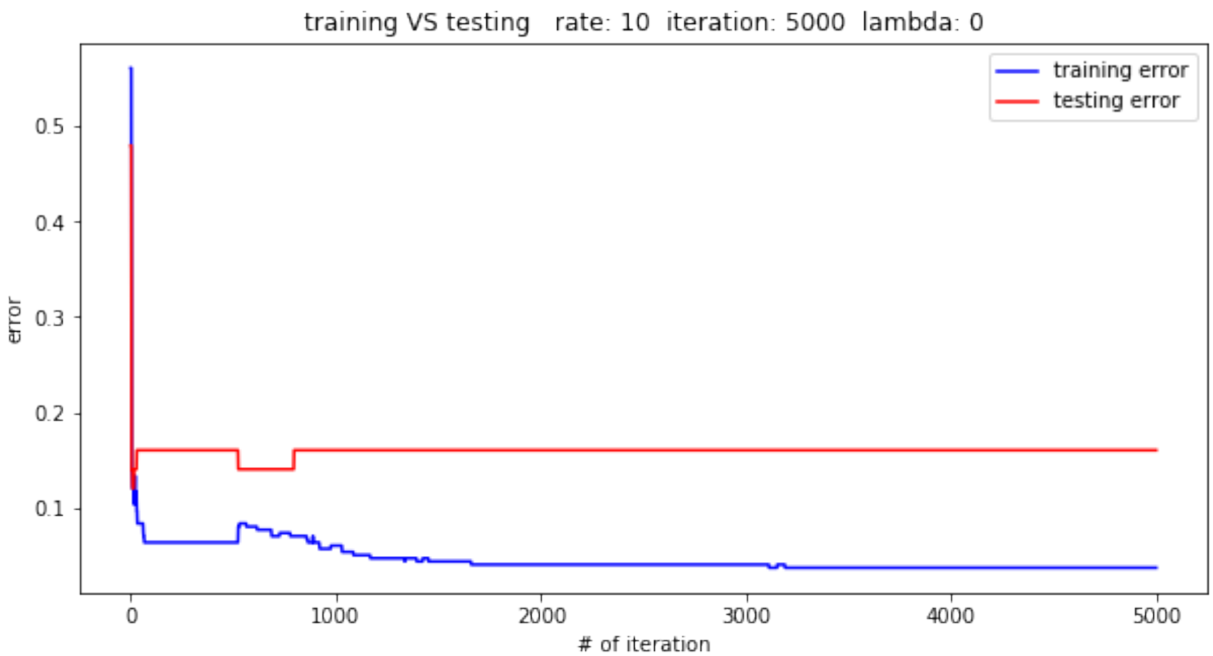
Below part is tring different different weights for the regularization term.

Plot the training and testing errors with respect of iterations.

In [3233]:
```
def display_error(train_acc_set, test_acc_set, rate, iteration, lambd
):
    train_error_set = 1 - np.array(train_acc_set)
    test_error_set = 1 - np.array(test_acc_set)
    name = 'training VS testing   rate: ' + str(rate) + '  iteration:
' + str(iteration) + '  lambda: '+ str(lambd)
    #create figure
    plt.figure(figsize=(10, 5))
    #title
    plt.title(name)
    #plot 1
    plt.plot(train_error_set, color='blue', label='training error')
    #plot 2
    plt.plot(test_error_set, color='red', label='testing error')
    #set aixs name
    plt.xlabel('# of iteration')
    plt.ylabel('error')
    #legned
    plt.legend()
    #display figure
    plt.show()
```

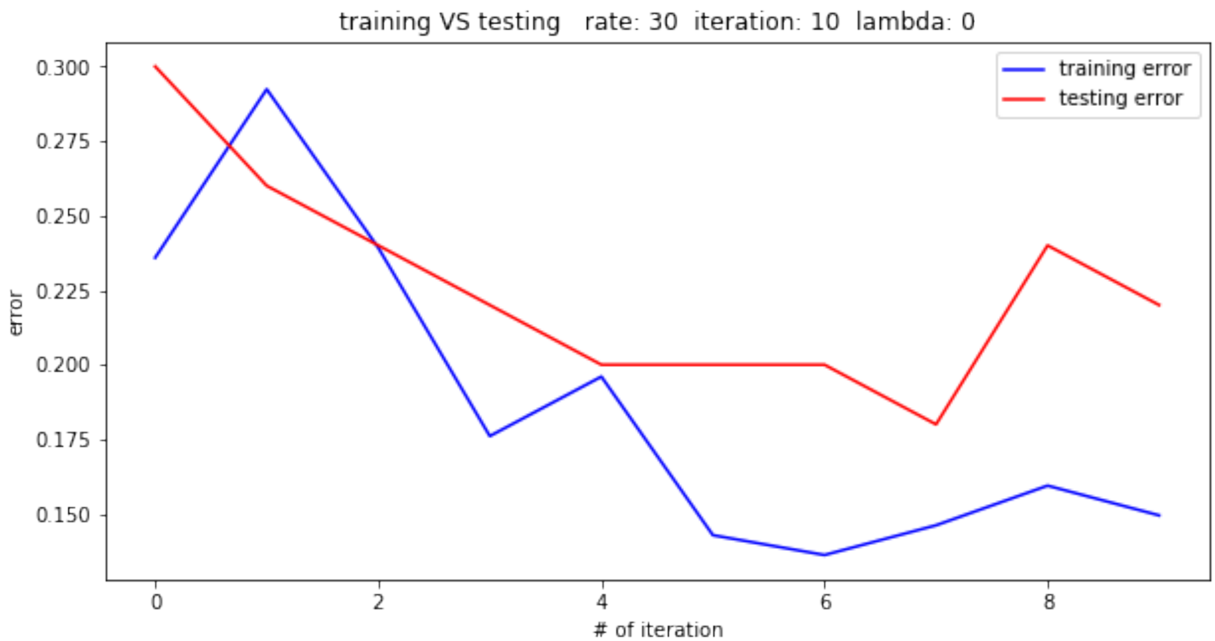
In [3234]:
```
lambd = 0
rate = 10
iteration = 5000
train_acc_set, test_acc_set = LR_GD(train_set_X, train_set_Y, test_se
t_X, test_set_Y, rate, iteration)
display_error(train_acc_set, test_acc_set, rate, iteration, lambd)
```

```
time >> 26.615890s
```



In [3235]:
```
lambd = 0
rate = 30
iteration = 10
train_acc_set, test_acc_set = LR_SGD(train_set_X, train_set_Y, test_s
et_X, test_set_Y, rate, iteration)
display_error(train_acc_set, test_acc_set, rate, iteration, lambd)
```

```
time >> 0.263014s
```



In [3236]:
```
lambd = 10
rate = 0.2
iteration = 5000
train_acc_set, test_acc_set = LR_GD_REG(train_set_X, train_set_Y, tes
t_set_X, test_set_Y, rate, iteration, lambd)
display_error(train_acc_set, test_acc_set, rate, iteration, lambd)
```

```
time >> 22.344032s
```