

1. Cook's membrane

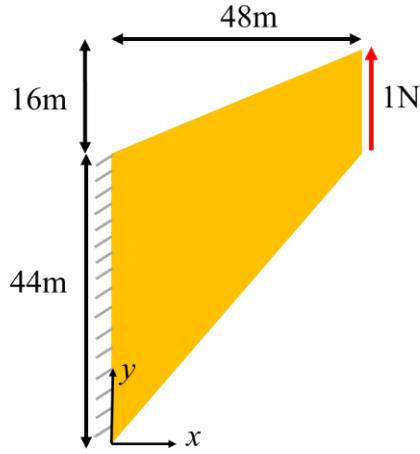


Figure 1: Cook Membrane: Geometric Configuration and Loading Conditions

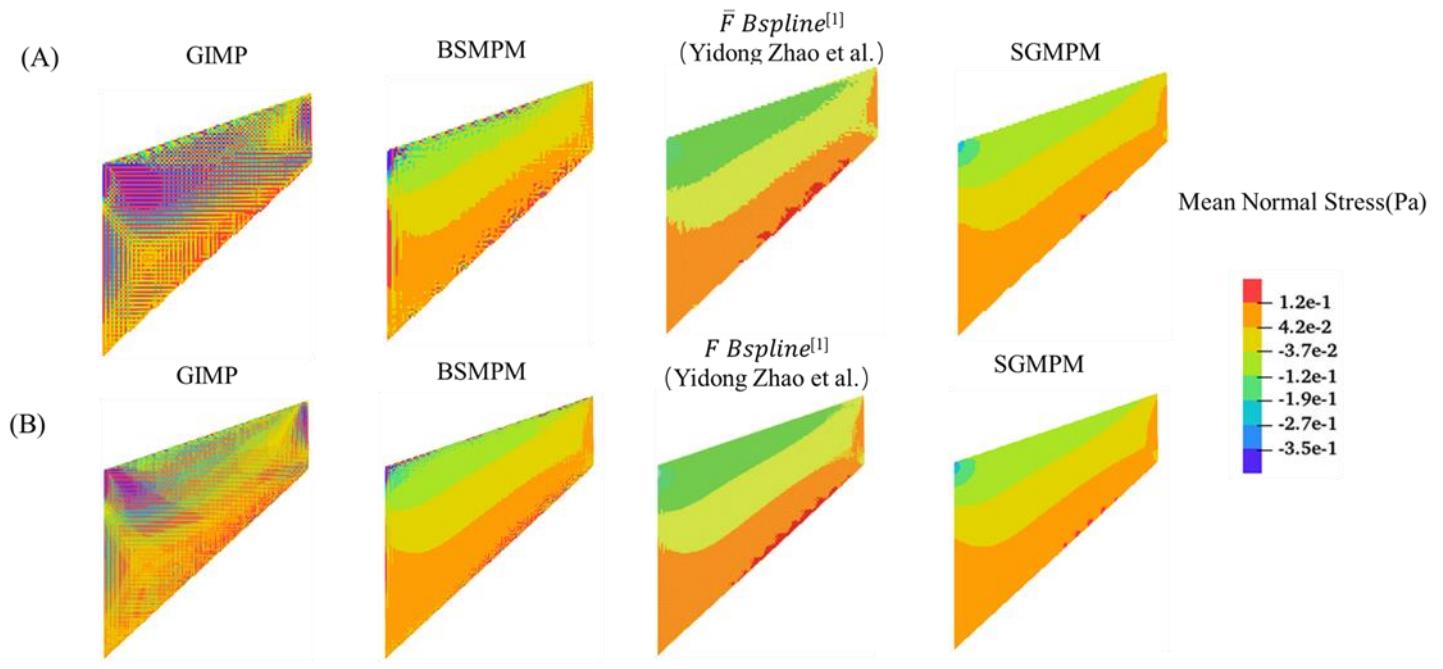
The problem is modeled as a plane strain problem, with relevant parameters referenced from[1]. The solution is computed using the MPM3D Fortran program. Since this problem is not a standard dynamic issue but rather a quasi-static one, the quasi-static loading and damping modules in the program must be activated via the keywords ‘QuLo’ and ‘Damp’. Running the input files in ‘VolumetricLockingEx\ cook_membrane’ yields the following displacement field results:

Method	Particle number	Tip vertical displacement(m)	Reference solution (m)	CPU time consumption(s)
MPM	5776	0.2489		39.84
GIMP	5776	0.2512		128.30
BSMPM	5776	0.2620		90.63
SGMPM	5776	0.2678	0.275	40.91
MPM	23072	0.2692		322.64
GIMP	23072	0.2688		1226.97
BSMPM	23072	0.2711		819.25
SGMPM	23072	0.2739		413.61

Table 1: Comparison of Solutions and CPU Time for Different Methods in Calculating Cook Membrane Problems

The calculation process of this simulation is carried out on an Intel (R) Xeon (R) Gold 6226R CPU @ 2.90GHz.

For the mean normal stress field, using the keyword ‘smoo’ to initiate the stress smoothing process based on grid node mapping in the program can enable SGMPM to achieve the best normal stress effect. Even if other methods use such smoothing processing, the effect is not as good as SGMPM. Running the various input files in ‘VolumetricLockingEx\cook_membrane’ yields the following pressure field results:



Cook's membrane: Mean normal stress fields in the standard GIMP、BSMPM 、 $\bar{F} \text{ Bspline}$ 、 SGMPM solutions
 (A) Coarse discretization (5776 material points). (B) Fine discretization (23,072 material points).

Figure 2: Comparison of pressure field results of Cook membrane problem calculated by different methods

2. Nearly incompressible free soft beam problem

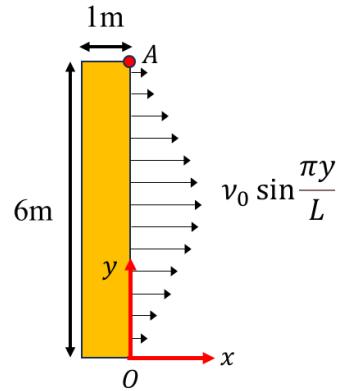


Figure 3: Nearly incompressible free soft beam problem calculation model: free soft beam

This example file is located in ‘VolumetricLockingEx\FreeSoftBeam’. The displacement field results of the example are compared as follows:

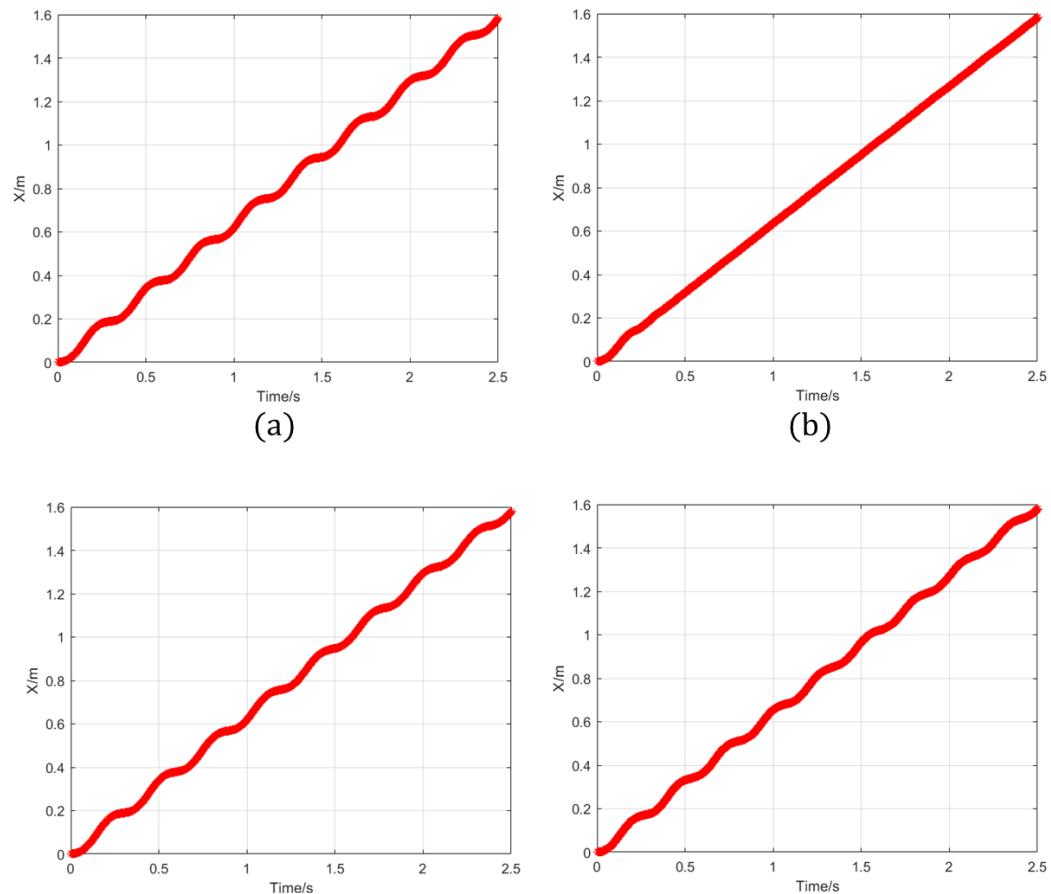


Figure 4: Two dimensional free soft beam problem: horizontal displacement time image of point A
 (a) $v=0.3$, MPM (b) $v=0.499$, MPM (c) $v=0.3$, SGMPM (d) $v=0.499$, SGMPM

The results of the kinetic energy field are shown in the following figure:

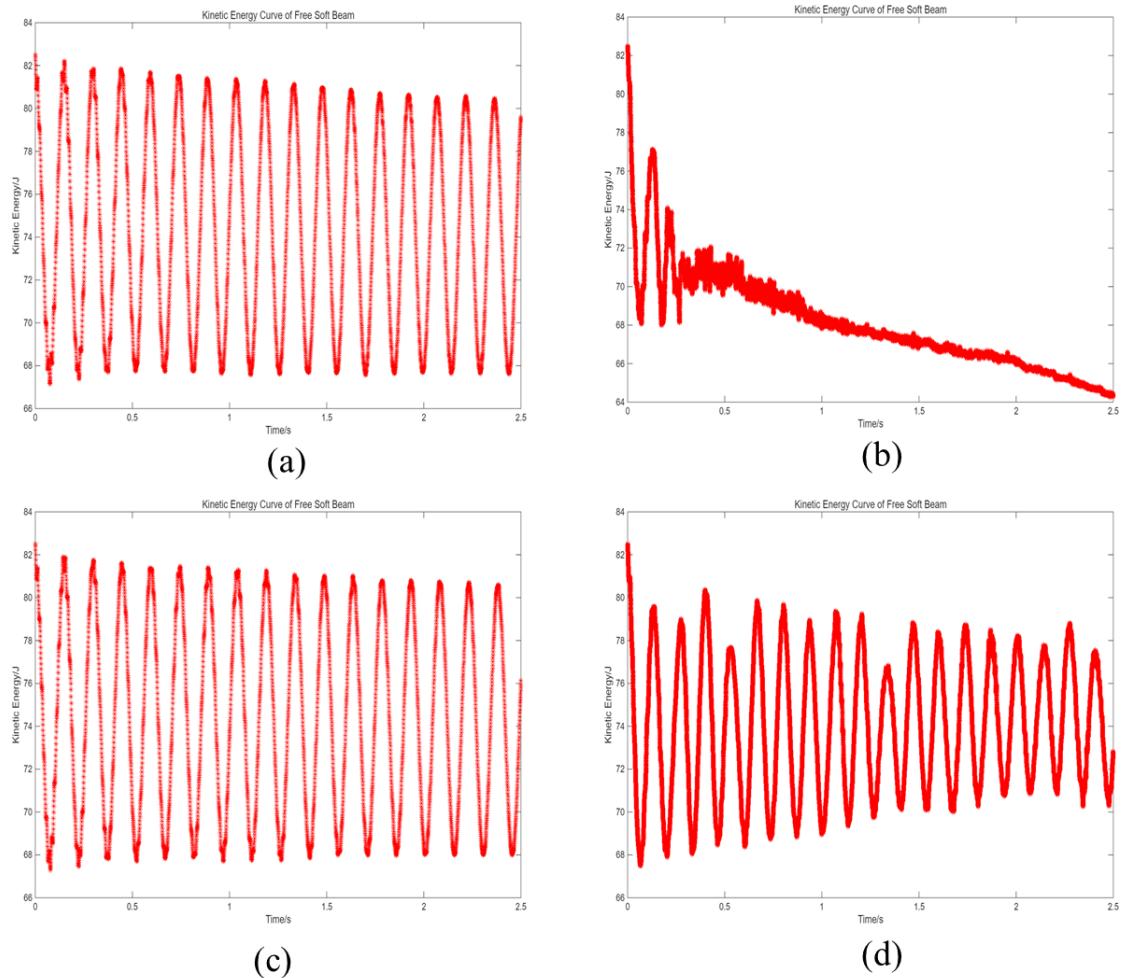


Figure 5: Two dimensional free soft beam problem: system kinetic energy time image

(a) $\nu=0.3$, MPM(b) $\nu=0.499$, MPM(c) $\nu=0.3$, SGMPM(d) $\nu=0.499$, SGMPM

3. Nearly incompressible fixed soft beam problem

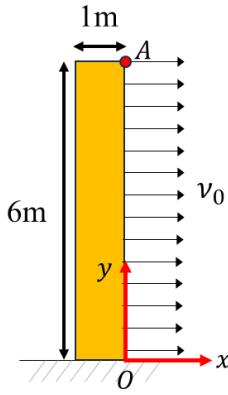


Figure 6: fixed soft beam model

Relevant parameters are referenced from [2]. This example file is located in ‘VolumetLockingEx \ SoftBeam’. To ensure energy conservation during the calculation process as much as possible, the TLMPM format is introduced, and the keyword TLmp is written in the input file to activate the TLMPM format for solving.

The following only provides the comparison results of the pressure field, and the displacement field results can be compared with relevant literature.

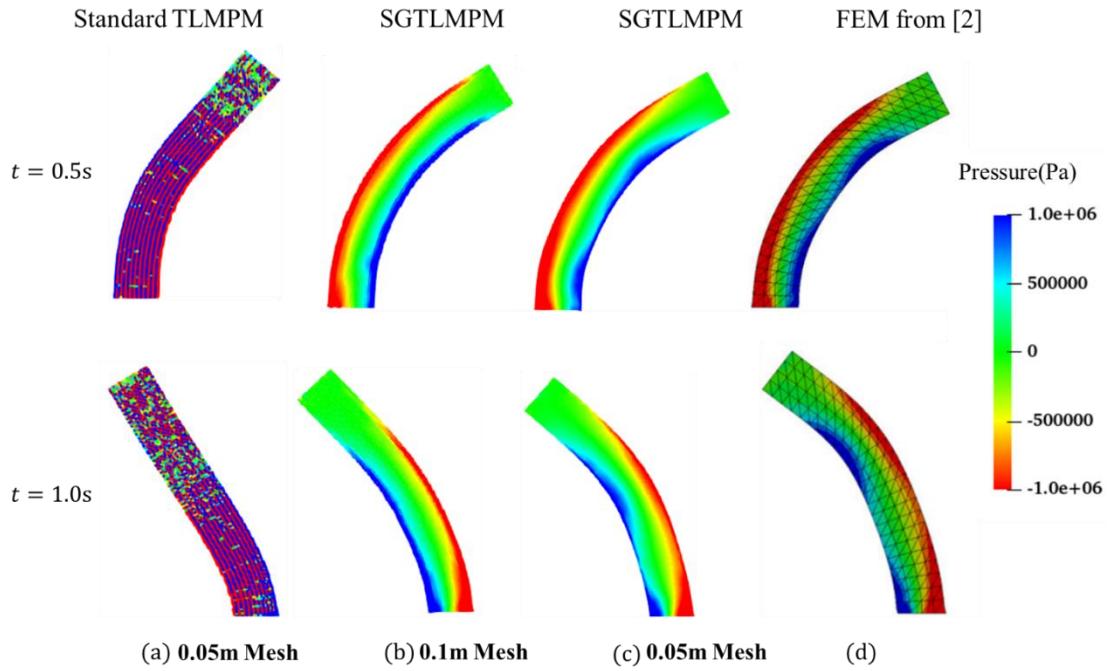


Figure 7: Comparison of Fixed Soft Beam Pressure Field

4. Dam Break

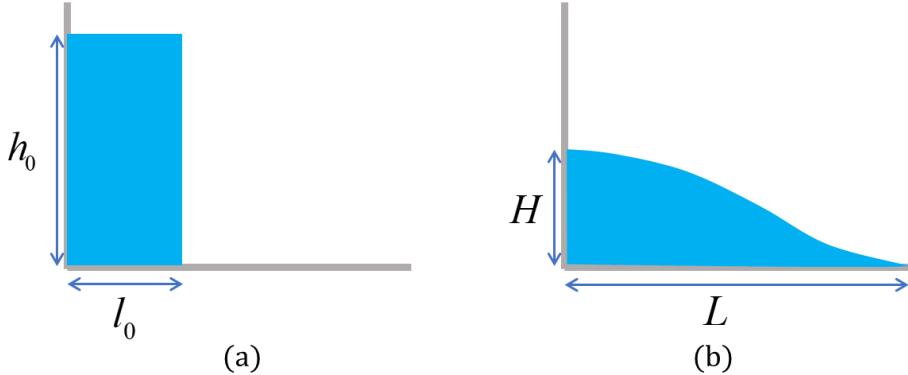


Figure 8: Model for dam break

The parameters of this example refer to reference [3]. The constitutive equation of water in this problem adopts Newtonian fluid constitutive theory and introduces artificial EOS to calculate pressure. The example file is located in ‘VolumetricLockingEx \ DamBreak’. By running the example files in SGMPM and MPM respectively, it can be compared and found that SGMPM has the function of eliminating volumetric locking. Quantitative comparison of displacement fields can refer to relevant literature. Note that the 0.68s runtime of the program corresponds to $T=3.0$ in the figure (converted using the dimensionless formula in the literature). At this time, the pressure of SGMPM still oscillates, possibly due to the excessive numerical sound velocity in the weakly compressible artificial EOS.

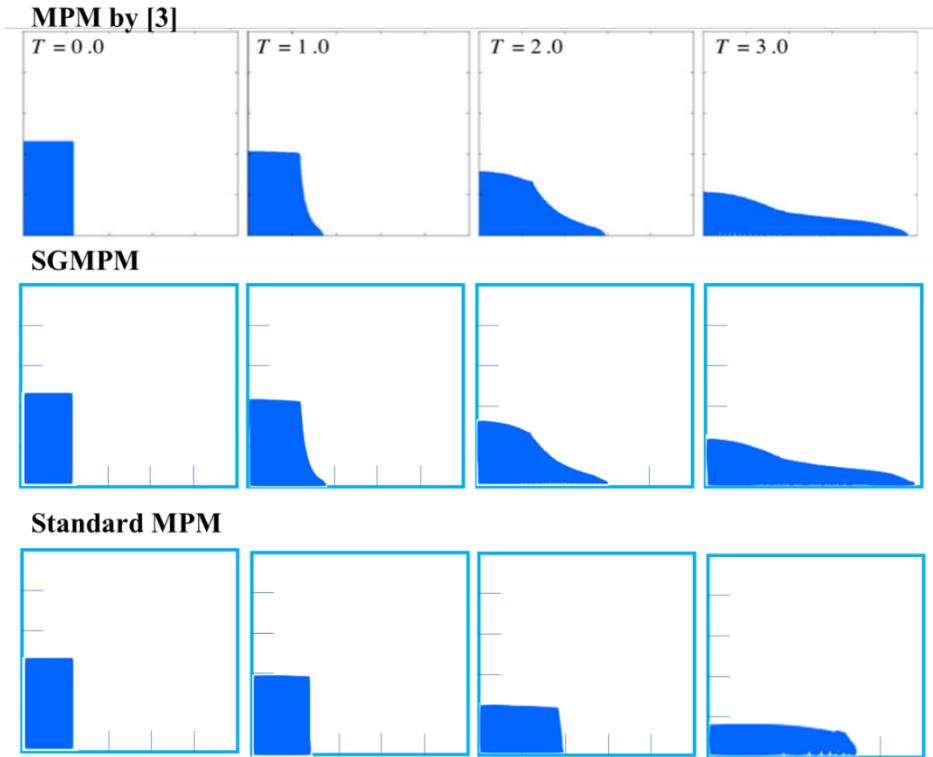


Figure 9: Results of dam break displacement field calculated by different methods

Run the example files from ‘VolumetricLockingEx\DamBreak\ SGMPMvsPressure’, using the bulk modulus and related dimensional parameters from reference [4]. The pressure results can be compared with reference [4], demonstrating that SGMPM can obtain smooth pressure in fluid calculations (note that time should also be treated with dimensionless parameters).

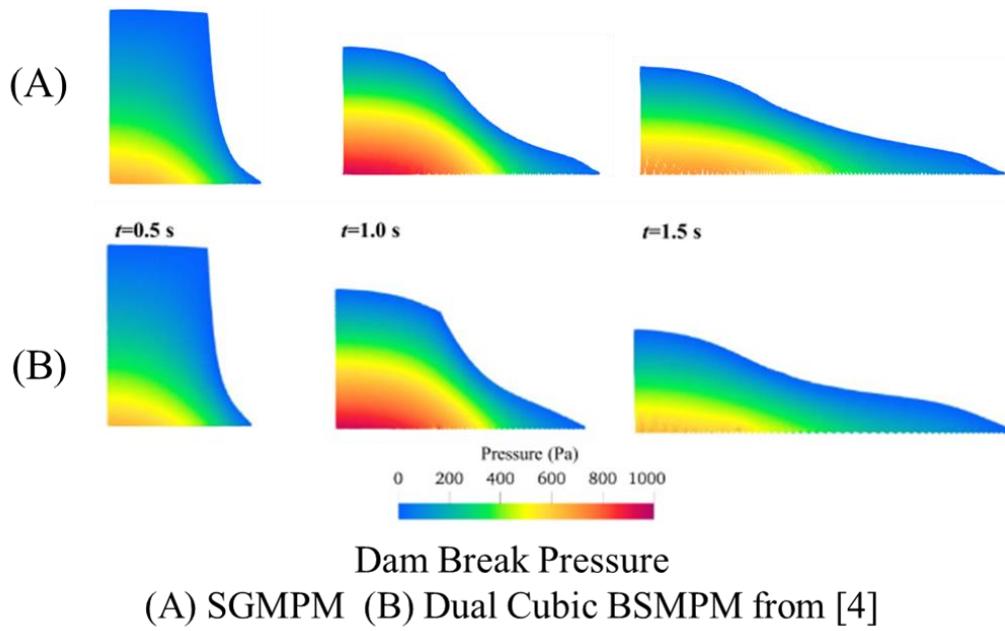


Figure 10: Comparison of pressure field for dam break problem

For more research details and theoretical achievements, please pay attention to the papers that the research group will publish in the future.

References

- [1] Zhao Y, Jiang C, Choo J. Circumventing volumetric locking in explicit material point methods: A simple, efficient, and general approach[J]. International Journal for Numerical Methods in Engineering, 2023, 124(23): 5334-5355
- [2] Kadapa, Chennakesava. Novel quadratic Bézier triangular and tetrahedral elements using existing mesh generators: Applications to linear nearly incompressible elastostatics and implicit and explicit elastodynamics[J]. International Journal for Numerical Methods in Engineering, 2019, 117(5): 543-573.
- [3] Mast C M, Mackenzie-Helnwein P, Arduino P, et al. Mitigating kinematic locking in the material point method[J]. Journal of Computational Physics, 2012, 231(16): 5351-5373.
- [4] Cheng Z, Zhao S, Chen H, et al. Stabilized explicit material point method for fluid flow and fluid-structure interaction simulations using dual high-order B-spline volume averaging[J]. Computer Methods in Applied Mechanics and Engineering, 2026, 448: 118428.