Name: _____ Period: _____

Review Worksheet 4.2-4.3

1. Determine whether the function $f(x) = \frac{x}{x+1}$ is one-to-one by examining the sign of $f'(x)$.

2. Find a formula for $f^{-1}(x)$ if $f(x) = \frac{5}{x^2+1}$ given that $x \geq 0$.

3. Find the derivative of $f^{-1}(x)$:
   a. $f(x) = 3 - x^2$ b. $f(x) = 5x - \sin(2x), -\frac{\pi}{4} < x < \frac{\pi}{4}$ c. $f(x) = -\frac{2}{x^3}$

4. Find $\frac{dy}{dx}$.
   a. $y = \ln(\ln x)$ b. $y = \ln(2 + \sqrt{x})$ c. $y = sin^2(\ln x)$

   d. $y = \ln(x \tan y)$ e. $y = \sin(x^{\ln x})$

5. Find $\frac{dy}{dx}$.
   a. $y = e^{\sqrt{1+5x^3}}$ b. $y = e^{e^{x^2}}$

   c. $y = \pi^{\cos x}$ d. $y = \frac{sin^3 x \cos x}{\sqrt[5]{x}}$

6. Find $\frac{d}{dx} log_{\ln x} e$

Find the derivative of the given function:

7. $f(x) = 8^x - log_6 x$	8. $f(x) = log_4 x + 16^x$
9. $f(x) = 4e^x - 4^x$	10. $f(x) = 6 \ln x$

Find the derivative of the function at the given point:

11. $f(x) = \ln (x^2 + 3x + 1)$ at $x = 1$	12. $f(x) = 2e^x - x$ at $x = 1$
13. $f(x) = x^3 - 5^x$ at $x = 2$	14. $\frac{d}{dx} e^{3x^2}\Big\|_{x=2}$
15. $\frac{d}{dx} 6 \cdot 5^x + \log x\Big\|_{x=2}$	16. $\frac{d}{dx} 10 \cdot e^x + 7x\Big\|_{x=0}$

Find the derivative of the given function.

17. $f(x) = e^{x^3+2x}$	18. $f(x) = \frac{e^{2x}}{x}$
19. $f(x) = \frac{(e^x)^4}{x^2}$	20. $f(x) = x^2 \ln(x^2 + 3x)$

21. Find the equation of the normal line to $y = 2e^x$ at $x = \ln 3$.

22.

(1999-AB4) Suppose that the function *f* has a continuous second derivative for all *x*, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let *g* be a function whose derivative is given by: $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all *x*.

a. Write an equation of the line tangent to the graph of *f* at the point where $x = 0$.
b. Given that $g(0) = 4$, write an equation of the line tangent to the graph of *g* at the point where $x = 0$.
c. Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$

Answers:

1. $f(x)$ is 1-1 2. $y = \sqrt{\frac{5}{x} - 1}; y \geq 0$

3. a. $\frac{dy}{dx} = -\frac{1}{2y}$ b. $\frac{dy}{dx} = \frac{1}{5-2\cos y}; -\frac{\pi}{4} < y < \frac{\pi}{4}$ c. $\frac{dy}{dx} = \frac{1}{6}y^4$

4. a. $\frac{dy}{dx} = \frac{1}{x\ln x}$ b. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}(2+\sqrt{x})}$ c. $\frac{dy}{dx} = \frac{2\sin(\ln x)\cos(\ln x)}{x}$
   d. $\frac{dy}{dx} = \frac{\tan y}{x(\tan y - sec^2 y)}$ or $\frac{dy}{dx} = \frac{1}{x(1-\sec y \csc y)}$
   e. $\frac{dy}{dx} = (\sin x)^{\ln x}\left(\frac{\ln \sin x}{x} + \ln x \cot x\right)$ f. $\frac{dy}{dx} = \frac{2\sin(\ln x)\cos(\ln x)}{x}$

5. a. $\frac{dy}{dx} = \frac{15x^2 e^{\sqrt{1+5x^3}}}{2\sqrt{1+5x^3}}$ b. $\frac{dy}{dx} = 2xe^{e^{x^2}}e^{x^2}$
   c. $\frac{dy}{dx} = -\pi^{\cos x}\sin x \ln \pi$ d. $\frac{dy}{dx} = \frac{sin^3 x \cos x}{\sqrt[5]{x}}\left(3\cot x - \tan x - \frac{1}{5x}\right)$

6. $\frac{dy}{dx} = -\frac{1}{x ln^2(\ln x)\ln x}$

7. $f'(x) = 8^x \ln 8 - \frac{1}{x ln\, 6}$ 8. $f'(x) = \frac{1}{x ln\, 4} + 16^x \ln 16$

9. $f'(x) = 4e^x - 4^x \ln 4$ 10. $f'(x) = \frac{6}{x}$

11. $f'(1) = 1$ 12. $f'(1) = 2e - 1$

13. $f'(2) = 12 - 25\ln 5$ 14. $12e^{12}$

15. $150\ln 5 + \frac{1}{2\ln 10}$ 16. 17

17. $(3x^2 + 2)e^{x^3+2x}$ 18. $\frac{e^{2x}(2x-1)}{x^2}$

19. $\frac{2e^{4x}(2x-1)}{x^3}$ 20. $x\left(2\ln(x^2+3x) + \frac{2x+3}{x+3}\right)$

21. $y - 6 = -\frac{1}{6}(x - \ln 3)$

22. a. $y = -3x + 2$ b. $y = 4$