BC CALCULUS PRACTICE 6.6B/7.6 **Name:** ______________________________ **Period** ________

Aisle: __________

6.6: pg. 406 – 51, 55, 59, 61, 64, 65, 71, 75

7.6: pg. 480 – 9,13, 17, 21, 24

pg. 406

51. (a) Let f be an odd function; that is, $f(-x) = -f(x)$. Invent a theorem that makes a statement about the value of an integral of the form

$$\int_{-a}^{a} f(x)\,dx$$

(b) Confirm that your theorem works for the integrals

$$\int_{-1}^{1} x^3\,dx \quad \text{and} \quad \int_{-\pi/2}^{\pi/2} \sin x\,dx$$

(c) Let f be an even function; that is, $f(-x) = f(x)$. Invent a theorem that makes a statement about the relationship between the integrals

$$\int_{-a}^{a} f(x)\,dx \quad \text{and} \quad \int_{0}^{a} f(x)\,dx$$

(d) Confirm that your theorem works for the integrals

$$\int_{-1}^{1} x^2\,dx \quad \text{and} \quad \int_{-\pi/2}^{\pi/2} \cos x\,dx$$

55. Use Part 2 of the FTC to find the derivatives.

(a) $\dfrac{d}{dx}\displaystyle\int_1^x \sin(\sqrt{t}\,)\,dt$ (b) $\dfrac{d}{dx}\displaystyle\int_1^x \sqrt{1+\cos^2 t}\,dt$

59. Let $F(x) = \int_4^x \sqrt{t^2+9}\,dt$. Find

(a) $F(4)$ (b) $F'(4)$ (c) $F''(4)$

61.

Let $F(x) = \displaystyle\int_0^x \frac{t-3}{t^2+7}\,dt$ for $-\infty < x < +\infty$.

(a) Find the value of x where F attains its minimum value.

(b) Find intervals over which F is only increasing or only decreasing.

(c) Find open intervals over which F is only concave up or only concave down.

64.

(a) Over what open interval does the formula

$$F(x) = \int_1^x \frac{1}{t^2-9}\,dt$$

represent an antiderivative of

$$f(x) = \frac{1}{x^2-9}?$$

(b) Find a point where the graph of F crosses the x-axis.

65. Find all values of x^* in the stated interval that satisfy Equation (8) in the Mean Value Theorem for Integrals, and explain what these numbers represent.

(a) $f(x) = \sqrt{x}$; $[0, 9]$

(b) $f(x) = 3x^2 + 2x + 1$; $[-1, 2]$

71. (a) If $h'(t)$ is the rate of change of a child's height measured in inches per year, what does the integral $\int_0^{10} h'(t)\,dt$ represent, and what are its units?

(b) If $r'(t)$ is the rate of change of the radius of a spherical balloon measured in centimeters per second, what does the integral $\int_1^2 r'(t)\,dt$ represent, and what are its units?

(c) If $H(t)$ is the rate of change of the speed of sound with respect to temperature measured in ft/s per °F, what does the integral $\int_{32}^{100} H(t)\,dt$ represent, and what are its units?

(d) If $v(t)$ is the velocity of a particle in rectilinear motion, measured in cm/h, what does the integral $\int_{t_1}^{t_2} v(t)\,dt$ represent, and what are its units?

75–76 Evaluate each limit by interpreting it as a Riemann sum in which the given interval is divided into n subintervals of equal width.

75. $\lim_{n\to+\infty} \sum_{k=1}^{n} \frac{\pi}{4n} \sec^2\left(\frac{\pi k}{4n}\right)$; $\left[0, \frac{\pi}{4}\right]$

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Find the average value of the function over the given interval.

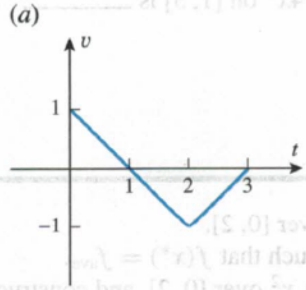
9.

$f(x) = 3x;\ [1, 3]$

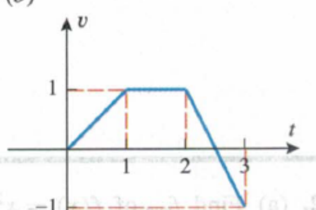
13. $f(x) = e^{-2x}$; [0,4]

17. In each part, the velocity versus time curve is given for a particle moving along a line. Use the curve to find the average velocity of the particle over the time interval $0 \leq t \leq 3$.

(*a*)



(*b*)



21.

(a) Suppose that the velocity function of a particle moving along a coordinate line is $v(t) = 3t^3 + 2$. Find the average velocity of the particle over the time interval $1 \leq t \leq 4$ by integrating.

(b) Suppose that the position function of a particle moving along a coordinate line is $s(t) = 6t^2 + t$. Find the average velocity of the particle over the time interval $1 \leq t \leq 4$ algebraically.

24.

(a) The temperature of a 10-m-long metal bar is 15°C at one end and 30°C at the other end. Assuming that the temperature increases linearly from the cooler end to the hotter end, what is the average temperature of the bar?

(b) Explain why there must be a point on the bar where the temperature is the same as the average, and find it.