AP Calculus BC – HW #________ Name: ______________________________ Period _______

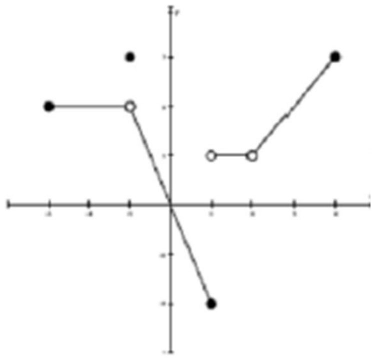
Limit and Continuity Review Worksheet (NO CALCULATOR)

Compute these limits algebraically. Show all your work with proper limit notations.

1	$\lim_{x\to 0} \frac{\sin 5x}{\cos 4x}$	2	$\lim_{x\to 0} \frac{\sin^2 3x}{x^2 \cos x}$
3	$\lim_{x\to 0} \frac{\sin 5x}{\sin \frac{1}{3} x}$	4	$\lim_{x\to 0} \frac{x}{\tan x}$
5	$\lim_{x\to \frac{\pi}{2}} \frac{1+\sin x}{1-\cos x}$	6	$\lim_{x\to \infty} \frac{\cos 2x}{x^2}$
7	$\lim_{x\to \infty} \frac{6x^3-5x}{x^2+4x^3}$	8	$\lim_{x\to b} \frac{4a^2-x^2}{2a+x}$

9	$\lim_{x\to-\infty} \frac{8x^3-5x}{x^2-3x}$	10	$\lim_{x\to\infty} \frac{x^2+x^4}{x^2+x^6}$
11	$\lim_{x\to 2} \frac{4x^3-32}{5x^2-20}$	12	$\lim_{x\to 4^-} \frac{5}{x-4}$
13	$\lim_{x\to 0} \frac{4}{x}\sin\frac{x}{5}$	14	$\lim_{x\to 1} \frac{4x^3-5}{5x^2-6}$
15	$\lim_{x\to 2^+} \frac{2x-2}{x-4}$	16	$\lim_{x\to\infty} \frac{8x^2-2x^3}{2x^2+4x}$
17	Write an equation of the line perpendicular to $3x-4y=12$ that passes through the point $(-3,1)$.	18	Graph $f(x)=\begin{cases} x^2-4, & x<-3 \\ x+3, & -3<x<2 \\ -x+2, & x\geq 2 \end{cases}$

Use the following graph of $f(x)$ to answer the next five questions. If the limit does not exist, state why. Each tick mark = 1.



19. $\lim_{x \to 1^+} f(x) =$ ____________

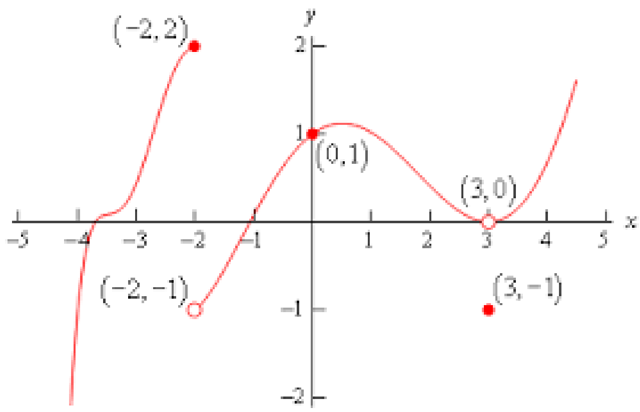
20. $\lim_{x \to 1^-} f(x) =$ ____________

21. $\lim_{x \to 1} f(x) =$ _______

22. $\lim_{x \to -1} f(x) =$ ____________

23. $\lim_{x \to 2} f(x) =$ _______

Use the following graph of $f(x)$ to answer 24 a-c.



24a. Use the 3-part definition of continuity to show if $f(x)$ is continuous at $x = 3$.

24b. What type(s) of discontinuity are shown in the graph of $f(x)$?

24c. Is there a removable discontinuity? If so, assign a value to remove it.

25	If $f(x) = \begin{cases} 2x - 1, & x \le 1 \\ -3x + 1, & x > 1 \end{cases}$, use the definition of continuity to show if $f(x)$ is continuous at $x = 1$.

26	If $f(x) = \begin{cases} \frac{x^2+6x+8}{x+2}, & x \neq -2 \\ 2, & x = -2 \end{cases}$, use the definition of continuity to show if $f(x)$ is continuous at $x = -2$.
27	If $f(x) = \frac{x^3+8}{x+2}$, use the definition of continuity to show if $f(x)$ is continuous at $x = -2$.
28	When $f(x) = \frac{x^3-64}{x-4}$, then $f(x)$ has point of discontinuity. Assign a value to $f(x)$ that removes the discontinuity. NOTATION!!!

29. If $f(x) = \frac{x^2-9}{x+3}$ is continuous at $x = -3$, then $f(-3) =$ ___________. Justify.

30. $\lim_{x \to 0} \frac{(3-x)^2}{x-3}$

31. $\lim_{x \to \pi} \sin 2x$

32. If a continuous function defined by $f(x) = \begin{cases} x^2 + bx, & x \leq 5 \\ 5 \sin \frac{\pi x}{2}, & x > 5 \end{cases}$. Find b.

33. If $y = 7$ is a horizontal asymptote of a rational function $f(x)$, then based on this information, which of the following must be true? Circle all that applies.

a. $\lim_{x\to 7} f(x) = \infty$ b. $\lim_{x\to\infty} f(x) = 7$ c. $\lim_{x\to 0} f(x) = 7$

d. $\lim_{x\to 7} f(x) = 0$ e. $\lim_{x\to -\infty} f(x) = -7$ f. $\lim_{x\to -\infty} f(x) = 7$

34. Which of the following is continuous at $x = 0$?

I. $f(x) = |x|$ II. $f(x) = e^x$ III. $f(x) = \ln(e^x - 1)$

a. I only b. II only c. I and II only d. II and III only e. none

35. Let $f(x) = \begin{cases} \ln x, & 0 < x \leq 2 \\ x^2 \ln 2, & 2 < x \leq 4 \end{cases}$, then $\lim_{x\to 2} f(x)$ is

a. $\ln 2$ b. $\ln 8$ c. $\ln 16$ d. 4 e. nonexsistent

36. Let $f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 3 - x, & 0 \leq x < 3 \\ (x-3)^2, & x > 3 \end{cases}$, then

a. $\lim_{x\to 0^+} f(x) =$ b. $\lim_{x\to 0^-} f(x) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3^+} f(x) =$ e. $\lim_{x\to 3^-} f(x) =$ f. $\lim_{x\to 3} f(x) =$

37. Let $f(x) = \begin{cases} \sqrt{x+1}, & 0 \leq x \leq 3 \\ 5 - x, & 3 < x \leq 5 \end{cases}$. Is $f(x)$ continuous at $x = 3$? Why or why not?

In each of the following, decide whether the Intermediate Value Theorem (IVT) can or cannot be used to justify the stated conclusion. Then explain why or why not.

38. The following table gives some function values for the continuous function $g(x)$:

x	0	2	4
$g(x)$	7	3	-6

Therefore, there exist some r in the interval $(0,2)$ for which $g(r) = 5$.

IVT applies

IVT does not apply

Why or why not?

39. Let k be the function defined by $k(x) = \begin{cases} x+2, & x \le 1 \\ x+4, & x > 1 \end{cases}$. There must exist some x between 0 and 2 for which $k(x) = 4$.

IVT applies

IVT does not apply

Why or why not?

40. Which of the following statements follows from the statement: "If f is continuous on the interval $[1, 4]$ with $f(1) = 3$ and $f(4) = 9$, then there exists at least one value of c in the interval $(1,4)$ for which $f(c) = 7$"?

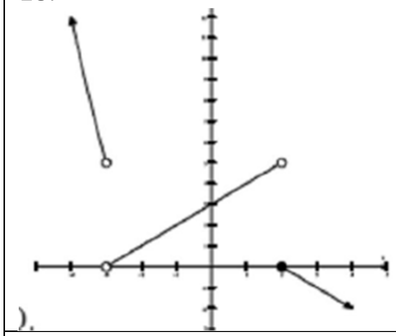
a) If f is not continuous on $[1,4]$ with $f(1) = 3$ and $f(4) = 9$, then there is no value of c in the interval $(1,4)$ for which $f(c) = 7$.

b) If f is defined on the interval $[1,4]$ with $f(1) = 3$ and $f(4) = 9$, and if there is no value of c in the interval $(1, 4)$ for which $f(c) = 7$, then f is not continuous.

Answers:

1. 0	2. 9	3. 15	4. 1	5. 2	6. 0	7. $\frac{3}{2}$
8. $2a - b$	9. $-\infty$	10. 0	11. $\frac{12}{5}$	12. $-\infty$	13. $\frac{4}{5}$	14. 1
15. -1	16. $-\infty$	17. $y - 1 = -\frac{4}{3}(x+3)$	18. See graph below	19. 1	20. -2	21. DNE
22. 2	23. 1					

18.



24.

a. I. $f(3) = -1$

II. $\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x)$; $\lim_{x\to3} f(x) = 0$

III. $f(3) \neq \lim_{x\to3} f(x)$

$\therefore f(x)$ is not continuous at $x = 3$.

b. Removable discontinuity at $x = 3$, non-removable at $x = -2$

c. $f(3) = 0$

25.

I. $\lim_{x\to1^-} f(x) \neq \lim_{x\to1^+} f(x)$; $\lim_{x\to1} f(x)$ DNE

$\therefore f(x)$ is not continuous at $x = 1$

26.

I. $f(-2) = 2$

II. $\lim_{x\to-2^-} f(x) = \lim_{x\to-2^+} f(x)$; $\lim_{x\to-2} f(x) = 2$

III. $f(-2) = \lim_{x\to-2} f(x)$

$\therefore f(x)$ is continuous at $x = -2$

27.

I. $f(-2)$ DNE

$\therefore f(x)$ is not continuous at $x = -2$

28.

$$\lim_{x\to4}\frac{x^3-64}{x-4} = \lim_{x\to4}\frac{(x-4)(x^2+4x+16)}{x-4}$$
$$= \lim_{x\to4}(x^2+4x+16) = 48$$

$\therefore f(4) = 48$

29.

$$\lim_{x\to-3}\frac{x^2-9}{x+3}$$
$$= \lim_{x\to-3}\frac{(x+3)(x-3)}{x+3}$$
$$= \lim_{x\to-3}(x-3)$$
$$= -6$$

$\therefore f(-3) = -6$

30. -3

31. 0

32. -4

33. b, f

34. c

35. e

36.

a. 3

b. 0

c. DNE

d. 0

e. 0

f. 0

37.

I. $f(3) = 2$

II. $\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x)$;

$\lim_{x\to3} f(x) = 2$

III. $f(3) = \lim_{x\to3} f(x)$

$\therefore f(x)$ is continuous at $x = 3$

38. IVT applies. The function g is continuous, and from the table, $g(0) = 7$ and $g(2) = 3$. Since 5 is between 7 and 3, the IVT applies, and it follows that there exists some r in the interval $(0,2)$ for which $f(r) = 5$.

39. IVT does not apply. In order for the IVT to apply, k would have to be continuous on the interval $[0,2]$. But k is discontinuous at $x = 1$, since $\lim_{x\to 1^-} k(x) \neq \lim_{x\to 1^+} k(x)$.

40.(b) If f is defined on the interval $[1,4]$ with $f(1) = 3$ and $f(4) = 9$, and if there is no value of c in the interval $(1, 4)$ for which $f(c) = 7$, then f is not continuous.