

<p>Use Theorem 6.4.2 to evaluate the sums.</p> <p>12. $\sum_{k=1}^{100} (7k + 1)$</p>	<p>15. $\sum_{k=1}^{30} k(k-2)(k+2)$</p>
<p>Evaluate the sums in closed form.</p> <p>19. $\sum_{k=1}^{n-1} \frac{k^3}{n^2}$</p>	<p>22. Solve the equation $\sum_{k=1}^n k = 465$.</p>
<p>In Exercises 23–26, express the function of n in closed form and then find the limit.</p> <p>23. $\lim_{n \rightarrow +\infty} \frac{1 + 2 + 3 + \cdots + n}{n^2}$</p>	<p>25. $\lim_{n \rightarrow +\infty} \sum_{k=1}^n \frac{5k}{n^2}$</p>
<p>In Exercises 29–32, divide the interval $[a, b]$ into $n = 4$ subintervals of equal length, and then compute</p> $\sum_{k=1}^4 f(x_k^*) \Delta x$ <p>with x_k^* as (a) the left endpoint of each subinterval, (b) the midpoint of each subinterval, and (c) the right endpoint of each subinterval.</p> <p>31. $f(x) = 3x + 1$; $a = 2, b = 6$</p>	<p>33. $f(x) = \cos x$; $a = 0, b = \pi$</p>

55. Use Definition 6.4.5 with x_k^* as the right endpoint of each subinterval to find the net signed area between the curve $y = f(x)$ and the specified interval.

$$y = x^2 - 1; a = 0, b = 2$$

59.

(a) Show that the area under the graph of $y = x^3$ and over the interval $[0, b]$ is $b^4/4$.

65.

63–66 Consider the sum

$$\begin{aligned} \sum_{k=1}^4 [(k+1)^3 - k^3] &= [5^3 - 4^3] + [4^3 - 3^3] \\ &\quad + [3^3 - 2^3] + [2^3 - 1^3] \\ &= 5^3 - 1^3 = 124 \end{aligned}$$

For convenience, the terms are listed in reverse order. Note how cancellation allows the entire sum to collapse like a telescope. A sum is said to **telescope** when part of each term cancels part of an adjacent term, leaving only portions of the first and last terms uncanceled. Evaluate the telescoping sums in these exercises.

$$\sum_{k=2}^{20} \left(\frac{1}{k^2} - \frac{1}{(k-1)^2} \right)$$

70. Let

$$S = \sum_{k=0}^n ar^k$$

Show that $S - rS = a - ar^{n+1}$ and hence that

$$\sum_{k=0}^n ar^k = \frac{a - ar^{n+1}}{1 - r}; r \neq 1$$