BC CALCULUS PRACTICE 10.1 **Name:** ______________________________ **Period** ________

Aisle __________ pg. 633; 3, 7, 11, 15, 17, 19, 23, 29, 33, 39, 46, 48

Show all necessary work neatly.

3. (a) Write out the first four terms of the sequence $\{1+(-1)^n\}$, starting with $n=0$.

(b) Write out the first four terms of the sequence $\{\cos n\pi\}$, starting with $n=0$.

(c) Use the results in parts (a) and (b) to express the general term of the sequence 4, 0, 4, 0, . . . in two different ways, starting with $n=0$.

7–22 Write out the first five terms of the sequence, determine whether the sequence converges, and if so find its limit.

7.
$\{2\}_{n=1}^{+\infty}$

11.
$\{1+(-1)^n\}_{n=1}^{+\infty}$

15.
$$\left\{\frac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{+\infty}$$

19. $\{n^2e^{-n}\}_{n=1}^{+\infty}$

23–30 Find the general term of the sequence, starting with $n=1$, determine whether the sequence converges, and if so find its limit.

23. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$

29. $(\sqrt{2}-\sqrt{3}), (\sqrt{3}-\sqrt{4}), (\sqrt{4}-\sqrt{5}), \ldots$

33.
(a) Starting with $n=1$, write out the first six terms of the sequence $\{a_n\}$, where

$$a_n=\begin{cases}1, & \text{if } n \text{ is odd}\\ n, & \text{if } n \text{ is even}\end{cases}$$

(b) Starting with $n=1$, and considering the even and odd terms separately, find a formula for the general term of the sequence

$$1, \frac{1}{2^2}, 3, \frac{1}{2^4}, 5, \frac{1}{2^6}, \ldots$$

(c) Starting with $n=1$, and considering the even and odd terms separately, find a formula for the general term of the sequence

$$1, \frac{1}{3}, \frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{7}, \frac{1}{7}, \frac{1}{9}, \frac{1}{9}, \ldots$$

(d) Determine whether the sequences in parts (a), (b), and (c) converge. For those that do, find the limit.

39. Consider the sequence $\{a_n\}_{n=1}^{\infty}$, where

$$a_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots \frac{n}{n^2}$$

(a) Find a_1, a_2, a_3, and a_4.

(b) Use numerical evidence to make a conjecture about the limit of the sequence.

(c) Confirm your conjecture by expressing a_n in closed form and calculate the limit.

46. Consider the sequence $\{a_n\}_{n=1}^{+\infty}$ whose nth term is

$$a_n = \frac{1}{n}\sum_{k=1}^{n} \frac{1}{1+(k/n)}$$

Show that $\lim_{n\to+\infty} a_n = \ln 2$ by interpreting a_n as the Riemann sum of a definite integral.

48.

The sequence whose terms are 1, 1, 2, 3, 5, 8, 13, 21, ... is called the ***Fibonacci sequence*** in honor of the Italian mathematician Leonardo ("Fibonacci") da Pisa (c. 1170–1250). This sequence has the property that after starting with two 1's, each term is the sum of the preceding two.

(a) Denoting the sequence by $\{a_n\}$ and starting with $a_1 = 1$ and $a_2 = 1$, show that

$$\frac{a_{n+2}}{a_{n+1}} = 1 + \frac{a_n}{a_{n+1}} \quad \text{if } n \geq 1$$

(b) Give a reasonable informal argument to show that if the sequence $\{a_{n+1}/a_n\}$ converges to some limit L, then the sequence $\{a_{n+2}/a_{n+1}\}$ must also converge to L.

(c) Assuming that the sequence $\{a_{n+1}/a_n\}$ converges, show that its limit is $(1+\sqrt{5})/2$.