AP Calculus BC – HW #________ Name: ____________________________ Period _______

Derivative Review Worksheet 2 (NO CALCULATOR)

Show all your work with proper notations.

1. If $g(x) = x^4 - 2$, find $g'(1)$ using the limit definition of the derivative and use it to find the equation of the tangent line of $g(x)$ at $x = 1$.

2. Name three ways in which a function can fail to be differentiable. Show each case using an example graph or function with a non-differentiable point.

a.

b.

c.

3. Sketch a graph of a function for which $f(0) = 0$, $f'(0) = -1$, $f(1) = 0$, and $f'(1) = -1$.

4. State the general form for these derivative rules:
   a. The power rule
   b. The constant multiple rule
   c. The sum rule
   d. The difference rule
   e. The product rule
   f. The quotient rule
   g. The chain rule

5. Find both the first and the second derivatives. Express in BOTH major notations.

$f(x) = 6x^{-5/3}$

6. Find the first derivative. Express in BOTH major notations.

a. $g(x) = \frac{x^2+x+1}{x}$

b. $y = 3p - \sqrt{p}$

Find the derivative:		
7. $f(x) = \frac{\sin x}{\cos x}$	8. $y = \cot(6x)$	9. $f(x) = sec^2x \cdot \tan x$
10. $g(x) = \tan(\cos x)$	11. $y = \sec x \cdot \sin(3x)$	12. $h(x) = tan^{10}3x$
13. $f(\theta) = \sin(\cos(\tan\theta)))$	14. $f(x) = ((3x^5 + x^4)^{12} + 2x)^3$	15. $y = \frac{1-\csc x}{\cot x}$
For #16-21, find $\frac{dy}{dx}$. Assume y is a differentiable function of x.		
16. $3y = x \cdot cos^5y$	17. $xy + y^2 + x^3 = 7$	18. $\frac{\sin y}{y^2+1} = 3x$
19. $\tan(x-y) = \frac{y}{1+x^2}$	20. $xy = \cot x^2 + x^3$	21. $x^3 + y^3 = 6xy$

22. Use implicit differentiation to find an equation of the tangent line to the curve $sin\,(x+y)=2x-2y$ at the point (π,π).		
If f and g are differentiable functions such that $f(2)=3$, $f'(2)=-1$, $g(2)=-5$, and $g'(2)=2$, find the answers to the following problems.		
23. $(g-f)'(2)$	24. $(fg)'(2)$	25. $\left(\frac{f}{g+f}\right)'(2)$
26. $(5f+3g)'(2)$	27. $(f\circ f)'(2)$	28. $(g\circ f)'(2)$

For #29-32, use the table to find the indicated value.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	-1	4	1
4	2	-1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	-2

29. $h(x)=f(x)+g(x)$, find $h'(2)$.	30. $d(x)=\frac{f(x)}{g(x)}$, find $d'(3)$.
31. $k(x)=[f(x)]^2$, find $k(4)$ and $k'(4)$.	32. $p(x)=g[f(x)]$, find $p'(6)$.

Answers:

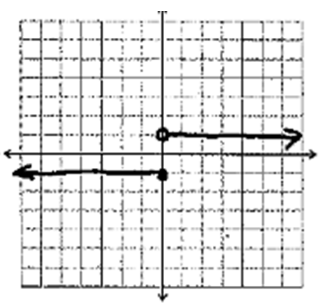
1.

$$\begin{aligned}
\lim_{h\to 0}\frac{g(x+h)-g(x)}{h} &= \lim_{h\to 0}\frac{(x+h)^4-2-(x^4-2)}{h}\\
&= \lim_{h\to 0}\frac{(x^2+2xh+h^2)^2-x^4}{h}\\
&= \lim_{h\to 0}\frac{x^4+2x^3h+x^2h^2+2x^3h+4x^2h^2+2xh^3+h^2x^2+2xh^3+h^4-x^4}{h}\\
&= \lim_{h\to 0}\frac{2x^3h+x^2h^2+2x^3h+4x^2h^2+2xh^3+h^2x^2+2xh^3+h^4}{h}\\
&= \lim_{h\to 0} 2x^3+x^2h+2x^3+4x^2h+2xh^2+hx^2+2xh^2+h^3\\
&= 2x^3+2x^3\\
&= 4x^3
\end{aligned}$$

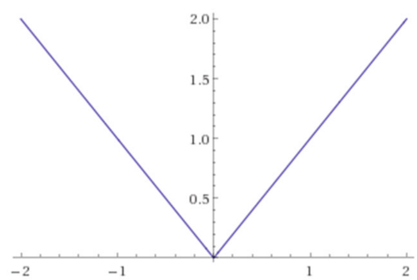
$g'(1)=4$; $g(1)=-1$; The equation of the tangent line to $g(x)$ at $x=1$ is $y+1=4(x-1)$

2. possible examples:

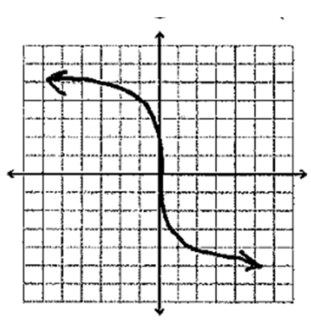
a. Jump discontinuity at $x=0$



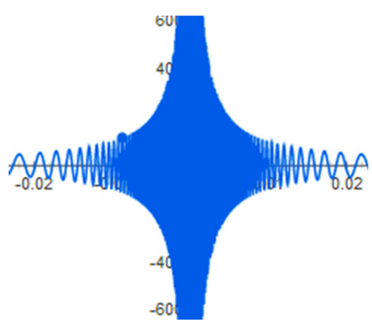
b. Corner at at $x=0$



c. Vertical tangent at $x=0$



d. Infinite oscillation at $x=0$



3. possible graph:

4.

a. $\frac{d}{dx}x^n = nx^{n-1}$for all real numbers n.

b. $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$

c,d. $\frac{d}{dx}[f(x)\pm g(x)] = \frac{d}{dx}f(x)\pm\frac{d}{dx}g(x)$

e. $\frac{d}{dx}[f(x)\cdot g(x)] = \frac{d}{dx}f(x)\cdot g(x)+\frac{d}{dx}g(x)\cdot f(x)$

f. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x)-g'(x)f(x)}{g^2(x)}$

g. $\frac{d}{dx}f[g(x)] = f'[g(x)]\cdot g'(x)$

5. $f'(x) = \frac{d}{dx} f(x) = -10x^{-8/3}$ $f"(x) = \frac{d^2 f(x)}{dx^2} = \frac{80}{3} x^{-11/3}$	6. a. $g'(x) = \frac{d}{dx} g(x) = 1 - x^{-2}$ b. $y' = \frac{dy}{dx} = 3 - \frac{1}{2} p^{-1/2}$
7. $y = 3x - 4$	