**BC CALCULUS PRACTICE 10.2** **Name:** ______________________________ **Period** ________

**Aisle** __________ pg. 641; 3, 5, 9, 11, 15, 17, 23, 25, 27

Show all necessary work neatly.

**1–6** Use the difference $a_{n+1} - a_n$ to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing. ■

| | |
|---|---|
| **3.** $\left\{\dfrac{n}{2n+1}\right\}_{n=1}^{+\infty}$ | **5.** $\{n - 2^n\}_{n=1}^{+\infty}$ |
| **7–12** Use the ratio $a_{n+1}/a_n$ to show that the given sequence $\{a_n\}$ is strictly increasing or strictly decreasing. ■<br>**9.** $\{ne^{-n}\}_{n=1}^{+\infty}$ | **11.** $\left\{\dfrac{n^n}{n!}\right\}_{n=1}^{+\infty}$ |
| 13-18 Use differentiation to show that the given sequence is strictly increasing or strictly decreasing.<br>15.<br>$\left\{\dfrac{1}{n + \ln n}\right\}_{n=1}^{\infty}$ | 17.<br>$\left\{\dfrac{\ln(n+2)}{n+2}\right\}_{n=1}^{+\infty}$ |

Show that the given sequence is eventually strictly increasing or eventually strictly decreasing.

**23.** $\left\{\dfrac{n!}{3^n}\right\}_{n=1}^{+\infty}$

25.

Suppose that $\{a_n\}$ is a monotone sequence such that $1 \le a_n \le 2$ for all $n$. Must the sequence converge? If so, what can you say about the limit?

Suppose that $\{a_n\}$ is a monotone sequence such that $a_n \le 2$ for all $n$. Must the sequence converge? If so, what can you say about the limit?

27.

The goal of this exercise is to establish Formula (5), namely,

$$\lim_{n\to+\infty} \frac{x^n}{n!} = 0$$

Let $a_n = |x|^n/n!$ and observe that the case where $x = 0$ is obvious, so we will focus on the case where $x \neq 0$.

(a) Show that

$$a_{n+1} = \frac{|x|}{n+1} a_n$$

(b) Show that the sequence $\{a_n\}$ is eventually strictly decreasing.

(c) Show that the sequence $\{a_n\}$ converges.

(d) Use the results in parts (a) and (c) to show that $a_n \to 0$ as $n \to \infty$

(e) Obtain Formula (5) from the result in part (d).