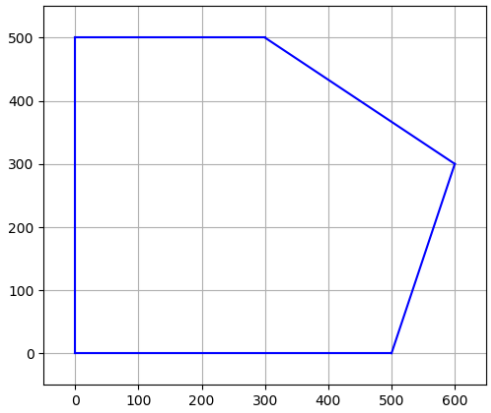
Homework 10: Nonlinear programs

Due date: 11:00pm on Friday May 3, 2019
See the course website for instructions and submission details.

Note: for this Homework set, we suggest using the `Ipopt` solver in JuMP. Be sure to make use of the macros `@NLconstraint` and `@NLobjective` when specifying nonlinear constraints or objectives respectively.

1. **Museum site planning revisit.** Recall the Museum site planning problem we did in Homework 3. A site is being investigated as a potential location for two new museums. An aerial plan of the site is shown in the figure below (in units of feet). Both the museums will have circular footprints.

   We have discussed that without dividing the site into subareas in advance we can have larger museums in terms of area. This time, we leave the entire site to you without dividing it. That is to say, the museums can be put anywhere you want inside the site.



   a) Where should the museums be located if we want the area of them to be as large as possible? Re-plot the figure above along with the optimally designed museums. What is the total area of your optimal designed museums. Looking at the figure, do you think you found the optimal solution?

   To get you started, the following Julia code plots the site and plots a circle:

```
using PyPlot
function plot_site()
    plot([0,0], [0,500], "b-")
    plot([0,500], [0,0], "b-")
    plot([500,600], [0,300],"b-")
    plot([300,600], [500,300],"b-")
    plot([0,300], [500,500],"b-")
    axis("image")
    axis([-.5,6.5,-.5,5.5]*100)
    grid()
end
figure(figsize=(6,5))
plot_site()
t = range(0,stop=2*pi,length=100)
x1 = [200,200];
r1 = 100
plot(x1[1] .+ r1*cos.(t), x1[2] .+ r1*sin.(t), "r")
```

b) Solve the problem while initializing the location of the centers of the museums to different values. Repeat until you find at least two or three different solutions. Which of them is the better one? Why do you get different solutions based on the starting value you set for the variables?

To set initial values of variable x to $(400, 100)$, you may want to use the following Julia code before solving the model:

```
setvalue(x, [400,100])
```

2. Fertilizer influence model. A series of experiments is conducted to determine the effect of a particular fertilizer on wheat crop yield. Here are the results: Such situations typically involve some sort

fertilizer rate (x)	crop yield (y)
-5	427
-3	451
-1	379
1	221
3	160
5	126

of "diminishing returns". So there is a limit to how much the fertilizer can boost the yield. A popular model for this relationship is the following:

$$y = k_1 + k_2 \exp(k_3 x^2 + k_4 x)$$

Use nonlinear least squares to determine the values of (k_1, k_2, k_3, k_4) that provide the best fit to the data. We are expecting coefficients in the neighborhood of $(500, -200, -0.1, 0.5)$.

To display your result, produce a plot of the original data points and the best-fit curve.