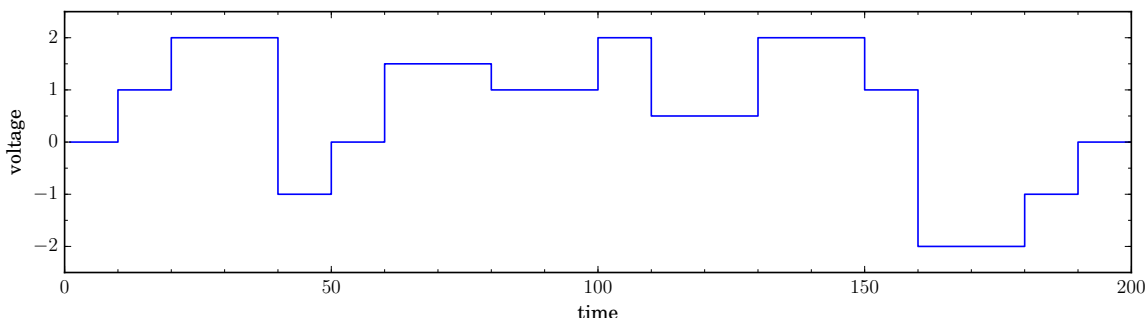


## Homework 6: Quadratic programs

Due date: 11:00pm on Friday March 8, 2019

See the course website for instructions and submission details.

- 1. Voltage smoothing.** We would like to send a sequence of voltage inputs to the manipulator arm of a robot. The desired signal is shown in the plot below (also available in `voltages.csv`).



Unfortunately, abrupt changes in voltage cause undue wear and tear on the motors over time, so we would like to create a new signal that is similar to the one above but with smoother transitions. If the voltages above are given by  $v_1, v_2, \dots, v_{200}$ , one way to characterize smoothness is via the sum of squared differences:

$$R(v) = (v_2 - v_1)^2 + (v_3 - v_2)^2 + \dots + (v_{200} - v_{199})^2$$

When  $R(v)$  is smaller, the voltage is smoother. Solve a regularized least squares problem that explores the tradeoff between matching the desired signal perfectly and making the signal smooth. Include a plot comparing the original signal to a few different smoothed versions obtained using regularized least squares and with varying degrees of smoothness.

- 2. Quadratic form positivity.** You're presented with the constraint:

$$2x^2 + 2y^2 + 9z^2 + 8xy - 6xz - 6yz \leq 1 \quad (1)$$

- a) Write the constraint (1) in the standard form  $v^T Q v \leq 1$ . Where  $Q$  is a symmetric matrix. What is  $Q$  and what is  $v$ ?

- b) It turns out the above constraint is *not* convex. In other words, the set of  $(x, y, z)$  satisfying the constraint (1) is not an ellipsoid. Explain why this is the case.

**Note:** you can perform an orthogonal decomposition of a symmetric matrix  $Q$  in Julia like this:

```
(L,U) = eig(Q)      # L is the vector of eigenvalues and U is orthogonal
U * diagm(L) * U'   # this is equal to Q (as long as Q was symmetric to begin with)
```

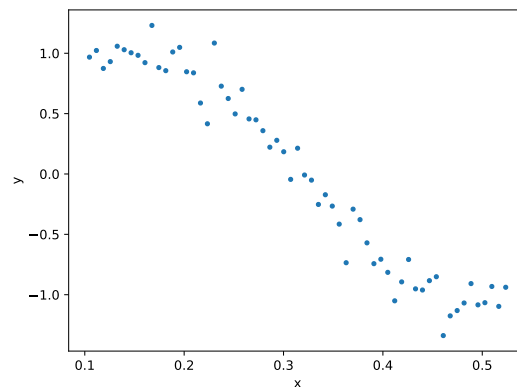
- c) We can also write the constraint (1) using norms by putting it in the form:

$$\|Av\|^2 - \|Bv\|^2 \leq 1$$

What is  $v$  and what are the matrices  $A$  and  $B$  that make the constraint above equivalent to (1)?

- d) Explain how to find a direction for vector  $(x, y, z)$  such that  $\|(x, y, z)\|_2$  (norm) is unbounded (can be made arbitrarily large) while satisfying the constraint (1).

**3. Lasso regression.** Consider the data  $(x, y)$  plotted below, available in `lasso_data.csv`.



In this problem, we will investigate different approaches for performing polynomial regression.

- a) Perform ordinary polynomial regression. Make plots that show the data as well as the best fit to the data for polynomials of degree  $d = 5$  and  $d = 15$ . Also comment on the magnitudes of the coefficients in the resulting polynomial fits. Are they small or large?
- b) In order to get smaller coefficients, we will use ridge regression ( $L_2$  regularization). Re-solve the  $d = 15$  version of the problem using a regularization parameter  $\lambda = 10^{-6}$  and plot the new fit. How does the fit change compared to the non-regularized case of part a? How do the magnitudes of the coefficients in the resulting polynomial fit change?
- c) Our model is still complicated because it has so many parameters. One way to simplify our model is to look for a sparse model (where many of the parameters are zero). Solve the  $d = 15$  problem once more, but this time use the Lasso ( $L_1$  regularization). Start with a large  $\lambda$  and progressively make  $\lambda$  smaller until you obtain a model with a small number of parameters that fits the data reasonably well. **Note:** due to numerical inaccuracy in the solver, you may need to round very small coefficients (say less than  $10^{-5}$ ) down to zero. Plot the resulting fit.