Homework 1: Linear programs

Due date: 11:00pm on Tuesday February 5, 2019
See the course website for instructions and submission details.

1. **Warm-up.** Model the following problem in JuMP.

$$\begin{aligned} \underset{x_1,x_2}{\text{maximize}} \quad & 5x_1 - x_2 \\ \text{subject to:} \quad & 2x_1 + x_2 \leq 5 \\ & x_1 + x_2 \geq 2 \\ & 0 \leq x_j \leq 3, \ j \in \{1,2\} \end{aligned}$$

   a) Given the choice of the solvers `Clp`, `ECOS`, and `SCS`, which one would you choose? Why?

   b) What is the optimal objective value? What is the optimal solution for the variables x_1, x_2?

   c) When you solve the problem, use the command
   ```
   status = solve(m)
   println(status)
   ```
   to check whether the solution you obtained was optimal. If `println(status)` returns `:Optimal`, it means that the problem was solved to optimality.

   d) What happens if you set the right hand side constant in the first inequality constraint to 0 (so the constraint becomes $2x_1 + x_2 \leq 0$)? Is the solution you obtain still optimal?

2. **Crop planning.** Farmer Jane owns 45 acres of land. She is going to plant each with wheat or corn. Each acre planted with wheat yields \$230 profit; each with corn yields \$190 profit. The labor and fertilizer used for each acre are given in the table below. 100 workers and 130 tons of fertilizer are available.

	Wheat	Corn
Labor	3 workers	2 workers
Fertilizer	2 tons	4 tons

   a) How should Jane plant her crops to maximize profit? Model and solve this problem using JuMP. Include your code in your solution.

   b) Code the same model once again, but this time separating the parameters from the solution as we did in class. Confirm that you obtain the same solution.

   c) Solve the problem graphically by plotting the feasible set and contour lines for the objective function. Confirm that you obtain the same solution as in the previous parts. (I suggest using the PyPlot package, but it is OK if you draw the plot by hand and submit a pdf scan.)