Homework 8: Integer programs and duality

Due date: 11:00pm on Friday April 12, 2019
See the course website for instructions and submission details.

1. **Lagrangian duality.** Consider the problem

$$\min_{x_1,x_2} \frac{1}{2}(x_1^2 + x_2^2) \quad \text{subject to } 1 - x_1 \leq 0.$$

(a) Write down the solution of this problem and the optimal primal value p^*.

(b) Derive the Lagrangian dual function $g(\lambda)$ for $\lambda \in \mathbb{R}$.

(c) Find the solution of the Lagrangian dual problem $\max_{\lambda \geq 0} g(\lambda)$ and write down the optimal dual objective d^*.

(d) Is the Slater condition satisfied for this problem? Does strong duality hold, that is, $p^* = d^*$?

2. **Thrift store.** Here are the weights of each coin:

Type of coin	penny	nickel	dime	quarter	half dollar
Weight (grams)	2.500	5.000	2.268	5.670	11.340

(a) How should you make change for 99 cents if the goal is to minimize the total weight of the coins used? You may use any number of each type of coin.

(b) How should you make change for 99 cents if the goal is to carry coins that weigh at most 35 grams, while minimizing the total number of coins? Compare the solution in terms of the chosen coins and their total weight with the answer you obtain for (a).

3. **Comquat Computers.** Comquat owns four production plants at which personal computers are produced. Comquat can sell up to 20,000 computers per year at a price of $3,500 per computer. For each plant the production capacity, cost per computer, and fixed cost of operating the plant for a year are given below. Determine how Comquat can maximize its yearly profit from computer production.

Plant	Production capacity	Plant fixed cost ($ Million)	Cost per computer ($)
1	10,000	9	1,000
2	8,000	5	1,700
3	9,000	3	2,300
4	6,000	1	2,900

4. **ABC Investments.** ABC Inc. is considering several investment options. Each option has a minimum and maximum investment allowed (only if the option is chosen). These restrictions, along with the expected return are summarized in the following table (figures are in millions of dollars):

Option	Minimum investment	Maximum investment	Expected return (%)
1	3	27	13
2	2	12	9
3	9	35	17
4	5	15	10
5	12	46	22
6	4	18	12

Because of the high-risk nature of Option 5, company policy requires that the total amount invested in Option 5 be no more that the combined amount invested in Options 2, 4 and 6. In addition, if an investment is made in Option 3, it is required that at least a minimum investment be made in Option 6. ABC has $80 million to invest and obviously wants to maximize its total expected return on investment. Which options should ABC invest in, and how much should be invested?