Homework 3: Minimax and network flows

Due date: 11:00pm on Friday February 15, 2019
See the course website for instructions and submission details.

1. **Doodle scheduling.** Doodle Inc. is looking to interview a candidate for a new software engineer position at their company. It works like this: the interview (10 AM to 3 PM) is divided into a number of 20-minute time slots that may be used for 1-on-1 meetings with the candidate. There is also a one-hour time slot in the middle of the day where 3 employees take the candidate out for lunch.

   It would be nice for all 15 senior employees to meet with the candidate at some point during the day, but everybody has a busy schedule so it's not clear whether this will be possible. A doodle poll (obviously) was sent to the 15 senior employees to figure out their availability. Here is the data:

	10:00	10:20	10:40	11:00	11:20	11:40	Lunch	1:00	1:20	1:40	2:00	2:20	2:40
Manuel	0	0	1	1	0	0	0	1	1	0	0	0	0
Luca	0	1	1	0	0	0	0	0	1	1	0	0	0
Jule	0	0	0	1	1	0	1	1	0	1	1	1	1
Michael	0	0	0	1	1	1	1	1	1	1	1	1	0
Malte	0	0	0	0	0	0	1	1	1	0	0	0	0
Chris	0	1	1	0	0	0	0	0	1	1	0	0	0
Spyros	0	0	0	1	1	1	1	0	0	0	0	0	0
Mirjam	1	1	0	0	0	0	0	0	0	0	1	1	1
Matt	1	1	1	0	0	0	0	0	0	1	1	0	0
Florian	0	0	0	0	0	0	0	1	1	0	0	0	0
Josep	0	0	0	0	0	0	1	1	1	0	0	0	0
Joel	1	1	0	0	0	1	1	1	1	0	0	1	1
Tom	1	1	1	0	1	1	0	0	0	0	0	1	1
Daniel	0	1	1	1	0	0	0	0	0	0	0	0	0
Anne	1	1	0	0	1	1	0	0	0	0	0	0	0

   In the table, a 1 means that the employee is available at the indicated time, while a 0 means that they are unavailable. Determine whether a feasible interview schedule exists. If so, print out a calendar for the candidate that lists who they will be meeting at each time slot. See `hw3data.ipynb` for the data.

2. **Car rental.** A small car rental company has a fleet of 94 vehicles distributed among its 10 agencies. The location of every agency is given by its geographical coordinates x and y in a grid based on miles. We assume that the road distance between agencies is approximately 1.3 times the Euclidean distance (as the crow flies). The following table indicates the coordinates of all agencies, the number of cars required the next morning, and the stock of cars in the evening preceding this day.

Agency number	1	2	3	4	5	6	7	8	9	10
x coordinate	0	20	18	30	35	33	5	5	11	2
y coordinate	0	20	10	12	0	25	27	10	0	15
Required cars	10	6	8	11	9	7	15	7	9	12
Cars present	8	13	4	8	12	2	14	11	15	7

   Supposing the cost for transporting a car is \$0.50 per mile, determine the movements of cars that allow the company to re-establish the required numbers of cars at all agencies, minimizing the total cost incurred for transport.

3. Building a stadium. A town council wishes to construct a small stadium in order to improve the services provided to the people living in the district. After the invitation to tender, a local construction company is awarded the contract and wishes to complete the task within the shortest possible time. All the major tasks are listed in the following table. Some tasks can only start after the completion of certain other tasks, as indicated by the "Predecessors" column. See `hw3data.ipynb` for the data.

Task	Description	Duration (in weeks)	Predecessors	Maximum reduction (in weeks)	Cost of reduction ($1k/wk)
1	Installing the construction site	2	none	0	–
2	Terracing	16	1	3	30
3	Constructing the foundations	9	2	1	26
4	Access roads and other networks	8	2	2	12
5	Erecting the basement	10	3	2	17
6	Main floor	6	4,5	1	15
7	Dividing up the changing rooms	2	4	1	8
8	Electrifying the terraces	2	6	0	–
9	Constructing the roof	9	4,6	2	42
10	Lighting of the stadium	5	4	1	21
11	Installing the terraces	3	6	1	18
12	Sealing the roof	2	9	0	–
13	Finishing the changing rooms	1	7	0	–
14	Constructing the ticket office	7	2	2	22
15	Secondary access roads	4	4,14	2	12
16	Means of signalling	3	8,11,14	1	6
17	Lawn and sport accessories	9	12	3	16
18	Handing over the building	1	17	0	–

a) What is the earliest possible date of completion for the construction? Note that the last two columns of the table are not relevant for this part of the problem.

b) The town council wants the builder to expedite the project. As an incentive, the council will pay a bonus of $30k/week for each week the work finishes early. To accomplish this, the builder may employ additional workers and rent more equipment to cut down on the total time. The last two columns of the table show the maximum number of weeks that can be saved per task and the associated additional cost per week incurred by the extra work. When will the project be completed if the builder is acting in a way that maximizes his profit?

4. Museum site planning. A site is being investigated as a potential location for two new museums. An aerial plan of the site is shown in the figure below (in units of feet). The space has already been divided in two subareas (by the red segement). Both the museums will have circular footprints, and there will be one of them in each subarea.

a) Where should the museums be located if we want the area of them to be as large as possible? Re-plot the figure below along with the optimally designed museums.

b) (You do not need to solve this task numerically in Julia.) If you should build two museum that together have the largest area without defining subareas ahead of time (i.e., take away the red line), how would you formulate the problem? Is it still a linear program?
Without solving the problem, can you determine whether the two museums would have a larger total area if there are no limitations (i.e. no red line)? Explain why / why not.