ESC103 Unit 22

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1 An example and a segue into MAT185

Find the inverse of matrix A using G.E. (assuming the inverse exists).

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} A \mid I \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & | 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & | 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & | 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & | 0 & 0 & 0 & 1 \end{bmatrix} R3 + 3R4$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 & | 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & | 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & | 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & | 0 & 0 & 0 & 1 \end{bmatrix} R2 + 2R3$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 0 & 0 & | 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & | 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 0 & | 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & | 0 & 0 & 0 & 1 \end{bmatrix} R1 + R2$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & |1 & 1 & 2 & 6 \\ 0 & 1 & 0 & 0 & |0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 0 & |0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & |0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Remember, if you don't get the identity matrix on the left side of the super augment matrix, then matrix A is not invertible.