1.11 By setting the derivatives of log likelihood function

$$\ln p(\mathbf{x} \mid \mu, \sigma^2) = -\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2}\ln\sigma^2 - \frac{N}{2}\ln(2\pi),$$

verify the results of

$$\mu_{\mathrm{ML}} = \frac{1}{N}\sum_{n=1}^{N} x_n,$$

$$\sigma^2_{\mathrm{ML}} = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu_{\mathrm{ML}})^2.$$

Answer. By setting the derivative with respect to μ to zero, we have

$$\frac{\partial}{\partial\mu}\left[-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2}\ln\sigma^2 - \frac{N}{2}\ln(2\pi)\right] = 0 \tag{1}$$

$$\frac{\partial}{\partial\mu}\left[-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2\right] = 0 \tag{2}$$

$$\frac{\partial}{\partial\mu}\sum_{n=1}^{N}(x_n^2 - 2\mu x_n + \mu^2) = 0 \tag{3}$$

$$\frac{\partial}{\partial\mu}\left[\sum_{n=1}^{N} x_n^2 - 2\sum_{n=1}^{N}\mu x_n + N\mu^2\right] = 0 \tag{4}$$

$$-2\sum_{n=1}^{N} x_n + 2N\mu = 0 \tag{5}$$

$$\mu = \frac{1}{N}\sum_{n=1}^{N} x_n. \tag{6}$$

We can conclude that the formula for μ_{ML} is correct.

Let $u = \sigma^2$, by setting the derivative with respect to u to zero, we have

$$\frac{\partial}{\partial\sigma^2}\left[-\frac{1}{2\sigma^2}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2}\ln\sigma^2 - \frac{N}{2}\ln(2\pi)\right] = 0$$

$$\frac{\partial}{\partial u}\left[-\frac{1}{2u}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2}\ln u - \frac{N}{2}\ln(2\pi)\right] = 0$$

$$\frac{1}{2u^2}\sum_{n=1}^{N}(x_n - \mu)^2 - \frac{N}{2u} = 0$$

$$u = \frac{1}{N}\sum_{n=1}^{N}(x_n - \mu)^2.$$

By substituting μ with μ_{ML}, we can conclude that the formula for σ^2_{ML} is correct.