Has Manipulation in the VIX Decreased?

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Manipulation in the VIX settlement can cause significant losses to investors. Analysing high-frequency data, we present indications of VIX manipulation accelerating since 2017. Deviations have an upward direction and average at around 6%. Specific effects accompany settlement days. The put/call ratio of underlying options surges by 10.9%. A time series decomposition demonstrates that this difference exceeds the day-specific variations of all other days by 80%. Data on open interest point towards leveraged funds, who systematically gather additional exposure in the seven days before settlement. All other players seem to reduce their VIX exposure before settlement. After 2017, the market seems to accustom itself and incorporate deviations more easily.

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1 Introduction

Market manipulation is a criminal act, asymmetrically transferring capital to few undeserving beneficiaries at the expense of honest investors. In addition to direct costs, there are indirect costs as such conduct undermines the trust in financial instruments and impedes the intermediary function of financial markets (Allen and Gale, 1992). In 2011, the Financial Times pointed towards conspicuous settlement prices that cast doubt on the integrity of the VIX settlement procedure (Kaminska, 2011). Several lawsuits (still pending) examine a potential market manipulation in the VIX fixing, which is supported by empirical evidence (Griffin and Shams, 2018).[1] Since the open interest in VIX derivatives is in the order of 5 to 10 billion dollars, rigging the fixing price can incur substantial damage to bearers of VIX derivatives.

In this paper, we examine indications of manipulation in the VIX. We are particularly interested in checking whether any potential rigging has slowed down since 2017, when Griffin and Shams (2018) drew attention to the vulnerabilities of the VIX. Using high-frequency data, we assess whether VIX settlement prices are in line with intraday trading and study to what extent settlement price outliers spill over into continuous trading. We further investigate what conditions are peculiar to settlement days in contrast to the other trading days. Considering open interest data, we point out how various trader groups behave differently in settlement weeks, aiming to reveal the actors behind a potential manipulation. Finally, we put the VIX into perspective to realized volatility since manipulation would distort the quality of the VIX as a volatility hedge. For the use case of holding a VIX exposure beyond a short time horizon, we address the difficulty of being dependent on potentially manipulated settlement prices. We also suggest strategies for investors to mitigate the costs of possibly manipulated settlement prices.

We find indications of upward manipulation in the VIX index: The settlement price is on average 5.5% above the first trading quote thereafter. In addition, the settlement price lies 6.9% higher than the mid-point of intraday high and low VIX

[1]In contrast, Saha et al. (2019) find no evidence for manipulation of the VIX index.

levels. Considering fixings outside the span of index quotes within the first 60 minutes of trading and last close, we observe such exceptions on more than 42% of the days with a tendency of strikingly high (as opposed to low) settlement prices. In the upward cases, the difference lies between 11% and 16% of the prevailing index level, on average. The downward cases are more moderate. These results are driven by a number of outliers, whereas most settlement prices remain within a reasonable range. We estimate that positive (negative) manipulation attempts take place on 13.4% (3.5%) of the days. Over time, the evidence for a rigged VIX index is most pronounced in the years 2017, 2018, and 2020, when it was well known by market participants how to influence the VIX in a cost-efficient manner.

Zooming in to the hours after settlement, we compare the VIX to its three-month adaption VIX3M which is analogously computed for a longer horizon. This comparison serves as a placebo test because there are no incentives for manipulating the VIX3M: Derivatives upon the VIX3N seem not sufficiently liquid and the options corresponding to the VIX3M are disjunct to those determining the VIX. The difference reaches 1.5% and dissolves over the first 90 minutes in the case of upward gaps.

On settlement days, the put/call ratio of the underlying S&P 500 options surges on average by 10.9%. The observation is in line with rigging the VIX index because the most cost-efficient manipulation is achieved by predominantly trading out-of-the-money (OTM) put options. A time series decomposition shows that the increased put/call ratio is particularly present on settlement days, exceeding the day-specific level of all other days by almost 80%. A comparable increase is not present in the VIX3M placebo index.

From the Commodity Futures Trading Commission's (CFTC) open interest data[2], we extract that leveraged funds increase their exposure on average by 10% within the seven days before settlement. Additionally, we find that this increase is not

[2]The CFTC is a regulatory agency of the U.S. government to which all large market participants report their open interest held every Tuesday. For instance, Ho and Lauwers (2017) provide background on the CFTC reports and consider the exposure of money managers in commodity futures as a predictor of commodity producers' stocks.

driven by market participants with the eight largest open interest positions, but rather by entities with smaller exposures.

Looking at changes after the publication of Griffin and Shams in 2017 (on SSRN), we find that deviations are more isolated (single days), while metrics like the number of outliers regarding the intraday span decrease. A decreasing gap width and fluctuation range point towards a market starting to accustom itself to and incorporating deviations more easily.

We regress the VIX index level on different measures of realized volatility. Residuals evaluated separately by settlement and non-settlement days reveal that the former are associated with higher VIX levels overall. However, settlement prices exceed these levels, suggesting that high settlement prices radiate into the day. A difference-in-difference estimator indicates an abnormal effect of 4.8% in the fixing price compared to realized volatility.

Analysing the term structure of the next five months' future contracts, we detect an abnormal effect of the front-month contract (to be settled) of an additional 1% premium. Contracts with longer maturities do not show a significant difference. Settlement prices tend to be more intensely abnormal when futures are in contango, linked to a more calm market environment, than otherwise.

In the literature, to the best of our knowledge, only two papers focus on manipulation in the VIX. Griffin and Shams (2018) analyse intraday option prices and volumes around the settlement. They find that the volume of options traded increases abnormally in the settlement auction and only in the OTM options relevant to the VIX formula. Option volumes furthermore exhibit a distribution proportional to optimal cost-effectivity for manipulative purposes. While Griffin and Shams (2018) discuss and rule out a battery of alternative explanations, they admit that they cannot reject all legitimate explanations.

The second paper regresses VIX index levels on moving windows of the S&P 500 level and volatility (Saha et al., 2019). The findings suggest that the VIX and its settlement prices are in line with market fundamentals.

We contribute to this strand of the literature by considering the settlement prices themselves and their relation to VIX intraday index values based on high-frequency data ranging from 1990 to 2021. In contrast, Griffin and Shams (2018) concentrate on intraday option data, while Saha et al. (2019) only have end-of-day data at their disposal. In addition, we include the VIX adaption VIX3M as placebo index since its underlying options have different expirations and would thus not be affected by a manipulation of the main VIX index. Based on this complementary data set and additional approaches, we present novel indications of manipulation. While existing literature primarily discusses whether the fixing prices are manipulated or not, we add characterisations of potentially manipulated settlements such as their scale and directions (upwards or downwards). We are also the first to study the development over time and identify the accelerating trend of growing deviations. Furthermore, this paper is – to the best of our knowledge – the first to investigate open interest data of the CFTC regarding VIX manipulation, shedding light on the behaviour of different trader groups around the VIX settlement. We are also the first to analyse the term structure of VIX futures with respect to patterns of potential manipulation. Finally, we suggest strategies for investors to circumvent the costs of manipulation while keeping a VIX exposure.

Related literature outlines the dynamics of the VIX relative to the general market and derive pricing methods for the VIX.[3] We contribute to this strand of the literature by characterising the VIX' relation to the VIX3M as well as by explaining the VIX' movements. Various papers discuss the VIX as an instrument in portfolios (Doran, 2020; Hood and Malik, 2013). Our proposed strategies to circumvent potentially distorted settlement prices while keeping a VIX exposure add to this literature. Considerable literature puts the VIX in perspective to realized volatility.[4] We refine these considerations by adjusting for settlement days.

A growing number of researchers studies market manipulation. In this field, considerable literature focuses on stock price manipulation (Allen and Gale, 1992;

[3] Whaley (2000), Carr and Wu (2006), Bardgett et al. (2019), Fernandez-Perez et al. (2019), and Hülsbusch and Kraftschik (2018).

[4] Andersen et al. (2003), Drechsler and Yaron (2011), and Engle and Gallo (2006).

Jarrow, 1992; Aggarwal and Wu, 2006; Comerton-Forde and Putniņš, 2011). For other asset classes, Merrick et al. (2005) analyse bond futures market manipulation and point out strategic trading behaviour with a focus on delivery squeezes. Other benchmark indices like the Libor proved to be fertile hunting grounds for manipulative practice (Abrantes-Metz et al., 2012; Fouquau and Spieser, 2015; Ashton and Christophers, 2015). Our paper adds to this literature by contributing to the discussion of whether manipulation takes place in the VIX as well as by characterising the magnitude of potential manipulation and its development over time. Moreover, we provide an approach to measure a normalisation effect of index values after the manipulation incentive has ceased.

Several studies review the term structure and volatility premium contained in the VIX (Johnson, 2017; Huang et al., 2019; Luo and Zhang, 2012; Cheng, 2019). We contribute to this strand of literature by sketching the role of settlement days and defining a contango magnitude to estimate an abnormal effect. Besides, we introduce distinctions of VIX gaps by contango and backwardation. Closely connected, Shu and Zhang (2003) elaborate on the relationship between the VIX and realized volatility. We extend this approach to use the model's predictions to measure an abnormal effect associated with settlement days.

2 Data

The data used in this paper comprises daily index data, intraday index data on a minute basis, and open interest data collected once per week.

2.1 Daily Index Data

The daily data consists of four data points for each time series per day, comprising the opening and closing quote as well as the high and low attained within the day. Cboe Exchange, Inc. provides such historical data going back to 1990 until 2021 for the VIX, VIX3M, and other relevant U.S. indices. Besides, the exchange publishes volume statistics of S&P 500 (SPX) and VIX options as well as the settlement prices of the VIX and the settlement dates. Concerning futures, Cboe Exchange, Inc. supplies end-of-day data of VIX futures for the next six months' expirations.

2.2 Intraday Index Data

Intraday data with one-minute granularity has been collected from Active Tick LLC and Barchart.com, Inc.[5] Including the years 2008 to 2021, there is one quote per minute of the VIX, VIX3M, and the S&P 500 indices, resulting in almost 1.5 million observations per index. The VIX3M is a variant of the VIX, which the Cboe computes by the same formula. However, it replaces the options serving as parameters by different expirations fitting the three-month time horizon.

2.3 CFTC Open Interest Data

The Commodity Futures Trading Commission (CFTC), a regulatory agency of the U.S. government, collects statistics from large market participants on their open interest held. Relevant market participants have a legal obligation to report to the CFTC every week. These reports comprise for any instrument – e.g., the VIX – the aggregated open interest (number of contracts) held in that instrument to Tuesday's record date (end of the day). Based on these reports, the CFTC publishes two sets of aggregated data divided by long and short exposure.

First, it provides commitment data on the open interest held in the groups of leveraged funds, asset managers, dealers, other reportables and the open interest in sum, including market participants not obligated to report (non-reportables). The values are expressed in USD 1000 * index value (contract value). The CFTC allocates each market participant to one category based on the following definitions:[6] Dealers comprise the sell-side, in particular banks, intermediaries and brokers. Asset managers include institutional investors like insurance companies, pension funds, mutual funds, endowments and portfolios managed predominantly for institutional accounts. Among leveraged funds, hedge funds and money managers are most prominent. Other market participants, which are obligated to report as they exceed the

[5] We use two data providers to cross-check the collected data.
[6] Commodity Futures Trading Commission, *Explanatory Notes: Traders in Financial Futures.*

minimum requirements but cannot be allocated to one of the categories as mentioned above, form the category of other reportables.

Second, the CFTC supplies data on open interest concentration, namely the net open interest held by the largest eight and four market participants.[7] The balanced interest held – neutralized by a reverse position – which the CFTC has subtracted, is reported separately as the "spread". We have retrieved the data used for this paper from Quandl Inc., a Nasdaq subsidiary. Quandl has pre-processed the data from the raw reports published by the CFTC.[8] The data horizon in this analysis ranges from 2006 to 2021.

3 Are Settlement Prices in line with Intraday Trading?

This section aims to put the monthly VIX settlement prices into perspective by comparing them with the intraday VIX quotes. We will answer the central question whether the settlement prices are regularly on abnormal levels compared to continuous trading after settlement.

The VIX is particularly susceptible to manipulation due to its cash-settlement (Kumar and Seppi, 1992). As opposed to a settlement in kind, cash settlement facilitates making profits from manipulation. The VIX is an index based on a formula representing a weighted sum of prices of S&P 500 *OTM* options, to capture implied volatility more adequately. However, these OTM options are highly illiquid compared to in-the-money-options (Griffin and Shams, 2018; Saha et al., 2019). In contrast, investors trade the VIX derivatives heavily, comprising futures and options on the index. They are settled in cash based on the same formula that serves to calculate the continuous VIX quotes.

The Cboe determines option prices in a special auction taking place on the monthly settlement days in the morning before regular trading begins. During the auction, there is no order execution except the order book clearing at its end.

[7]It is based on each market participant's open interest, which is netted concerning the excess long and short exposure.

[8]Data quality has been cross-checked manually with randomly drawn raw reports.

These execution prices serve as parameters to the VIX formula, yielding the VIX derivatives' settlement price.

The natural approach to take advantage of this procedure is as follows: A manipulator enters a considerable position in VIX options or futures, regardless of whether this is long or short exposure. This transaction can be executed at low premiums to the market consensus of implied volatility as spreads are small in the liquid market for VIX derivatives. In the auction on the settlement day, the manipulator can drive the prices of the underlying illiquid OTM-S&P 500 options to an abnormal level by buying or selling them to move their prices in the direction favourable to the cash settlement he is going to receive on his VIX derivatives. The cost representing the unjustified premiums paid on S&P 500 options is negligible compared to the difference obtained in the cash settlement of the VIX derivatives. However, outside of the settlement auction, there is no direct incentive to manipulate option prices in advance to or after the settlement because solely the clearing price matters. As a result, we compare the settlement days to all other days to detect manipulative patterns.

3.1 Opening Quote

Settlement prices can be most directly compared to opening quotes. In the settlement auction, traders can provide orders from 8.30 a.m. until 9.15 a.m. (Eastern Time). In the following 15 minutes, the order books of options included in the VIX formula are frozen.[9] The Cboe executes SPX option orders at 9.30 a.m. These execution prices are the parameters to the VIX formula yielding the settlement price. Continuous trading begins directly after that.

To gauge whether there is a bias between the settlement price and opening quote, we consider their difference (settlement price minus opening quote). To facilitate interpretation, we divide this difference by the VIX opening quote. We assume that this difference should be relatively small, given that the amount of new information is limited due to the 15 minutes time difference. The average difference should

[9]Market participants can still submit orders in other options.

also hover around zero in absence of manipulation, because positive and negative information events should be as likely.

The first column of Table 1 shows the empirical results. The settlement price was 5.5% higher than the quote not being subject to potential interventions, on average. The median difference (1.5%) is lower but with a high standard deviation (59.2%). This contrast can be traced back to the positive skew (0.7), pointing out that single days drive the higher mean. If manipulation is the reason for these differences, it implies that interventions are conducted to a substantial extent in isolated cases, rather than more frequent and more cautious rigging. These single occurrences can be observed in Figure 1. There are multiple differences exceeding one index point. However, there are also several substantial negative deviations. The main difficulty in further quantifying the scale of manipulation is that it is not apparent to what extent, if any, these numbers are regular market effects or random deviations instead of manipulative biases.

To grasp how the difference between settlement price and open develops over time, we aggregate them for half-year periods. We employ the absolute value of the difference such that positive and negative differences do not cancel each other out. Figure 2 presents this development. We observe a growing trend starting in 2015, moving upwards within a difference of 2 to 6 index points per half-year. These findings are consistent with an increasing extent of manipulation conducted. The accelerating trend might be driven by imitators, who have learnt about the possibility to manipulate and subsequently taken part in the conduct.

3.2 Range of Daily High and Low

However, another explanation of this observation might be that in fact – for some reason apart from manipulation – the opening quotes are biased. To obtain further information on what happens after the settlement took place, a further natural reference are the highest and lowest index values after settlement. We use a scale that is defined by the low – fixing the point of 0% – and the high marking the point of 100%. In a second step, the settlement price is mapped accordingly to this scale

$$\frac{\text{Settlement Price} - \text{Low}}{\text{High} - \text{Low}}$$

expressing the position of the settlement price on this scale as a percentage[10]. These high and low values refer to continuous trading and exclude the settlement auction. For this reason, settlement prices may be below 0% or above 100 % on this scale if they are outside the range in which continuous trading takes place. However, such occurrences should be relatively infrequent as the continuous trading starts right after the fixing of the settlement price. Furthermore, by symmetry of information arrival and the law of large numbers, we expect the position of settlement prices on the scale to be around 50%.

The second column of Table 1 shows the empirical results. On average, the settlement prices are 6.9% above the mid-point of 50%. We test the realizations against the null hypothesis given this mid-point. Both a standard t-test (for the mean) and the Wilcoxon signed-rank test (for the median) indicate that realizations are higher than the mid-point. 13.4% of the settlement prices are above the intrady high, while 3.5% of the settlement prices are below the intraday low. These findings are consistent with manipulators rather biasing the VIX upwards.

Figure 3 depicts a histogram of the intraday position between high and low, highlighting that the cases above the span are – in addition to the higher frequency – also more pronounced in magnitude. This impression supports the hypothesis that manipulation only occurs on selective dates. Overall, our observations are remarkable because they reveal that settlement prices strongly differ from subsequent intraday trading levels.

3.3 Gaps

We next take into account the preceding day's close. Both close and open prices are particularly reliable benchmarks as they are usually determined under high liquidity

[10]relative to the low at 0% and the high at 100%

and without an incentive to manipulate them related to the cash settlement. The settlement price is recorded in the time between the closing price of the previous day and the settlement day's open. Therefore, it should be likely that in the majority of cases the settlement price is also in the interval between the two values (last close and following open). To count how often this is actually the case, we put settlement prices into three categories. First, settlement prices are seemingly artificially high when they are above the last close and above the following open. This situation implies a "positive gap", i.e., the settlement price jumps above the close and reverses by falling again right after settlement. We define the magnitude of this gap as the settlement price minus the arithmetic mean of the open and the last day's close. Vice versa, "negative gaps" are recorded if the settlement price is below both the open and the close of the preceding day. If so, we define the magnitude of the gap in the same manner. Third, a settlement day falls into the category "no gap" if the settlement price is between the open and the last close. For the category "no gap", we do not define any magnitude.

As a potential criticism of the previous approach may be a dependence on the opening quote, we also use later quotes during continuous trading. Concretely, we use the reference times in five-minute steps until 9:45 a.m., i.e., 9:35, 9:40, 9:45, and then in fifteen-minute steps until 10:30 a.m. Table 2 shows the results. Panel A reports the frequency of positive and negative gaps, which is the number of positive and negative gaps divided by the total number of settlement days. Across all reference times, we notice a gap on a substantial share of the settlement days. The highest frequency (57.8%) occurs at 9:30 a.m., while we also observe a high fraction of gaps for later reference times. There appear to be more positive gaps, which is again in line with the previous findings that positive abnormal deviations occur more often compared to negative ones.

Panel B presents the magnitude of positive gaps. The gap reaches its highest mean (16.1%) at the open (9:30 am). As before, we find a positive skew and accordingly medians that tend to be lower than the corresponding means, being nevertheless primarily above 10%. The standard deviations reveal a higher dispersion for the opening time at 9:30 (29.0%) and 9:35 (12.8%), while they seem to begin to normalize after 9:40 a.m. Panel C reports results for negative gaps.[11] The mean and median gap magnitudes are both in the range of -9%, again pointing towards a less pronounced negative bias.

3.4 Watermarks within first 15 Minutes

As a final investigation on whether settlement prices are in line with intraday trading, we zoom in to the first 15 minutes of each settlement day and consider the intraday data in one-minute granularity. We compute the highest watermark, denoted by h_n, up to the n-th minute ($n \in \{1, ..., 15\}$) of trading on each settlement day, and analogously the "lowest watermark", denoted by l_n, in the opposite direction. Our approach implies that the highest watermark of x_1 (x_n denoting any time series) is itself, $h_1 = x_1$, and for any $n \geq 2$ the watermark of x_n is $h_n = \max(x_n, h_{n-1})$ recursively. We define the "lowest" watermark as the equivalent in the negative direction. Formally, again $l_1 = x_1$ and for $n \geq 2$ the "lowest" watermark of x_n is $l_n = \min(x_n, l_{n-1})$. These watermark values define a range, the interval $[\, l_n, h_n \,]$, for any n-th minute on a given settlement day. By the same approach as in Section 3.2, we map the settlement price to this scale defined by 0% at l_n and 100% at h_n.

Figure A1 shows the histograms of the settlement price position for every n-th minute, $n \in \{1, ..., 15\}$. Dotted vertical lines in the plot highlight the thresholds of 0% and 100% and mark the watermark range attained so far by index values. The histogram's solid vertical lines represent the median.

There are significant deviations for the first three minutes until the histograms normalize for the remaining minutes. For the first minute, there is a symmetric histogram around the median of approx. 200% of the $[\, l_1, h_1 \,]$ range, indicating that settlement prices are systematically higher than opening quotes. The median shifts to 150% in the second minute, then remaining close to the 100% threshold and crossing it in the sixth minute. Nevertheless, medians stay in the upper quartile up to the 15th minute after the market opens. All in all, the histograms support the

[11] Note that there is a negative sign by the definition of this measure.

notion that settlement prices are systematically higher than intraday VIX values. This effect does not depend on the open quote and also holds when considering the first minutes after the open instead of the open quote.

3.5 Do Index Values normalize after Settlement?

It seems natural that a shock of settlement price manipulation continues to influence the index also after manipulation has ceased as the effect on supply and demand endures for a certain amount of time before the index arrives at an unbiased level again. A striking correction effect at the beginning of the continuous trading may indicate potential manipulation.

To study the intraday index values of the VIX on settlement days, we consider the deviation of the VIX in relation to the VIX3M in percent.[12] To aggregate these time series over the different days, we plot their means split by four categories. These categories are "positive gap", "negative gap" and "no gap" defined as above. Recall that for positive (negative) gaps the settlement price is above (below) both open and preceding close, and else there is no gap. For comparison purposes we add non-settlement days as a fourth category.

Figure 4 reports the results. Most notably, we see in the case of positive gaps that starting with strong first 30 minutes, the overall correction takes roughly 90 minutes. The plot also indicates that the scale of the correction is approximately 1.5%. There is a weaker effect for negative and no gaps, although also these categories arrive at an average level after 90 minutes. There is a systematic difference enduring throughout the whole day between the three categories of settlement days which the gaps' definition might (at least partly) explain. However, the non-settlement days serving for comparison are indeed almost constant around 0%.

Alternatively to considering a difference, we finally construct a stopping time approach: For each settlement day, we record the time it takes until the index value reaches the settlement price until its close. We also note whether the index value does

[12] More precisely, we use the centred and standardized difference of VIX and VIX3M. We consider the percentage deviation from the mean by dividing by the mean of their means and multiplying by their average standard deviation.

not reach the settlement price of the same trading day. Figure A2 in the Appendix shows a histogram of the results. While around half of the settlement prices are met within the first 30 minutes, even 13% are not reached at all on the settlement day. The remaining settlement prices are mainly evenly distributed throughout the day. These findings speak again in favour of the conjecture mentioned earlier that manipulation is conducted selectively in single cases, while it is not in line with manipulation on a majority of settlement days.

4 What Settlement Day Effects do occur?

This section shows to what extent settlement days differ from the other days. Specifically, we review the external conditions, being possibly consequences of manipulation. We focus on the put/call ratio of SPX options, the fluctuation range of the VIX and VIX3M, and the open interest in VIX derivatives.

4.1 S&P 500 Options

Characteristically to the VIX formula, put options receive such a high weight that the call options' influence is negligible. Specifically, the VIX formula is[13]

$$\mathrm{VIX} = 100 \cdot \sqrt{\sum_i \frac{K_{i+1} - K_{i-1}}{T_i \cdot K_i^2} \cdot 2 \cdot e^{R \cdot T} \cdot Q(K_i) - \frac{1}{T} \cdot \left(\frac{F}{K_0} - 1\right)^2}$$

where the variables represent:

T_i	i^{th} option's expiration time;
F	forward VIX level implied by index option prices;
K_0	highest option strike price below F;
K_i	i^{th} out-of-the-money option's strike price;
R	risk-free interest rate to expiration (U.S. Treasury);
$Q(K_i)$	trade price of the option at settlement.

Only out-of-the money (OTM) options are included, while all other options are ignored. Each individual option receives a weight proportional to $\frac{1}{K_i^2}$, implying the higher weight of put options: Put options are deeper OTM the lower the strike price is (implying a higher $\frac{1}{K_i^2}$). Excluding in-the-money (call) options removes the call

[13]Cboe Exchange (2019).

options whose summand would otherwise be influential, as the low K_i, corresponding to a high $\frac{1}{K_i^2}$, are in-the-money.

For instance, assume a hypothetical S&P 500 index level of 4000. Suppose further that there are put and call options with strikes ranging from 2000 to 6000 (in 100 index point steps). Due to including only OTM options, the strikes between 2000 and 3900 are put options, while the other options with strikes between 4100 and 6000 are call options. In this numeric example, put options account for 76% of the sum of all weights, whereas call options make up 24%. These numbers signify that put options are three-times more relevant than call options.

This mechanism incentivizes manipulators to concentrate on put options. Such behaviour would leave footprints in the put/call ratio of these SPX options, representing the ratio of the volume traded in puts divided by the volume traded in calls. Therefore, manipulation – being conducted by primarily trading put options – would result ceteris paribus in a higher put/call ratio.

Panel A of Table 3 sketches the statistics of the put/call ratio of SPX options. It outlines descriptive statistics on non-settlement days in the first column and the same measures for settlement days in the second column. There is a higher put/call ratio on settlement days (mean: 1.94, median: 1.90) compared to the other days (mean: 1.74, median: 1.71). The difference of 0.19 is significant at the 1% level both under a t-test and the Wilcoxon signed-rank test. Figure A3 in the appendix discloses a histogram, in particular underlining that there is an evenly distributed deviation between the two kernel density estimates. These findings are consistent with manipulative trading on settlement days. The difference indicates that the extra trading in put options amounts to approximately 10%.

Table 3's Panel B presents the results for an adjusted time series of the put/call ratio. We divide each day's put/call ratio by its moving two-week average (ten trading days) and subtract 1 for centring. This approach aims at including the market environment that prevailed in the two weeks before. We choose a length of two weeks since it comprises half of the four-week period between settlements. A shorter period would make the measure more volatile, while a sufficient distance to the previous settlement day is necessary to avoid a spillover (from the earlier settlement day). Thus, Panel B in Table 3 implies that under this transformation the difference found is 14% (mean) or 12% (median), both significant at the 1% level. These observations show that discrepancies of settlement days exceed 10%, which would be in line with manipulation.

We also employ a time series decomposition of the SPX options' put/call ratio, by decomposing daily put/call ratios additively into three components. Every period lasts four weeks (20 trading days) and settlement days are fixed as day 20.[14] The first component is a repetitive series of length 20, assuming the same value on any day n, $n \in \{1, ..., 20\}$. The second component constitutes a trend, deduced as the moving average of the preceding ten days. Finally, the third component is the residual, incorporating the remainder such that the put/call ratio equals the sum of the three components. The appendix provides further details.

The repetitive component is the most relevant part as it exposes the systematic effect to settlement days and reveals what happens on the days before it. Figure A4 depicts this repetitive component whose fluctuations range roughly within -0.1 and $+0.1$ and peaking on settlement days almost at $+0.18$. Notably, the minimum at -0.14 is attained on day 19, one day before settlement.

Hence, the time series decomposition confirms the relevance of the settlement day as the magnitude of the repetitive component on settlement days is approximately 80% higher than on non-settlement days. Moreover, this result implies that the difference of the SPX put/call ratio in Table 3 is exclusive to settlement days (i.e., isolating any other day instead of the settlement day does not yield similar results). Overall, we find a settlement day effect in the options which determine the VIX. This effect is consistent with manipulation as cost-efficient manipulation would entail a higher put/call ratio.

[14]If there are more than 20 trading days within a month, the first day(s) is (are) dropped since our procedure requires periods of equal length. The omitted days are of little relevance as the emphasis lies on the days before settlement days.

4.2 Index Value Fluctuation Range

Next, we consider the range in which the VIX index values move within the day. We define this fluctuation range for every day as the difference of the highest minus the lowest value. To reveal what effect is characteristic of the VIX settlement, we compare the results to the fluctuation range of the VIX3M.[15]

Table 4 provides the results. For the VIX, Panel A points out that the fluctuation range mean of 1.58 index points broadens to 1.92 on settlement days. This difference of 0.35 index points is significant at the 5% level under a t-test. The median difference (0.27) is significant at the 1% level using a Wilcoxon signed-rank test. Hence, the most extreme fluctuations occur on the settlement day, i.e., after the special opening auction. The VIX3M placebo index does not exhibit significant deviations (Panel B), which is again in line with manipulation in the VIX index.

4.3 Open Interest

There is weekly data available on the overall open interest from the CFTC. The CFTC collects the statistics for the record date of Tuesday's end-of-day. This point in time is directly before the next morning's settlement. Consequently, a week in which a monthly settlement happens shows up in the data based on the exact settled volume plus the open interest with different expirations. To eliminate a general trend, we subtract – in each settlement week – the arithmetic mean of the preceding three weeks from the open interest. The resulting time series provides the excess (difference) for each settlement week to the moving average of three non-settlement weeks, expressed in 1000 USD * index value.

Figure A5 plots this time series. From roughly 2011 to 2017, there is an increasing trend which normalizes slightly from 2018 onwards. With a few exceptions, the majority of differences is positive. This aspect implies that in settlement weeks the open interest is larger than in other weeks. Suppose there is no systematic difference in the distribution of expirations within the open interest. Then, this observation implies a net plus in the open interest to be settled within the seven days

[15]Table A1 in the appendix gives descriptive statistics of the two VIX-versions themselves.

before settlement. One might assume that market participants tend to reduce their exposure in the week before settlement to circumvent the settlement price fixing. This behaviour is rational if traders are aware of potential costs of unrealistic fixings. However, we observe the opposite. This phenomenon might occur as investors trust the settlement procedure and go openly into it with their positions. However, upright investors might also be net reducers of their positions before settlement, following a more risk-averse approach. Then, the extra exposure could possibly be additional exposure entered into by manipulators.

5 Who drives the VIX Market on Settlement days?

In this section we try to reveal the market participants behind the potential market manipulation by analyzing CFDC open interest information.

5.1 Concentration: Largest Four and Eight Market Participants

At first, we review data on open interest concentration comprising information about the largest four and eight market participants' exposure. We focus on the market participants' position relative to the overall open interest by considering it as a percentage of the overall open interest. All calculations are made separately for the largest four and eight parties for settlement and non-settlement days. We also differentiate between long and short positions and analyse week-on-week changes with respect to the week before settlement.

Table 5 reports the statistics. Concerning the largest four open interest volumes (first three columns), the long share of open interest is on average 2.2 percentage points lower on settlement days, which is significant at the 10% level. The analogous difference in the short direction is more moderate and statistically insignificant. Related to week-on-week changes (Panels C and D), there is a more substantial effect: The open interest falls on average by 6.1 (long) and 2.6 (short) percentage points, which is significant at the 1% and 10% levels, respectively. Results are qualitatively similar for the largest eight open interest volumes (last three columns).

Tables A2 and A3 in the appendix provide further summary statistics on the concentration data. Figures A7 and A8 in the appendix outline the development over time in greater detail, showing separate boxplots for each year. For every year, there are two boxplots for settlement and non-settlement days. Both exposures of the largest four and eight decline, with moderate negative outliers in 2017 and 2018 and subsequently reverting slightly in a positive direction.

As the information gathered so far does not distinguish between a potential upward or downward manipulation, we investigate the correlation to positive and negative gaps. In this context, we focus on the largest eight as this group already includes the largest four and is – if in doubt – the more interesting one. Which market participants finally reach the largest four is rather random and thus more volatile, making conclusions more prone to errors. As before, we record a positive (negative) gap if both VIX open and last close are below (above) the settlement price, and otherwise we record no gap. This definition implies that we only consider settlement days and drop all other days. Hence, we compare "gap"-days to no "no gap"-days similarly to comparing settlement days to non-settlement days. That is, after first separating positive and negative gaps, we compute descriptive statistics on the subgroups of "gap"-days as opposed to "no gap"-days.

Table 6 describes the results. Independently from whether we consider absolute exposure or week-on-week changes, positive gaps come with slightly less long exposure. On the other hand, negative gaps are associated with additional long exposure compared to "no gap" settlement days. However, these differences are not significant at the 10% level. Furthermore, concerning the short exposure, positive gaps correlate with materially less exposure: The largest eight are on average 3.41% less engaged than the week before connected to gaps, significant at the 5% level (t-test) or 10% level (Wilcoxon signed-rank test). On the contrary, the short exposure is – linked to gaps – by 7.93% higher, significant at the 1% level, and 3.69% higher than the preceding week. In conclusion, there is no significant deviation connected to gaps in the long direction. In contrast, negative gaps come with a substantial surplus in short exposure, albeit we also record significant distortions concerning positive gaps. Under the hypothesis of manipulation, the observations would indicate that the largest eight manipulate downwards, if at all. However, the more relevant upward manipulation direction associated with positive gaps reveals a negative difference. This number tells that the largest eight were systematically lower engaged in the direction manipulators would enter a position in and indicates that they suffered more than they earned linked to the material deviations, possibly due to specific manipulation. Below the line, the largest eight seem to be likely not involved in manipulation.

5.2 Commitment of Investor Groups

Now turning to the open interest held by groups, Figure A6 portrays the development of the absolute exposure split by direction (long or short). The vertical lines highlight the publication of Griffin and Shams (2018), the first rendering the journal's website posting and the second standing for the distribution of the paper version. Concerning long exposure, we see first a simultaneous ascending trend across the groups. While there is no reversal of the trend apparent linked to the publication of Griffin and Shams (2018), leveraged funds' exposure reverts from its peak, which occurs at the time of publication. Assessing the short exposure, asset managers' exposure remains principally constant. Dealers show an ascending trend, predominated by a threefold more intense rise in leveraged funds' short exposure. Connected to the publication, there is a somewhat slighter decline.

In brief, VIX exposure overall rose over the years. Linked to the publication, leveraged funds' exposure trended downwards, while no material changes occurred in the case of the other groups. This exploration would be consistent with leveraged funds being active in settlement price manipulation, therefore tending to refrain from this strategy after the publication and the corresponding media attention.

However, to analyse groups' commitment, we spotlight the week-on-week changes which Table 7 states. Given that this change from week to week reflects the chronological sequence of the commitment's development, it is more relevant than the absolute figures. For reference, the appendix' Table A4 contributes the absolute exposure. Strikingly, leveraged funds are by 10.1% (mean; median: 7.2%) more heavily exposed to long VIX derivatives on settlement days. This variation is significant at the 10% level (t-test) and the 1% level (Wilcoxon signed-rank test). This difference is relevant to the overall market as leveraged funds represent the second largest group after dealers by long positions. This deviation is the only outstanding one on the long side. Conversely, leveraged funds possess a smaller commitment in the short direction associated with the settlement. This reduced short exposure intensifies the effect on the long side. Other reportables signify the second most substantial difference, being by 8.8% (mean; median: 5.7%) more intensely committed in the short direction. The other differences are close to zero. The appendix contains summary statistics on the CFTC commitment data for further reference (Tables A6 and A7). Figures A9 to A11 in the appendix supply boxplots for every year, complementing the evolution over time and contributing separate boxplots for the settlement days of each year.

In sum, leveraged funds exhibit a conspicuous deviation on settlement days, being vastly stronger committed in the long direction. The other groups expose no special effect except other reportables. However, Table A4 points out that the absolute open interest of other reportables is negligible, being the smallest and representing less than 5% of the overall open interest. Besides, this group – being the fallback category – is the most heterogeneous one. In connection with the notion emerging from the previous sections, the findings concerning leverage funds are consistent with with a preference for upward manipulation. Overall, the observations indicate that leveraged funds might be the manipulators, being mostly active in long positions. For this reason, we consider this group's long exposure in the following in greater detail.

For the sake of carving out how leveraged funds' behave on settlement days, we consider the difference of open interest held on the settlement day minus the moving three-week average. This measure signals the excess of open interest owned relative to the settlement, expressed in 1000 USD * VIX index value. Hence, it is undistorted by changing VIX levels. Panel A of Figure 5 illustrates this measure. Until 2012, there are negligible fluctuations around zero. After that, variations emerge, whose majority is in a positive direction. These cases entail a higher commitment on settlement than the moving average, albeit there is a minority of substantial negative deflections. An increasing trend until 2018 becomes apparent, returning after that towards visibly lower levels, although not entirely dissolving.

Besides, we proceed with another approach that takes into account leveraged funds' relative changes from week to week. To this end, we pick settlement days and divide their exposure by the open interest held in the week before. This quotient yields the percentage excess in comparison to the week before. Aiming to put this percentage change into perspective to the overall trend of the open interest, we subtract the arithmetic mean of these changes over all years considered. Finally, we obtain the relative surplus of settlement days proportionately to the week before, adjusted by the long-term trend. Panel B of Figure 5 plots the results. The majority of bars is positive, although other bars reveal material negative deviations. This fact demonstrates that – in comparison to the preceding week – the exposure increases rather abruptly in the seven days before settlement. Advanced information on what happens within these seven days is not available due to the granularity in which the CFTC collects and publishes the data.

Summing up, leveraged funds are more substantially engaged in the long direction of VIX derivatives before settlement than the three weeks before. Furthermore, the funds enter into a relevant portion of this extra exposure in the seven days ahead of settlement. These observations further support the hypothesis that leveraged funds might bear responsibility for a suspected manipulation.

6 What has changed since 2017?

The paper of Griffin and Shams (2018) was uploaded on SSRN in 2017, directing broader attention to the vulnerability of the VIX. Raising this awareness may trigger opposite effects: (i) The public inspection may frighten manipulators, who would consequently stop engaging in criminal conduct. (ii) The paper delivered the recipe on how to conduct manipulation in a cost-efficient manner. As a result, free-riders might have learnt about the possibility and have started to manipulate the VIX.

Accordingly, this section examines the changes between the period until (including) 2017 and the years after that up to 2021. Table 8 depicts key measures from the above sections, split by the two time periods.

Panel A shows metrics on the settlement price itself. The immediate jump between settlement and open quote almost doubles (5% until 2017 compared to 9% after that). In contrast, the count of settlement prices lying outside the intraday trading range decreases in both directions: strongly (15% to 3%) in the upward direction and moderately in the downward direction (4% to 3%). Also, the average gap width decreases on average from 7% to −9%. These observations indicate that the higher sums of deviations are driven by fewer, more pronounced days. This notice would be in line with single more aggressive manipulations.

In Panel B, we report the difference in the fluctuation range. This span between each day's high and low enlarges both on non-settlement days (1.47 to 2.74) and settlement days (1.85 to 2.26). The difference for settlement days (0.41) is underproportional to the other days (1.27). Regarding the difference between settle and other days within the periods, we find a weakening effect after 2017; the fluctuation range is smaller on average on settlement days than on the other days. This observation is consistent with market participants starting to incorporate deviations in settlement prices more easily.

Panel C presents the development of the put/call ratio of S&P 500 options. We focus on the week-on-week change compared to the preceding week because this measure seems most robust when considering the comparably short second period, independent of a systematic rise (particularly regarding the COVID-19 pandemic effects in the second time period). The put/call ratio change on other days (-0.05 to -0.02) and settlement days (0.09 to 0.08) come closer to each other. Concerning the difference within the periods between settlement and other days, there is a lower average after 2017 (0.10 versus 0.14), although statistical significance increases after

2017 (p-value $< 10\%$). This relation would be well explicable with manipulators further concentrating on single days.

Moving on to open interest data, Panel D shows the absolute long exposure of leveraged funds. While their position increases slightly on other days after 2017 (additionally 147 units after 40 286 before), there is an accelerating increase on settlement days (additionally 2 416 units after 43 963 before). The difference within each period of settlement versus other days increases notably (3 677 to 5 946). For comparison, Panel E depicts the long open interest of all other market participants combined, which tend to rise similarly to the positions of leveraged funds.

In total, the observations point towards hypothesis (ii): A potential fear of getting caught seemingly does not outweigh the advancing development in which free-riders are joining the culprits. However, manipulation attempts might start to be conducted in a more focused manner, choosing particular days and trading aggressively. There seems to be some habituation effect, implying that the market also begins to incorporate deviations in settlements prices more easily from 2017 onwards.

7 How do Settlement Prices compare to Realized Volatility?

We consider the relationship of the VIX and realized volatility, distinguishing between settlement and other days. As manipulators are likely concentrating on SPX options, we assume that realized volatility in the S&P 500 is vastly unaffected by manipulation. Therefore, realized volatility is a suitable benchmark to reveal artificial price levels of the VIX linked to settlement.

The VIX incorporates the implied volatility of the S&P 500 looking into the future, whereas realized volatility reflects the past. Thus, comparing the two is non-trivial. Nevertheless, in an efficient market the two are closely linked. In the VIX construction, a central goal has been to establish an effective hedge against the volatility materialising in the S&P 500 (Whaley, 2009). We employ the framework proposed by Shu and Zhang (2003) who focus explicitly on this relationship with regard to the S&P 500.

To grasp the relationship between the VIX and realized volatility in the S&P 500, we estimate a linear regression:

$$\text{volatility}_{\text{realized}}(t) = \alpha + \beta \cdot \text{VIX}(t) \,/\, 100 + \epsilon(t) \tag{1}$$

This equation aims to explain the realized volatility by the VIX. The VIX needs to be divided by 100 to match the scale of the volatility measures. Akin to the framework of Shu and Zhang (2003), we use different measures of realized volatility: First, using daily log returns, $r(t)$, we calculate the closing price volatility on day t for a three-day window, i.e.,

$$\text{volatility}_{\text{ClosingPrice}}(t) = \sqrt{\frac{252}{n} \sum_{n=0}^{2} r_{t-n}^2}\,.$$

Equivalently to the definition of Parkinson (1980), the Parkinson-volatility is

$$\text{volatility}_{\text{Parkinson}}(t) = \sqrt{\frac{252}{n} \sum_{n=0}^{2} \frac{(\ln H_{t-n} - \ln L_{t-n})^2}{4\ln(2)}}.$$

H_t and L_t denote the highest and lowest index values on day t.

The volatility measure established by Yang and Zhang (2000) captures daily high, low, open and close index values as well as their difference to the preceding day. The appendix contains the more complex formula to calculate this $\text{volatility}_{\text{YangZhang}}(t)$. The latter two measures come with the advantage of taking into account more dimensions of the data than the canonical closing price volatility. For each of the measures of realized volatility, we implement a separate regression in which the corresponding proxy is the dependent variable in Equation 1. Table A8 in the appendix presents the results, which replicate the stylized fact that implied is typically higher than realized volatility.

We reorder the regression model of Equation 1 to gauge a potential artificial bias in the VIX on settlement days in the following way:

$$0 = \underbrace{100 \cdot \alpha + \beta \cdot \text{VIX}(t) - 100 \cdot \text{volatility}_{\text{realized}}(t)}_{=:\text{ abnormal component formula}} + \epsilon(t) \tag{2}$$

We use Equation 2 without the residuals to carve out an abnormal component in the VIX. We analyze this abnormal component separately for the subgroups of non-settlement and settlement days. These underlying index values refer exclusively to continuous trading. In particular, they do not contain the settlement price fixed in the special opening auction. For this reason, we consider the settlement prices themselves individually.

Table 9 supplies the results. There are three panels, each referring to one of the measures of realized volatility. There is a surplus of the abnormal component in continuous trading on settlement days across all three realized volatility measures. There is one exception in closing price volatility slightly less (-0.03) in the mean, although the median is by 0.45 substantially higher. Results are qualitatively similar if we replace the settlement day VIX levels with settlement prices (column 3).

All in all, settlement days seem to suffer from an abnormal upward bias in the VIX index. If we relate the abnormal effect to the VIX median of 17.47 (comp. Table A1), the settlement price difference reach a maximum deviation of 4.8%.

8 What happens to the Futures Term Structure on Settlement Days?

We explore the term structure of VIX futures, i.e., the relationship between VIX futures of different expirations. The contracts different from the front-month contract are settled in the following months and are more closely linked to SPX options likewise expiring later. Accordingly, these futures are likely largely independent from manipulation.

Every month, there is one monthly settlement date. The VIX future expiring on the next settlement date is called the front-month contract. Let f_1 denote the closing price of the front-month contract. However, there are futures traded already more than half a year in advance. Let $f_2, f_3, ...$ denote the closing prices of these futures, expiring in the months after the front-month contract. VIX futures prices are typically in contango, i.e., $f_1 < f_2 < ... < f_n$ (Fernandez-Perez et al., 2019).[16] Cheng (2019) argues that these price differences are mainly risk premiums as there is no cost of carry – as with physical goods – and low margin requirements.

If the futures are not in contango, they are in backwardation. In these cases, market participants expect return variance to shrink, and/or that exposure to short-term variance risk requires a larger risk premium than exposure to long-term variance risk (Johnson, 2017). Backwardation implies that there is a particularly high demand for VIX contracts settled earlier than the following months.

We compare these risk premia with different maturity. A natural measure is the percentage difference between the future prices of two months: $\frac{f_{t+1}-f_t}{f_t}$. We call this term contango magnitude. Its sign indicates whether the futures are in contango (positive) or backwardation (negative). The contango magnitude and the VIX are negatively correlated. This relationship suggests that, with rising VIX levels, short-term demand for VIX futures rises, whereas the effect is more moderate for longer time horizons. The front-month contract is closely linked to the options relevant to manipulators. In contrast, the following months' contracts can be assumed to be vastly unaffected by manipulation.

These considerations put forward to consider the contango magnitude between consecutive months. We calculate descriptive statistics of these contango magnitudes split by non-settlement and settlement days. The difference between the groups serves to reveal an effect characteristic to the pair of months. In this context, we want to compare the first pair to the other pairs. The first pair is most directly affected by a potential manipulation, while the other pairs should be relatively independent. As the first five months are more liquid than later expiries, we focus on those.

Table 10 states the results. We find that the difference of contango magnitudes between the first two months' futures is negative (mean: -0.81%, median: -1.18%). This discrepancy demonstrates that settlement days are associated with a higher

[16]Table A5 presents summary statistics of f_1 to f_5, confirming that contango prevails on average in our observation period.

price of f_t than f_{t+1}. As stated in previous sections, this higher price level may be a result of manipulation. However, we explore to what extent this difference may have a legitimate reason. To do so, we consider the difference in the contango magnitude of the months 2 to 3 and the following months. We display these results in Panels B to D. We observe that the differences stay negative, increasing towards zero, though. Moreover, all other differences are *not* significant at the 10% level. In conclusion, we notice an abnormal effect regarding the front-month contract, which is likely the contract most affected by manipulation. Other expiries manifest a more moderate effect with a yet also negative difference.

As the last approach in this paper, we evaluate the relationship between gaps (as defined in Section 3.3) and contango/backwardation. As we define gaps only for settlement days, we do not consider any non-settlement days in the following. Initially, we compute the frequency of occurrences of contango and backwardation. Panel A of Table 11 points out that in 90.9% of the (settlement) days the futures were in contango. Next, we combine the contango/backwardation information with the occurences of a gap. As before, we say a settlement price implies a positive (negative) gap if it is above (below) both VIX open and last close. Panel B of Table 11 presents the empirical probabilities: Out of the 90.9% of contango days, most settlement prices induced a gap (66.8%), whereas no gaps only occurred on only 24.0% of the days. Within the minority of backwardation days (9.1%), there were more gaps (5.3%) than not (3.8%).

Subsequently, we zoom in to the subgroups of contango/backwardation days and consider the gaps' direction. To calculate the difference between those two categories of days, we first compute the conditional probabilities. The rationale is to adjust the probabilities by dividing them by the occurrence probability of contango (90.9%) or backwardation (9.1%).

Panel C reports the results. If the futures are in contango, the (conditional) probabilities are as follows: In one quarter (26.4%) of the cases, no gap arises. In the cases of gaps (73.5%) a slight majority (41.3%) are in the positive direction as opposed to the negative direction (32.5%). Vice versa, for futures in backwardation, there are more "no gap" events (41.6%). The remainder consists of again more positive (33.3%) than negative (25%) gaps. These numbers highlight that gaps correlate more closely with contango (15.2 points more). The share of positive and negative gaps remains vastly stable between contango and backwardation, with slenderly more positive gaps in backwardation.

In summary, gaps occur more often in contango. This observation is in keeping with manipulation being conducted during calm market environments (contango). The abnormally many gaps in the positive direction are consistent with manipulators preferring driving prices upwards.

9 Conclusion

We find a vast amount of empirical findings that potential manipulation in the VIX has not slowed down since the financial press and academic papers (Griffin and Shams, 2018) started raising concerns in this regard. We show that settlement prices are often not in line with continuous trading. Upward deviations occur more often and are substantially more intense than downward distortions. Investors trade substantially more put options on settlement days, which is in line with cost-efficient VIX manipulation. Leveraged funds go into the settlement with 10% more exposure compared to the week before.

The Cboe Exchange, being responsible for the settlement design, failed to refute the massive criticism. A high share of Cboe's earnings attributable to the VIX may create an incentive against admitting weak spots in its settlement procedure. Until this issue is resolved, the main VIX index appears to be ill-suited for volatility hedging. The VIX3M seems a better alternative as the Cboe Exchange also issues derivatives on that index.

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Tables

Table 1: VIX Settlement Prices

This table reports statistics on VIX settlement prices in comparison to intraday index quotes. The first column shows the difference between the settlement price and the opening quote as a percentage of the average VIX open on all days. In the second column, the position of the settlement price within the intraday trading range – spanning from the intraday low to the intraday high – is reported as a percentage of this span (defining the scale from 0%, the low, to 100%, the high).

As the quotes in continuous trading fix this scale, the settlement is excluded. As a result, it is possible that the intraday trading range does not contain the settlement price, resulting in values below 0% or above 100%. The number of these settlement days as a percentage of the total number of settlement days is shown in the last two rows. *, **, *** denote significance at the 10%, 5% and 1% level.

	Difference to Open	Intraday Position within High and Low
mean	5.5%	56.9%**
median	1.5%	50.9%***
standard deviation	59.2%	37.7%
skew	0.7	0.3
# values $\leq 0\%$		3.5%
# values $\geq 100\%$		13.4%

Table 2: Intraday Gaps around Settlement

This table reports on gaps between the settlement price and the last day's VIX close and intraday VIX values shortly after settlement. A gap is recorded if the settlement price is higher than the previous day's close but trades shortly after settlement below the settlement price again (positive direction) or vice versa (negative direction), having the character of a "reversal". For the exact point in time after settlement, different reference times are chosen and separately reported. Panel A reports the empirical probability that such a gap occurs. Panels B and C report descriptive statistics on the gaps after non-gap days have been deleted.

reference time	**9:30**	**9:35**	**9:40**	**9:45**	**10:00**	**10:15**	**10:30**
Panel A: Empirical Probability of Gaps (Settle Price not between Close and Open)							
Gap Probability	57.8%	42.0%	42.7%	43.9%	45.5%	49.8%	52.3%
thereof:							
positive	28.7%	22.0%	22.7%	22.3%	25.5%	26.2%	28.8%
negative	29.1%	20.0%	20.0%	21.6%	20.0%	23.6%	23.5%
Panel B: Width of Upward (positive) Gaps – in percent of last VIX Close							
mean	16.1%	13.0%	11.3%	11.8%	12.3%	13.9%	14.1%
median	1.9%	10.6%	9.9%	10.2%	10.6%	12.3%	12.1%
standard dev.	29.0%	12.8%	10.3%	10.4%	9.6%	10.3%	10.3%
skew	5.5	1.7	1.7	1.8	1.8	1.8	1.7
Panel C: Width of Downward (negative) Gaps – in Percent of last VIX Close							
mean	−9.3%	−9.5%	−9.6%	−9.8%	−9.5%	−8.9%	−9.5%
median	−9.0%	−9.4%	−9.2%	−9.4%	−9.0%	−9.2%	−9.0%
standard dev.	5.7%	5.4%	5.4%	6.0%	6.0%	5.7%	5.5%
skew	−0.3	−0.3	−0.3	−0.5	−0.8	−0.6	−0.4

Table 3: Underlying Options' Put/Call Ratio

This table shows descriptive statistics of the put/call ratio of the SPX options. The statistics are reported separately for settlement days and all other days. The VIX formula is based on a weighted sum of SPX options prices. As put options receive significantly higher weight, cost efficient manipulation will – ceteris paribus – result in a higher put/call ratio. Panel A shows a the descriptive statistics directly, whereas Panel B is computed from an adjusted and zero-centered time series ("excess multiple"). The adjustment is conducted by dividing each day's value by the ten day moving average. *, **, *** denote significance at the 10%, 5% and 1% level.

	other days	settle days	difference
Panel A: Put/Call Ratio			
mean	1.74	1.94	0.19***
median	1.71	1.90	0.19***
standard deviation	0.38	0.43	
skew	0.57	0.39	
Panel B: Excess Multiple of Put/Call Ratio			
mean	0.01	0.15	0.14***
median	−0.01	0.11	0.12***
standard deviation	0.21	0.27	
skew	0.71	0.76	

Table 4: Index Value Fluctuation Range

This table shows descriptive statistics of the intraday range in which index values fluctuate, defined as highest index value minus lowest index value of each day. In the second column, the monthly settlement days are isolated. The last column shows their absolute difference. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. Characteristically, no derivatives are issued upon the three-month counterpart VIX3M of the VIX whose index values are calculated by the same formula. This fact makes them irrelevant to manipulators. Therefore, the VIX3M is a suitable comparison to validate whether the fluctuation range of the VIX is abnormally enlarged on settlement days. *, **, *** denote significance at the 10%, 5% and 1% level.

	other days	settle days	difference
Panel A: Fluctuation Range of VIX			
mean	1.58	1.92	0.35**
median	1.15	1.42	0.27***
standard deviation	1.65	1.88	
skew	5.97	3.91	
Panel B: Fluctuation Range of VIX3M			
mean	1.32	1.37	0.05
median	0.88	1.00	0.12
standard deviation	1.69	1.54	
skew	6.49	5.39	

Table 5: Concentration of Large Positions

Panels A and B show the net exposure held by the four resp. eight largest parties in sum as a percentage of the overall open interest. Panels C and D evaluate the exposure share week-on-week change (multiplicatively, i.e., a week's share is divided by the share of the week before, and centered by subtracting 100%). The raw information is collected on a weekly basis and published by the American Commodity Futures Trading Commission (CFTC). The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. *, **, *** denote significance at the 10%, 5% and 1% level.

	Largest 4			**Largest 8**		
	other days	settle days	difference	other days	settle days	difference
Panel A: Long Share of Open Interest (%)						
mean	34.0	31.8	-2.2^{*}	40.8	38.3	-2.5^{**}
median	32.4	29.3	-3.1^{*}	39.7	37.3	-2.4^{**}
st. dev.	14.1	13.6		12.9	12.5	
Panel B: Short Share of Open Interest (%)						
mean	27.6	26.8	-0.9	36.8	35.4	-1.4
median	22.3	21.5	-0.8	31.6	30.1	-1.5^{*}
st. dev.	13.7	13.2		13.9	13.3	
Panel C: Long Share week-on-week change (%)						
mean	0.9	-5.2	-6.1^{***}	0.7	-4.8	-5.5^{***}
median	-1.0	-5.5	-4.5^{***}	-0.6	-5.0	-4.4^{***}
st. dev.	14.2	12.3		12.5	10.3	
Panel D: Short Share week-on-week change (%)						
mean	0.6	-2.0	-2.6^{*}	0.4	-2.9	-3.3^{***}
median	-0.7	-3.1	-2.3^{**}	-1.0	-2.9	-1.9^{***}
st. dev.	12.8	12.3		10.6	9.4	

Table 6: The Role of the Largest 8 Parties regarding Positive and Negative Gaps

This table reports the correlation of gaps around the settlement of VIX derivatives and the largest 8 VIX longs and shorts, respectively. A gap is recorded if the settlement price is higher than the previous day's close but the VIX trades shortly after settlement below the settlement price again (positive direction) or vice versa (negative direction). If there is a gap in that direction, the corresponding day is classified into the "gap" category; if there is no gap in that direction, it is classified into the "no gap" group. Panel A and C show the absolute market values in USD multiples, whereas Panels B and D report the percentage change compared to the value of the week before. *, **, *** denote significance at the 10%, 5% and 1% level.

	Positive			**Negative**		
	no gap	gap	difference	no gap	gap	difference
Panel A: Largest 8's ***Long*** *Exposure absolute*						
mean	38.68	38.46	−0.22	38.02	39.55	1.53
median	38.10	36.00	−2.1	37.00	38.95	1.95
st. dev.	12.69	12.18		11.82	13.48	
Panel B: Largest 8's ***Long*** *Exposure week-on-week change (%)*						
mean	−2.85	−4.01	−1.16	−4.20	−1.79	2.41
median	−4.73	−4.96	−0.23	−4.86	−4.79	0.07
st. dev.	16.80	9.17		8.83	20.45	
Panel C: Largest 8's ***Short*** *Exposure absolute*						
mean	33.57	37.24	3.67^{*}	32.85	40.78	7.93^{***}
median	31.00	29.50	-1.50^{*}	29.65	32.05	2.40^{*}
st. dev.	14.65	10.94		9.72	16.91	
Panel D: Largest 8's ***Short*** *Exposure week-on-week change (%)*						
mean	−0.93	−4.34	-3.41^{**}	−3.60	0.09	3.69^{**}
median	−1.63	−3.56	-1.93^{*}	−3.37	−1.20	2.17^{*}
st. dev.	10.62	7.91		8.47	11.38	

Table 7: Commitment by Trader Group, Week-on-Week Change

This table shows the **week-on-week change in percent** of the net open interest in VIX derivatives (in 1000 USD * index value) held by the respective (disjunct) groups in Panels A to D. For the groups, reporting their exposure to the American Commodity Futures Trading Commission (CFTC), who publishes the aggregated data, is mandatory. The groups make up the majority of the open interest in VIX derivatives; smaller traders are not obliged to report. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. *, **, *** denote significance at the 10%, 5% and 1% level.

	Long			**Short**		
	other days	settle days	difference	other days	settle days	difference
Panel A: Leveraged Funds (%)						
mean	6.0	16.0	10.1*	1.6	0.8	−0.7
median	−0.5	6.8	7.2***	1.0	1.0	−0.1
st. dev.	52.3	71.4		16.0	14.9	
Panel B: Asset Managers (%)						
mean	0.1	0.0	−0.1	0.0	0.0	0.0
median	0.0	0.0	0.0**	0.0	0.0	0.0
st. dev.	0.6	0.5		0.7	0.3	
Panel C: Dealer (%)						
mean	4.6	4.9	0.3	3.4	4.2	0.8
median	0.6	0.7	0.1	0.7	1.3	0.6
st. dev.	36.0	30.6		28.7	24.0	
Panel D: Other Reportables (%)						
mean	7.4	11.0	3.5	12.3	21.1	8.8
median	0.0	1.4	1.4	0.3	6.0	5.7*
st. dev.	56.6	59.0		140.5	139.6	

Table 8: Changes after Public Attention in 2017

This table outlines the changes between the periods until (including) 2017 and starting from 2018 until 2021 based on The paper by Griffin and Shams became publicly available in this year. The measures are defined as in the Tables 1, 3, 4, and 7. In Panel E, Other Market Participants refer to all market participants (total open interest) minus Leveraged Funds. *, **, *** denote significance at the 10%, 5% and 1% level.

	until 2017	after 2017	difference
Panel A: VIX Settlement Price			
Settle Price − Open	5%	9%	4%
# Settle Prices > Intraday High	15%	3%	−12%***
# Settle Prices < Intraday Low	4%	3%	−1%
Gap Width	7%	−9%	−16%
Panel B: Fluctuation Range			
Other Days	1.47	2.74	1.27***
Settle Days	1.85	2.26	0.41
Difference	0.38***	−0.48	−0.86
Panel C: Put/Call Ratio Week-on-Week Change			
Other Days	−0.05	−0.02	0.03*
Settle Days	0.09	0.08	−0.01
Difference	0.14	0.10*	−0.04
Panel D: Leveraged Funds' Absolute Longs			
Other Days	40 286	40 433	147
Settle Days	43 963	46 379	2416
Difference	3677	5946	2269
Panel E: Other Market Participants' Absolute Longs			
Other Days	240 027	315 353	75 326***
Settle Days	157 252	354 575	197 323
Difference	−82 775	39 222	121 997

Table 9: VIX versus Realized Volatility

This table reports descriptive statistics on the difference between the VIX and its forecast based on realized volatility, computed with the parameters α and β of the regressions in Table A8:

$$100 \cdot \alpha + \beta \cdot \mathrm{VIX}(t) - 100 \cdot \mathrm{volatility}_{\mathrm{realized}}(t)$$

The three Panels refer to regressions 1 to 3. Realized Volatility and α are multiplied by 100 to match the VIX' scale. Columns (1) and (2) are based on the regression forecast and refer to continuous trading, which excludes the settlement price. For column (3), settlement prices are inserted into the equation instead of the VIX.

	(1) non-settle days	(2) settle days	(3) settle price	(4) difference (2)−(1)	(5) difference (3)−(1)
Panel A: VIX difference to S&P 500 Realized Closing Price Volatility					
mean	0.12	0.09	0.16	−0.03	0.04
median	0.67	1.12	0.93	0.45	0.26
st. dev.	7.71	6.44	6.48		
skew	−1.25	−1.54	−1.58		
Panel B: VIX difference to S&P 500 Realized Parkinson-Volatility					
mean	0.10	0.40	0.16	0.30	0.06
median	0.37	0.50	0.93	0.13	0.56
st. dev.	4.79	4.21	6.48		
skew	−1.77	−0.81	−1.58		
Panel C: VIX difference to S&P 500 Realized Yang-Zhang-Volatility					
mean	0.15	0.90	0.99	0.75	0.84
median	0.80	1.06	1.11	0.26	0.31
st. dev.	8.23	6.60	6.74		
skew	−2.14	−0.70	−0.63		

Table 10: VIX Futures Term Structure

This table reports descriptive statistics of the magnitude of the contango of VIX futures *in percent*. The magnitude is calculated as the price difference of the later to the earlier month's expiry relative to (divided by) the earlier month's expiry price, i.e., $\frac{f_{t+1}-f_t}{f_t}$ where f_t denotes the price of the VIX future expiring in month t. Month 1 refers to the front-month contract with the nearest expiry.

In the second column, the monthly settlement days are isolated. The last column shows their absolute difference. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. *, **, *** denote significance at the 10%, 5% and 1% level.

	other days	settle days	difference
Panel A: Contango Magnitude Month 1 to Month 2 (%)			
mean	6.36	5.55	-0.81^{**}
median	6.92	5.74	-1.18^{**}
st. dev.	4.58	6.76	
Panel B: Contango Magnitude Month 2 to Month 3 (%)			
mean	3.76	3.26	-0.50
median	4.22	3.60	-0.62
st. dev.	4.10	3.28	
Panel C: Contango Magnitude Month 3 to Month 4 (%)			
mean	2.48	2.29	-0.19
median	2.90	2.62	-0.28
st. dev.	2.76	3.02	
first quartile	0.96	1.17	0.21
Panel D: Contango Magnitude Month 4 to Month 5 (%)			
mean	2.03	1.91	-0.12
median	2.24	2.19	-0.05
st. dev.	2.51	2.54	

Table 11: Gaps in Contango and Backwardation

This table presents the occurrence of contango and backwardation in VIX futures and its correlation to gaps around the settlement. On any day, a contango in the VIX futures is reported if the front-month contract is cheaper than the contract expiring one month later. Otherwise, they are in backwardation. Panel A depicts how often these cases occur.

Panel B reports the empirical probability of the four disjunct events of contango or backwardation and a gap or not. A gap is recorded if the settlement price is higher than the previous day's close but the VIX trades shortly after settlement below the settlement price again (positive direction) or vice versa (negative direction). The probability of a gap is further split into its summands referring to the corresponding direction.

Panel C shows the probabilities of a gap occurring or not given the futures are in contango or backwardation, respectively (conditional probability). These conditional probabilities can be compared by their difference as calculated in the third column.

	contango	backwardation	difference
Panel A: Distribution of Contango and Backwardation			
probability	90.9%	9.1%	
Panel B: Overall Probability			
No Gap	24.0%	3.8%	
Gap	66.8%	5.3%	
thereof:			
positive direction	37.5%	3.0%	
negative direction	29.27%	2.3%	
Panel C: Conditional Probability (given contango / backwardation)			
No Gap	26.4%	41.6%	15.2%
Gap	73.5%	58.3%	−15.2%
thereof:			
positive direction	41.3%	33.3%	−8.0%
negative direction	32.2%	25.0%	−7.2%

Figures

Figure 1: VIX Settlement Price Minus Open

The histogram shows the distribution of the difference between the VIX settlement price and the opening quote fixed minutes later in absolute index points (x-axis) and the corresponding frequency in percent (y-axis). The dotted line indicates the arithmetic mean.

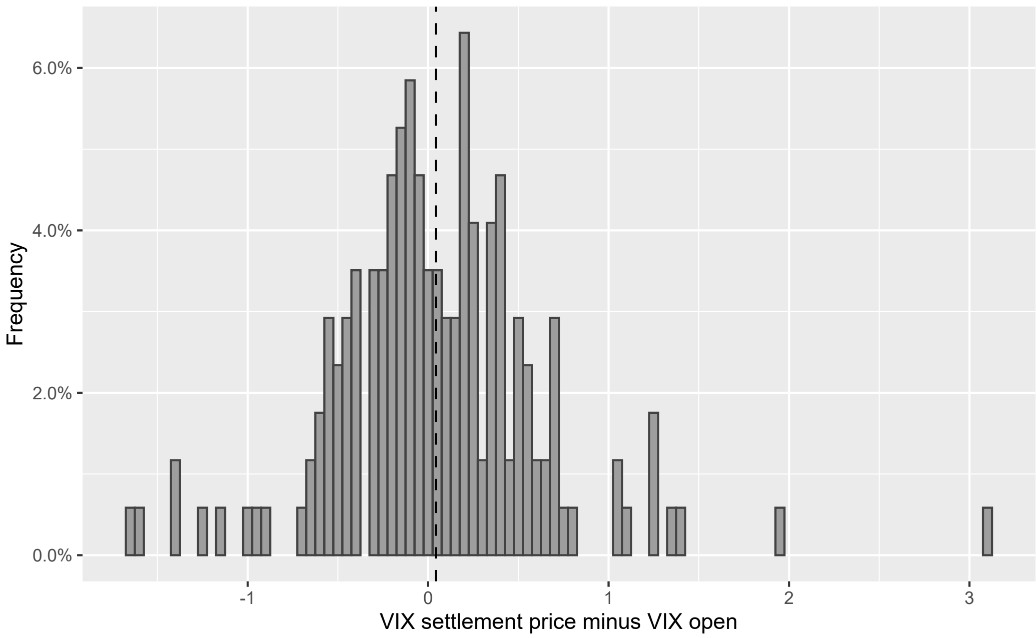


Figure 2: Difference of Settlement Price and Open

Summed up for each half year, this plot depicts the sum of the absolute values of the difference "VIX Settlement Price minus VIX Open" in index points. The dotted line represents the smoothed trend.

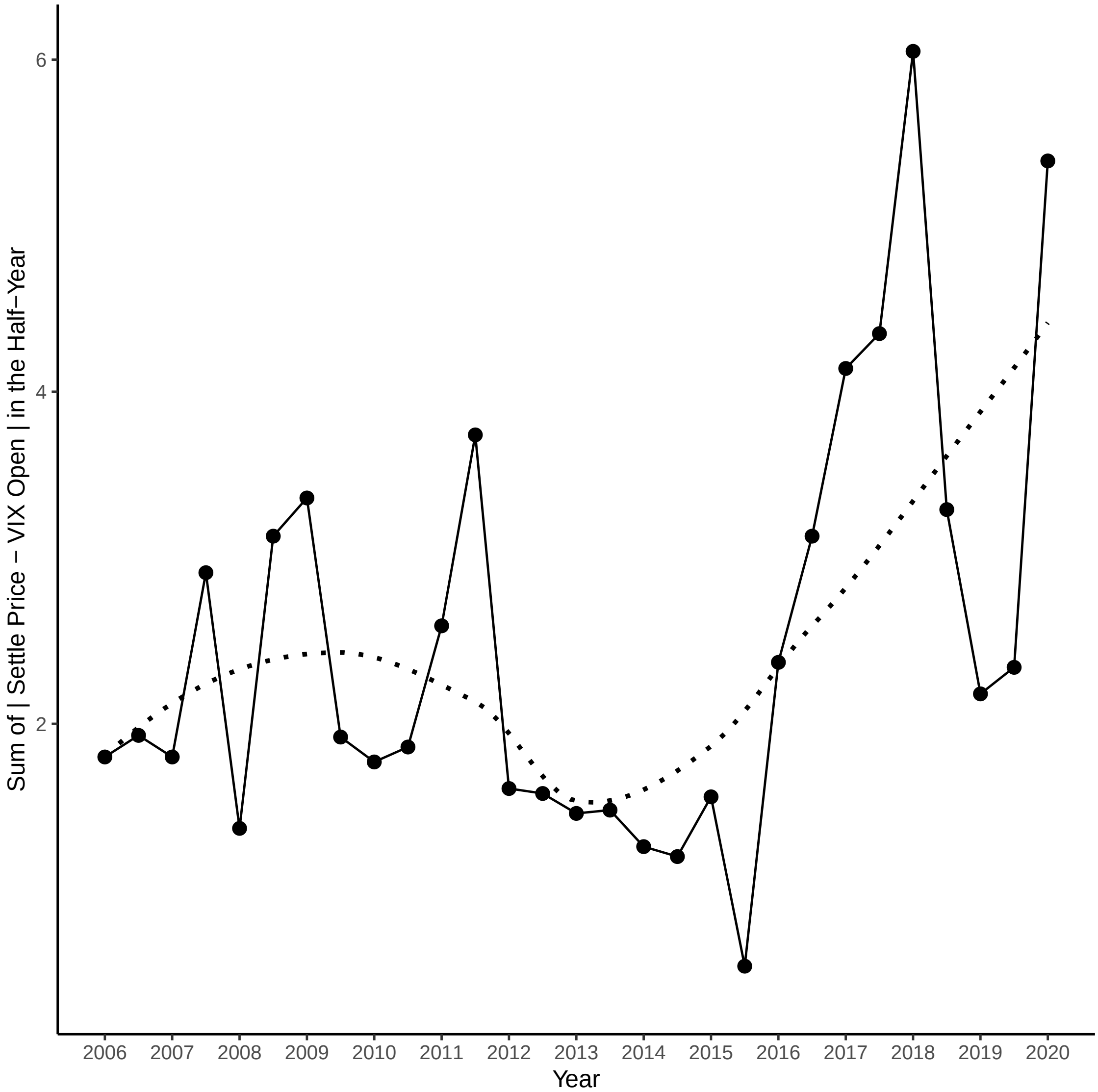


Figure 3: VIX Settlement Price in Fluctuation Range

The histogram depicts the distribution of the settlement price within the fluctuation range mapped to 0 to 1. For each settlement day, the settlement price position within the intraday fluctuation range is calculated, where the intraday low (excluding the settlement auction) serves as 0 and the high as 1. As the auction is excluded, values may be outside the interval [0; 1].

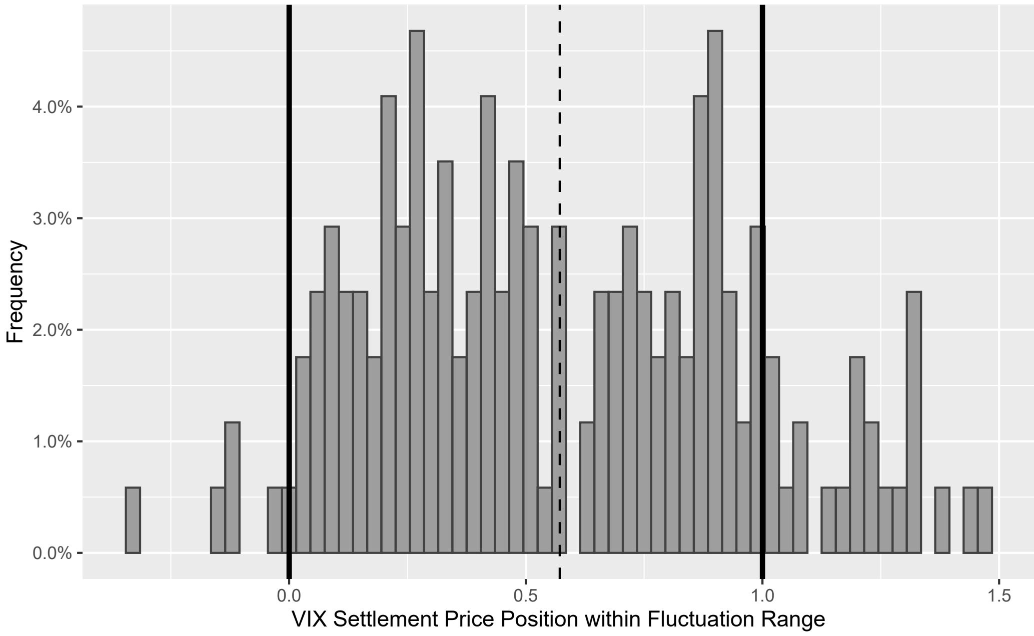


Figure 4: Intraday Difference of VIX and VIX3M

This figure presents the average development of the standardized difference of the VIX and VIX3M intraday in the following sense: A settlement day is a “positive gap”-day if both the VIX open and previous day’s close are below the settlement price and it is a “negative gap”-day the settlement price is above. Other days, with a settlement price between VIX open and the previous day’s close are “no gap”-days. For non-settle days there is no distinction.

Within these four categories, a variance-standardized difference (expressed in percent) of the VIX minus the VIX3M serves as basis for minutely means calculated from all index values after the n-th minute after market open at 9:30 a.m.

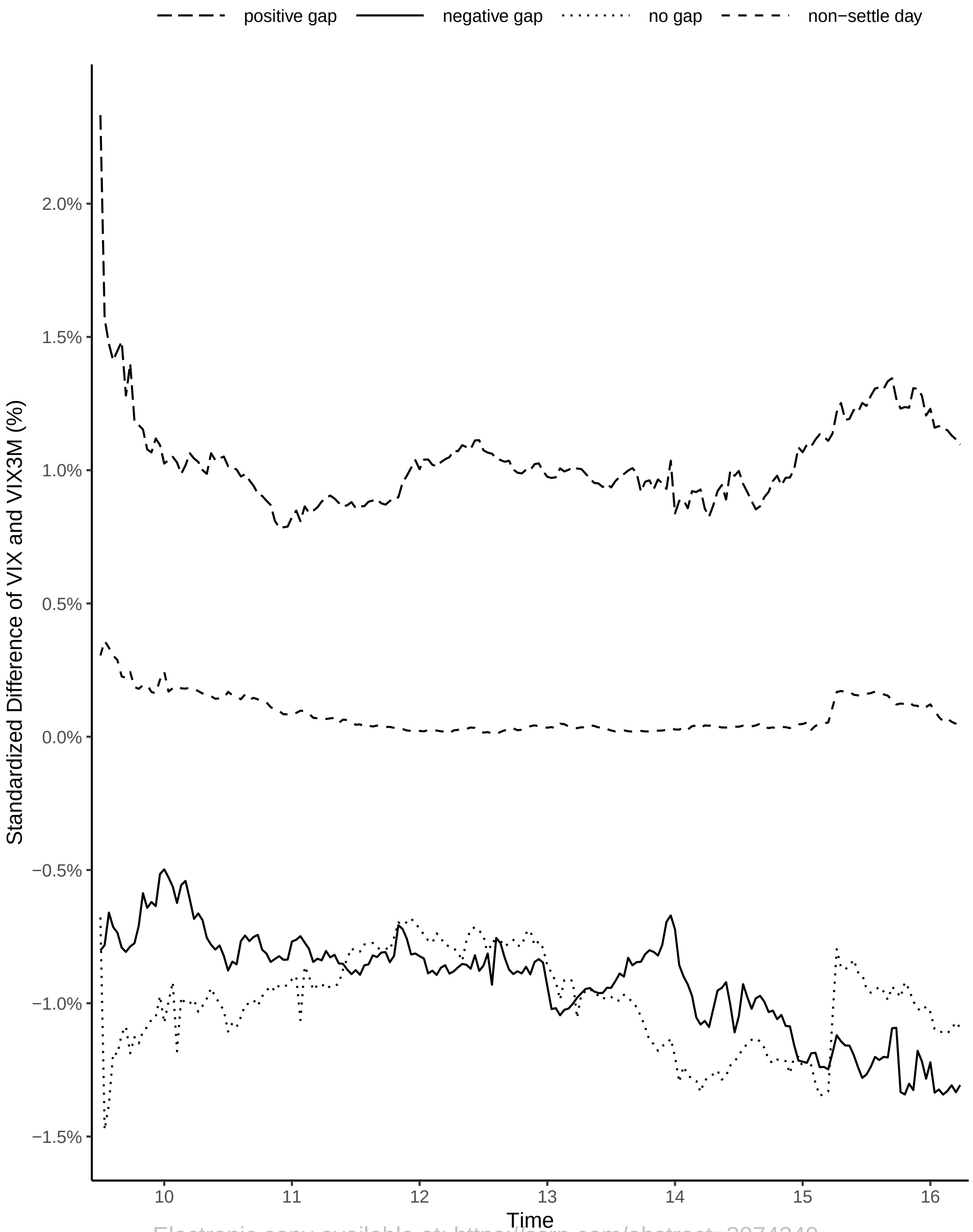
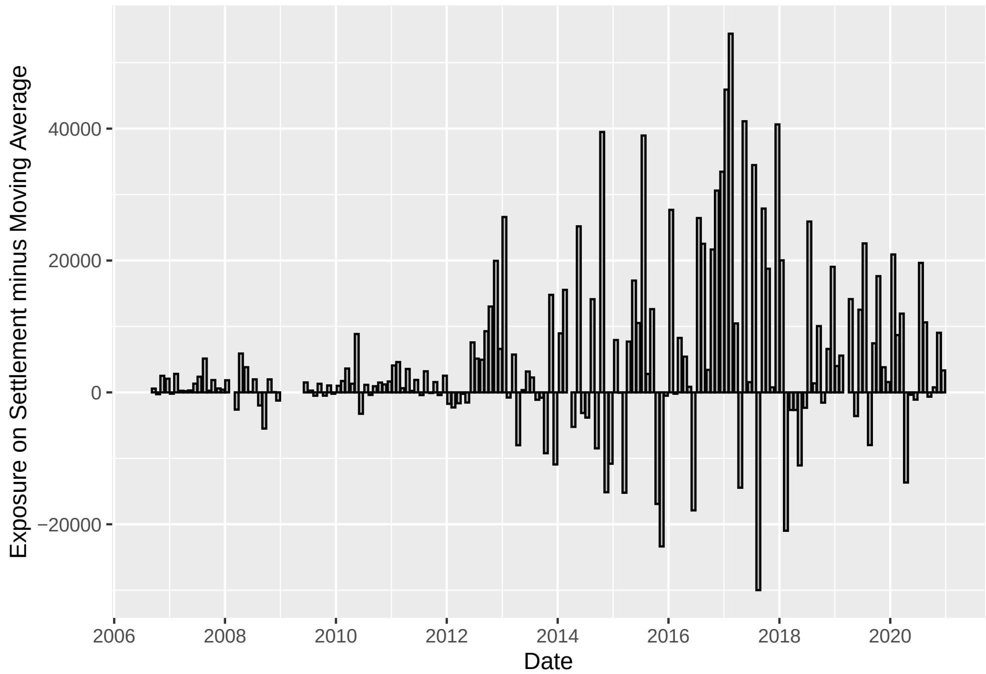


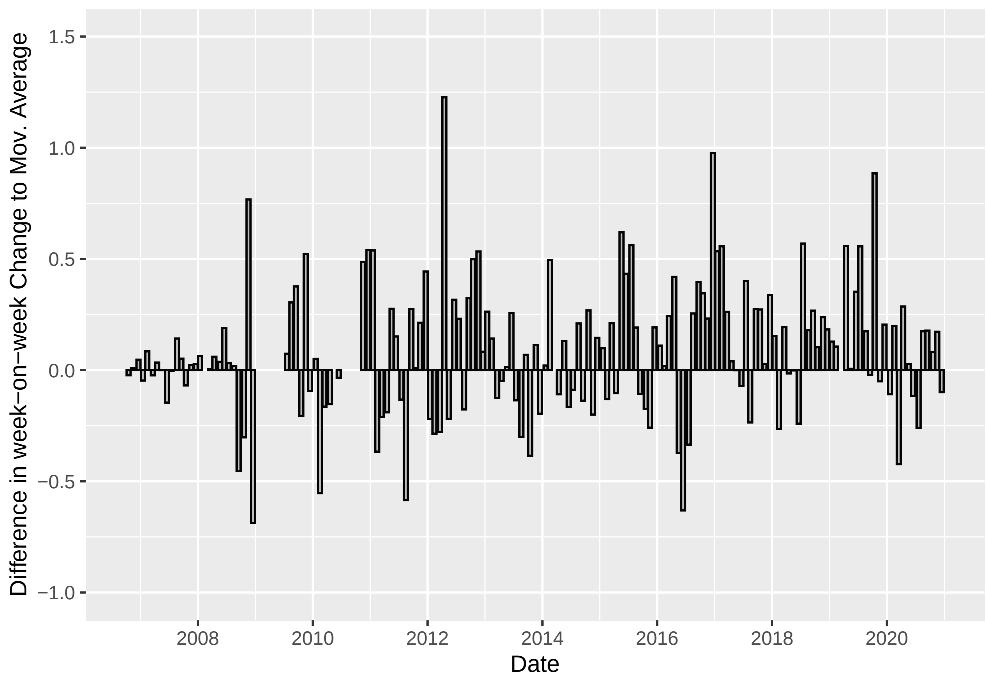
Figure 5: Commitment of Leveraged Funds

Panel A shows the long exposure to VIX derivatives of leveraged funds on (monthly) settlement days minus the three-week moving average. Panel B shows the difference between the long exposure's week-on-week change of settlement days minus the three-week moving average.

Panel A: Absolute Difference to the Average of the three Weeks before



Panel B: Relative Difference to the Percentage Change Average



Appendix

Deduction of the Time Series Decomposition

The detailed deduction of the time series decomposition components is as follows:

Let p_t denote the put/call ratio of the options on the S&P 500 on day t. Our aim is to decompose p_t such that

$$p_t = \text{trend}_t + \text{seasonal}_t + \text{random}_t.$$

For this purpose, let $a_i(p)$ denote the moving average of $p := (p_t)$ of the ten day window ending in $i-1$, i.e.,

$$a_i(p) = \frac{1}{10} \sum_{k=i-10}^{i-1} p_k.$$

Now define the trend as the moving average,

$$\text{trend}_t := a_t(p).$$

To deduce the seasonal component, consider $p_t - \text{trend}_t = \text{seasonal}_t + \text{random}_t$. Let $s_1, s_2, s_3, ...$ denote the indices of the settlement days in p_t. As there are approximately ≥ 20 trading days in each month, consider (approximate) months represented by their time indices $m_i := (s_i - 19, s_i - 18, ..., s_i)$ and write for example $(m_i)_2 = s_i - 18$ for the index of the second day in month i. Let M be the number of (full) months in the data considered and delete, if necessary, the data of incomplete months. (Days not allocated to any month in this representation are also deleted.)

Based on this notation, we can easily define the (uncentered) seasonal component at time t as the arithmetic mean of the t-th day over all months $(i = 1, ..., M)$,

$$\text{uncenteredseasonal}_t := \frac{1}{M} \sum_{i=1}^{M} p_{(m_i)_t}.$$

The seasonal component is obtained from this series as its centered version,

$$\text{seasonal}_t := \text{uncenteredseasonal}_t - \text{mean}(\text{uncenteredseasonal})$$

Finally, the remainder is put into the random component,

$$\text{random}_t := p_t - \text{trend}_t - \text{seasonal}_t,$$

yielding the decomposition.

Yang-Zhang-Volatility

Yang and Zhang (2000) define a volatility measure based on daily high (H_t), low (L_t), open (O_t) and close (C_t). The formula in its equivalently rewritten form of Shu and Zhang (2003) is as follows:

$$\text{volatility}_{\text{YangZhang}}(t) = \sqrt{252} \cdot \sqrt{V_O + kV_C + (1-k)V_{RS}},$$

where

$$\begin{aligned} k &= \frac{0.34}{1.34 + \frac{n+1}{n-1}} \\ V_O &= \frac{1}{n-1}\sum_{t=1}^{n}(o_t - \bar{o})^2, \\ o_t &= \ln O_t - \ln O_{t-1}, \\ \bar{o} &= \frac{1}{n}\sum_{n}^{t=1} o_t, \end{aligned}$$

and analogously

$$\begin{aligned} V_C &= \frac{1}{n-1}\sum_{t=1}^{n}(c_t - \bar{c})^2, \\ c_t &= \ln C_t - \ln C_{t-1}, \\ \bar{c} &= \frac{1}{n}\sum_{n}^{t=1} c_t, \end{aligned}$$

and finally

$$V_{RS} = \frac{1}{n}\sum_{t=1}^{n}\Big[(\ln H_t - \ln O_t)\cdot(\ln H_t - \ln C_t) + (\ln L_t - \ln O_t)\cdot(\ln L_t - \ln C_t)\Big].$$

Table A1: Summary Statistics of VIX Indices

This table reports summary statistics of the VIX and VIX3M daily data (open, high, low, close) and the intraday time series. Note that the intraday data comprises less years.

					Quantiles						
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%	Skew	Kurtosis
VIX											
VIX Open	19.46	8.36	9.01	82.69	11.22	13.47	17.23	22.99	34.55	2.19	7.88
VIX Close	19.48	8.11	9.14	82.69	11.28	13.66	17.47	22.89	33.77	2.19	8.26
VIX High	20.30	8.85	9.31	89.53	11.70	14.11	17.96	23.86	36.19	2.35	9.14
VIX Low	18.69	7.82	8.56	72.76	10.90	13.00	16.55	22.14	32.91	2.03	6.61
VIX intraday	19.55	10.40	8.56	89.03	10.47	13.23	16.19	21.87	42.25	2.53	7.79
VIX3M											
VIX3M Open	21.31	8.57	11.87	73.14	13.16	15.62	18.66	23.93	39.55	2.14	5.60
VIX3M Close	21.28	8.58	11.85	72.98	13.16	15.57	18.71	23.86	39.96	2.14	5.67
VIX3M High	21.97	9.07	11.91	86.61	13.54	15.97	19.30	24.63	41.57	2.21	6.25
VIX3M Low	20.65	8.05	11.03	65.15	12.93	15.32	18.09	23.11	38.19	2.06	5.01
VIX3M intraday	20.39	5.87	11.03	44.56	13.60	15.73	18.90	24.03	32.18	0.98	0.33

Table A2: Summary Statistics of CFTC Concentration Data

This table reports summary statistics on the CFTC concentration data.

					Quantiles						
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%	Skew	Kurtosis
Largest 4 Longs Gross	43.50	13.64	16.90	80.60	25.70	32.38	41.75	52.00	71.33	0.62	−0.18
Largest 4 Longs Net	34.28	14.03	10.30	75.90	15.60	22.50	32.85	44.00	60.41	0.58	−0.17
Largest 4 Shorts Gross	35.14	16.05	16.00	89.20	19.10	24.00	29.45	40.02	69.91	1.29	0.70
Largest 4 Longs Net	27.63	13.69	11.20	85.40	15.60	18.88	22.30	31.20	57.91	1.72	2.49
Largest 8 Longs Gross	54.52	12.54	26.50	85.60	37.50	44.40	53.40	63.10	79.10	0.40	−0.46
Largest 8 Longs Net	41.15	12.95	16.30	80.80	23.20	30.60	40.10	49.90	64.90	0.49	−0.31
Largest 8 Shorts Gross	48.70	16.90	27.40	96.80	31.50	36.70	42.50	54.23	84.60	1.16	0.14
Largest 8 Longs Net	36.76	13.88	17.30	92.30	23.50	27.90	31.65	40.20	67.41	1.57	1.88

Table A3: Summary Statistics of CFTC Concentration Week-on-Week Change

This table reports summary statistics on the week-on-week change of CFTC concentration data. The week-on-week change is computed as the time series divided by its one-week lagged version, and finally 1 is subtracted from the result.

					Quantiles						
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%	Skew	Kurtosis
Largest 4 Longs Gross	0.00	0.08	−0.47	0.44	−0.11	−0.04	−0.01	0.04	0.15	0.57	3.54
Largest 4 Longs Net	0.01	0.14	−0.79	1.16	−0.17	−0.07	−0.01	0.07	0.23	1.78	12.11
Largest 4 Shorts Gross	0.00	0.10	−0.37	0.62	−0.13	−0.06	0.00	0.05	0.15	1.12	5.54
Largest 4 Longs Net	0.01	0.13	−0.44	1.09	−0.16	−0.07	−0.01	0.06	0.22	1.60	9.50
Largest 8 Longs Gross	0.00	0.06	−0.44	0.27	−0.09	−0.04	0.00	0.04	0.11	0.00	3.97
Largest 8 Longs Net	0.01	0.13	−0.77	1.51	−0.14	−0.06	−0.01	0.05	0.20	3.09	33.87
Largest 8 Shorts Gross	0.00	0.07	−0.25	0.36	−0.10	−0.04	0.00	0.04	0.11	0.76	3.26
Largest 8 Longs Net	0.00	0.11	−0.36	0.85	−0.14	−0.06	−0.01	0.05	0.19	1.27	6.91

Table A4: Commitment by Trader Group (Absolute Exposure)

This table shows which net open interest in VIX derivatives (in 1000 USD * index value) is held by the respective (disjunct) groups in Panels A to D. For the groups, reporting their exposure to the American Commodity Futures Trading Commission (CFTC), who publishes the aggregated data, is mandatory. The groups make up the majority of the open interest in VIX derivatives; smaller traders are not obliged to report. In light of the notion that the ultimate counterparties of leveraged funds are asset managers, Panel E reports the difference of their open interest. The stars indicate significance based on a t-test (for the mean) and a Wilcoxon signed-rank test (for the median), respectively. *, **, *** denote significance at the 10%, 5% and 1% level.

	Long			**Short**		
	other days	settle days	difference	other days	settle days	difference
Panel A: Leveraged Funds						
mean	42 234	46 118	3884	94 215	94 363	148
median	36 293	38 557	2264	94 445	93 449	−996
st. dev.	32 274	35 902		70 714	72 694	
Panel B: Asset Managers						
mean	47 802	43 445	−4357	27 127	26 501	−627
median	42 288	38 362	−3926	18 488	17 596	−892
st. dev.	43 116	40 093		32 032	32 838	
Panel C: Dealer						
mean	66 292	67 986	1695	34 510	36 395	1884
median	64 900	66 587	1687	27 873	27 776	−97
st. dev.	44 546	45 745		20 953	22 954	
Panel D: Other Reportables						
mean	7181	7414	232	5960	6336	375
median	6152	5840	−312	4562	4762	200
st. dev.	6715	6897		5867	6322	
Panel E: Leveraged Funds Minus Asset Managers						
mean	−5568	2673	8241**	67 087	67 862	775
median	1178	3415	2237**	46 332	43 335	−2997
st. dev.	38 447	35 997		56 056	58 220	

Table A5: Summary Statistics of VIX Futures Term Structure

This table reports summary statistics on the term structure of VIX futures. For every VIX future, the time series of its prices is partitioned into subsets, each representing the prices observed when the expiration is in $n \in \{0, 1, ..., 4\}$ months. Any such n is the basis for one row of following table, summarizing statistics on the union of the corresponding subsets of all futures.

					Quantiles						
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%	Skew	Kurtosis
front-month expiration	18.66	6.57	9.88	72.62	11.98	14.45	16.68	20.90	32.10	2.20	8.38
expiration in 1 month	19.62	6.00	11.32	61.42	12.97	15.57	17.64	21.80	32.05	1.66	4.17
expiration in 2 months	20.22	5.56	12.22	51.50	13.82	16.40	18.27	22.75	31.79	1.25	1.38
expiration in 3 months	20.62	5.26	12.97	44.30	14.57	16.93	18.69	23.40	31.49	1.07	0.38
expiration in 4 months	20.98	5.08	13.47	37.48	15.28	17.32	19.02	24.10	31.40	0.95	−0.14

Table A6: Summary Statistics of CFTC Commitment Data

This table reports summary statistics on the CFTC commitment data.

					Quantiles				
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%
Open Interest	292 947	174 494	21 309	704 831	42 806	108 339	330 054	418 399	578 651
Dealer Longs	65 533	44 294	200	175 665	1509	29 015	62 600	102 690	139 228
Dealer Shorts	34 164	20 843	2468	129 967	11 409	19 107	27 634	45 664	74 087
Dealer Spreads	25 197	13 781	1644	67 810	4100	14 450	23 606	36 208	47 688
Asset Manager Longs	49 388	43 853	0	151 065	671	4134	43 570	83 020	126 072
Asset Manager Shorts	27 864	32 052	0	145 856	0	740	19 584	41 610	97 524
Asset Manager Spreads	19 907	23 155	0	124 611	0	631	15 764	27 882	60 268
Leveraged Funds Longs	41 982	31 944	0	145 577	1806	17 232	36 020	61 712	102 573
Leveraged Funds Shorts	94 495	69 933	2474	269 305	8108	20 526	96 573	149 603	213 209
Leveraged Funds Spreads	45 804	37 422	81	184 257	1167	10 338	42 896	68 440	115 579
Other Reportable Longs	7253	6656	0	43 391	343	1346	6340	10 924	19 234
Other Reportable Shorts	5891	5816	0	40 939	0	1570	4401	8500	15 693
Other Reportable Spreads	17 208	16 793	0	79 702	116	2147	13 404	23 506	53 123
Non-Reportable Longs	20 674	8332	4225	49 873	7219	14 025	21 872	26 104	33 646
Non-Reportable Shorts	22 416	10 925	793	92 871	3939	13 535	25 594	30 493	35 822

Table A7: Summary Statistics of CFTC Commitment Week-on-Week Change

This table reports summary statistics on the week-on-week changes of the CFTC commitment data. The week-on-week change is computed as the time series divided by its one-week lagged version, and finally 1 is subtracted from the result.

					Quantiles						
Variable	Mean	SD	Min	Max	5%	25%	50%	75%	95%	Skew	Kurtosis
Open Interest	0.01	0.09	−0.57	0.31	−0.14	−0.04	0.02	0.06	0.13	−0.91	3.50
Dealer Longs	0.05	0.36	−0.91	4.38	−0.26	−0.06	0.01	0.09	0.39	5.67	51.01
Dealer Shorts	0.03	0.29	−0.88	2.57	−0.33	−0.10	0.01	0.11	0.46	2.57	15.90
Dealer Spreads	0.04	0.28	−0.87	1.81	−0.40	−0.10	0.04	0.17	0.47	0.96	4.79
Asset Manager Longs	0.06	0.55	−1.00	8.63	−0.37	−0.08	0.00	0.10	0.50	8.65	108.33
Asset Manager Shorts	0.04	0.67	−1.00	13.80	−0.47	−0.08	0.01	0.11	0.42	14.81	296.45
Asset Manager Spreads	0.01	0.30	−1.00	3.80	−0.28	−0.10	0.00	0.08	0.37	3.44	45.27
Leveraged Funds Longs	0.06	0.52	−1.00	8.41	−0.34	−0.12	0.00	0.13	0.58	8.66	115.36
Leveraged Funds Shorts	0.02	0.16	−0.56	1.05	−0.23	−0.06	0.01	0.09	0.25	0.73	5.11
Leveraged Funds Spreads	0.05	0.32	−0.91	3.09	−0.31	−0.09	0.02	0.13	0.44	3.27	22.00
Other Reportable Longs	0.07	0.56	−1.00	7.79	−0.44	−0.13	0.00	0.14	0.73	6.24	64.97
Other Reportable Shorts	0.12	1.39	−1.00	29.62	−0.57	−0.18	0.00	0.20	0.72	16.51	323.16
Other Reportable Spreads	0.24	5.66	−1.00	149.00	−0.39	−0.12	0.01	0.14	0.54	26.18	685.77
Non-Reportable Longs	0.02	0.21	−0.66	1.88	−0.28	−0.09	0.01	0.10	0.35	1.77	12.37
Non-Reportable Shorts	0.04	0.31	−0.82	4.93	−0.33	−0.08	0.00	0.10	0.45	6.15	83.09

Table A8: Regressions of Realized Volatility and VIX

This table reports the results of three linear regressions, all of which have the VIX as independent variable:

$$\text{volatility}_{\text{realized}}(t) = \alpha + \beta \cdot \text{VIX}(t) \,/\, 100 + \epsilon(t)$$

Different measures of realized volatility as dependent variable distinguish the three regressions. Realized volatility is computed with regard to a moving three-day window. The VIX is divided by 100 to match the volatility scale. *, **, *** denote significance at the 10%, 5% and 1% level.

	Regression 1	**Regression 2**	**Regression 3**
Dependent Variable	S&P 500 Realized Closing Price Volatility	S&P 500 Realized Parkinson-Volatility	S&P 500 Realized Yang-Zhang-Volatility
α	-0.07^{***}	-0.05^{***}	-0.07^{***}
(t-statistic)	(-22.96)	(-27.51)	(-22.58)
β	1.13^{***}	0.89^{***}	1.33^{***}
(t-statistic)	(77.74)	(109.80)	(92.48)
Adjusted R^2	0.65	0.77	0.71
F-Statistic	6043^{***}	$12\,055^{***}$	8552^{***}

Figure A1: Intraday Settlement Price Position within 15 Minutes

This figure presents histograms of the position of the settlement price relative to the highest (and analogously lowest) watermark range in which trading has taken place in the first 1 to 15 minutes. Settlement prices are mapped to a scale of 0 (lower bound) to 1 (upper bound) of intraday trading. The histogram's vertical lines depict the median.

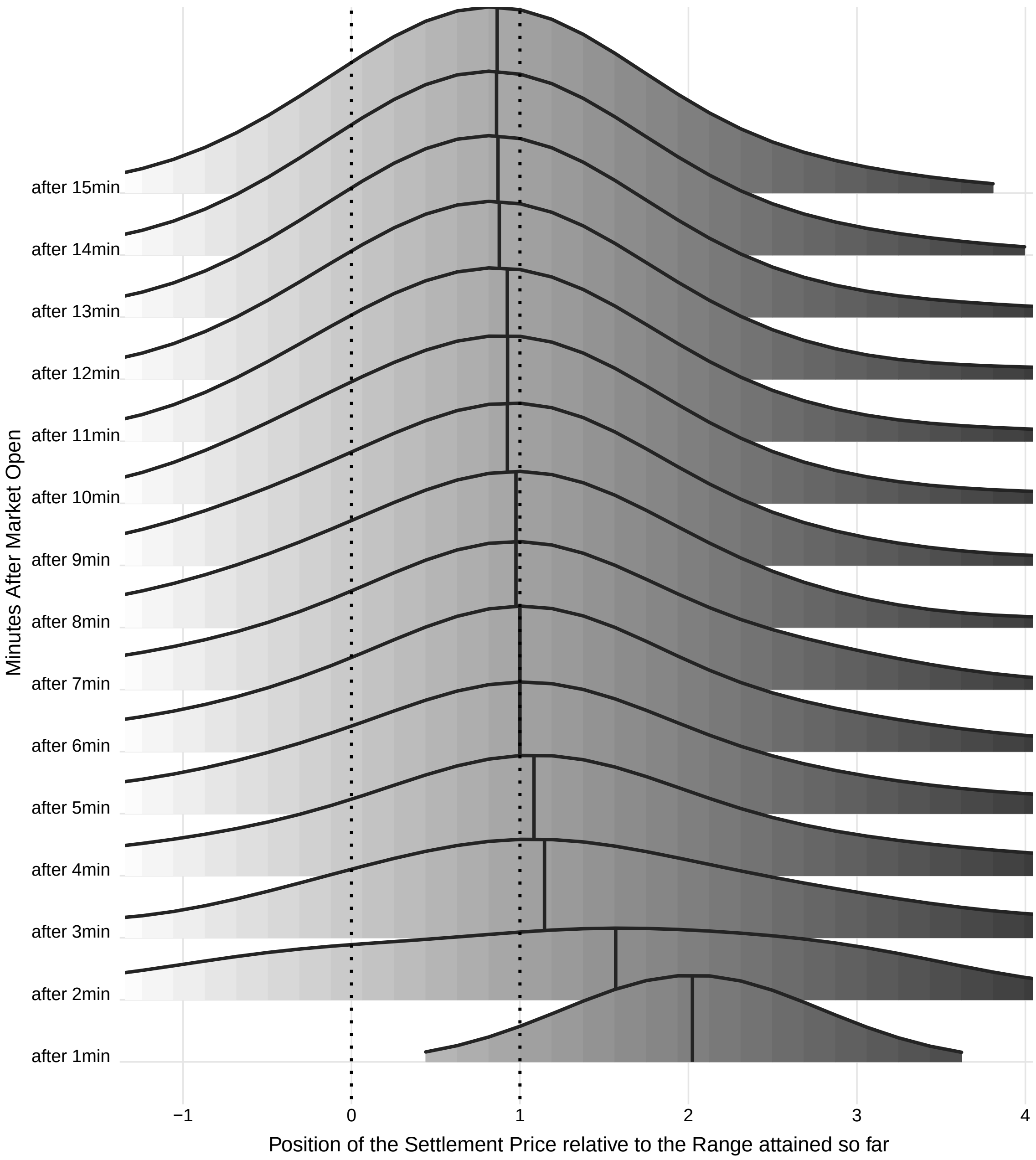


Figure A2: Time until the Settlement Price is reached

The histogram shows the time in hours until the settlement price can be observed within the settlement day (after settlement took place which is before markets open). If the settlement price is above the open quote, then the first time the VIX exceeds the settlement price is taken, and vice versa. Each hour is divided into six bars, each standing for a ten-minute interval, starting at zero right after markets open up to 6.5 hours when markets close. If there is no such point in time at all within the settlement day, the day is represented in the last bar on the right.

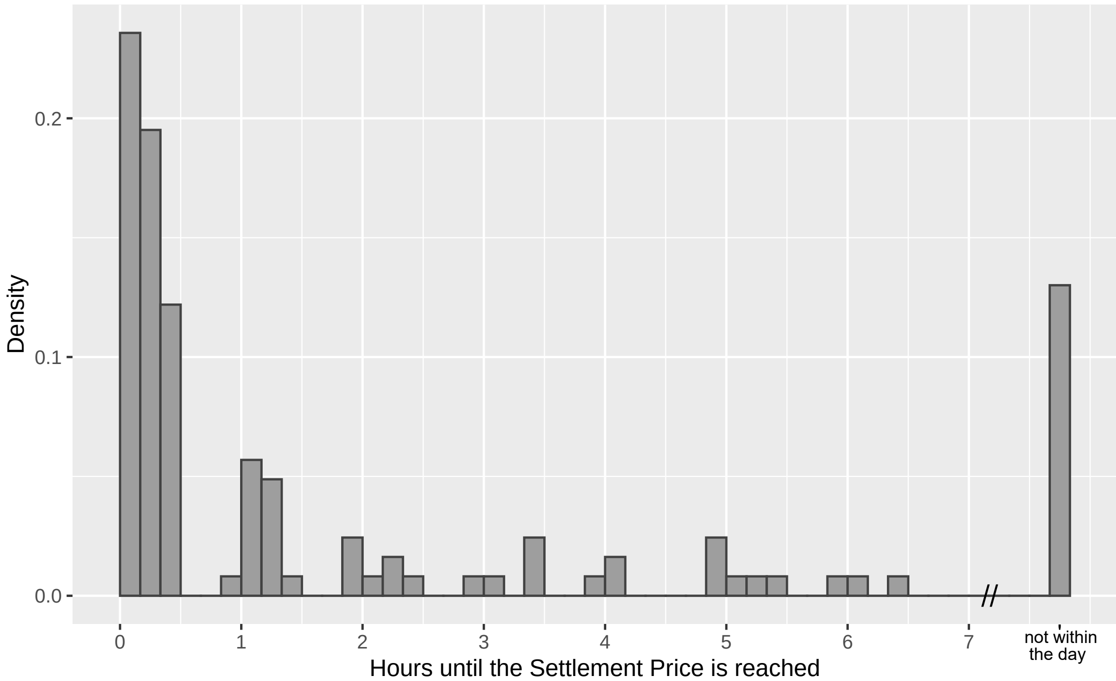


Figure A3: Put/Call Ratio of SPX options

The plot shows the kernel density estimates of the underlying S&P 500 options for settlement days (dashed line) and the other days (solid line). The corresponding vertical lines show the arithmetic mean.

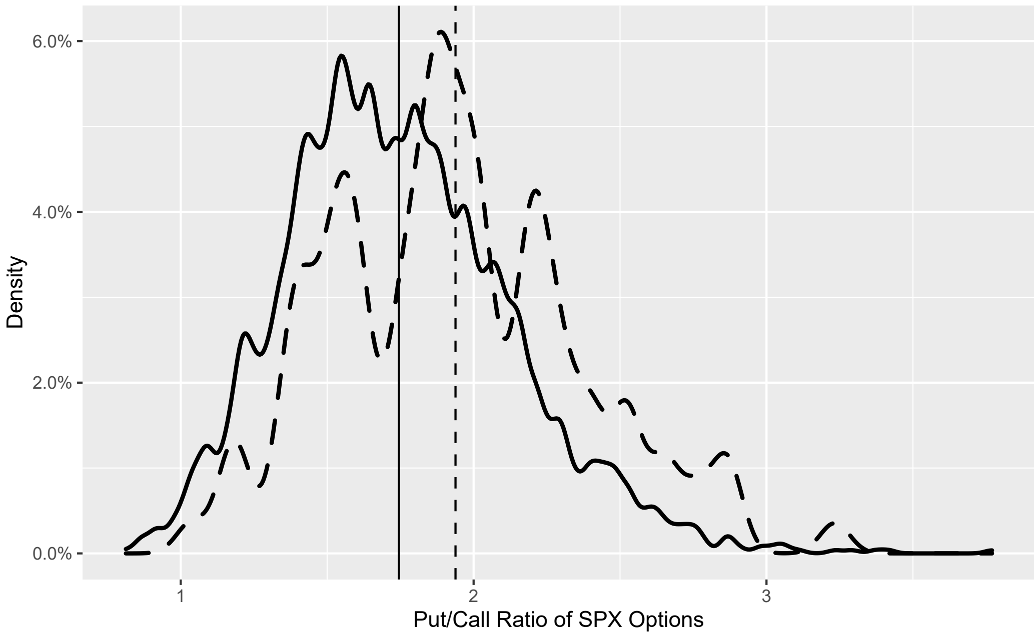


Figure A4: Repetitive Component of the Put/Call Ratio of SPX options

Based on a time series decomposition into three components (trend, repetitive and a residual), the plot shows the repetitive component of the put/call ratio of the underlying SPX options. The twentieth day is the settlement day of each one-month interval highlighted with the vertical points.

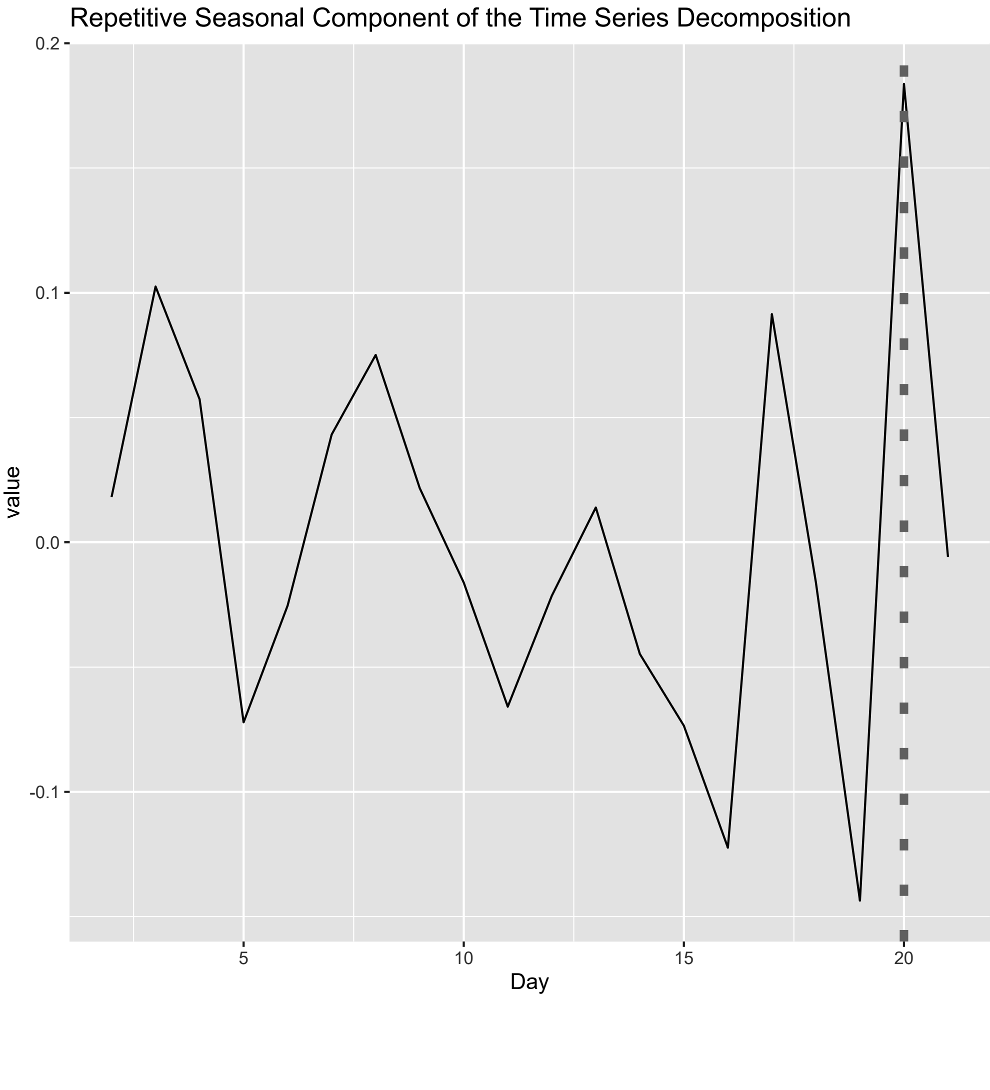


Figure A5: Open Interest

The plot shows the open interest in VIX derivatives on (monthly) settlement days minus the three-week moving average. The numbers are expressed in market values in 1000 USD * index value.

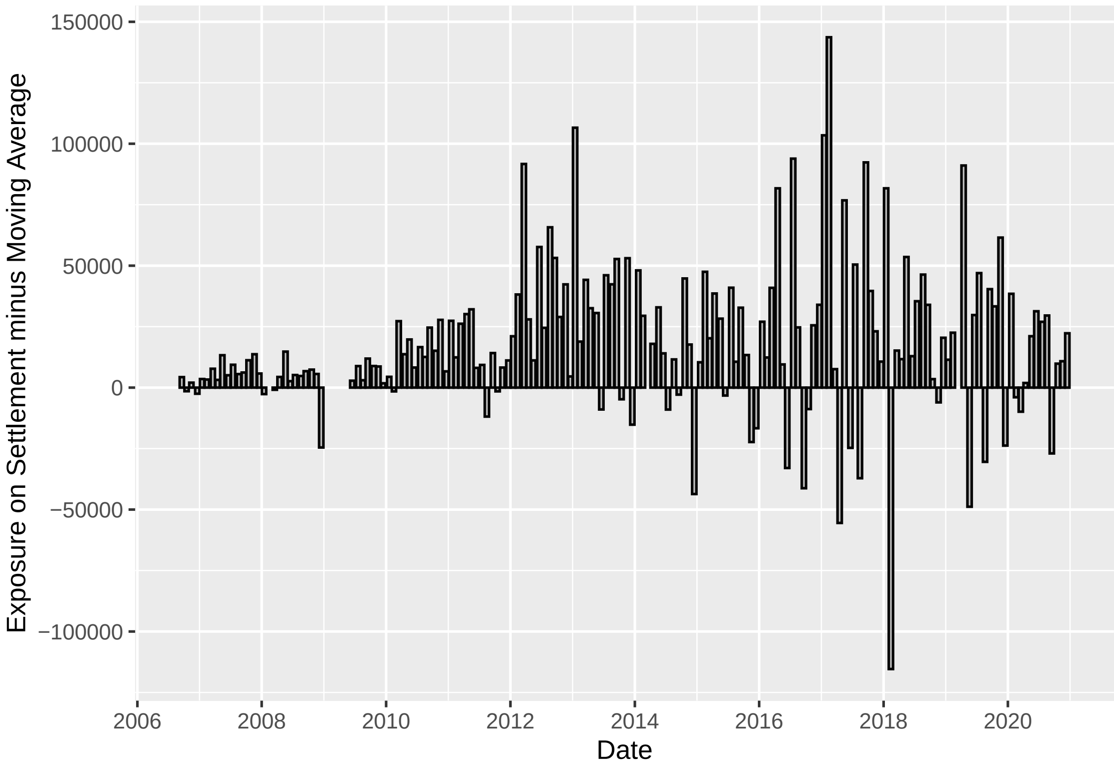
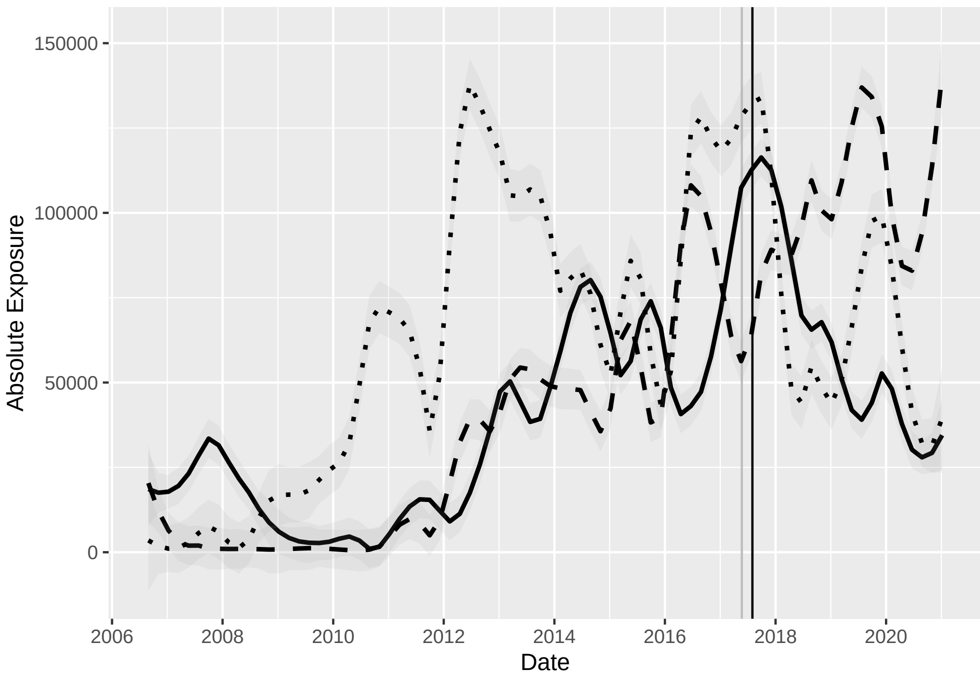


Figure A6: Absolute Exposure by Group

The plots show the absolute exposure (in VIX index value * 1000 USD) to VIX derivatives by group (Leveraged Funds with a solid line, Asset Manager dashed, Dealer dotted). The lines are smoothed and the bands represent the confidence interval to the 95% level. The vertical lines show the point in time of the publication (journal website and print version) of Griffin and Shams (2018).

Panel A: Long Exposure



Panel B: Short Exposure

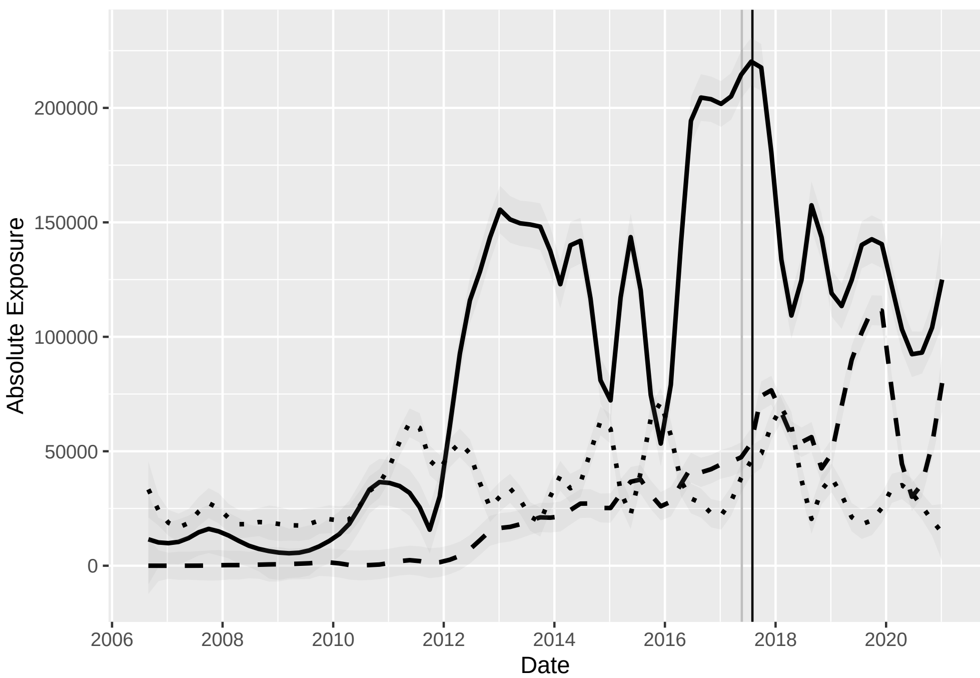


Figure A7: Development of the Largest 8's Exposure

This figure reports the development of the exposure of the largest **8** market participants split into two panels by long and short exposure, respectively. Every year, the weekly exposures induce two boxplots, for the subgroups of monthly settlement weeks and the other weeks. The data source is the Commodity Futures Trading Commission (CFTC).

Panel A: Largest **8's Long** Exposure **Panel B:** Largest **8's Short** Exposure

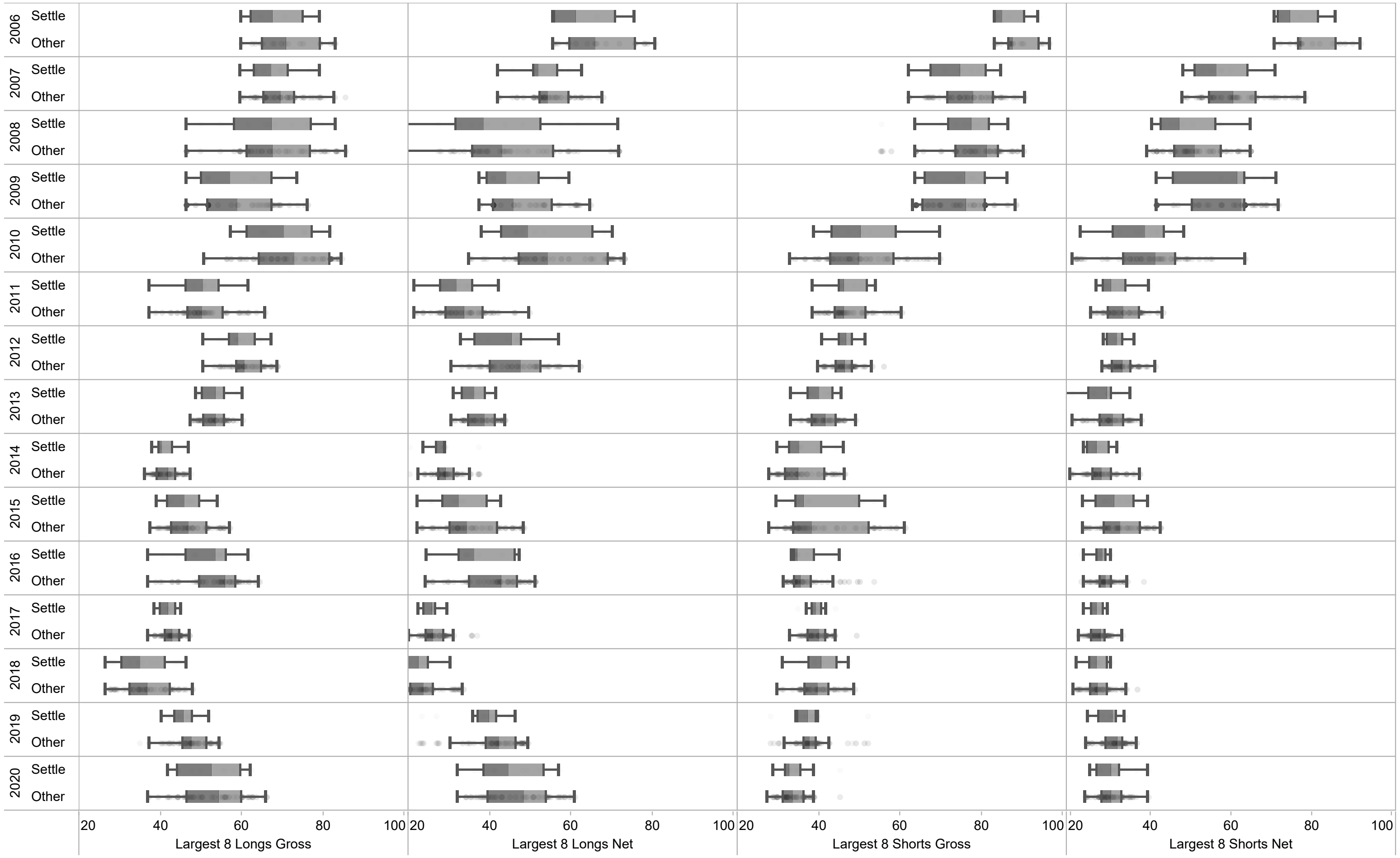


Figure A8: Development of the Largest 4's Exposure

This figure reports the development of the exposure of the largest **4** market participants split into two panels by long and short exposure, respectively. Every year, the weekly exposures induce two boxplots, for the subgroups of monthly settlement weeks and the other weeks. The data source is the Commodity Futures Trading Commission (CFTC).

Panel A: Largest **4's Long** Exposure **Panel B:** Largest **4's Short** Exposure

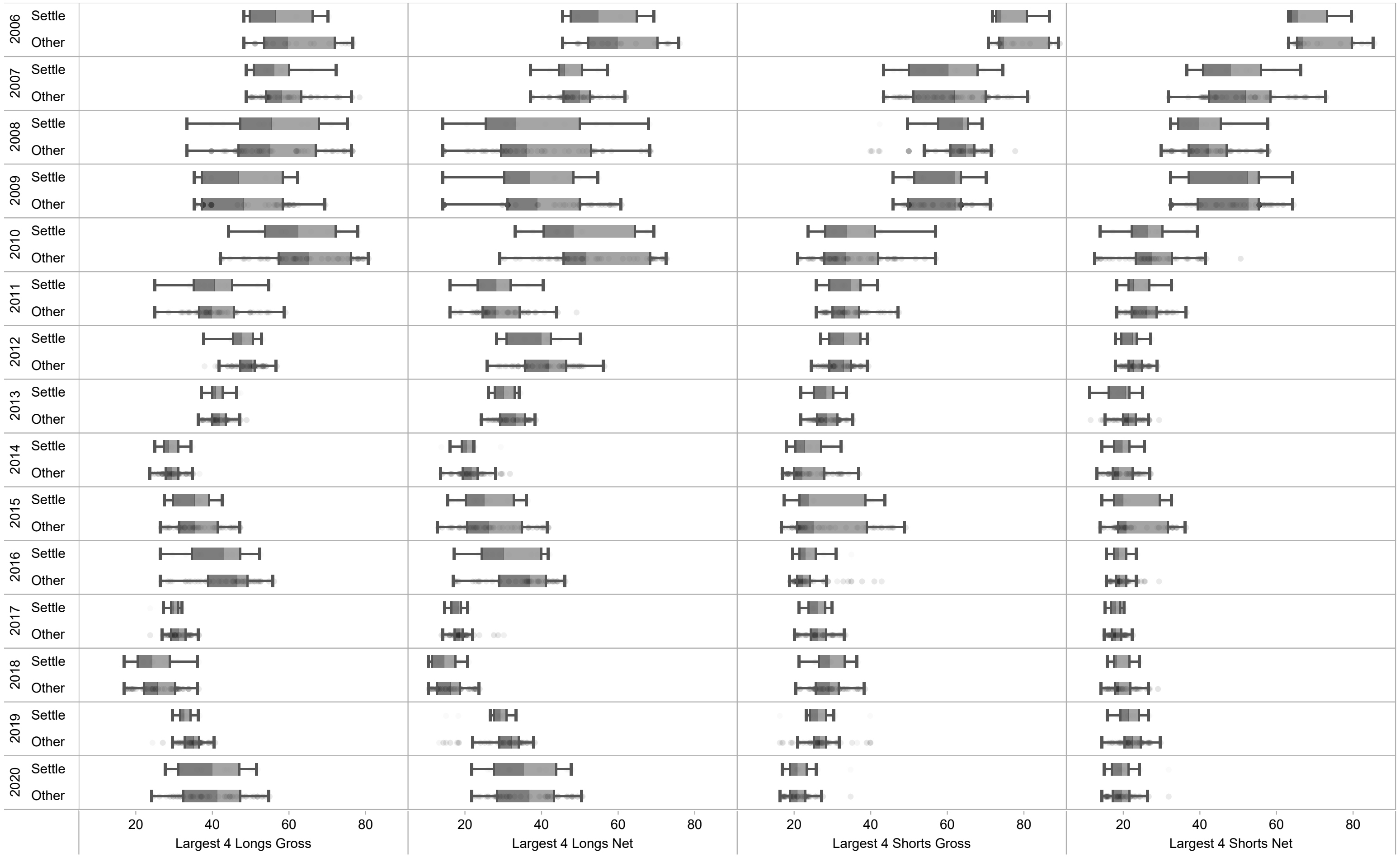


Figure A9: Development of Leveraged Funds' Exposure

This figure reports the development of the exposure of Leveraged Funds split into three panels by (A) long exposure, (B) short exposure and (C) the additional gross exposure to which the individual trader holds a neutralizing position ("spreads"). Every year, the weekly exposures induce two boxplots, for the subgroups of monthly settlement weeks and the other weeks. The data source is the Commodity Futures Trading Commission.

Panel A: Long **Panel B:** Short **Panel C:** Spreads

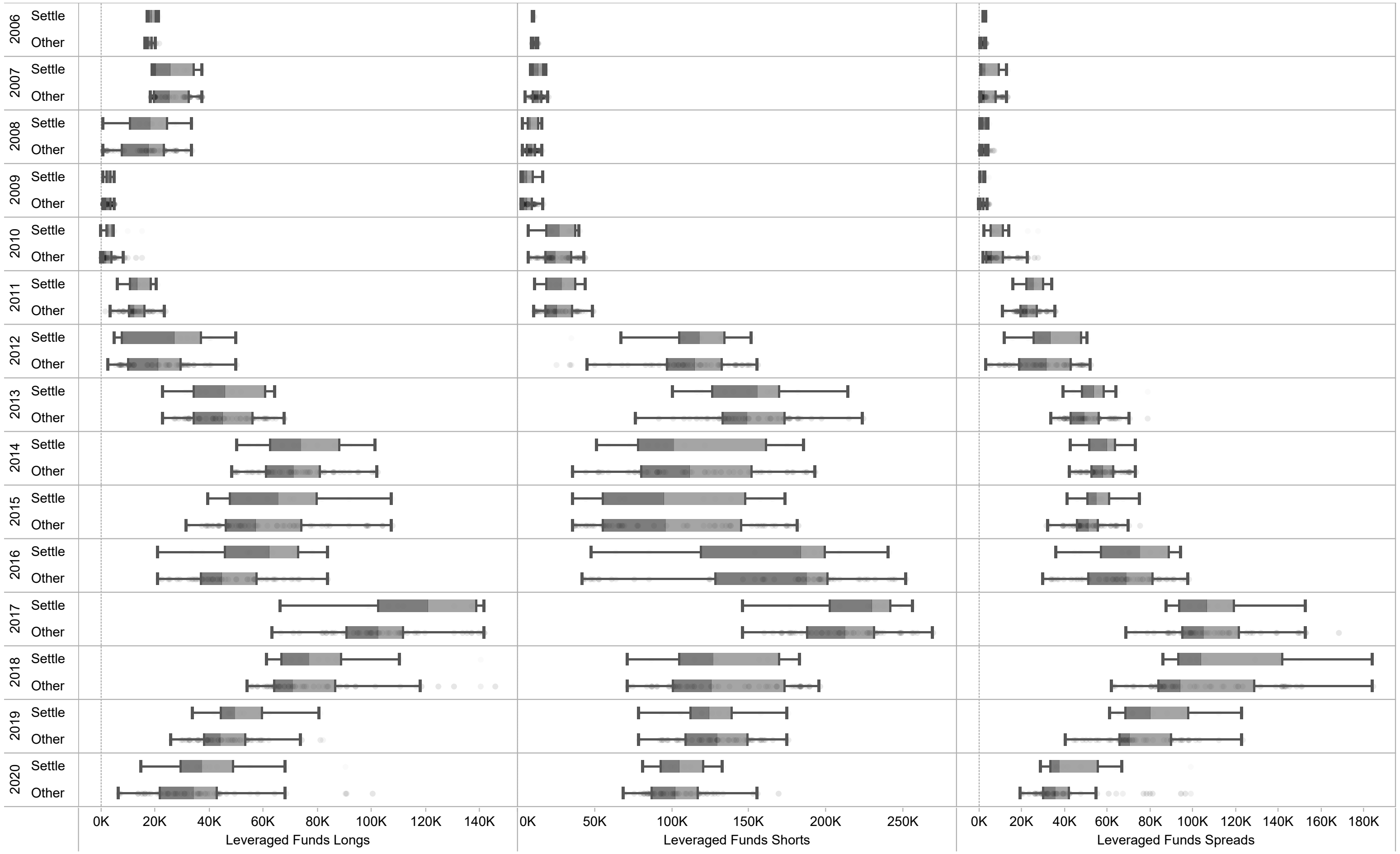


Figure A10: Development of Dealers' Exposure

This figure reports the development of the exposure of Dealers split into three panels by (A) long exposure, (B) short exposure and (C) the additional gross exposure to which the individual trader holds a neutralizing position ("spreads"). Every year, the weekly exposures induce two boxplots, for the subgroups of monthly settlement weeks and the other weeks. The data source is the Commodity Futures Trading Commission (CFTC).

Panel A: Long **Panel B:** Short **Panel C:** Spreads

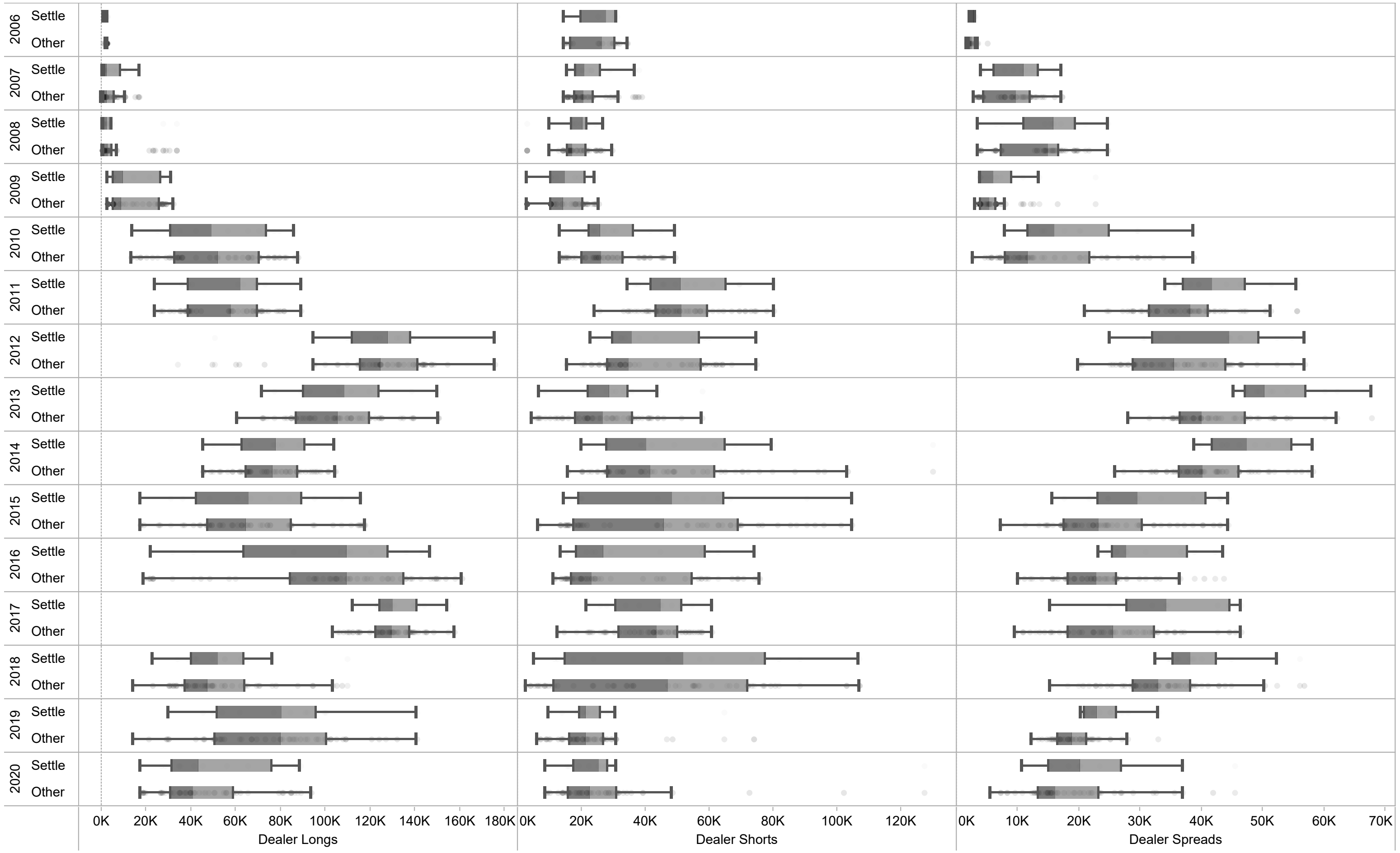


Figure A11: Development of Asset Managers' Exposure

This figure reports the development of the exposure of Asset Managers split into three panels by (A) long exposure, (B) short exposure and (C) the additional gross exposure to which the individual trader holds a neutralizing position ("spreads"). Every year, the weekly exposures induce two boxplots, for the subgroups of monthly settlement weeks and the other weeks. The data source is the Commodity Futures Trading Commission.

Panel A: Long **Panel B:** Short **Panel C:** Spreads