Stochastic Simulation (MIE1613H) - Homework 2

Due: February 24, 2022

- Submit your homework on Quercus as a single PDF file by the deadline. Late submissions are penalized 10% each day the homework is late.
- At the top of your homework include your name, student number, department, and program.
- You may discuss the assignment with other students in general terms, but each student must solve the problems, write the code and the solutions individually.
- The simulation models must be programmed in Python. You must include both the source code (including comments to make it easy to follow) and the output of the simulation in your submission.
- **It is very important to explain your answers and solution approach clearly and not just provide the code/results**. Full marks will only be given to correct solutions that are fully and clearly explained.

Problem 1. (10 Pts.) Use the Lindely's recursion simulation of the $M/G/1$ queue to provide an estimate (including a 95% CI) for $E[Y_{10}]$, i.e., the expected waiting time of the 10th customer to enter the system. Write down a mathematical expression for your proposed estimator and state whether it is biased or not. Keep the parameters the same as in the example from the lecture.

Problem 2. (20 Pts.) Another variation of European options are barrier options. Denote the stock price at time t by $X(t)$ and assume that it is modelled as a Geometric Brownian Motion (GBM). An up-and-out call option with barrier B and strike price K has payoff

$$I\left\{\max_{0\leq t\leq T} X(t) < B\right\}(X(T)-K)^+,$$

where $I\{A\}$ is the indicator function of event A. This means that if the value of the asset goes above \$$B$ before the option matures then the option is worthless. Hence, the value of the option is

$$E\left[e^{-rT} I\left\{\max_{0\leq t\leq T} X(t) < B\right\}(X(T)-K)^+\right].$$

Using the same parameters as in the Asian option example, i.e., $T = 1$, $X(0) = \$50$; $K = \$55$; $r = 0.05$ and $\sigma^2 = (0.3)^2$, estimate the value of this option for barriers $B = 60, 65, 70$ and provide an intuitive reason for the effect of increasing the barrier on the value of the option. Use $m = 64$ steps when discretizing the GBM and use $n = 40,000$ replications. Report a 95% confidence interval for your estimates.

Problem 3. (20 Pts.) Beginning with the PythonSim event-based $M/G/1$ simulation, implement the changes necessary to make it an $M/G/s$ simulation (a single queue with s servers). Keep the average service time at $\tau = 0.8$, and set the arrival rate to the number of servers, i.e., $\lambda = s$. Simulate the system for $s = 10, 20, 30$. Use 10 replications with run-length of 5500, and set the warmup to 500.

(a) Report the estimated steady-state expected system time, and expected utilization (average number of busy servers divided by the number of servers) in each case.

(b) Compare the results and state clearly what you observe. What you're doing is investigating the impact of system size on performance.

HINT: You need to modify the logic, not just set the number of available servers to s. The attribute "NumberOfUnits" of the Resource object returns the number of available units for any instance of the object.

Problem 4. (20 Pts.) Modify the PythonSim event-based simulation of the $M/G/1$ queue to allow for customer returns. This means that after finishing service, a customer may return to the system with probability p and after a random amount of time which is exponentially distributed with mean $1/\gamma$. Assume that return probability is $p = 0.1$ and expected time to return is $1/\gamma = 2$ and provide an estimate for the expected steady-state number of customers in system (waiting in queue or in service). For other parameters use the same values as in the $M/G/1$ example.

Problem 5. **(20 Pts.)** (Chapter 4, Exercise 15) The phone desk for a small office is staffed from 8 a.m. to 4 p.m. by a single operator. Calls arrive according to a Poisson process with rate 6 per hour, and the time to serve a call is uniformly distributed between 5 and 12 min. Callers who find the operator busy are placed on hold, if there is space available; otherwise, they receive a busy signal and the call is considered "lost." In addition, 10% of callers who do not immediately get the operator decide to hang up rather than go on hold; they are not considered lost, since it was their choice. Because the hold queue occupies resources, the company would like to know the smallest capacity (the number of callers) for the hold queue that keeps the daily fraction of lost calls under 5%. In addition, they would like to know the long-run utilization of the operator to make sure he or she will not be too busy. Use PythonSim to simulate this system and find the required capacity for the hold queue. Model the callers as class Entity, the hold queue as class FIFOQueue and the operator as class Resource. Use the PythonSim functions Expon and Uniform for random-variate generation. Use class DTStat to estimate the fraction of calls lost (record a 0 for calls not lost and a 1 for those that are lost so that the sample mean is the fraction lost). Use the statistics collected by class Resource to estimate the utilization.

Problem 6. (10 Pts.) For the AR(1) model discussed in Section 3.3 of the textbook, obtain the asymptotic variance γ^2. **HINT:** Start with expression (5.7) on page 102 of the textbook.