

## I. APPENDIX

### A. Proof of Lemma 1

**Lemma 1:** When historical gradient compensation is applied, the weight for each client in each round is bounded by

$$p_{k,r} \leq \frac{1}{1 + (K-1)(2\tau_{max} + K)^{-\alpha_s}}. \quad (1)$$

**Proof:** Given the application of historical gradient compensation, for a client not in  $S_r$ , if its last upload round is  $r'$ , then its staleness  $\tilde{\tau}_{k,r}$  evolves as follows:

- If client  $k$  does not receive the updated global model between its last upload round  $r'$ , then  $\tilde{\tau}_{k,r} = r - r' + \tau_{k,r'}$ .
- If client  $k$  receives the updated global model at round  $r^*$  after round  $r'$ , then  $\tilde{\tau}_{k,r} = \tau_{k,r} + r^* - r' + \tau_{k,r'}$ .

Because satellites arrive sequentially, after a satellite's last update, if it does not receive the latest model during its communication window, it must wait until the next visible window. During the interval between two windows, other satellites may upload their models and update the global model. Thus, there can be at most  $K-1$  global updates between two communication windows for a given satellite. Therefore, the staleness of actual aggregated model satisfies:

$$\begin{aligned} \tilde{\tau}_{k,r} &\leq \tau_{k,r} + r^* - r' + \tau_{k,r'} \\ &\leq \tau_{k,r} + K - 1 + \tau_{\max} \\ &\leq 2\tau_{\max} + K - 1 \end{aligned} \quad (2)$$

Note that  $\tilde{\tau}_{k,r} \geq 0$ . Therefore, we have

$$\begin{aligned} p_{k,r} &= \frac{q_{k,r}}{q_{k,r} + \sum_{j \neq k} q_{j,r}} \\ &\leq \frac{q_{k,r}}{q_{k,r} + (K-1)(2\tau_{max} + K)^{-\alpha_s}} \\ &\leq \frac{1}{1 + (K-1)(2\tau_{max} + K)^{-\alpha_s}} \end{aligned} \quad (3)$$

For brevity of notation, the subsequent analysis denotes  $p_{k,r} \leq C$ , where  $C = \frac{1}{1 + (K-1)(2\tau_{max} + K)^{-\alpha_s}}$ .

### B. Proof of Lemma 2

**Lemma 2:** If Assumption 1, 2 and 4 are satisfied, the deviation of local mode from the global model after  $i$  iterations is bounded by

$$\mathbb{E}\|\mathbf{w}_{k,r}^i - \mathbf{w}_r\|^2 \leq 4\eta^2(I-1)(2IG^2 + \varsigma^2). \quad (4)$$

**Proof:** Note that Lemma 2 is satisfied when  $i = 0$  due to  $\mathbf{w}_{k,r}^0 = \mathbf{w}_r$ . For the case of  $i \geq 1$ , we have

$$\begin{aligned} \mathbb{E}\|\mathbf{w}_{k,r}^i - \mathbf{w}_r\|^2 &= \mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \eta \nabla \tilde{F}_k(\mathbf{w}_{k,r}^{i-1}) - \mathbf{w}_r\|^2 \\ &= \mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \eta \nabla F_k(\mathbf{w}_{k,r}^{i-1}) - \mathbf{w}_r\|^2 \\ &\quad + \eta^2 \mathbb{E}\|\nabla \tilde{F}_k(\mathbf{w}_{k,r}^{i-1}) - \nabla F_k(\mathbf{w}_{k,r}^{i-1})\|^2 \\ &\leq \mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \eta \nabla F_k(\mathbf{w}_{k,r}^{i-1}) - \mathbf{w}_r\|^2 + \eta^2 \varsigma^2 \end{aligned} \quad (5)$$

where the second equation is due to the unbiased stochastic gradient assumption. The last inequation is due to Assumption 2. For the first term of the RHS of (5), we have

$$\begin{aligned} &\mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \eta \nabla F_k(\mathbf{w}_{k,r}^{i-1}) - \mathbf{w}_r\|^2 \\ &= \mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r\|^2 + \eta^2 \mathbb{E}\|\nabla F_k(\mathbf{w}_{k,r}^{i-1})\|^2 \\ &\quad - 2\mathbb{E}\langle \frac{1}{\sqrt{I-1}}(\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r), \sqrt{I-1} \eta \nabla F_k(\mathbf{w}_{k,r}^{i-1}) \rangle \\ &\leq (1 + \frac{1}{I-1})\mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r\|^2 + \eta^2 I \mathbb{E}\|\nabla F_k(\mathbf{w}_{k,r}^{i-1})\|^2 \\ &\leq (1 + \frac{1}{I-1})\mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r\|^2 + 2\eta^2 I \mathbb{E}\|\nabla F_k(\mathbf{w}_r)\|^2 \\ &\quad + 2\eta^2 I \mathbb{E}\|\nabla F_k(\mathbf{w}_{k,r}^{i-1}) - \nabla F_k(\mathbf{w}_r)\|^2 \\ &\leq (1 + \frac{1}{I-1} + 2\eta^2 IL^2)\mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r\|^2 + 2\eta^2 IG^2 \end{aligned} \quad (6)$$

where the first inequation holds by the Young's Inequality and the second holds by triangle inequality. The last inequation comes from Assumption 1 and 4. Substituting (6) into (5) obtains

$$\begin{aligned} \mathbb{E}\|\mathbf{w}_{k,r}^i - \mathbf{w}_r\|^2 &\leq (1 + \frac{1}{I-1} + 2\eta^2 IL^2)\mathbb{E}\|\mathbf{w}_{k,r}^{i-1} - \mathbf{w}_r\|^2 \\ &\quad + 2\eta^2 IG^2 + \eta^2 \varsigma^2 \end{aligned} \quad (7)$$

Telescoping the above inequation obtains

$$\begin{aligned} \mathbb{E}\|\mathbf{w}_{k,r}^i - \mathbf{w}_r\|^2 &\leq (2\eta^2 IG^2 + \eta^2 \varsigma^2) \frac{(1 + \frac{1}{I-1} + \frac{1}{2K^2 I})^{I-1} - 1}{\frac{1}{I-1} + \frac{1}{2K^2 I}}, \end{aligned} \quad (8)$$

where the first inequation holds by  $K\eta \leq \frac{1}{2IL}$ . Because  $a = \frac{1}{I-1} + \frac{1}{2K^2 I} \geq \frac{1}{I-1}$ ,  $(1+a)^{I-1} = e^{(I-1)\ln(1+a)} \leq e^{(I-1)a}$ . Then, we have

$$\begin{aligned} (I-1)a &= (I-1)(\frac{1}{I-1} + \frac{1}{2K^2 I}) \\ &= 1 + \frac{I-1}{2K^2 I} \\ &\leq 1 + \frac{1}{2K^2} \leq \frac{3}{2} \end{aligned} \quad (9)$$

Therefore, we have

$$\frac{(a+1)^{I-1} - 1}{a} \leq \frac{e^{\frac{3}{2}} - 1}{a} \leq (e^{\frac{3}{2}} - 1)(I-1) \leq 4(I-1). \quad (10)$$

Substituting (10) into (8) obtains

$$\mathbb{E}\|\mathbf{w}_{k,r}^i - \mathbf{w}_r\|^2 \leq 4\eta^2(I-1)(2IG^2 + \varsigma^2). \quad (11)$$

### C. Proof of Lemma 3

**Lemma 3:** If Assumptions 1, 2, and 4 is satisfied, the drift between the global models in two rounds, i.e.,  $r_1$  and  $r_2$  ( $r_1 > r_2$ ), is bounded by

$$\mathbb{E}\|\mathbf{w}_{r_1} - \mathbf{w}_{r_2}\|^2 \leq 3\eta^2(r_1 - r_2)^2 C^2 D_1, \quad (12)$$

where  $D_1 = (I^2 K^2 + I - 1)\varsigma^2 + (I^2 K^2 + 2I(I-1))G^2$ .

**Proof:** For the two different rounds, i.e.,  $r_1$  and  $r_2$  ( $r_1 > r_2$ ), we have

$$\begin{aligned}
& \mathbb{E} \|\mathbf{w}_{r_1} - \mathbf{w}_{r_2}\|^2 \\
&= \mathbb{E} \left\| \sum_{r=r_2}^{r_1-1} (\mathbf{w}_{r+1} - \mathbf{w}_r) \right\|^2 \\
&= \eta^2 \mathbb{E} \left\| \sum_{r=r_2}^{r_1-1} \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2 \\
&\leq \eta^2 (r_1 - r_2) K I \sum_{r=r_2}^{r_1-1} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq \eta^2 (r_1 - r_2) K I C^2 \sum_{r=r_2}^{r_1-1} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2
\end{aligned} \tag{13}$$

where the first inequation comes from using Cauchy-Schwarz Inequality and , and the last inequation is due to Lemma 1.

$$\begin{aligned}
& \mathbb{E} \|\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&= \mathbb{E} \left\| \underbrace{\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)}_{A_1} \right. \\
&\quad \left. + \underbrace{\nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{r-\tilde{\tau}_{k,r}})}_{A_2} \right. \\
&\quad \left. + \underbrace{\nabla F_k(\mathbf{w}_{r-\tilde{\tau}_{k,r}})}_{A_3} \right\|^2 \\
&\leq 3\mathbb{E} \|A_1\|^2 + 3\mathbb{E} \|A_2\|^2 + 3\mathbb{E} \|A_3\|^2 \\
&\leq 3\zeta^2 + 12\eta^2 (I-1) (2IG^2 + \zeta^2) L^2 + 3G^2
\end{aligned} \tag{14}$$

where the first inequation comes from using the triangle inequality, and the second inequation is due to Assumption 1, Assumption 4 and Lemma 2. Substituting (14) into (13), considering  $K\eta \leq \frac{1}{2IL}$ , we have

$$\begin{aligned}
\mathbb{E} \|\mathbf{w}_{r_1} - \mathbf{w}_{r_2}\|^2 &\leq \eta^2 (r_1 - r_2)^2 K^2 I^2 C^2 \\
&\quad (3\zeta^2 + 12\eta^2 (I-1) (2IG^2 + \zeta^2) L^2 + 3G^2), \\
&\leq 3\eta^2 (r_1 - r_2)^2 C^2 D_1
\end{aligned} \tag{15}$$

where  $D_1 = (I^2 K^2 + I - 1) \zeta^2 + (I^2 K^2 + 2I(I-1)) G^2$ .

#### D. Proof of Theorem 1

**Theorem 1:** If Assumptions 1-4 is satisfied, and  $K\eta \leq \frac{1}{2IL}$ , the R-round convergency bound of SC-SFL is

$$\begin{aligned}
\mathbb{E}[F(\mathbf{w}_R) - F^*] &\leq c_1^R \mathbb{E}[F(\mathbf{w}_0) - F^*] + c_2 \sum_{r=0}^{R-1} c_1^r \\
&\quad + c_3 \sum_{r=0}^{R-2} c_1^{R-r-2} \sum_{k=1}^K (1 - s_{k,r})(\tilde{\tau}_{k,r} + 1)^2
\end{aligned} \tag{16}$$

where  $c_1 = (1 - \eta I \mu + 2CKI^2 L \mu \eta^2)$ ,  $c_2 = 12\eta^3 L^2 KI(I-1)(2IG^2 + \zeta^2)(\frac{1}{2}C + CIL\eta) + \frac{3}{2}CK\eta I\Gamma^2 + \frac{1}{2}\eta KC^2\zeta^2 + \frac{1}{2}CKI^2\eta^2 L\zeta^2 + 3CKI^2\eta^2 L\Gamma^2$  and  $c_3 = 3\eta^3 C^2 D_1 IL^2(\frac{3}{2}C + \frac{7}{2} + 3CIL\eta L)$ .

**Proof:** According to Assumption 1, we have

$$\begin{aligned}
& \mathbb{E}[F(\mathbf{w}_{r+1}) - F(\mathbf{w}_r)] \\
&\leq \mathbb{E} \langle \nabla F(\mathbf{w}_r), \mathbf{w}_{r+1} - \mathbf{w}_r \rangle + \frac{L}{2} \mathbb{E} \|\mathbf{w}_{r+1} - \mathbf{w}_r\|^2 \\
&= -\eta \mathbb{E} \langle \nabla F(\mathbf{w}_r), \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \rangle \\
&\quad + \frac{\eta^2 L}{2} \mathbb{E} \left\| \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2
\end{aligned} \tag{17}$$

For the first term of (17), we have

$$\begin{aligned}
& \mathbb{E} \langle \nabla F(\mathbf{w}_r), \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \rangle \\
&= \mathbb{E} \langle \nabla F(\mathbf{w}_r), \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \\
&\quad - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) + \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \rangle \\
&= \mathbb{E} \langle \nabla F(\mathbf{w}_r), \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \rangle \\
&\quad + \mathbb{E} \langle \nabla F(\mathbf{w}_r), b_1 \rangle
\end{aligned} \tag{18}$$

where  $b_1 = \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)]$ . Substituting (18) into (17), we have

$$\begin{aligned}
& \mathbb{E}[F(\mathbf{w}_{r+1}) - F(\mathbf{w}_r)] \\
&\leq -\eta \mathbb{E} \langle \nabla F(\mathbf{w}_r), \underbrace{\sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)}_{B_1} \rangle \\
&\quad - \underbrace{\eta \mathbb{E} \langle \nabla F(\mathbf{w}_r), b_1 \rangle}_{B_2} + \frac{\eta^2 L}{2} \underbrace{\mathbb{E} \left\| \sum_{k=1}^K \sum_{i=0}^{I-1} p_{k,r} \nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2}_{B_3}
\end{aligned} \tag{19}$$

For term  $B_1$ , we have

$$\begin{aligned}
B_1 &= -\eta I \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2 \\
&\quad + \eta I \mathbb{E} \langle \nabla F(\mathbf{w}_r), \nabla F(\mathbf{w}_r) - \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \rangle \\
&\leq -\eta I \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2 + \frac{\eta I}{2} \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2 \\
&\quad + \frac{\eta I}{2} \mathbb{E} \left\| \nabla F(\mathbf{w}_r) - \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2 \\
&= -\frac{\eta I}{2} \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2 \\
&\quad + \frac{\eta I}{2} \mathbb{E} \left\| \nabla F(\mathbf{w}_r) - \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2
\end{aligned} \tag{20}$$

where the first equation is due to the unbiased stochastic gradient assumption and the first inequation holds by triangle inequality. For the last term on the RHS of (20), we have

$$\begin{aligned}
& \mathbb{E} \left\| \nabla F(\mathbf{w}_r) - \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) \right\|^2 \\
&= \mathbb{E} \left\| \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} [\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \right\|^2 \\
&\leq \mathbb{E} \left[ \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{p_{k,r}}{I} \|\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \right] \quad , \quad (21) \\
&\leq \mathbb{E} \left[ \sum_{k=1}^K \sum_{i=0}^{I-1} \frac{C}{I} \|\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \right] \\
&\leq \frac{C}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2
\end{aligned}$$

where the first inequation holds by Jensen's inequality and the second inequation holds by Lemma 1. According to Assumption 1 and 3, we have

$$\begin{aligned}
& \frac{C}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq \frac{3C}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_r)\|^2 \\
&+ \frac{3C}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F_k(\mathbf{w}_r) - \nabla F_k(\mathbf{w}_{r-\tilde{\tau}_{k,r}})\|^2 \quad , \quad (22) \\
&+ \frac{3C}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F_k(\mathbf{w}_{r-\tilde{\tau}_{k,r}}) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq 3CK\Gamma^2 + 3CL^2 \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_r - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \\
&+ \frac{3CL^2}{I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\mathbf{w}_{r-\tilde{\tau}_{k,r}} - \mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i\|^2
\end{aligned}$$

Substituting (21) and (22) into (20), we have

$$\begin{aligned}
B_1 &\leq -\frac{\eta I}{2} \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2 + \frac{3}{2} CK\eta I\Gamma^2 \\
&+ \frac{3}{2} C\eta IL^2 \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_r - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \quad , \quad (23) \\
&+ \frac{3}{2} C\eta L^2 \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\mathbf{w}_{r-\tilde{\tau}_{k,r}} - \mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i\|^2
\end{aligned}$$

For the term  $B_2$ , we have

$$\begin{aligned}
B_2 &= -\eta I \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \langle \nabla F(\mathbf{w}_r), \\
&\quad \frac{p_{k,r}}{I} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \rangle \\
&= -\eta I \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \langle \nabla F(\mathbf{w}_r) - \nabla F(\mathbf{w}_{r-\tilde{\tau}_{k,r}}), \\
&\quad \frac{p_{k,r}}{I} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \rangle
\end{aligned}$$

$$\begin{aligned}
& \frac{p_{k,r}}{I} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \rangle \\
&\leq \frac{\eta I^2}{2} \sum_{k=1}^K \mathbb{E} \|\nabla F(\mathbf{w}_r) - \nabla F(\mathbf{w}_{r-\tilde{\tau}_{k,r}})\|^2 \quad , \\
&+ \frac{\eta I}{2} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \left\| \frac{p_{k,r}}{I} [\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)] \right\|^2 \\
&\leq \frac{\eta I^2 L^2}{2} \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_r - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \\
&+ \frac{\eta C^2}{2I} \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq \frac{\eta I^2 L^2}{2} \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_r - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 + \frac{1}{2} \eta K C^2 \zeta^2 \quad (24)
\end{aligned}$$

where the first inequation holds by Young's inequality and the last two inequation is due to Assumption 1 and 2. For the term  $B_3$ , we have

$$\begin{aligned}
B_3 &\leq CI \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla \tilde{F}_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq CKI^2 \zeta^2 + CI \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)\|^2 \\
&\leq CKI^2 \zeta^2 \quad , \quad (25) \\
&+ \underbrace{2CI \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i) - \nabla F(\mathbf{w}_r)\|^2}_{B_4} \\
&+ 2CKI^2 \mathbb{E} \|\nabla F(\mathbf{w}_r)\|^2
\end{aligned}$$

where the first inequation holds by Jensen's inequality and Lemma 1, and the second inequation is due to Assumption 2. The third inequation is by adding and subtracting  $\nabla F(\mathbf{w}_r)$  and  $\nabla F_k(\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i)$ . For the term  $B_4$ , we have

$$\begin{aligned}
B_4 &\leq 6CIL^2 \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \\
&+ 6CI^2 L^2 \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_{r-\tilde{\tau}_{k,r}} - \mathbf{w}_r\|^2 \\
&+ 6CI^2 \sum_{k=1}^K \mathbb{E} \|\nabla F_k(\mathbf{w}_r) - \nabla F(\mathbf{w}_r)\|^2 \\
&\leq 6CIL^2 \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E} \|\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \quad , \quad (26) \\
&+ 6CI^2 L^2 \sum_{k=1}^K \mathbb{E} \|\mathbf{w}_{r-\tilde{\tau}_{k,r}} - \mathbf{w}_r\|^2 \\
&+ 6CKI^2 \Gamma^2
\end{aligned}$$

where the first inequation is by adding and subtracting  $\nabla F_k(\mathbf{w}_{r-\tilde{\tau}_{k,r}})$  and  $\nabla F_k(\mathbf{w}_r)$  and using Assumption 1. The

second inequation is due to Assumption 3. Substituting (26) into (25), we have

$$\begin{aligned}
B_3 &\leq CKI^2\zeta^2 + 6CKI^2\Gamma^2 + 2CKI^2\mathbb{E}\|\nabla F(\mathbf{w}_r)\|^2 \\
&+ 6CIL^2 \sum_{k=1}^K \sum_{i=0}^{I-1} \mathbb{E}\|\mathbf{w}_{k,r-\tilde{\tau}_{k,r}}^i - \mathbf{w}_{r-\tilde{\tau}_{k,r}}\|^2 \\
&+ 6CI^2L^2 \sum_{k=1}^K \mathbb{E}\|\mathbf{w}_{r-\tilde{\tau}_{k,r}} - \mathbf{w}_r\|^2
\end{aligned} \quad (27)$$

Substituting (23), (24) and (27) into (19), and using Assumption 1, Lemma 2 and Lemma 3, we have

$$\begin{aligned}
&\mathbb{E}[F(\mathbf{w}_{r+1}) - F(\mathbf{w}_r)] \\
&\leq (-\eta I\mu + 2CKI^2L\mu\eta^2)\mathbb{E}[F(\mathbf{w}_r) - F^*] \\
&+ c_2 + c_3 \sum_{k=1}^K \tilde{\tau}_{k,r}^2,
\end{aligned} \quad (28)$$

where  $c_2 = 12\eta^3L^2KI(I-1)(2IG^2 + \zeta^2)(\frac{1}{2}C + CIL\eta) + \frac{1}{2}\eta CK((C + I^2\eta L)\zeta^2 + (3I + 6I^2\eta L)\Gamma^2)$  and  $c_3 = 3\eta^3C^2D_1IL^2(\frac{3}{2}C + \frac{I}{2} + 3CIL\eta L)$ .

Subtracting  $F^*$  to  $F(\mathbf{w}_{r+1})$  and  $F(\mathbf{w}_r)$ , we have

$$\begin{aligned}
&\mathbb{E}[F(\mathbf{w}_{r+1}) - F^*] \\
&\leq (1 - \eta I\mu + 2CKI^2L\mu\eta^2)\mathbb{E}[F(\mathbf{w}_r) - F^*] \\
&+ c_2 + c_3 \sum_{k=1}^K \tilde{\tau}_{k,r}^2.
\end{aligned} \quad (29)$$

Note that  $\tilde{\tau}_{k,r} \leq \tau_{k,r} + \tau_{\max} + K - 1$ , we have

$$\begin{aligned}
c_3 \sum_{k=1}^K \tilde{\tau}_{k,r}^2 &\leq 2c_3 \sum_{k=1}^K \tau_{k,r}^2 + 2c_3 \sum_{k=1}^K (\tau_{\max} + K - 1)^2 \\
&\leq 2c_3 \sum_{k=1}^K \tau_{k,r}^2 + 2c_3 K(\tau_{\max} + K - 1)^2
\end{aligned} \quad (30)$$

Considering the impact of satellite communication windows, a satellite may not be able to obtain the latest global model immediately after a global update. Therefore for the model updated in round  $r$ , we have

- If  $s_{k,r-1} = 1$ , it indicates that the satellite was selected for upload in the previous round, meaning that the model updated in round  $r$  must be trained based on global model  $\mathbf{w}_r$ , where its staleness  $\tau_{k,r} = 0$ .
- If  $s_{k,r-1} = 0$ , there are two cases: 1) if there was a model trained in round  $r-1$ , then the staleness  $\tau_{k,r} = \tau_{k,r-1} + 1$ . 2) if there was no model trained in round  $r-1$ , then the model updated in round  $r$  must be trained based on global model  $\mathbf{w}_r$ , where its staleness  $\tau_{k,r} = 0$ .

Therefore, we have  $\tau_{k,r} \leq (1 - s_{k,r-1})(\tau_{k,r-1} + 1)$ . Noticing that the  $\tau_{k,0} = 0$ , and telescoping the above inequation, we have

$$\begin{aligned}
&\mathbb{E}[F(\mathbf{w}_R) - F^*] \\
&\leq c_1^R \mathbb{E}[F(\mathbf{w}_0) - F^*] \\
&+ [c_2 + 2c_3K(\tau_{\max} + K - 1)^2] \sum_{r=0}^{R-1} c_1^r \\
&+ 2c_3 \sum_{r=1}^{R-1} c_1^{R-r-1} \sum_{k=1}^K (1 - s_{k,r-1})(\tau_{k,r-1} + 1)^2, \\
&= c_1^R \mathbb{E}[F(\mathbf{w}_0) - F^*] \\
&+ [c_2 + 2c_3K(\tau_{\max} + K - 1)^2] \sum_{r=0}^{R-1} c_1^r \\
&+ 2c_3 \sum_{r=0}^{R-2} c_1^{R-r-2} \sum_{k=1}^K (1 - s_{k,r})(\tau_{k,r} + 1)^2
\end{aligned} \quad (31)$$

where  $c_1 = (1 - \eta I\mu + 2CKI^2L\mu\eta^2)$ .