Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

2 The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

3 **(a)** Show that the equation $\dfrac{7\tan\theta}{\cos\theta}+12=0$ can be expressed as

$$12\sin^2\theta-7\sin\theta-12=0.$$ [3]

$\dfrac{7\tan\theta}{\cos\theta}+12=0$

$\Rightarrow 7\dfrac{\sin\theta}{\cos\theta}\times\dfrac{1}{\cos\theta}+12=0$

$\Rightarrow 7\dfrac{\sin\theta}{\cos^2\theta}+12=0$

$\Rightarrow 7\sin\theta+12\cos^2\theta=0$

$\Rightarrow 7\sin\theta+12(1-\sin^2\theta)=0$

$\Rightarrow 7\sin\theta+12-12\sin^2\theta=0$

$\Rightarrow 12\sin^2\theta-7\sin\theta-12=0$

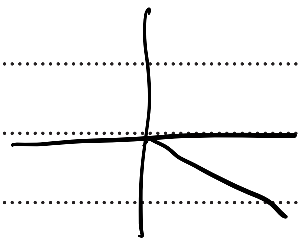
(b) Hence solve the equation $\dfrac{7\tan\theta}{\cos\theta}+12=0$ for $0^\circ\leqslant\theta\leqslant360^\circ$. [3]

$\dfrac{7\tan\theta}{\cos\theta}+12=0$

$\Rightarrow 12\sin^2\theta-7\sin\theta-12=0$

let $\sin\theta=x$

$\Rightarrow 12x^2-7x-12=0$



$3x \quad -4$

$4x \quad 3$

$\Rightarrow (3x-4)(4x+3)=0$

$\Rightarrow x=\frac{4}{3}$ or $x=-\frac{3}{4}$

$\because x=\sin\theta$

$\therefore \sin\theta=\frac{4}{3}$ or $\sin\theta=-\frac{3}{4}$

$\Rightarrow \times$ $\quad\Rightarrow \theta=-0.8481$

$\Rightarrow \theta=-48.6^\circ$

$\therefore \theta=180+48.6$, $\theta=360-48.6$

$\Rightarrow \theta=228.6^\circ$ $\quad\Rightarrow \theta=311.4^\circ$

4 The function f is defined as follows:

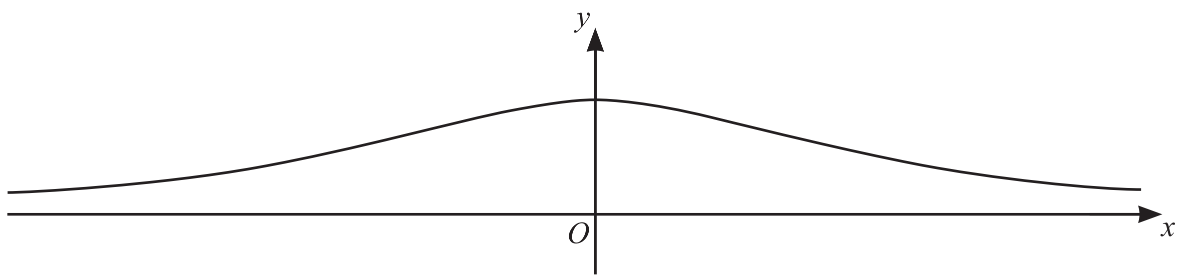
$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$. [1]

$f^{-1}(x)$: $x = \sqrt{y} - 1$

$\Rightarrow (x+1)^2 = y$

$\Rightarrow y = (x+1)^2$



The diagram shows the graph of $y = g(x)$ where $g(x) = \dfrac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

when $x = 0$ $g(x) = \frac{1}{2}$

$\therefore$ range of g : $0 < g \leq \frac{1}{2}$

g^{-1} does not exist because $g(x)$ is a many-one function.

The function h is defined by $h(x) = \dfrac{1}{x^2+2}$ for $x \geqslant 0$.

(c) Solve the equation $hf(x) = f\left(\dfrac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [4]

5 The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

first : $\tan \frac{1}{4}\pi = 1$

second : $\sin \frac{1}{4}\pi = \frac{\sqrt{2}}{2}$ $= \frac{1}{2}n\{2a + (n-1)d\}$

$U_n = a + (n-1)d$

$\Rightarrow \frac{\sqrt{2}}{2} = 1 + d$

$\Rightarrow d = \frac{-2+\sqrt{2}}{2}$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$\Rightarrow S_{40} = \frac{1}{2} \times 40 \times \left(2 + 39 \times \frac{-2+\sqrt{2}}{2}\right)$

$\Rightarrow S_{40} = 20 \times \frac{-74+39\sqrt{2}}{2}$

$\Rightarrow S_{40} = -740 + 390\sqrt{2}$

The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) **(i)** Find the sum to infinity of the progression in terms of θ. [2]

$U_n = ar^{n-1}$ $\qquad S_\infty = \frac{a}{1-r}$

$\Rightarrow \sin\theta = \tan\theta \, r$ $\qquad \Rightarrow S_\infty = \frac{\tan\theta}{1-\cos\theta}$

$\Rightarrow \sin\theta = \frac{\sin\theta}{\cos\theta} r$

$\Rightarrow r = \cos\theta$

(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

first: $\tan\frac{\pi}{3} = \sqrt{3}$

second: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\because r = \cos\theta$, $\cos\theta = \cos\frac{\pi}{3} = \frac{1}{2}$

$\therefore r = \frac{1}{2}$

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_{10} = \frac{\sqrt{3}\times(1-\frac{1}{2}^{10})}{1-\frac{1}{2}}$

$\Rightarrow S_{10} = \sqrt{3}\times(1-\frac{1}{2}^{10})\times 2$

$\Rightarrow S_{10} = 3.46$

6 The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at A and intersects the positive x-axis at B.

(a) Find the coordinates of A and B. [4]

$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}}$

when stationary, $\frac{dy}{dx} = 0$

$\therefore 2 - 4x^{-\frac{1}{2}} = 0$

$\Rightarrow 4x^{-\frac{1}{2}} = 2$

$\Rightarrow x^{-\frac{1}{2}} = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2}$

$\Rightarrow \sqrt{x} = 2$

$\Rightarrow x = 4$

when $x = 4$

$y = 2 \times 4 - 8 \times 4^{\frac{1}{2}}$

$\Rightarrow y = 8 - 8 \times 2$

$\Rightarrow y = -8$

$\therefore A : (4, -8)$

when indersect at positive x-axis

$2x - 8x^{\frac{1}{2}} = 0$

$\Rightarrow 2x = 8x^{\frac{1}{2}}$

$\Rightarrow x = 4x^{\frac{1}{2}}$

$\Rightarrow x - 4x^{\frac{1}{2}} = 0$

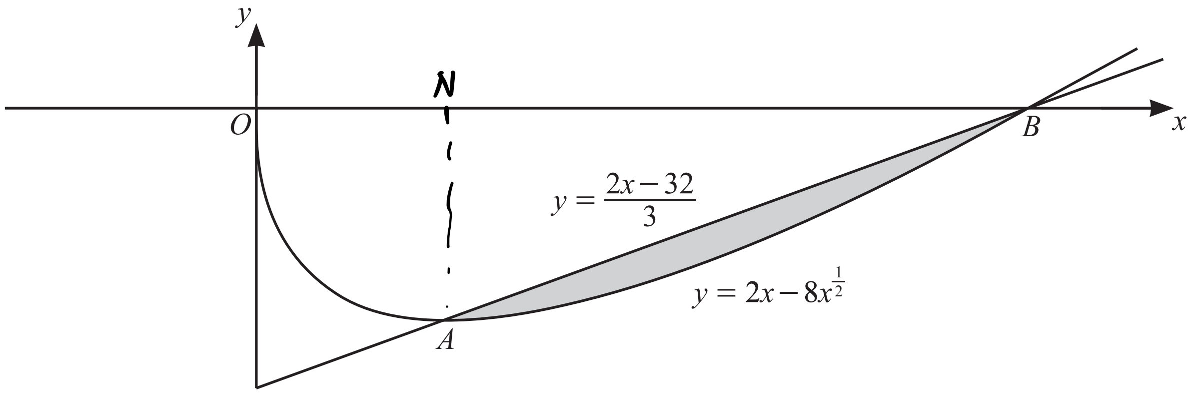
$\Rightarrow x^{\frac{1}{2}}(x^{\frac{1}{2}} - 4) = 0$

$\Rightarrow x^{\frac{1}{2}} = 0$ or $x^{\frac{1}{2}} - 4 = 0$

$\Rightarrow x = 0$ or $x = 16$

$B : (16, 0)$

(b)



The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB. It is given that the equation of AB is $y = \frac{2x-32}{3}$.

Find the area of the shaded region between the curve and the line. [5]

A: (4, -8)

B: (16, 0)

$\int_4^{16} (2x - 8x^{\frac{1}{2}})\,dx$

$= [x^2 - 8 \times \frac{2}{3} x^{\frac{3}{2}} + c]_4^{16}$

$= [x^2 - \frac{16}{3} x^{\frac{3}{2}} + c]_4^{16}$

$= (16^2 - \frac{16}{3} \times 16^{\frac{3}{2}}) - (4^2 - \frac{16}{3} \times 4^{\frac{3}{2}})$

$= -\frac{176}{3}$

$\therefore S_{NAB} = \frac{176}{3}$

$\because$ A: (4, -8), B(16, 0)

$\therefore NB = 16 - 4 = 12$, $NA = 8$

$\therefore S_{\triangle NAB} = 12 \times 8 \times \frac{1}{2} = 48$

$\therefore$ shaded area $= \frac{176}{3} - 48 = \frac{32}{3}$

7 The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

$(y+a)^2 = 18-(x-6)^2$

$\Rightarrow y+a = \pm\sqrt{18-(x-6)^2}$

$\Rightarrow y = \pm\sqrt{18-(x-6)^2} - a$

equation is a tangent to the circle.

$\therefore$ $2a-x = \sqrt{18-(x-6)^2} - a$ ① or $2a-x = -\sqrt{18-(x-6)^2} - a$ ②

②: $3a-x = -\sqrt{18-(x-6)^2}$

$\Rightarrow (x-3a)^2 = 18-(x-6)^2$

$\Rightarrow x^2-6ax+9a^2 = ①$

①: $(3a-x)^2 = 18-(x-6)^2$

$\Rightarrow 9a^2-6ax+x^2 = 18-(x^2-12x+36)$

$\Rightarrow 9a^2-6ax+x^2 = 18-x^2+12x-36$

$\Rightarrow 2x^2-(6a+12)x+9a^2+18=0$

$b^2-4ac=0$

$\Rightarrow (6a+12)^2-4\times2\times(9a^2+18)=0$

$\Rightarrow 36a^2+144a+144-72a^2-144=0$

$\Rightarrow 36a^2-144a=0$

$\Rightarrow a^2-4a=0$

$\Rightarrow a(a-4)=0$

$\Rightarrow a=0$ or $a=4$

(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

$y = 2a - x$

gradient of the normal is 1

$(x-6)^2 + (y+a)^2 = 18$

when $a = 4$: $(x-6)^2 + (y+4)^2 = 18$ ②, $y = 8 - x$ ①

center of the circle: $(6, -4)$

Substitute ① into ②:

$(x-6)^2 + (8-x+4)^2 = 18$

$\Rightarrow (x-6)^2 + (12-x)^2 = 18$

$\Rightarrow x^2 - 12x + 36 + 144 - 24x + x^2 = 18$

$\Rightarrow 2x^2 - 36x + 162 = 0$

$\Rightarrow x^2 - 18x + 81 = 0$

$x \quad -9$

$x \quad -9$

$\Rightarrow (x-9)^2 = 0$

$\Rightarrow x = 9$

when $x = 9$, $y = 8 - 9$

$\Rightarrow y = -1$

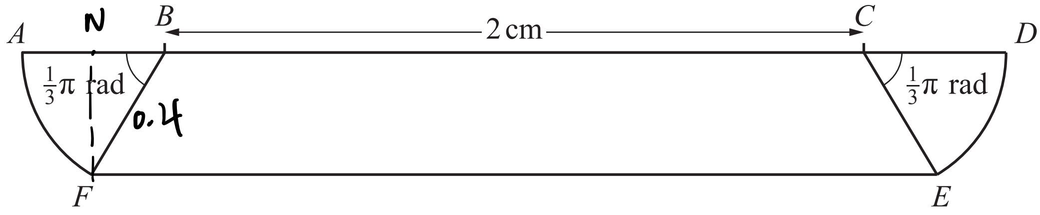
$\therefore (9, -1)$

$y + 1 = 1 \times (x - 9)$

$\Rightarrow y + 1 = x - 9$

$\Rightarrow y = x - 10$

8



The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius *r* cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

(i) Show that the length $EF = 2.4$ cm. [2]

rad : $\frac{1}{3}\pi$, $r = 0.4$

$NB = 0.4\cos\frac{1}{3}\pi = \frac{1}{5} = 0.2$

$NB \times 2 + BC = EF$

$\Rightarrow EF = 2 + 0.2 \times 4 = 2.4$

(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

S plate $= (2+2.4) \times 0.4\sin(\frac{1}{3}\pi) \times \frac{1}{2} + 2 \times \frac{1}{2} \times 0.4^2 \times \frac{1}{3}\pi$

$= 4.4 \times \frac{\sqrt{3}}{5} \times \frac{1}{2} + \frac{4}{25} \times \frac{1}{3}\pi$

$= \frac{11\sqrt{3}}{25} + \frac{4}{75}\pi$

$= 0.930$ cm^2

(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r. Give your answer correct to 3 significant figures. [4]

$(r\cos\frac{\pi}{3})\times 2 + 2\times 2 + 2r + 2\times \frac{\pi}{3}r = 6$

$\Rightarrow r + 4 + 2r + \frac{2\pi}{3}r = 6$

$\Rightarrow 3r + \frac{2\pi}{3}r = 2$

$\Rightarrow \frac{9+2\pi}{3}r = 2$

$\Rightarrow r = 0.393$

9 A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

(a) Determine the set of values of x for which $f(x)$ is decreasing. [4]

when sdationary

$f'(x) = 0$

$\Rightarrow 6(2x-3)^2 - 6x = 0$

$\Rightarrow 6 \times (4x^2 - 12x + 9) - 6x = 0$

$\Rightarrow 24x^2 - 72x + 54 - 6x = 0$

$\Rightarrow 24x^2 - 78x + 54 = 0$

$\Rightarrow 12x^2 - 39x + 27 = 0$

$\Rightarrow 4x^2 - 13x + 9 = 0$

$1x \quad -1$
$4x \quad -9$

$\Rightarrow (x-1)(4x-9) = 0$

$\Rightarrow x = 1$ or $x = \frac{9}{4}$

$f''(x) = 12(2x-3) \times 2 - 6$
$= 48x - 72 - 6$
$= 48x - 78$

when $x = 1$: $48 \times 1 - 78 < 0$
$\therefore$ function is in increasing
when $x = \frac{9}{4}$: $48 \times \frac{9}{4} - 78 > 0$
$\therefore$ function is decreasing.

when $x > \frac{9}{4}$, function is decreasing.

(b) Given that f(1) = −1, find f(x). [4]

$\int [6(2x-3)^2 - 6x]\,dx$

$= \int [6(4x^2 - 12x + 9) - 6x]\,dx$

$= \int (24x^2 - 72x + 54 - 6x)\,dx$

$= \int (24x^2 - 78x + 54)\,dx$

$= 8x^3 - 39x^2 + 54x + C$

when $f(1) = -1$: $8 \times 1 - 39 \times 1 + 54 \times 1 + C = -1$

$\Rightarrow 8 - 39 + 54 + C = -1$

$\Rightarrow C = -24$

$\therefore f(x) = 8x^3 - 39x^2 + 54x - 24$

10 The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

(a) A point P is moving along the curve in such a way that the y-coordinate of point P is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is increasing when $y = 32$. [4]

$\frac{dy}{dt} = -5$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{3}{2}(5-2x)^{\frac{1}{2}} \times (-2)$

$= -3(5-2x)^{\frac{1}{2}}$

when $y = 32$:

$32 = (5-2x)^{\frac{3}{2}} + 5$

$\Rightarrow 27 = (5-2x)^{\frac{3}{2}}$

$\Rightarrow 5-2x = 9$

$\Rightarrow 2x = -4$

$\Rightarrow x = -2$

when $x = -2$:

$-3(5+4)^{\frac{1}{2}} = -9 = \frac{dy}{dx}$

$-9 = -5 \times \frac{dt}{dx}$

$\Rightarrow \frac{dx}{dt} = \frac{5}{9}$

(b) Point A on the curve has y-coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3.

Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

when point B's gradient is -3:

$\frac{dy}{dx} = -3(5-2x)^{\frac{1}{2}} = -3$

$\Rightarrow (5-2x)^{\frac{1}{2}} = 1$

$\Rightarrow 5-2x = 1$ when $x = \frac{5}{2}$:

$\Rightarrow x = \frac{5}{2}$ $y = 5$

$B: (\frac{5}{2}, 5)$

when point A's $y = 32$:

$32 = (5-2x)^{\frac{3}{2}} + 5$

$\Rightarrow 27 = (5-2x)^{\frac{3}{2}}$

$\Rightarrow 9 = 5-2x$

$\Rightarrow 2x = -4$ $A: (-2, 32)$

$\Rightarrow x = -2$

gradient for line AB: $\frac{32-5}{\frac{5}{2}+2} = 6$

$\therefore$ line AB: $y-32 = 6(x+2)$

$\Rightarrow y-32 = 6x+12$

$\Rightarrow y = 6x+44$

mid point of AB: $x: \frac{1}{4}$, $y: \frac{37}{2}$ mid: $(\frac{1}{4}, \frac{37}{2})$

Normal for gradient AB: $-\frac{1}{6}$

$\therefore$ line of perpendicular bisector:

$y - \frac{37}{2} = -\frac{1}{6}(x - \frac{1}{4})$

$\Rightarrow y - \frac{37}{2} = -\frac{1}{6}x + \frac{1}{24}$ $\Rightarrow y = -\frac{1}{6}x + \frac{445}{24}$

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.