Development of an optimal filtering method for Ultra-Light Dark Matter search with GW detectors

Takumi Fujimori (Osaka Metropolitan University)

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Frontiers in Gravity and Fundamental Physics (YU Workshop 2025)

2025/03/05 Yonezawa, Yamagata, JAPAN

Contents

✳ Dark Matter search with Gravitational Wave detectors

- Ultra-Light Dark Matter and ULDM models
- A problem with data analysis

✳ Approach with an optimal filtering method

- Correlation of the ULDM signals
- Optimal filter for the signals
- Application to simulated data and comparison with a previous method

✳ Discussion

- DM model selection

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DM search with GW detectors

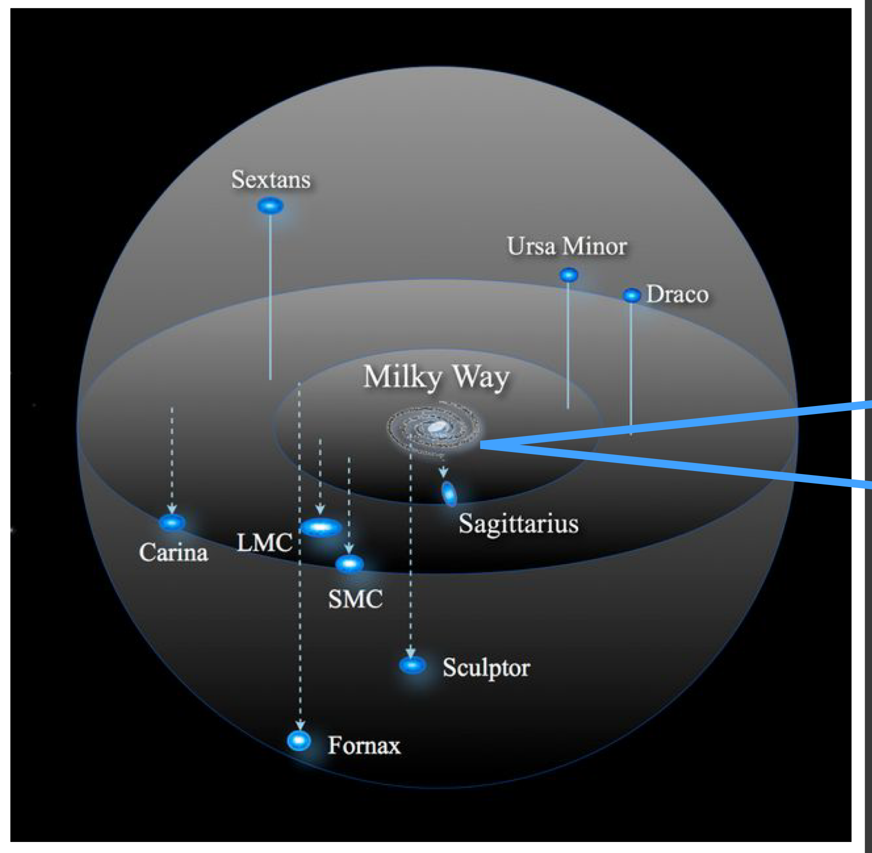
✳ Huge discovery space: 90 orders of magnitude!

10^{-22} eV | eV | $\sim$ GeV | $m_{\rm DM}$

Excluded… | **Ultra-Light DM (wave-like DM)** | Particle DM | more heavier models from WIMPs to PBH

↔ : The mass region that GW detectors can search for ($10^{-19} \sim 10^{-11}$ eV)

✳ Local DM distribution



Local DM energy density

$\rho_{\rm DM} \sim 0.4\ {\rm GeV/cm^3}$

cite: de Martino, I.+ Universe **2020**, 6, 107.

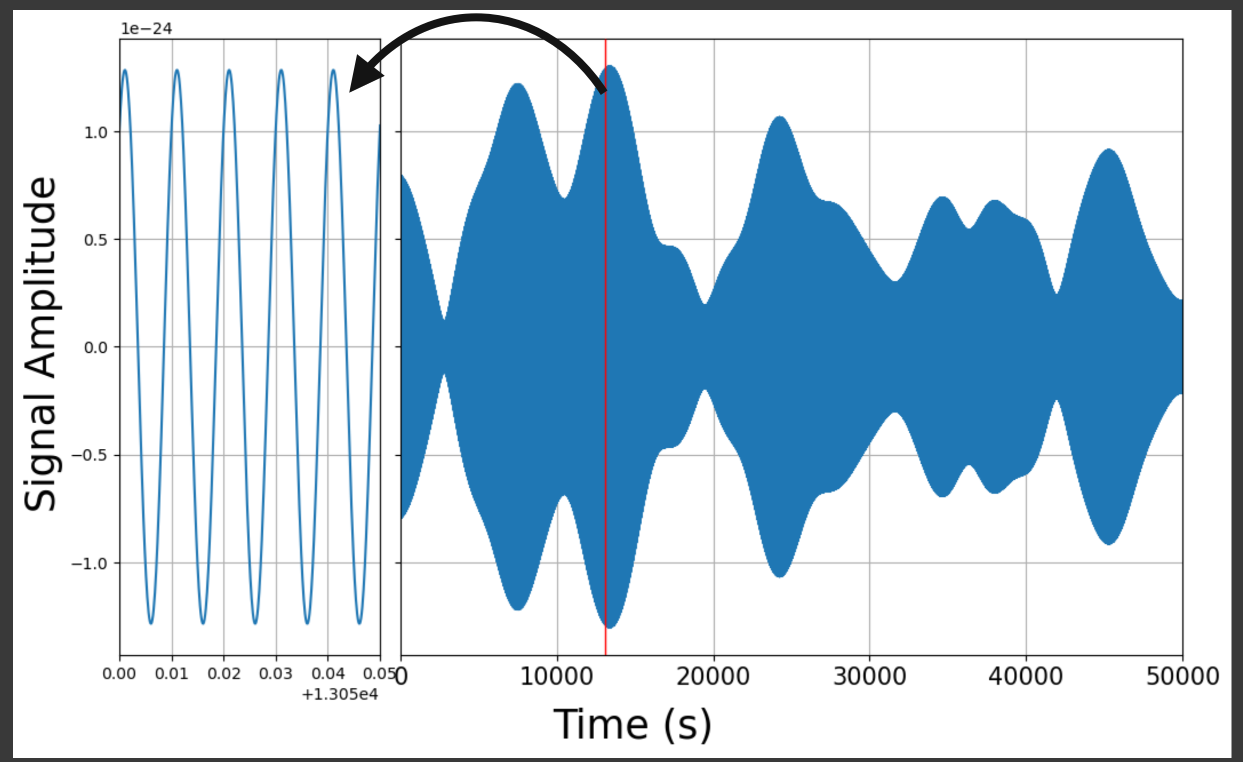
When the halo is composed of ULDM

$f_{\rm DM} \propto m_{\rm DM}/2\pi$

→ number density >> 1

→ DM halo behaves as a classical wave field

→ The amplitude fluctuates **stochastically**



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ULDM models

✳ According to the number density, $m_{\rm DM} \leq 1\ \mathrm{eV}$ is allowed for **bosons**

<table>
<tr>
<td>

spin-0 candidate:

axion (or ALPs)

- coupling to photon

$$\mathcal{L} \supset \frac{g_{a\gamma}}{4} a F_{\mu\nu} F^{\mu\nu}$$

1. K. Nagano+, PRL **123**, 111301 (2019)
2. D. Martynov+, PRD **101**, 095034 (2020)
3. K. Nagano+, PRD **104**, 062008 (2021)

</td>
<td>

spin-1 candidate:

dark photon

- coupling to fermions

$$\mathcal{L} \supset e\epsilon_D J^{\mu}_D A_\mu$$

※ We especially focused on the $D = B - L$ case (so-called "$U(1)_{B-L}$" gauge boson)

1. Y. Michimura+, PRD **102**, 102001 (2020)
2. S. Morisaki+, PRD **103**, L051702 (2021)
3. LVK Collab., PRD **110**, 042001 (2024)

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<td>

spin-2 candidate:

spin-2 DM

- coupling to matter field

$$\mathcal{L} \supset -\frac{\alpha}{2M_{\rm Pl}} \Phi_{ij} T^{ij}$$

1. Y. Manita+, PRD **107**, 104007 (2023)
2. Y. Manita+, PRD **109**, 095012 (2024)

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✳ Each model interacts with GW detectors and **produces similar signals**

Data analysis in the previous study

✳ Analysis method for ULDM

Time series data: $d(t)$

Cut into N_{ch} chunks & Fourier Transform by each

$\tilde{d}(f, t_1)$ $\tilde{d}(f, t_2)$ $\tilde{d}(f, t_i)$

✳ **Incoherent analysis**

- detection statistic: $\rho\left(f_{\mathrm{DM}}\right)$

incoherently sum up ρ_i ,

$$\rho(f) = \sum_{i}^{N_{\mathrm{ch}}} \rho_i(f), \quad \rho_i(f) = \frac{4|\tilde{d}(f; t_i)|^2}{T_{\mathrm{ch}} S(f; t_i)}$$

then search for the DM with the threshold determined by the frequentist method

Cutting in chunks is to;

- consider **the directional change** of the detectors
- Ensure **the stationarity** of the detectors

➡ **shorter chunk is better**

T_{ch} : duration of each chunk

$S(f; t_i)$: one-sided noise power spectral density

A Problem of Incoherent Analysis

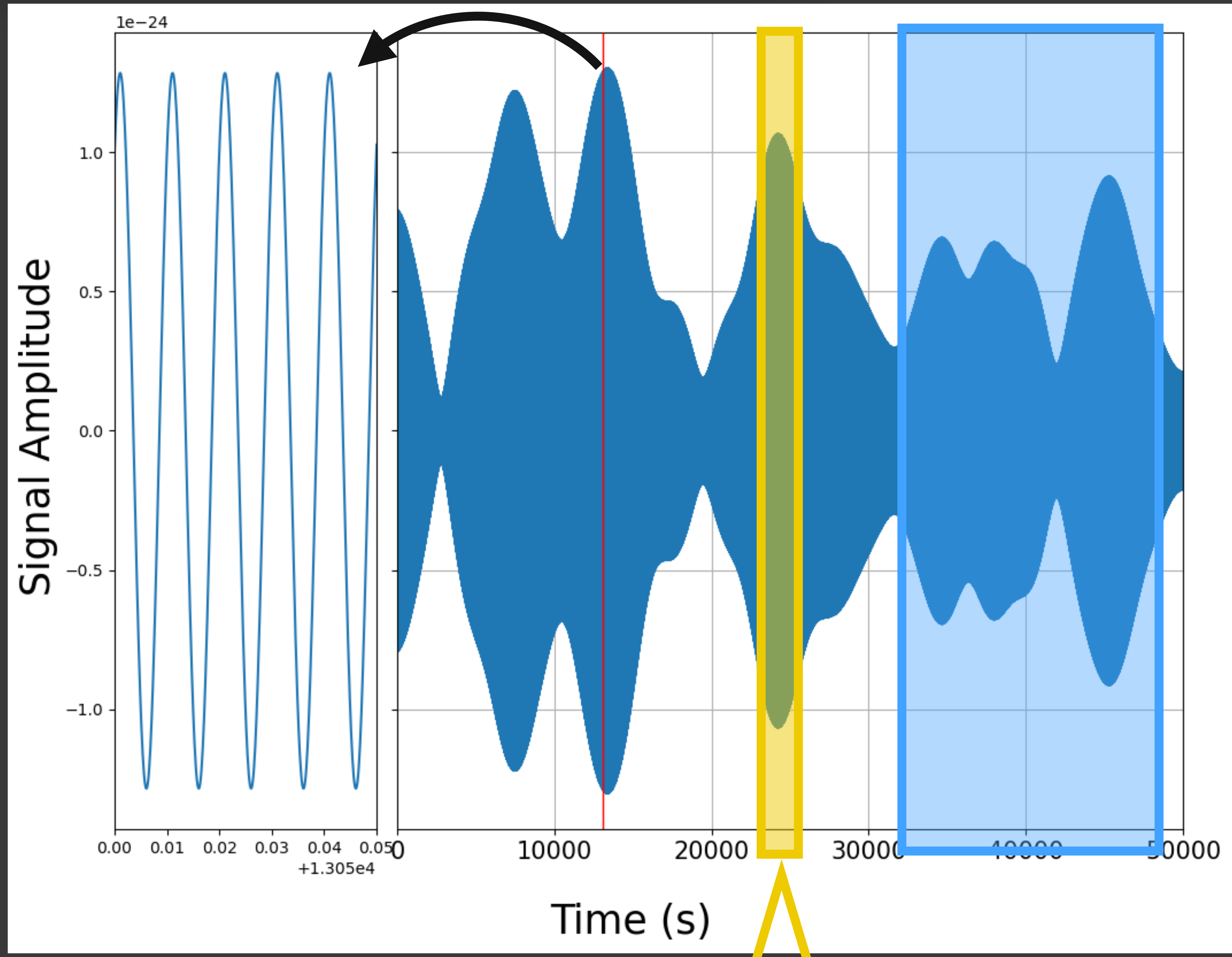
- Criteria of the detection or upper bound estimation: **Signal-to-Noise Ratio(SNR)**

$$\mathrm{SNR} \propto \begin{cases} (T_{\mathrm{ch}} T_{\mathrm{tot}})^{1/4} & \text{for } T_{\mathrm{ch}} < \tau \text{ (or low } m_{\mathrm{DM}}\text{)} \\ (\tau T_{\mathrm{tot}})^{1/4} & \text{for } T_{\mathrm{ch}} > \tau \text{ (or high } m_{\mathrm{DM}}\text{)} \end{cases}$$

$\tau \equiv 2\pi / m_{\mathrm{DM}} \langle v^2 \rangle$
Coherent time (typical timescale of variation of field amplitude)

- Incoherent search loses phase information
- SNR improvement is limited by the duration of the chunk instead of the coherent time

➡ **Worse efficiency for search for lower mass ULDM**



Shorter chunks fail to integrate over the coherence time

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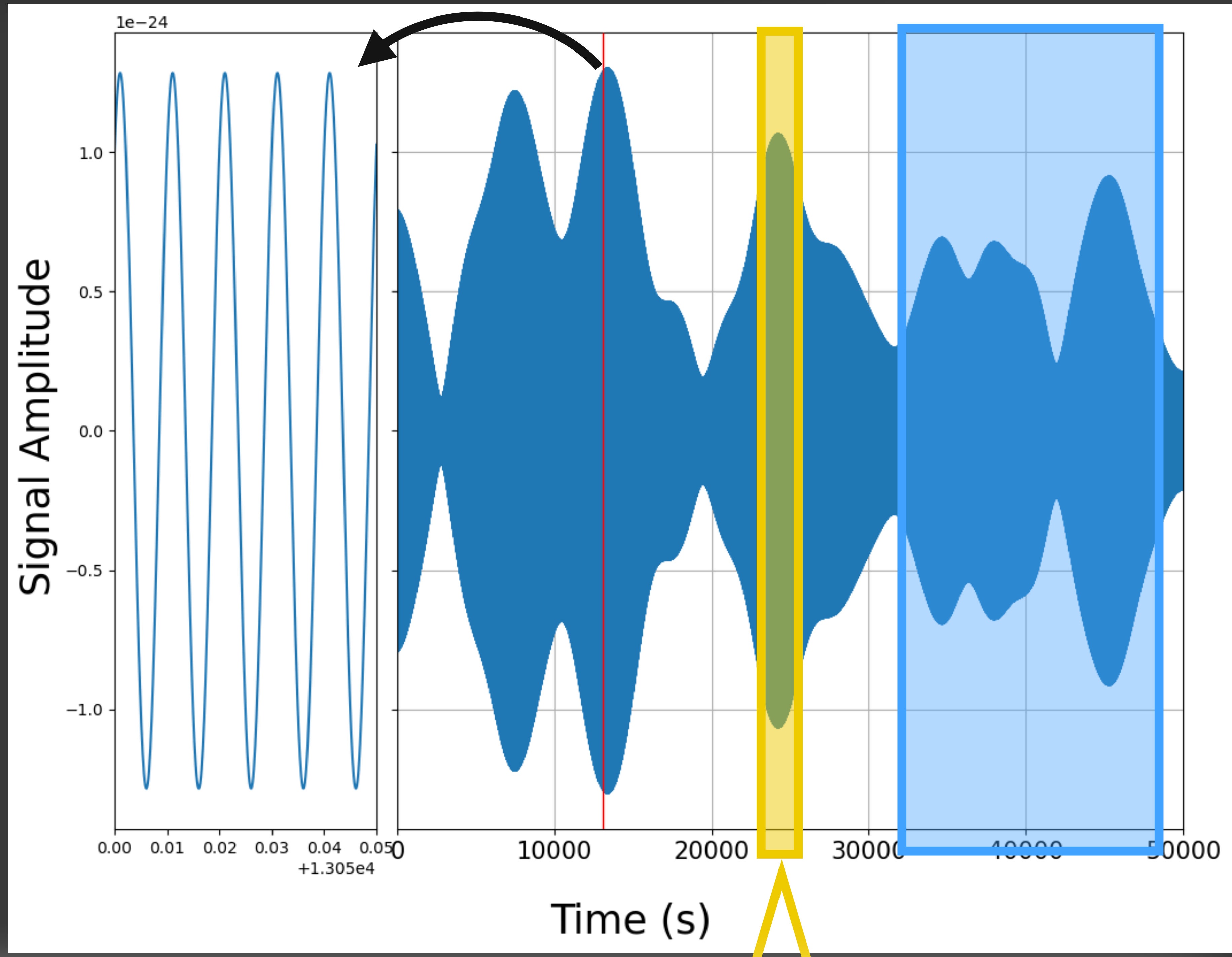


✳ Incoherent search loses phase information

✳ SNR improvement is limited by the duration of the chunk instead of the coherent time

➡ **Worse efficiency for search for lower mass ULDM**

A more effective analysis method is needed!

Contents

Correlation between chunks

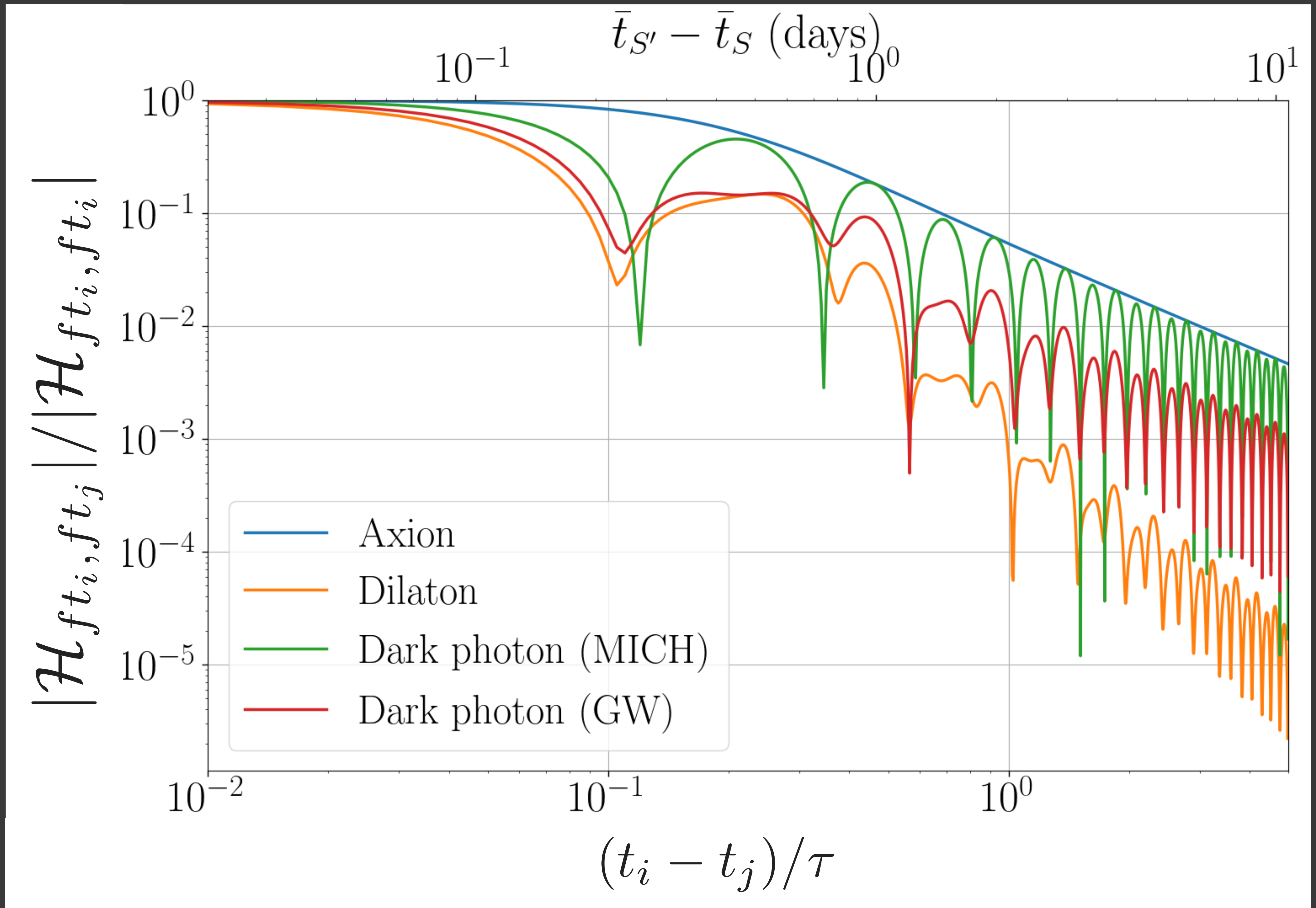
✳ DM signal in each chunk has a correlation

$$\tilde{d}(f;t_i) = \tilde{n}(f;t_i) + \epsilon\tilde{h}(f;t_i)$$

$$\mathcal{H}_{ft_i,f't_j} \equiv \langle \tilde{h}(f;t_i), \tilde{h}(f';t_j)\rangle$$

✳ **Correlation functions are different for each DM models**

✳ The difference comes from the mechanism of the interaction with detectors



Effect of direction of relative velocity

✳ The case that the correlation depends on the relative motion

- Assuming the DM velocity distribution is isotropic
- Consider the relative velocity as $\vec{v} = \vec{v}_{\rm vir} + \vec{v}_{\odot}$
  (ignoring the velocity of the Earth's orbit and rotation)

$\vec{v}_{\rm vir}$: virial velocity of DM
$\vec{v}_{\odot}$: velocity of the solar system relative to the galactic center

✳ **dark photon** (spin-1 (vector) DM)

signal: $h(t) = h_1(t) + h_2(t)$

$$\boxed{\begin{aligned} h_2(t) = &\frac{\epsilon_D e}{m_{\rm DM}} \frac{Q_D}{M} \sum_i A_i ((\vec{n} \cdot \vec{e}_i)(\vec{n} \cdot \underline{\vec{k}_i}) \\ &- (\vec{m} \cdot \vec{e}_i)(\vec{m} \cdot \underline{\vec{k}_i})) \cos(\omega_i (t - L) + \phi_i) \end{aligned}}$$

$\vec{n}, \vec{m}$: unit vector of the GW detector
$\vec{e}_i, \vec{k}_i$: unit polarization vector and wave number vector of i-th wave

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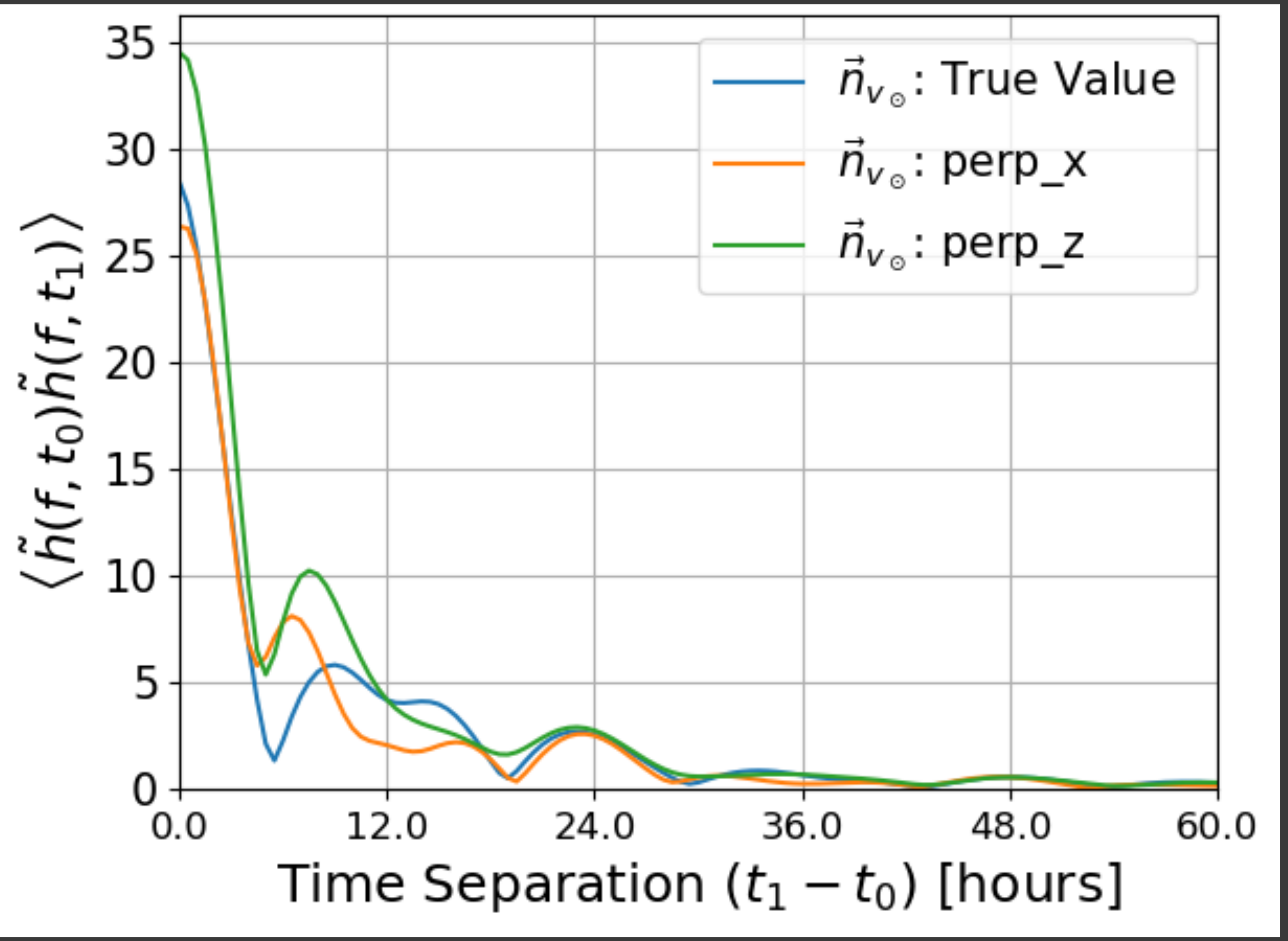

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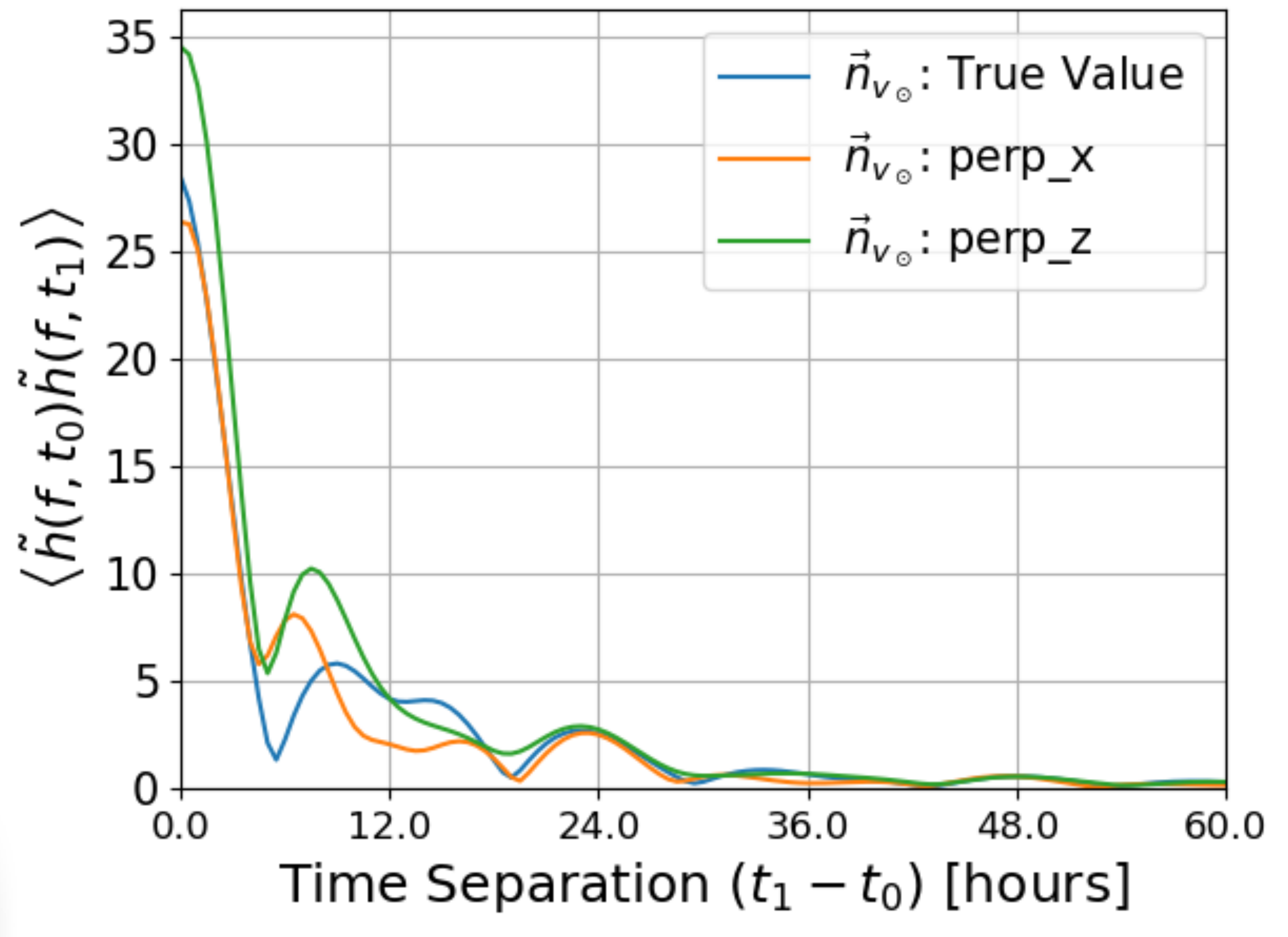
Accurate $\vec{v}_{\odot}$ must be taken into account!



Optimal Filtering Approach

✳ Construction of Detection Statistics

$$\rho = \vec{d}^{\dagger} \mathcal{N}^{-1} \mathcal{H} \mathcal{N}^{-1} \vec{d}$$

$\vec{d}$: Fourier components of data segments

$\mathcal{N}$: noise covariance $\mathcal{H}$: signal covariance

Incoherent analysis

$$\rho(f) = \sum_{i}^{N_{\mathrm{ch}}} \rho_i(f),$$

$$\rho_i(f) = \frac{4|\tilde{d}(f;t_i)|^2}{T_{\mathrm{ch}} S(f;t_i)}$$

for maximizing SNR; $\mathrm{SNR} \equiv \langle \rho \rangle / \sqrt{\mathrm{Var}[\rho]}$

- Considering the correlation of each chunk in $\mathcal{N}$ and $\mathcal{H}$
- The effect of the relative velocity is also included
- **This can be applied to any model and other ULDM searches**

Application to the simulation data

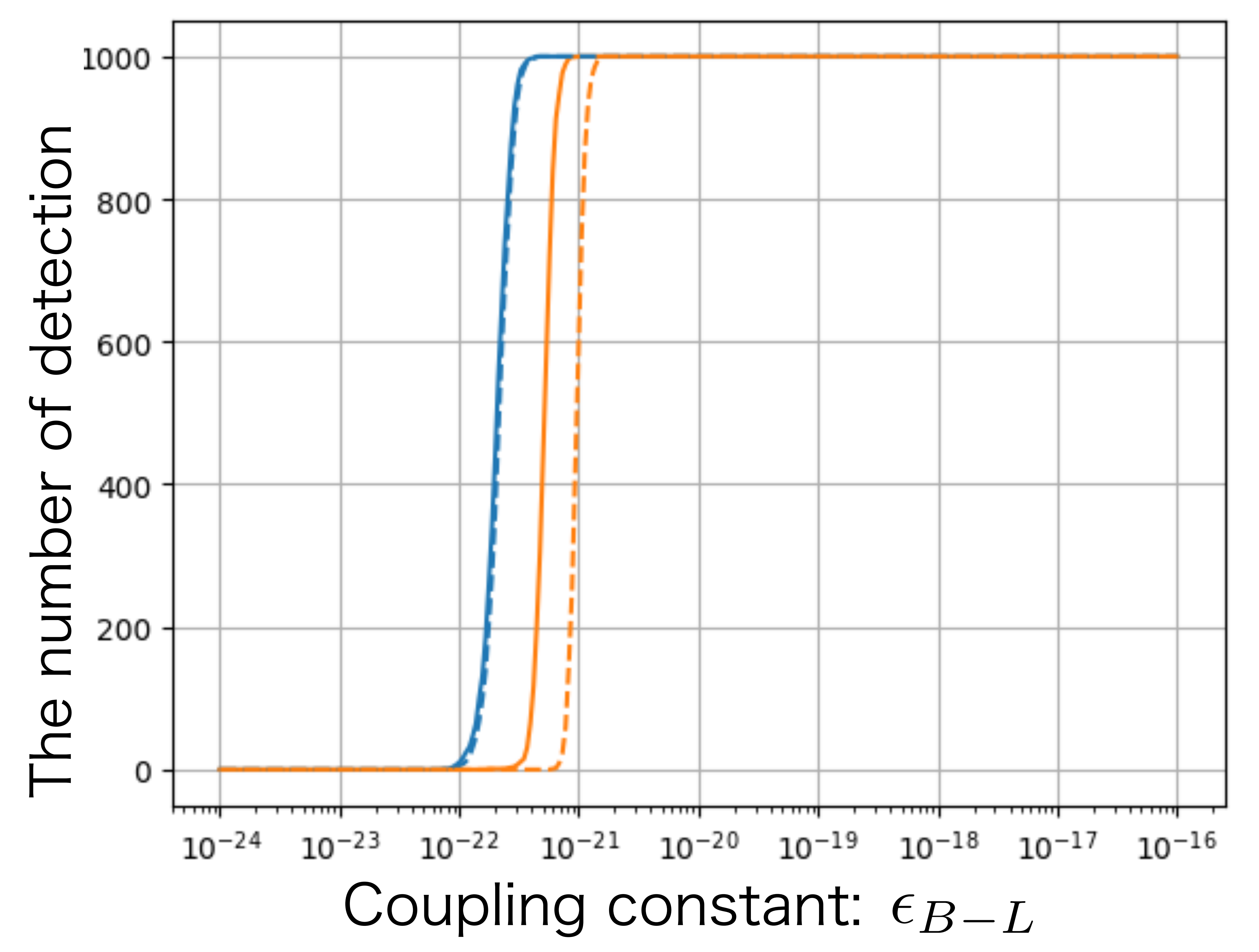
✳ Search for 1000 simulated data

Signal: $U(1)_{B-L}$ gauge boson (100 Hz)

$$\tau \sim \frac{2\pi}{m_{\mathrm{DM}}\langle v^2\rangle} \sim 2.8 \text{ hours}$$

Characterizes the strength of the interaction

$$\tilde{d} = \tilde{n} + \epsilon_{B-L}\tilde{h}$$



: OF, 100 sec.
: OF, 25 sec.
: Incoherent search, 100 sec.
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※OF: Optimal Filtering

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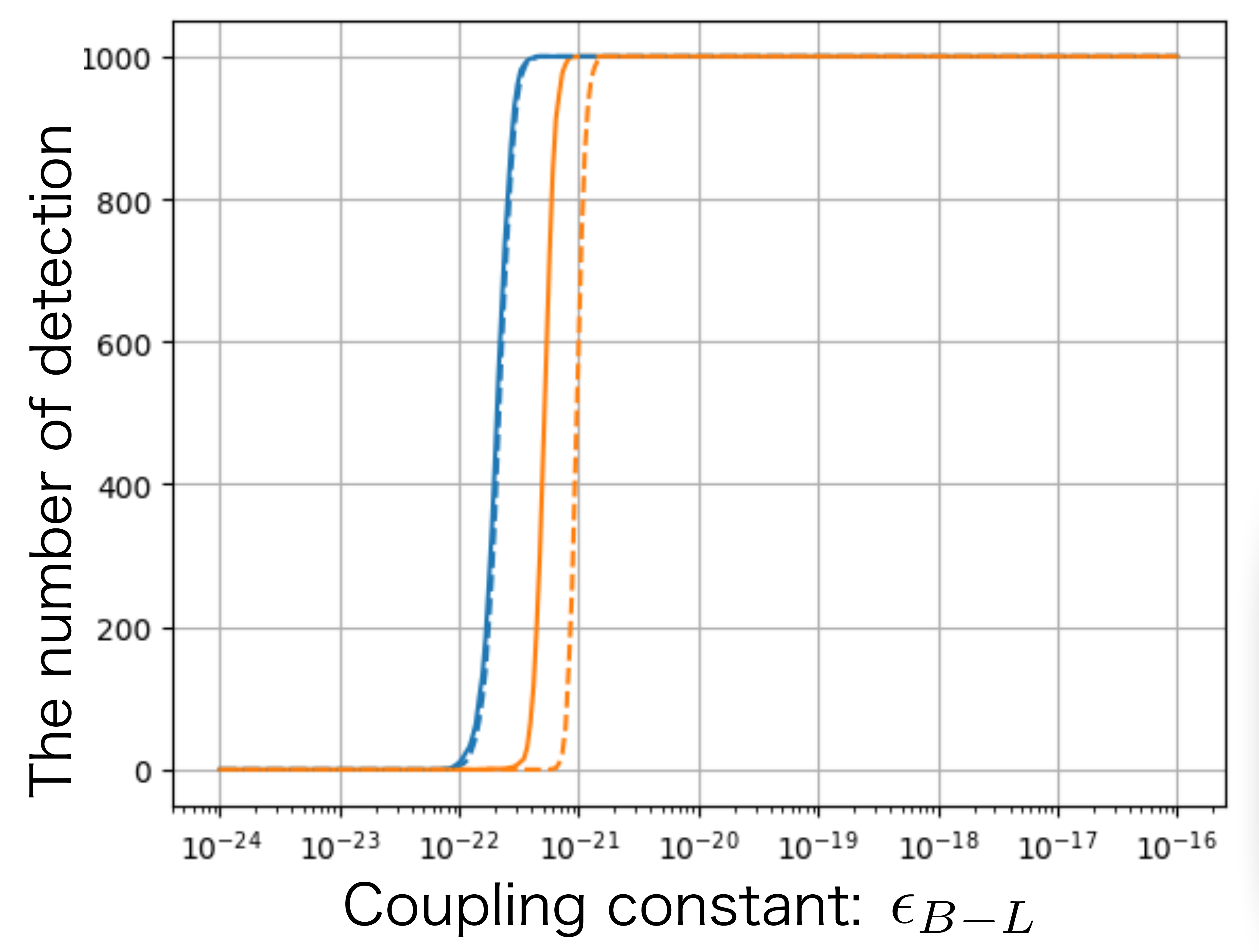

: OF, 100 sec.
: OF, 25 sec.
: Incoherent search, 100 sec.
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※OF: Optimal Filtering

The optimal filtering method;

✓ can **detect much smaller signal**

✓ is **independent of the duration of the chunk**

Application to the simulation data

✳ Estimate the 95% upper bound with 1000 data

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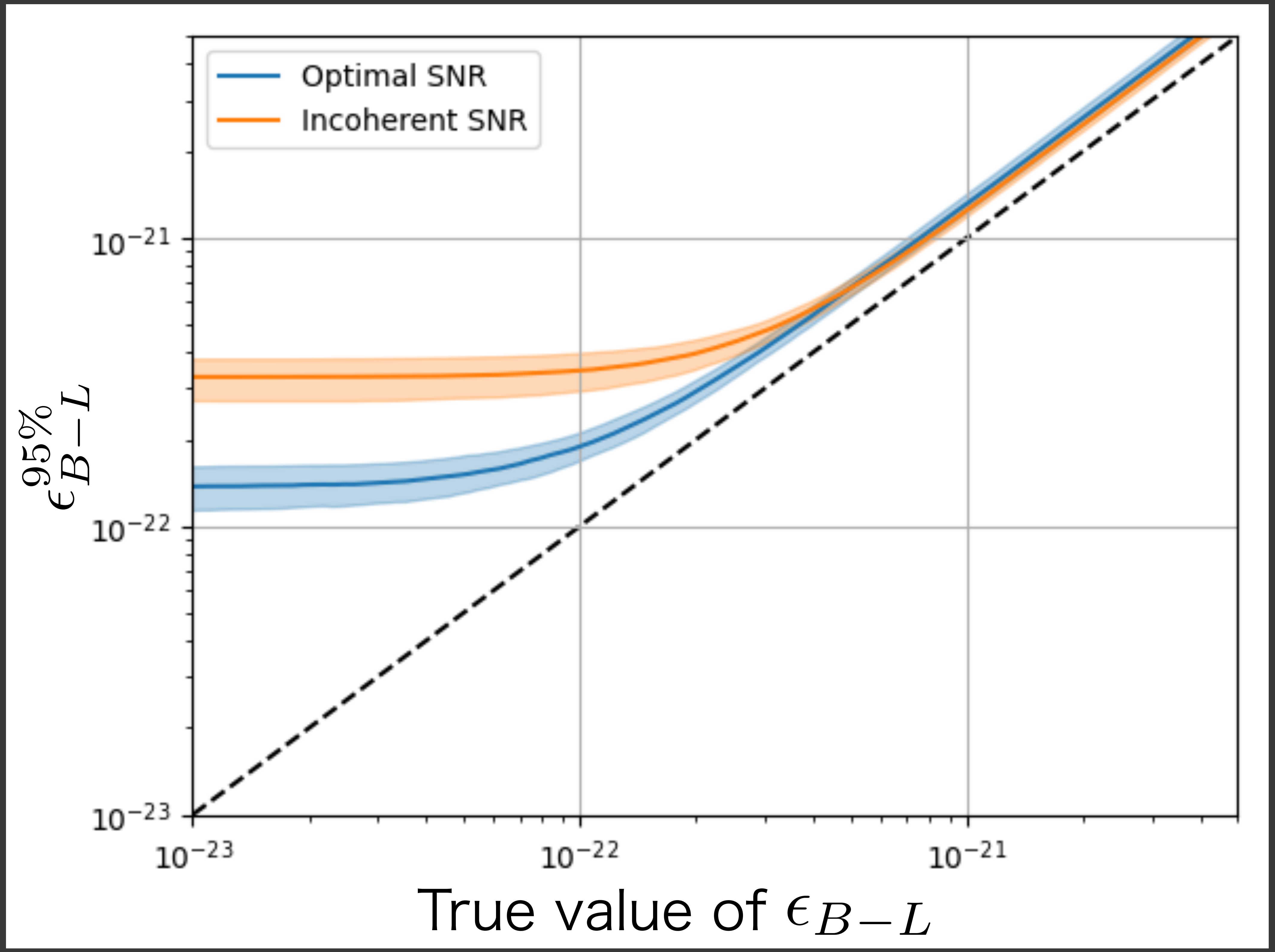

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※Chunk duration: 100 sec.

Application to the simulation data

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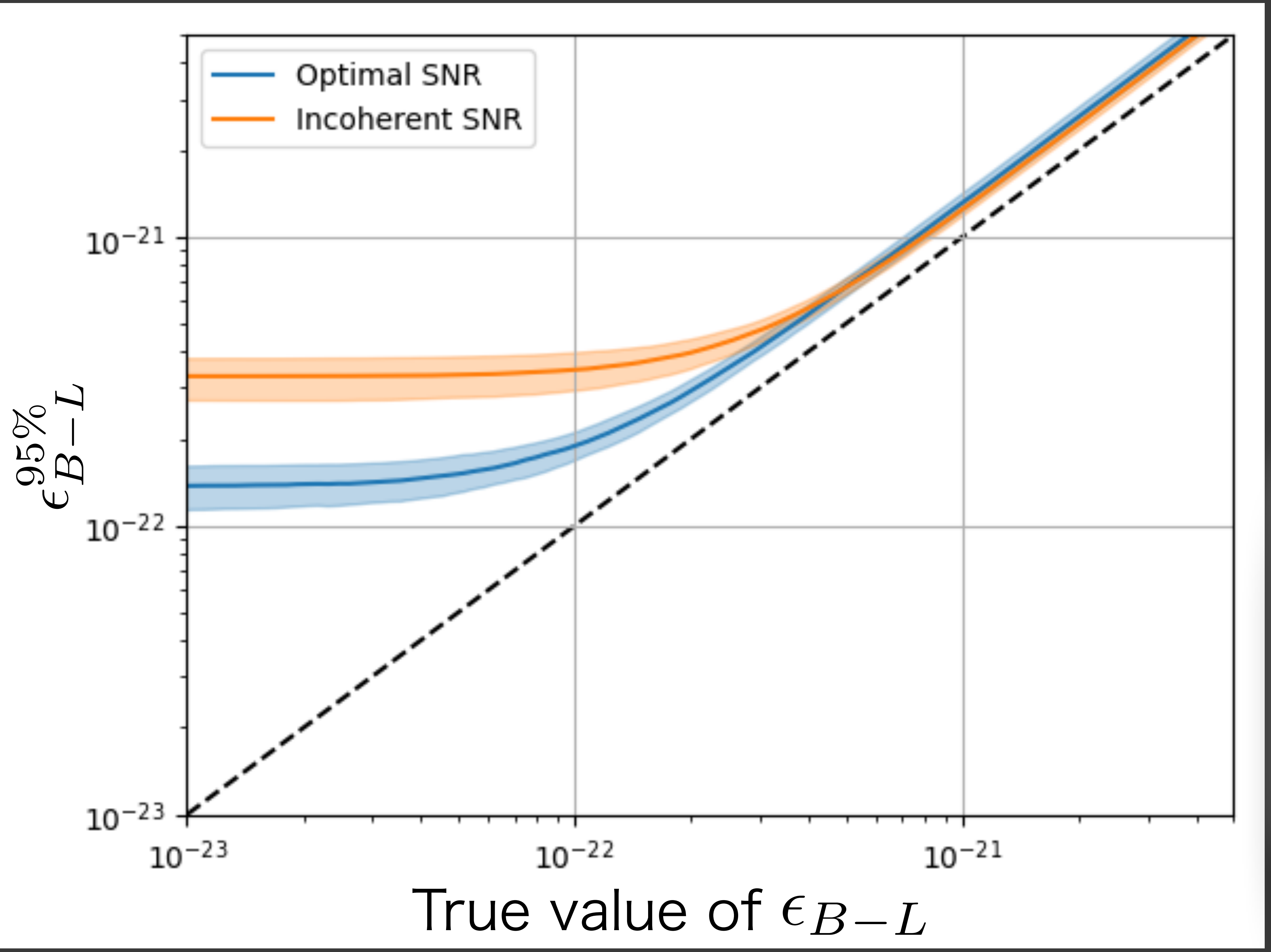

: OF : Incoherent search

※Chunk duration: 100 sec.

The optimal filtering (OF) method;

✓ can **set a much tighter bound when $T_{\rm ch} < \tau$**

Conclusion and Future Works

✳ Optimal Filtering(OF) method

- New data analysis method for ULDM search
- OF is constructed based on the general nature of the ULDM field
  ➡ can be applied to any other ULDM searches
- **More detection numbers** and **Tighter bound on coupling constants** than those with the previous method

✳ Future Works

- Application for real data

✳ Discussion

- DM model selection

$$\rho = \vec{d}^{\dagger}\mathcal{N}^{-1}\mathcal{H}\mathcal{N}^{-1}\vec{d}$$

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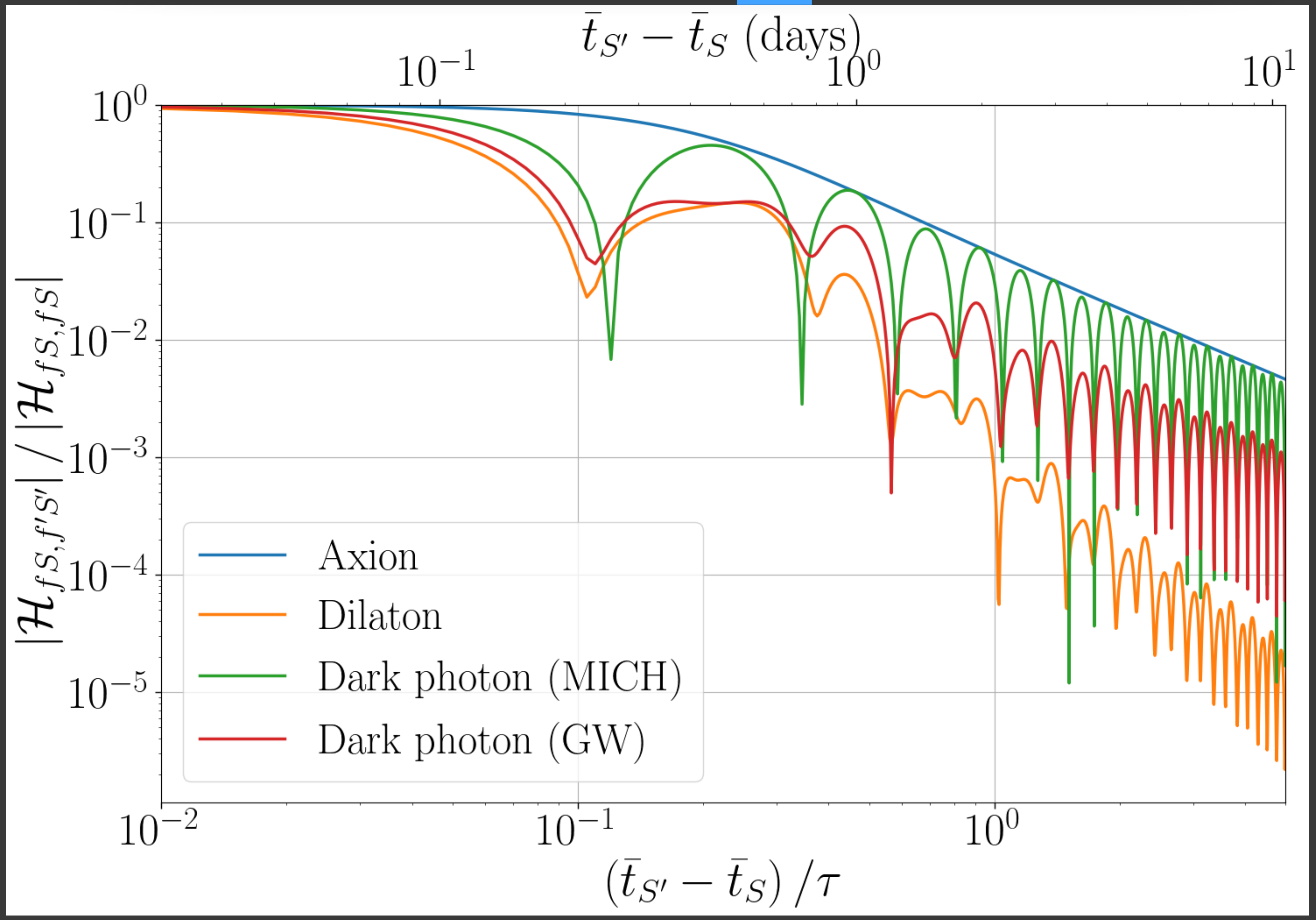
DM Model Selection

✳ The nature of the ULDM field (quasi-monoclonal classical wave field, stochastic amplitude fluctuation) is **common to all the models**

✳ On the other hand, the correlation matrix $\mathcal{H}$ changes with the interaction mechanism

🤔 We will be able to distinguish the origin of the DM signal by focusing on the statistical difference of ρs???

$$\rho = \vec{d}^{\dagger}\mathcal{N}^{-1}\mathcal{H}\mathcal{N}^{-1}\vec{d}$$



Appendix

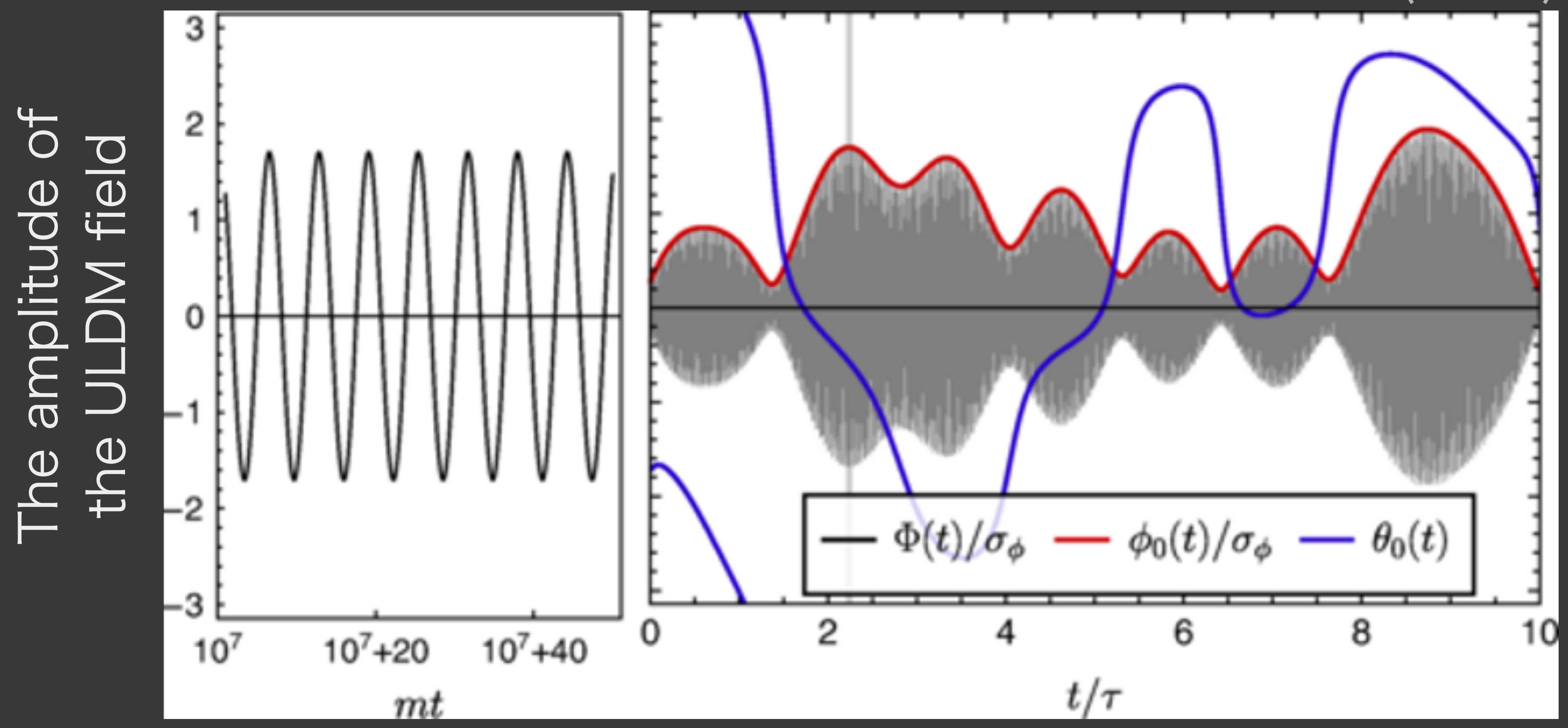
DM search with GW detectors

✳ The description of the ULDM field

$$\vec{A}(t) = \frac{A}{\sqrt{N}} \sum_{n=1}^{N} \vec{e}_n \cos\left(m_{\rm DM}\left(1 + \frac{v_n^2}{2}\right) t - m_{\rm DM}\vec{v}_n \cdot \vec{x} + \theta_n \right)$$

N : number of partial waves
$\vec{v}_n$: velocity of n-th wave
$\vec{v} = \vec{v}_{\rm vir} + \vec{v}_{\odot}$
θ_n : random phase of n-th wave

H. Nakatsuka+ (2023)



Time-domain field generated by ULDM

$\vec{v} \sim 10^{-3} \ll 1$

Frequency: $f \sim \dfrac{m_{\rm DM}}{2\pi}$

Frequency width: $\Delta f \sim \dfrac{m_{\rm DM}\langle v^2 \rangle}{2\pi}$

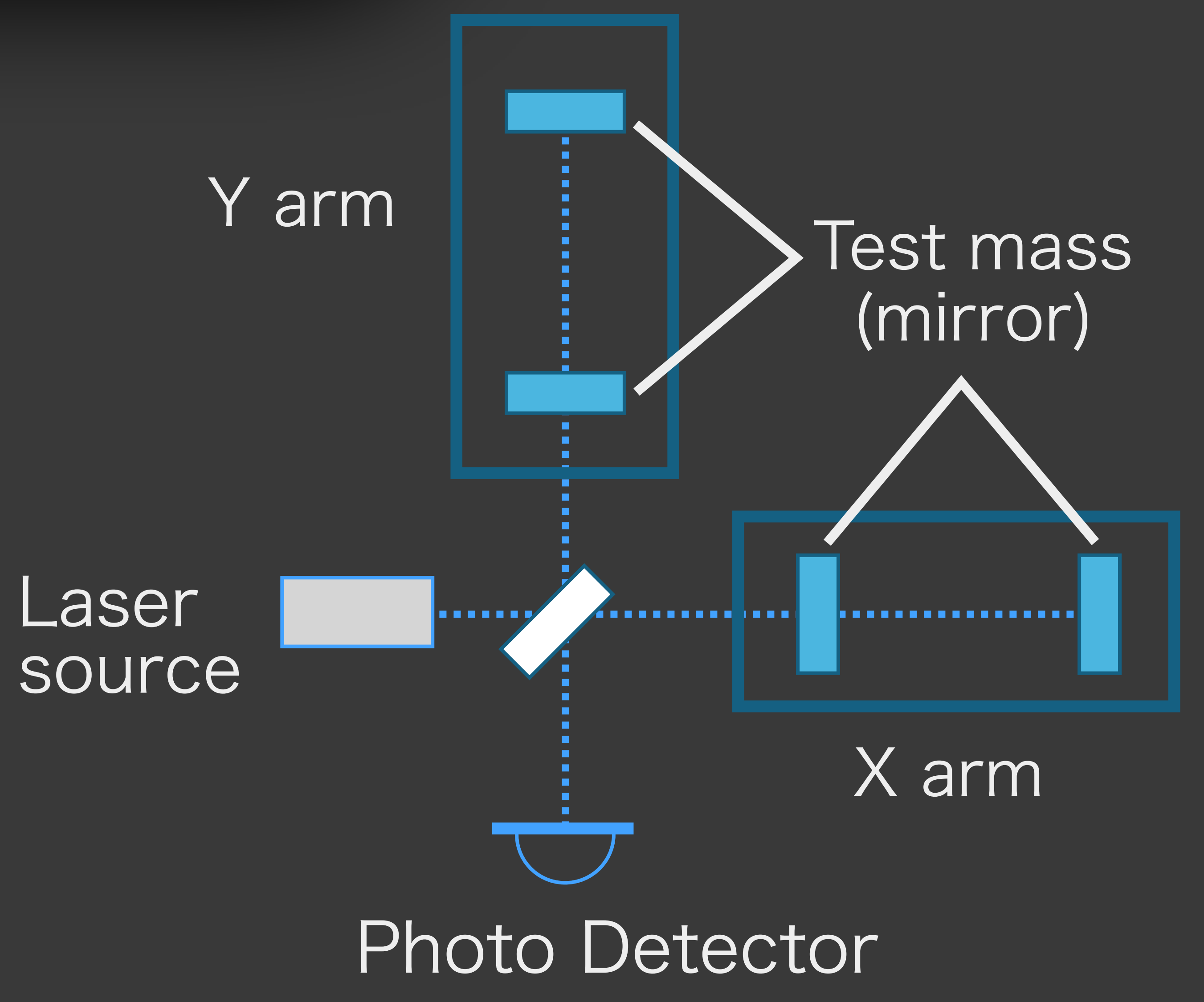
Coherent time: $\tau \sim \dfrac{2\pi}{m_{\rm DM}\langle v^2 \rangle} \sim \dfrac{10^6}{f}$

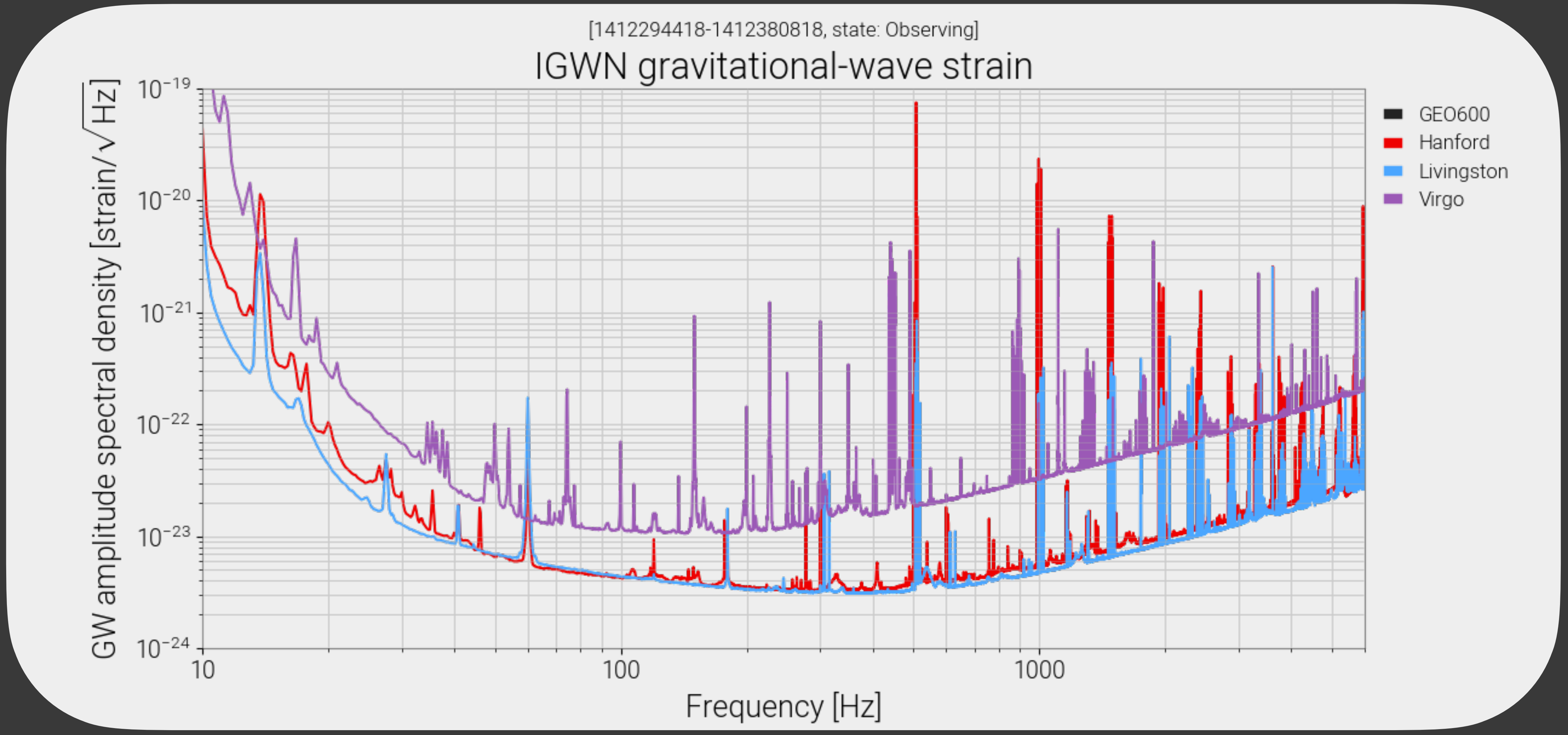
Gravitational Wave Detector

- Laser Michelson interferometer
- Observe the change in the difference between the lengths of the two arms.

$$\frac{\Delta L}{L} \sim 10^{-23}$$

- Arm length: 4 km (LIGO, Virgo), 3 km(KAGRA)





Obtaining the threshold

✳ For obtaining the detection threshold

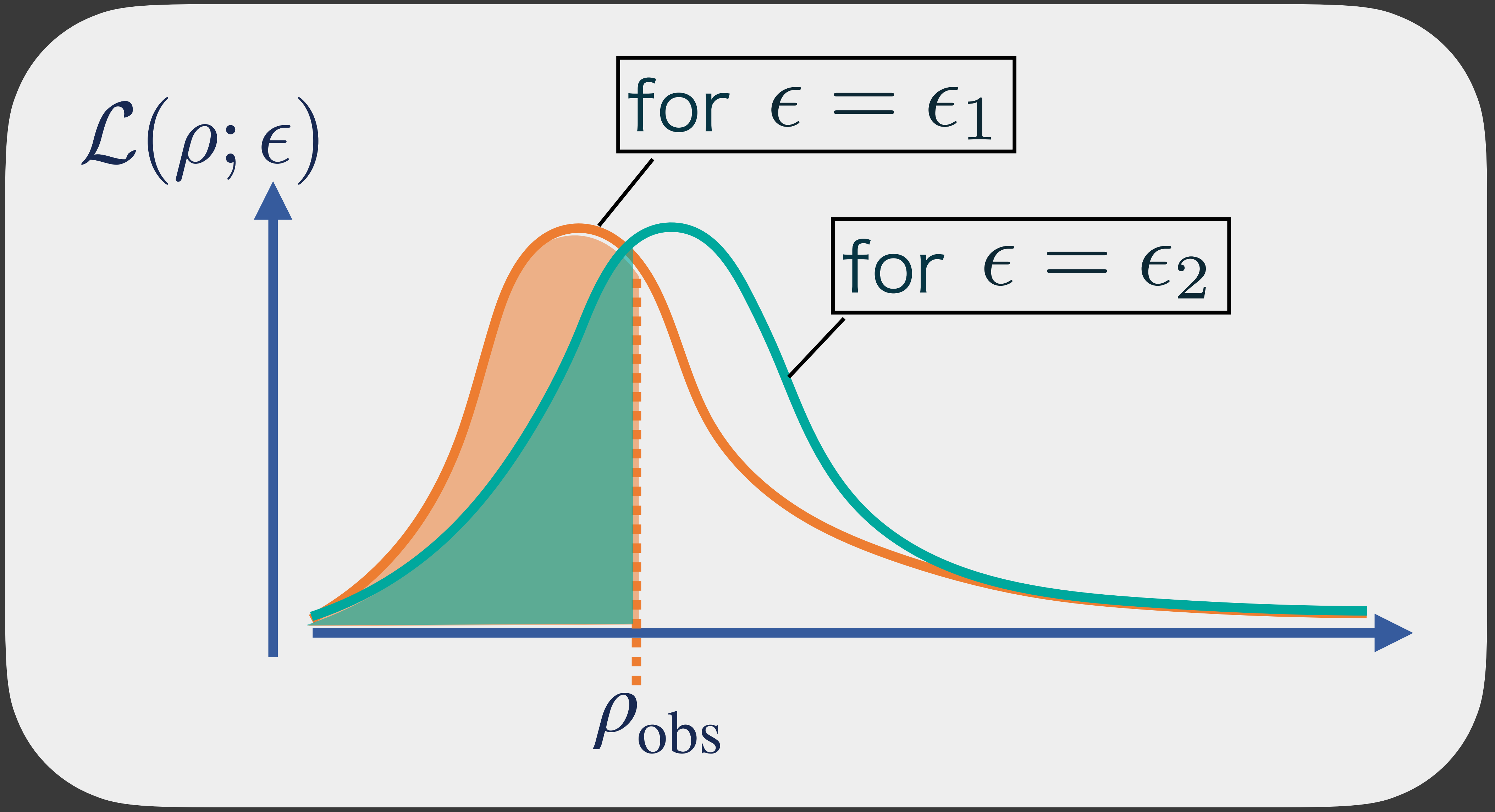
- α is the false alarm rate
  (in this presentation $\alpha = 1 - 10^{-4}$)

$$\alpha = \int_0^{\rho_{\rm thr}} \mathcal{L}(\rho(f)|\epsilon = 0)d\rho$$

✳ For setting the upper bound

- $\beta = 95$ is often used

$$1 - \frac{\beta}{100} = \int_0^{\rho_{\rm obs}} \mathcal{L}(\rho(f)|\epsilon^{\beta\%})d\rho$$

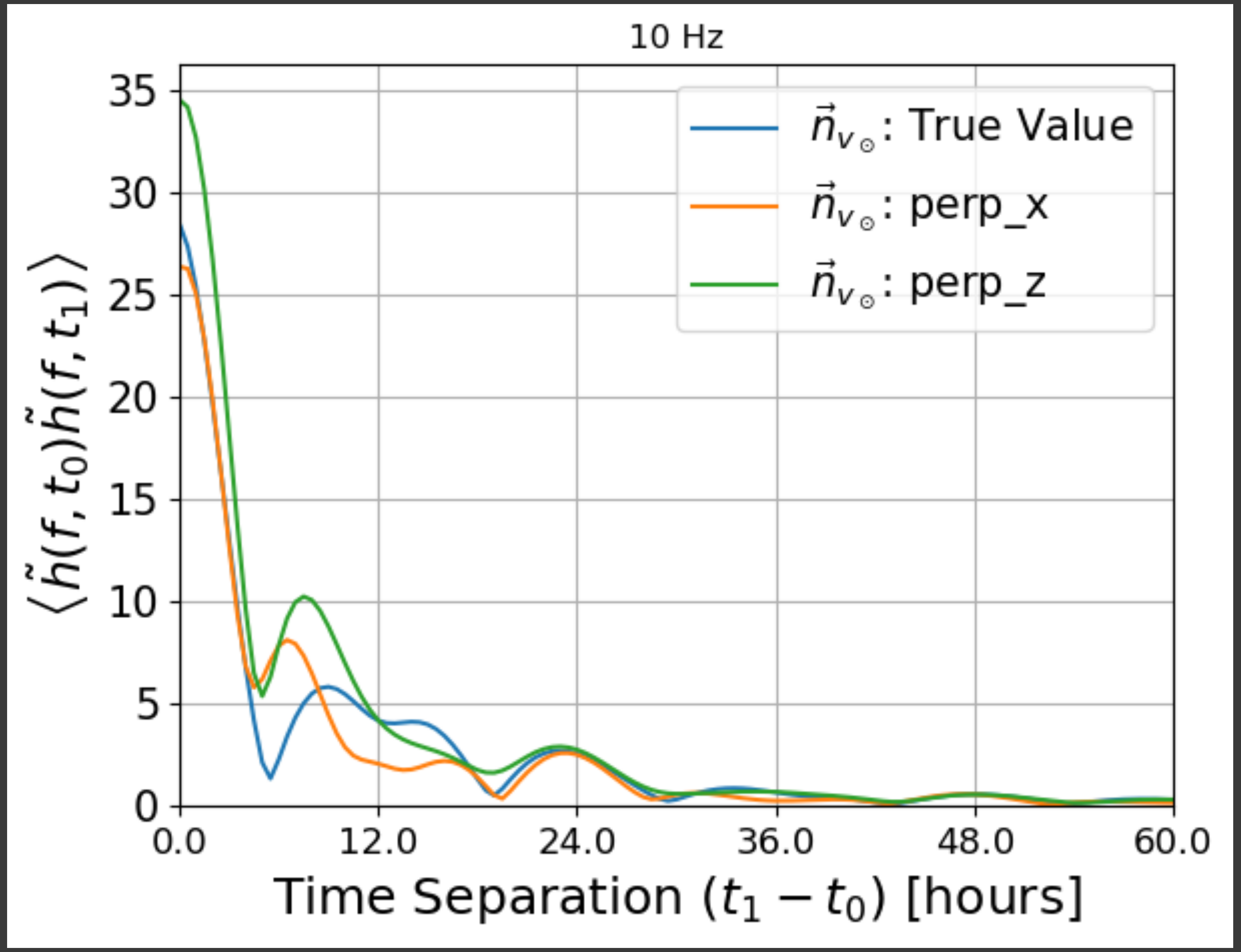


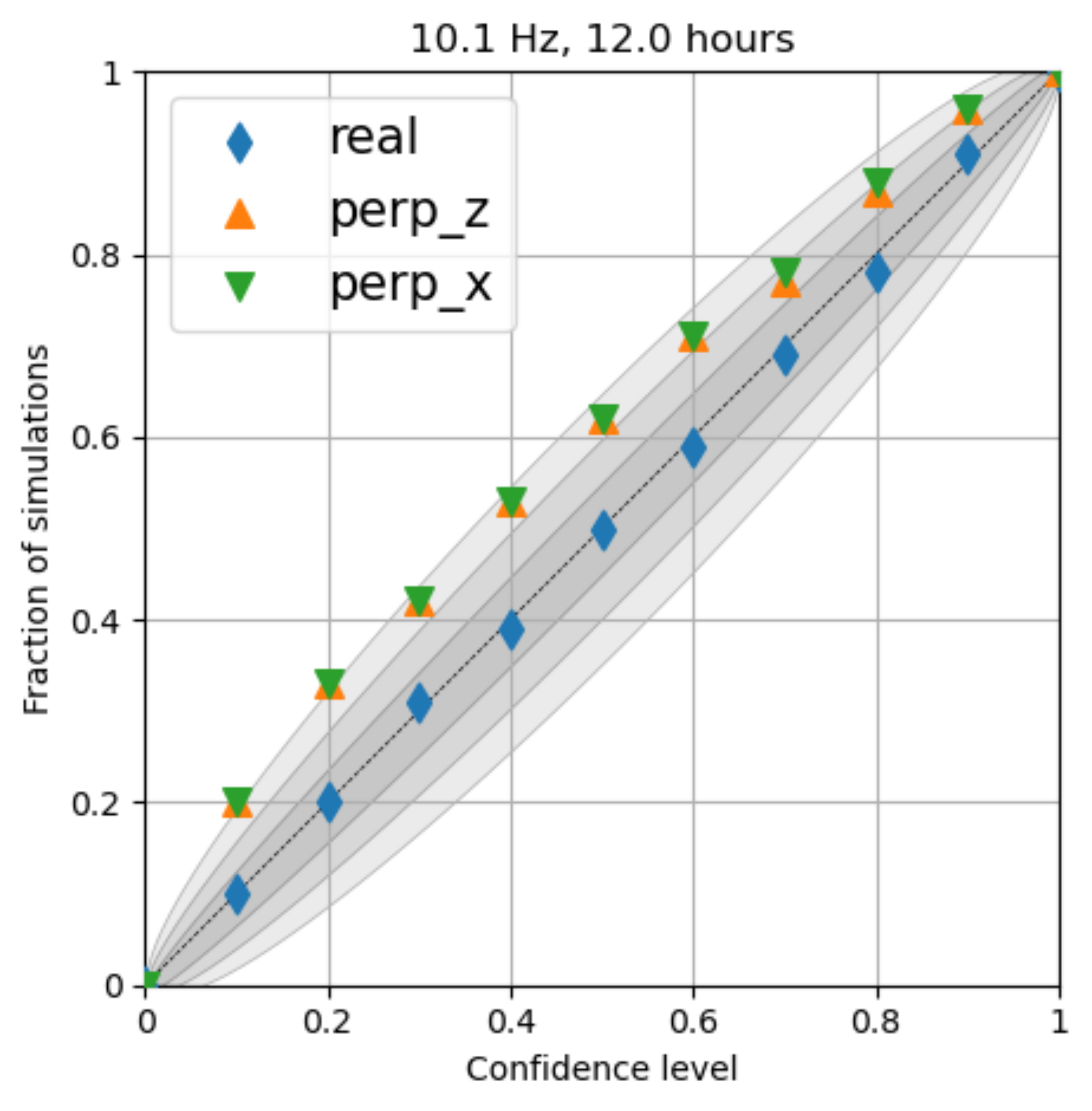
Effect of direction of relative velocity

✳ Analysis with incoherent method

$$\mathcal{L} \supset e\epsilon_D J^{\mu}_D A_\mu$$

- Coupling constant estimation for **dark photon** with simulated data





$$\vec{v} = \vec{v}_{\rm vir} + \vec{v}_\odot$$

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