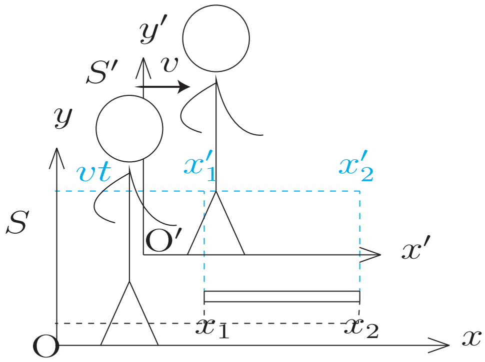
Modern Physics - Lecture Exercises Solutions
Lecture 1
Student Answers by Youssef Samy

Ex. 1 Show that although observers in S and S' measure different coordinates for the ends of a stick at rest in S, they agree on the length of the stick.

Sol. 1



- Known:

$$\text{System } S \text{ is stationary} \quad \text{(a)}$$
$$\text{System } S' \text{ is moving with velocity } v \text{ relative to } S \quad \text{(b)}$$
$$L = x_2 - x_1 \quad \text{(c)}$$
$$L' = x'_2 - x'_1 \quad \text{(d)}$$

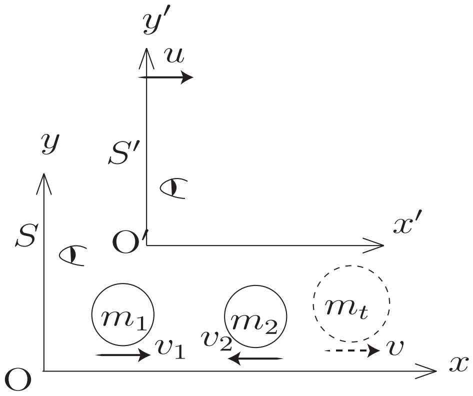
- Showing that $L = L'$ using Galilean Transformation:

$$x'_1 = x_1 - vt \quad (1)$$
$$x'_2 = x_2 - vt \quad (2)$$
$$L' = x'_2 - x'_1 \quad \text{(d)}$$
$$L' = (x_2 - vt) - (x_1 - vt)$$
$$L' = x_2 - x_1 \quad (3)$$
$$\therefore L' = L \quad \text{(from (c), (d), and (3))}$$

Ex. 2 *Conservation of Linear Momentum Is Covariant Under the Galilean Transformation.*

Assume that two masses m_1 and m_2 are moving in the positive x direction with velocities v_1 and v_2 as measured by an observer in S before a collision. After the collision, the two masses stick together and move with a velocity v in S. Show that if an observer in S finds momentum to be conserved, so does an observer in S'.

Sol. 2



We perform Galilean transformation.

- Known:

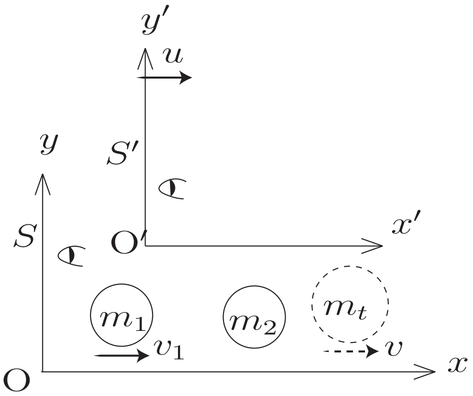
$$\text{System } S \text{ is stationary} \quad \text{(a)}$$
$$\text{System } S' \text{ is moving with velocity } u \text{ relative to } S \quad \text{(b)}$$
$$m_1 = m'_1, \quad m_2 = m'_2 \text{ (invariance of mass)} \quad \text{(c)}$$
$$\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} \quad \text{(d)}$$
$$\text{Direction of positive velocity} \quad \text{(e)}$$
$$v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} \quad \text{(f)}$$
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v \text{ (conserved momentum)} \quad \text{(g)}$$

- Showing that the momentum is conserved under S':

$$m'_1 v'_1 + m'_2 v'_2 = (m'_1 + m'_2)v' \quad \text{(to be proven)}$$
$$m_1(v_1 - u) + m_2(v_2 - u) = (m_1 + m_2)(v - u) \text{ (Galilean transformation)} \quad (1)$$
$$m_1 v_1 - \underline{m_1 u} + m_2 v_2 - \underline{m_2 u} = m_1 v + m_2 v - \underline{m_1 u} - \underline{m_2 u} \quad (2)$$
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v \quad (3)$$
$$\therefore \text{Momentum is conserved} \quad \text{(from (g) and (3))}$$

Ex. 3 A 2000-kg car moving with a speed of 20 m/s collides with and sticks to a 1500-kg car at rest at a stop sign. Show that because momentum is conserved in the rest frame, momentum is also conserved in a reference frame moving with a speed of 10 m/s in the direction of the moving car.

Sol. 3



We perform Galilean transformation.

• Known:

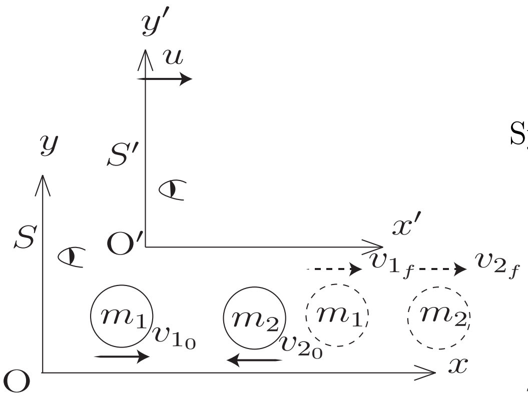
$$\text{System } S \text{ is stationary} \tag{a}$$
$$\text{System } S' \text{ is moving with velocity } u = 10\text{m/s relative to } S \tag{b}$$
$$m_1 = m_1', \quad m_2 = m_2' \text{ (invariance of mass)} \tag{c}$$
$$\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} \tag{d}$$
$$\text{Direction of positive velocity} \tag{e}$$
$$v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} \tag{f}$$
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v \text{ (conserved momentum)} \tag{g}$$
$$m_1 = 2000\text{kg}, m_2 = 1500\text{kg}, v_1 = 20\text{m/s}, v_2 = 0\text{m/s} \tag{h}$$
$$2000 \times 20 + 1500 \times 0 = 3500 \times v \tag{i}$$
$$v \approx 11.43 \text{ up to the nearest } 2^{nd} \text{ decimal point} \tag{j}$$

• Showing that the momentum is conserved under S':

$$m_1'v_1' + m_2'v_2' = (m_1' + m_2')v' \tag{to be proven}$$
$$m_1(v_1 - u) + m_2(v_2 - u) = (m_1 + m_2)(v - u) \text{ (Galilean transformation)} \tag{1}$$
$$2000 \times (20 - 10) + 1500 \times (-10) = 3500 \times (11.43 - 10) \tag{2}$$
$$\text{Left hand side} = 5000 \tag{3}$$
$$\text{Right hand side} = 5005 \tag{4}$$
$$\text{Left hand side} \approx \text{Right hand side} \tag{5}$$
$$\therefore \text{Momentum is conserved} \tag{from (5)}$$

Ex. 4 A billiard ball of mass 0.3 kg moves with a speed of 5 m/s and collides elastically with a ball of mass 0.2 kg moving in the opposite direction with a speed of 3 m/s. Show that because momentum is conserved in the rest frame, it is also conserved in a frame of reference moving with a speed of 2 m/s in the direction of the second ball.

Sol. 4



We perform Galilean transformation.

• Known:

$$\text{System } S \text{ is stationary} \tag{a}$$
$$\text{System } S' \text{ is moving with velocity } u = 2\text{m/s relative to } S \tag{b}$$
$$m_1 = m_1', \quad m_2 = m_2' \text{ (invariance of mass)} \tag{c}$$
$$\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} \tag{d}$$
$$\text{Direction of positive velocity} \tag{e}$$
$$v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} \tag{f}$$
$$m_1 v_{1_0} + m_2 v_{2_0} = m_1 v_{1_f} + m_2 v_{2_f} \text{ (conserved momentum)} \tag{g}$$

Solution continued in the following page

• Showing that the momentum is conserved under S' parametrically:

$$m'_1 v'_{1_0} + m'_2 v'_{2_0} = m'_1 v'_{1_f} + m'_2 v'_{2_f} \quad \text{(to be proven)}$$
$$m_1(v_{1_0} - u) + m_2(v_{2_0} - u) = m_1(v_{1_f} - u) + m_2(v_{2_f} - u) \text{ (Galilean transformation)} \quad (1)$$
$$m_1 v_{1_0} + m_2 v_{2_0} - \underline{u(m_1 + m_2)} = m_1 v_{1_f} + m_2 v_{2_f} - \underline{u(m_1 + m_2)} \quad (2)$$
$$m_1 v_{1_0} + m_2 v_{2_0} = m_1 v_{1_f} + m_2 v_{2_f} \quad (3)$$
$$\therefore \text{Momentum is conserved} \quad \text{(from (3) and (g))}$$

• Alternative solution - to substitute with numbers, assume conservation of kinetic energy due to elastic collision.

In S:

$$\frac{1}{2} m_1 v_{1_0}^2 + \frac{1}{2} m_2 v_{2_0}^2 = \frac{1}{2} m_1 v_{1_f}^2 + \frac{1}{2} m_2 v_{2_f}^2 \text{ (conserved kinetic energy)} \quad \text{(h)}$$
$$0.3 \times 5 + 0.2 \times -3 = 0.3 v_{1_f} + 0.2 v_{2_f} \text{ (substitution in (g))} \quad \text{(i)}$$
$$3 v_{1_f} = 9 - 2 v_{2_f} \quad \text{(j)}$$
$$v_{1_f} = -1.4, \quad v_{2_f} = 6.6 \text{ (from substituting with (i) in (h))} \quad \text{(k)}$$

In S':

$$m'_1 v'_{1_0} + m'_2 v'_{2_0} = m'_1 v'_{1_f} + m'_2 v'_{2_f} \quad \text{(to be proven)}$$
$$m_1(v_{1_0} - u) + m_2(v_{2_0} - u) = m_1(v_{1_f} - u) + m_2(v_{2_f} - u) \quad (1)$$
$$0.3 \times (5 - 2) + 0.2 \times (-3 - 2) = 0.3 \times (-1.4 - 2) + 0.2 \times (6.6 - 2) \quad (2)$$
$$\text{Left hand side} = -0.1 \quad (3)$$
$$\text{Right hand side} = -0.1 \quad (4)$$
$$\text{Left hand side} = \text{Right hand side} \quad (5)$$
$$\therefore \text{Momentum is conserved} \quad \text{(from (5))}$$