**Modern Physics - Lecture Exercises Solutions**
**Lecture 1**
**Student Answers by Youssef Samy**

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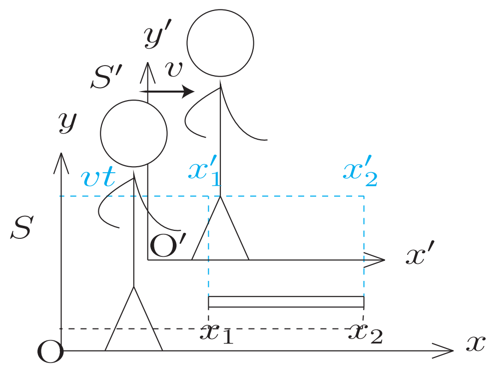
**Ex. 1** Show that although observers in $S$ and $S'$ measure different coordinates for the ends of a stick at rest in $S$, they agree on the length of the stick.

**Sol. 1**




- Known:

$$\begin{aligned} &\text{System } S \text{ is stationary} && \text{(a)}\\ &\text{System } S' \text{ is moving with velocity } v \text{ relative to } S && \text{(b)}\\ &L = x_2 - x_1 && \text{(c)}\\ &L' = x'_2 - x'_1 && \text{(d)} \end{aligned}$$

- Showing that $L = L'$ using Galilean Transformation:

$$\begin{aligned} x'_1 &= x_1 - vt && (1)\\ x'_2 &= x_2 - vt && (2)\\ L' &= x'_2 - x'_1 && \text{(d)}\\ L' &= (x_2 - vt) - (x_1 - vt) &&\\ L' &= x_2 - x_1 && (3)\\ \therefore L' &= L && \text{(from (c), (d), and (3))} \end{aligned}$$

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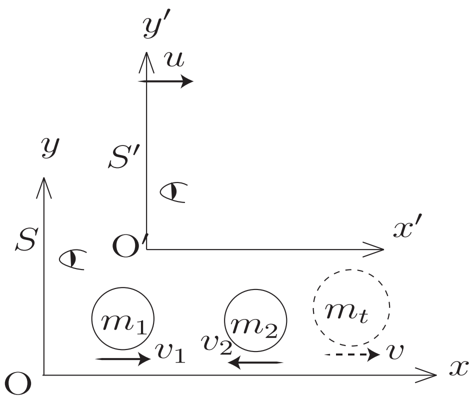
**Ex. 2** *Conservation of Linear Momentum Is Covariant Under the Galilean Transformation.*

Assume that two masses $m_1$ and $m_2$ are moving in the positive $x$ direction with velocities $v_1$ and $v_2$ as measured by an observer in $S$ before a collision. After the collision, the two masses stick together and move with a velocity $v$ in $S$. Show that if an observer in $S$ finds momentum to be conserved, so does an observer in $S'$.

**Sol. 2**




We perform Galilean transformation.

- Known:

$$\begin{aligned} &\text{System } S \text{ is stationary} && \text{(a)}\\ &\text{System } S' \text{ is moving with velocity } u \text{ relative to } S && \text{(b)}\\ &m_1 = m'_1, \quad m_2 = m'_2 \text{ (invariance of mass)} && \text{(c)}\\ &\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} && \text{(d)}\\ &\text{Direction of positive velocity} && \text{(e)}\\ &v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} && \text{(f)}\\ &m_1v_1 + m_2v_2 = (m_1 + m_2)v \text{ (conserved momentum)} && \text{(g)} \end{aligned}$$

- Showing that the momentum is conserved under $S'$:

$$\begin{aligned} m'_1v'_1 + m'_2v'_2 &= (m'_1 + m'_2)v' && \text{(to be proven)}\\ m_1(v_1 - u) + m_2(v_2 - u) &= (m_1 + m_2)(v - u) \text{ (Galilean transformation)} && (1)\\ m_1v_1 - \underline{m_1u} + m_2v_2 - \underline{m_2u} &= m_1v + m_2v - \underline{m_1u} - \underline{m_2u} && (2)\\ m_1v_1 + m_2v_2 &= (m_1 + m_2)v && (3)\\ \therefore \text{Momentum is conserved} & && \text{(from (g) and (3))} \end{aligned}$$

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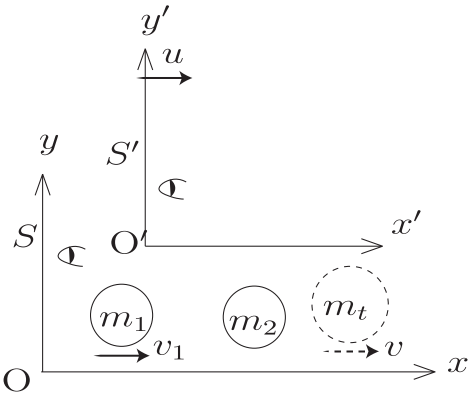
**Ex. 3** A 2000-kg car moving with a speed of 20 m/s collides with and sticks to a 1500-kg car at rest at a stop sign. Show that because momentum is conserved in the rest frame, momentum is also conserved in a reference frame moving with a speed of 10 m/s in the direction of the moving car.

**Sol. 3**




We perform Galilean transformation.

• Known:

$$\text{System } S \text{ is stationary} \quad \text{(a)}$$
$$\text{System } S' \text{ is moving with velocity } u = 10\text{m/s relative to } S \quad \text{(b)}$$
$$m_1 = m_1', \quad m_2 = m_2' \text{ (invariance of mass)} \quad \text{(c)}$$
$$\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} \quad \text{(d)}$$
$$\text{Direction of positive velocity} \quad \text{(e)}$$
$$v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} \quad \text{(f)}$$
$$m_1v_1 + m_2v_2 = (m_1 + m_2)v \text{ (conserved momentum)} \quad \text{(g)}$$
$$m_1 = 2000\text{kg}, m_2 = 1500\text{kg}, v_1 = 20\text{m/s}, v_2 = 0\text{m/s} \quad \text{(h)}$$
$$2000 \times 20 + 1500 \times 0 = 3500 \times v \quad \text{(i)}$$
$$v \approx 11.43 \text{ up to the nearest } 2^{nd} \text{ decimal point} \quad \text{(j)}$$

• Showing that the momentum is conserved under $S'$:

$$m_1'v_1' + m_2'v_2' = (m_1' + m_2')v' \quad \text{(to be proven)}$$
$$m_1(v_1 - u) + m_2(v_2 - u) = (m_1 + m_2)(v - u) \text{ (Galilean transformation)} \quad (1)$$
$$2000 \times (20 - 10) + 1500 \times (-10) = 3500 \times (11.43 - 10) \quad (2)$$
$$\text{Left hand side} = 5000 \quad (3)$$
$$\text{Right hand side} = 5005 \quad (4)$$
$$\text{Left hand side} \approx \text{Right hand side} \quad (5)$$
$$\therefore \text{Momentum is conserved} \quad \text{(from (5))}$$

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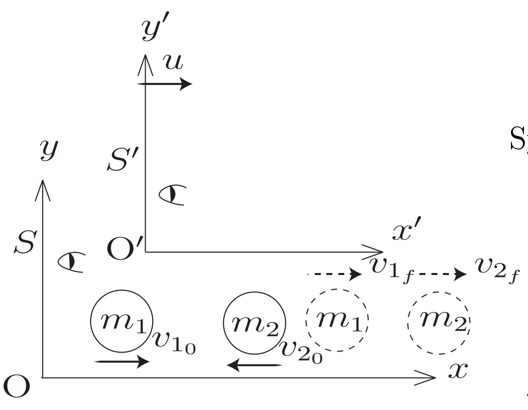
**Ex. 4** A billiard ball of mass 0.3 kg moves with a speed of 5 m/s and collides elastically with a ball of mass 0.2 kg moving in the opposite direction with a speed of 3 m/s. Show that because momentum is conserved in the rest frame, it is also conserved in a frame of reference moving with a speed of 2 m/s in the direction of the second ball.

**Sol. 4**




We perform Galilean transformation.

• Known:

$$\text{System } S \text{ is stationary} \quad \text{(a)}$$
$$\text{System } S' \text{ is moving with velocity } u = 2\text{m/s relative to } S \quad \text{(b)}$$
$$m_1 = m_1', \quad m_2 = m_2' \text{ (invariance of mass)} \quad \text{(c)}$$
$$\text{Motion of observer in } S' \text{ is in the positive } x\text{-direction} \quad \text{(d)}$$
$$\text{Direction of positive velocity} \quad \text{(e)}$$
$$v'_{\text{event}} = v_{\text{event}} - u \text{ for all events (covariance of velocity)} \quad \text{(f)}$$
$$m_1v_{1_0} + m_2v_{2_0} = m_1v_{1_f} + m_2v_{2_f} \text{ (conserved momentum)} \quad \text{(g)}$$

*Solution continues in the following page*

- Showing that the momentum is conserved under $S'$:

$$
\begin{aligned}
m_1' v_{1_0}' + m_2' v_{2_0}' &= m_1' v_{1_f}' + m_2' v_{2_f}' && \text{(to be proven)} \\
m_1(v_{1_0} - u) + m_2(v_{2_0} - u) &= m_1(v_{1_f} - u) + m_2(v_{2_f} - u) \text{ (Galilean transformation)} && (1) \\
m_1 v_{1_0} + m_2 v_{2_0} - \underline{u(m_1 + m_2)} &= m_1 v_{1_f} + m_2 v_{2_f} - \underline{u(m_1 + m_2)} && (2) \\
m_1 v_{1_0} + m_2 v_{2_0} &= m_1 v_{1_f} + m_2 v_{2_f} && (3)
\end{aligned}
$$

$\therefore$ Momentum is conserved (from (3) and (g))