**Midterm SSP Exam - Linear Algebra - Fall 2024**
**No. of Questions : 28**

[Qu.] [2 x 1 mark(s)]
For the following sets

$$S_1 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

$$S_2 = \left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$$

$$S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$S_5 = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

1. Which of the above sets is a basis for $\mathbb{R}^3$?
   (a) all (b) all except $S_2$
   (c) only $S_3$ & $S_5$ (d) only $S_3$ & $S_4$
   (e) only $S_1$ & $S_3$ & $S_4$ (f) only $S_3$

2. Which of the above sets is a spanning set for $\mathbb{R}^3$?
   (a) only $S_3$ & $S_4$ (b) only $S_1$ & $S_3$ & $S_4$
   (c) all except $S_2$ (d) all
   (e) only $S_3$ & $S_5$ (f) only $S_3$

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[Qu.] [1 mark(s)]
Given the set of 5 vectors $S = \{v_1, v_2, v_3, v_4, v_5\}$ where:

$$v_1 = \begin{bmatrix} 2 \\ -4 \\ -4 \\ 6 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ 6 \\ 18 \end{bmatrix}, v_3 = 2v_1 - 3v_2,$$

$$v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} -1 \\ 5 \\ 2 \\ -3 \end{bmatrix}$$

3. A basis of $H = \text{span}(S)$ is ...
   (a) $\{v_1, v_2, v_5\}$
   (b) $\{v_1, v_2, v_3, v_4\}$
   (c) $\{v_1, v_2, v_3, v_5\}$
   (d) $\{v_1, v_2, v_3\}$
   (e) $\{v_1, v_2, v_3, v_4, v_5\}$
   (f) All are bases

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[Qu.] [1 mark(s)]

4. The set of all $x \in \mathbb{R}$ for which the vectors $\langle 1, x, 0 \rangle$, $\langle 0, x^2, 1 \rangle$, and $\langle 0, 1, x \rangle$ are linearly dependent is

   (a) $\mathbb{R} - \{0\}$ (b) $\{-1\}$

   (c) $\{0\}$ (d) $\mathbb{R} - \{-1\}$

   (e) $\mathbb{R} - \{1\}$ (f) $\{1\}$

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[Qu.] [1 mark(s)]

5. Which of the following statements is **FALSE**?

   (a) A set containing a single non-zero vector is linearly independent.

   (b) Every linearly dependent set contains the zero vector.

   (c) The set of vectors $v, kv$ is linearly dependent for every scalar $k$.

   (d) Every set containing the zero vector is linearly dependent.

[Qu.] [1 mark(s)]

6. Given the linear system of equations $AX = B$, where $B \neq 0$ and $A$ of size $m \times n$. This system could have a unique solution if
   (a) $m \neq n$ only (b) $m \geq n$
   (c) $m \leq n$ (d) $m < n$ only
   (e) $m > n$ only (f) $m = n$ only

[Qu.] [1 mark(s)]

7. Let the vector $u = \langle k, 3, 0 \rangle \in \mathbb{R}^3$ be a linear combination of the vectors $v = \langle 1, 0, 2 \rangle$ and $w = \langle -1, 1, 2 \rangle$ such that $u = \alpha v + \beta w$. The values of $\alpha, \beta, k$ are ...
   (a) $\alpha = 2, \beta = -2, k = 6$
   (b) $\alpha = -3, \beta = 3, k = -6$
   (c) $\alpha = 1, \beta = -1, k = 6$
   (d) $\alpha = 3, \beta = -3, k = -6$

[Qu.] [3 x 1 mark(s)]

Given the set $V = \left\{ \begin{bmatrix} x \\ x^3 \\ 0 \end{bmatrix} ; x \in \mathbb{R} \right\}$

8. The set $V$ is closed under scalar multiplication.
   (a) False (b) True

9. The set $V$ is closed under addition.
   (a) False (b) True

10. The set $V$ is ...
    (a) a subset of $\mathbb{R}^3$
    (b) a subspace of $\mathbb{R}^2$
    (c) a subspace of $\mathbb{R}^3$
    (d) an independent set
    (e) a subset of $\mathbb{R}^2$

[Qu.] [3 x 1 mark(s)]

For $W = \{[a - 3b \ , \ b - a \ , \ a \ , \ b]^T : a, b \in \mathbb{R}\}$

11. $W$ is a subspace of ...
    (a) $\mathbb{R}^3$ (b) $\mathbb{R}^2$
    (c) $\mathbb{R}^4$ (d) not a subspace

12. A basis of $W$ is ...

    (a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

    (b) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

    (c) $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

    (d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

    (e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

13. The dimension of $W$ is ...
    (a) 1 (b) 4 (c) 3 (d) 2 (e) 0

[Qu.] [1 mark(s)]

14. Let $B$ and $C$ denote subsets of a vector space $V$. Select the **CORRECT** statement.

    (a) If $B \subseteq C$ and $C$ is independent, then $B$ is independent.

    (b) If $B \subseteq C$ and $C$ spans $V$, then $B$ spans $V$.

    (c) If $B \subseteq C$, then $\text{span}(C) = \text{span}(B)$.

    (d) If $B \subseteq C$ and $C$ is linearly dependent, then $B$ is linearly dependent.

[Qu.] [1 mark(s)]

15. Let $v_1, v_2, v_3, v_4, v_5, v_6$ be six vectors in $\mathbb{R}^4$. Which one of the following statements is **FALSE**?

    (a) These vectors are not linearly independent.

    (b) It is not necessary that these vectors span $\mathbb{R}^4$.

    (c) If the set $\{v_1, v_3, v_5, v_6\}$ spans $\mathbb{R}^4$, then it forms a basis for $\mathbb{R}^4$.

    (d) Any four of these vectors form a basis for $\mathbb{R}^4$.

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[Qu.] [1 mark(s)]

16. Let $A$ be an $n \times n$ matrix. Consider the linear system $AX = 4X$, where $X \in \mathbb{R}^n$. This system has a unique solution if and only if ...... is an invertible matrix.

    (a) $A - 4I$ (b) $A$ (c) $X - 4I$
    (d) $X$ (e) $4A$ (f) $4X$

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[Qu.] [1 mark(s)]

17. For the system $A_{4\times 3}X_{3\times 1} = B$. If the rank of the matrix $A = 3$, and the augmented matrix of $A$ ($Aug(A)$) is invertible, then the system
    (a) has no solution
    (b) has unique solution
    (c) has infinite number of solutions
    (d) not enough information

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[Qu.] [2 x 1 mark(s)]
Given the system of equations

$$\begin{aligned} x + 2y + z &= 3 \\ 2x - y - 3z &= 5 \\ 4x + 3y - z &= k \end{aligned}$$

where $k$ is a scalar value.

18. The set of all values of $k$ for which the system of linear equations has a single solution is
    (a) $\{4\}$ (b) $\{11\}$
    (c) $\mathbb{R} - \{11\}$ (d) $\mathbb{R} - \{4\}$
    (e) $\mathbb{R} - \{2\}$ (f) $\phi$

19. The set of all values of $k$ for which the system of linear equations has no solution is
    (a) $\{4\}$ (b) $\mathbb{R} - \{2\}$
    (c) $\{11\}$ (d) $\mathbb{R} - \{4\}$
    (e) $\phi$ (f) $\mathbb{R} - \{11\}$

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[Qu.] [1 mark(s)]

20. Let $S = \{A_1, A_2, \ldots, A_n\}$ be a linearly independent set of matrices in $\mathbb{M}_{n\times n}; n \geq 2$. Which of the following statements is **TRUE**?

    (a) The only way to write 0 as a linear combination of the elements of $S$ is the zero combination.

    (b) $S$ spans $\mathbb{M}_{n\times n}$.

    (c) $S$ could be a basis of $\mathbb{M}_{n\times n}$.

    (d) All are true

[Qu.] [2 x 1 mark(s)]

Let the set $W$ be the solution set of the system

$x + y - z + w = 0$

$2x - 3y - 2w = 0$

21. $W$ is ...
    (a) a subspace of $\mathbb{R}^2$
    (b) a subspace of $\mathbb{R}^3$
    (c) a subspace of $\mathbb{R}^4$
    (d) not a subspace
    (e) a subspace of $\mathbb{M}_{2\times 4}$
    (f) a subspace of $\mathbb{M}_{4\times 2}$

22. $\dim(W) = \ldots$
    (a) 0 (b) 2 (c) 3
    (d) 5 (e) 4 (f) 1

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[Qu.] [1 mark(s)]

23. Which of the following is **NOT** a subspace of $\mathbb{M}_{2\times 2}$?

    (a) $\left\{ \begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix} ;\ a \in \mathbb{R} \right\}$

    (b) $\left\{ \begin{bmatrix} a-b & b-1 \\ a & b \end{bmatrix} ;\ a, b \in \mathbb{R} \right\}$

    (c) $\left\{ \begin{bmatrix} a & -a \\ -a & a \end{bmatrix} ;\ a \in \mathbb{R} \right\}$

    (d) $\left\{ \begin{bmatrix} a & a \\ a & a-b \end{bmatrix} ;\ a, b \in \mathbb{R} \right\}$

    (e) $\left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} ;\ a, b, c \in \mathbb{R} \right\}$

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[Qu.] [1 mark(s)]

24. The maximum possible rank of an $m \times n$ matrix that is not square is
    (a) $m - n$ (b) $n$
    (c) $m$ (d) Max of $m, n$
    (e) $m/n$ (f) Min of $m, n$

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[Qu.] [1 mark(s)]

25. Let $A$ be an $n \times n$ matrix. Suppose that the system $AX = B$ is inconsistent for some $B \in \mathbb{R}^n$, then the system $AX = 0$, has
    (a) Unique trivial solution
    (b) Infinite number of solutions
    (c) Unique non-trivial solution
    (d) No solutions

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[Qu.] [2 x 1 mark(s)]

Given the set of polynomials

$H = \{ax^2 + bx + b,\ a \geq b,\ a, b \in \mathbb{R}\}$

26. The set $H$ is closed under addition.
    (a) False (b) True

27. The set $H$ is a vector space.
    (a) False (b) True

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[Qu.] [1 mark(s)]

28. If the system $A_{n\times n} x_{n\times 1} = b_{n\times 1}$ has no solution for some $b$, then the columns of $A$ ...
    (a) form a basis for $\mathbb{R}^n$
    (b) are linearly independent.
    (c) are linearly dependent.
    (d) form a spanning set for $\mathbb{R}^n$

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Total marks = 28