Midterm SSP Exam - Linear Algebra - Fall 2024
RE-CREATED
No. of Questions : 28

[Qu.] [2 x 1 mark(s)]
For the following sets

$$S_1 = \left\{ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

$$S_2 = \left\{ \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$$

$$S_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$S_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$S_5 = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$$

1. Which of the above sets is a basis for $\mathbb{R}^3$?
   (a) all (b) all except S_2
   (c) only S_3 & S_5 (d) only S_3 & S_4
   (e) only S_1 & S_3 & S_4 (f) only S_3

2. Which of the above sets is a spanning set for $\mathbb{R}^3$?
   (a) only S_3 & S_4 (b) only S_1 & S_3 & S_4
   (c) all except S_2 (d) all
   (e) only S_3 & S_5 (f) only S_3

[Qu.] [1 mark(s)]
Given the set of 5 vectors $S = \{v_1, v_2, v_3, v_4, v_5\}$ where:

$$v_1 = \begin{bmatrix} 2 \\ -4 \\ -4 \\ 6 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 9 \\ 6 \\ -18 \end{bmatrix}, v_3 = 2v_1 - 3v_2,$$

$$v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, v_5 = \begin{bmatrix} -1 \\ 5 \\ 2 \\ -3 \end{bmatrix}$$

3. A basis of $H = \text{span}(S)$ is ...
   (a) $\{v_1, v_2, v_5\}$
   (b) $\{v_1, v_2, v_3, v_4\}$
   (c) $\{v_1, v_2, v_3, v_5\}$
   (d) $\{v_1, v_2, v_3\}$
   (e) $\{v_1, v_2, v_3, v_4, v_5\}$
   (f) All are bases

[Qu.] [1 mark(s)]

4. The set of all $x \in \mathbb{R}$ for which the vectors $\langle 1, x, 0 \rangle$,$\langle 0, x^2, 1 \rangle$, and $\langle 0, 1, x \rangle$ are linearly independent is

   (a) $\mathbb{R} - \{0\}$ (b) $\{-1\}$

   (c) $\{0\}$ (d) $\mathbb{R} - \{-1\}$

   (e) $\mathbb{R} - \{1\}$ (f) $\{1\}$

[Qu.] [1 mark(s)]

5. Which of the following statements is **FALSE**?

   (a) A set containing a single non-zero vector is linearly independent.

   (b) Every linearly dependent set contains the zero vector.

   (c) The set of vectors v, kv is linearly dependent for every scalar k.

   (d) Every set containing the zero vector is linearly dependent.

[Qu.] [1 mark(s)]

6. Given the linear system of equations $AX = B$, where $B \neq 0$ and A of size $m \times n$. This system could have a unique solution if
   (a) $m \neq n$ only (b) $m \geq n$
   (c) $m \leq n$ (d) $m < n$ only
   (e) $m > n$ only (f) $m = n$ only

[Qu.] [1 mark(s)]

7. Let the vector $u = \langle k, 3, 0 \rangle \in \mathbb{R}^3$ be a linear combination of the vectors $v = \langle 1, 0, 2 \rangle$ and $w = \langle -1, 1, 2 \rangle$ such that $u = \alpha v + \beta w$. The values of α, β, k are ...
   (a) $\alpha = 2, \beta = -2, k = 6$
   (b) $\alpha = -3, \beta = 3, k = -6$
   (c) $\alpha = 1, \beta = -1, k = 6$
   (d) $\alpha = 3, \beta = -3, k = -6$

[Qu.] [3 x 1 mark(s)]

Given the set $V = \left\{ \begin{bmatrix} x \\ x^3 \\ 0 \end{bmatrix} ; x \in \mathbb{R} \right\}$

8. The set V is closed under scalar multiplication.
   (a) False (b) True

9. The set V is closed under addition.
   (a) False (b) True

10. The set V is ...
   (a) a subset of $\mathbb{R}^3$
   (b) a subspace of $\mathbb{R}^2$
   (c) a subspace of $\mathbb{R}^3$
   (d) an independent set
   (e) a subset of $\mathbb{R}^2$

[Qu.] [3 x 1 mark(s)]

For $W = \{[a - 3b\ ,\ b - a\ ,\ a\ ,\ b]^T : a, b \in \mathbb{R}\}$

11. W is a subspace of ...
   (a) $\mathbb{R}^3$ (b) $\mathbb{R}^2$
   (c) $\mathbb{R}^4$ (d) not a subspace

12. A basis of W is ...

   (a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

   (e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

13. The dimension of W is ...
   (a) 1 (b) 4 (c) 3 (d) 2 (e) 0

[Qu.] [1 mark(s)]

14. Let B and C denote subsets of a vector space V. Select the **CORRECT** statement.

   (a) If $B \subseteq C$ and C is independent, then B is independent.

   (b) If $B \subseteq C$ and C spans V, then B spans V.

   (c) If $B \subseteq C$, then $\text{span}(C) = \text{span}(B)$.

   (d) If $B \subseteq C$ and C is linearly dependent, then B is linearly dependent.

[Qu.] [1 mark(s)]

15. Let $v_1, v_2, v_3, v_4, v_5, v_6$ be six vectors in $\mathbb{R}^4$. Which one of the following statements is **FALSE**?

(a) These vectors are not linearly independent.

(b) It is not necessary that these vectors span $\mathbb{R}^4$.

(c) If the set $\{v_1, v_3, v_5, v_6\}$ spans $\mathbb{R}^4$, then it forms a basis for $\mathbb{R}^4$.

(d) Any four of these vectors form a basis for $\mathbb{R}^4$.

[Qu.] [1 mark(s)]

16. Let A be an $n \times n$ matrix. Consider the linear system $AX = 4X$, where $X \in \mathbb{R}^n$. This system has a unique solution if and only if ...... is an invertible matrix.

(a)	$A - 4I$	(b)	A	(c)	$X - 4I$
(d)	X	(e)	$4A$	(f)	$4X$

[Qu.] [1 mark(s)]

17. For the system $A_{4\times3}X_{3\times1} = B$. If the rank of the matrix $A = 3$, and the augmented matrix of A ($Aug(A)$) is invertible, then the system

(a) has no solution
(b) has unique solution
(c) has infinite number of solutions
(d) not enough information

[Qu.] [2 x 1 mark(s)]

Given the system of equations

$$x + 2y + z = 3$$
$$2x - y - 3z = 5$$
$$4x + 3y - z = k$$

where k is a scalar value.

18. The set of all values of k for which the system of linear equations has a single solution is

(a) $\{4\}$ (b) $\{11\}$
(c) $\mathbb{R} - \{11\}$ (d) $\mathbb{R} - \{4\}$
(e) $\mathbb{R} - \{2\}$ (f) ϕ

19. The set of all values of k for which the system of linear equations has no solution is

(a) $\{4\}$ (b) $\mathbb{R} - \{2\}$
(c) $\{11\}$ (d) $\mathbb{R} - \{4\}$
(e) ϕ (f) $\mathbb{R} - \{11\}$

[Qu.] [1 mark(s)]

20. Let $S = \{A_1, A_2, \ldots, A_n\}$ be a linearly independent set of matrices in $\mathbb{M}_{n\times n}; n \geq 2$. Which of the following statements is **TRUE**?

(a) The only way to write 0 as a linear combination of the elements of S is the zero combination.

(b) S spans $\mathbb{M}_{n\times n}$.

(c) S could be a basis of $\mathbb{M}_{n\times n}$.

(d) All are true

[Qu.] [2 x 1 mark(s)]

Let the set W be the solution set of the system

$x + y - z + w = 0$

$2x - 3y - 2w = 0$

21. W is ...
    (a) a subspace of $\mathbb{R}^2$
    (b) a subspace of $\mathbb{R}^3$
    (c) a subspace of $\mathbb{R}^4$
    (d) not a subspace
    (e) a subspace of $\mathbb{M}_{2\times 4}$
    (f) a subspace of $\mathbb{M}_{4\times 2}$

22. $\dim(W) = \ldots$
    (a) 0 (b) 2 (c) 3
    (d) 5 (e) 4 (f) 1

[Qu.] [1 mark(s)]

23. Which of the following is **NOT** a subspace of $\mathbb{M}_{2\times 2}$?

    (a) $\left\{ \begin{bmatrix} a & 0 \\ a & 0 \end{bmatrix} ; \, a \in \mathbb{R} \right\}$

    (b) $\left\{ \begin{bmatrix} a-b & b-1 \\ a & b \end{bmatrix} ; \, a, b \in \mathbb{R} \right\}$

    (c) $\left\{ \begin{bmatrix} a & -a \\ -a & a \end{bmatrix} ; \, a \in \mathbb{R} \right\}$

    (d) $\left\{ \begin{bmatrix} a & a \\ a & a-b \end{bmatrix} ; \, a, b \in \mathbb{R} \right\}$

    (e) $\left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} ; \, a, b, c \in \mathbb{R} \right\}$

[Qu.] [1 mark(s)]

24. The maximum possible rank of an $m \times n$ matrix that is not square is
    (a) $m - n$ (b) n
    (c) m (d) Max of m, n
    (e) m/n (f) Min of m, n

[Qu.] [1 mark(s)]

25. Let A be an $n \times n$ matrix. Suppose that the system $AX = B$ is inconsistent for some $B \in \mathbb{R}^n$, then the system $AX = 0$, has
    (a) Unique trivial solution
    (b) Infinite number of solutions
    (c) Unique non-trivial solution
    (d) No solutions

[Qu.] [2 x 1 mark(s)]

Given the set of polynomials

$H = \{ax^2 + bx + b, \; a \geq b, \; a, b \in \mathbb{R}\}$

26. The set H is closed under addition.
    (a) False (b) True

27. The set H is a vector space.
    (a) False (b) True

[Qu.] [1 mark(s)]

28. If the system $A_{n\times n} x_{n\times 1} = b_{n\times 1}$ has no solution for some b, then the columns of A ...
    (a) form a basis for $\mathbb{R}^n$
    (b) are linearly independent.
    (c) are linearly dependent.
    (d) form a spanning set for $\mathbb{R}^n$

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Total marks = 28