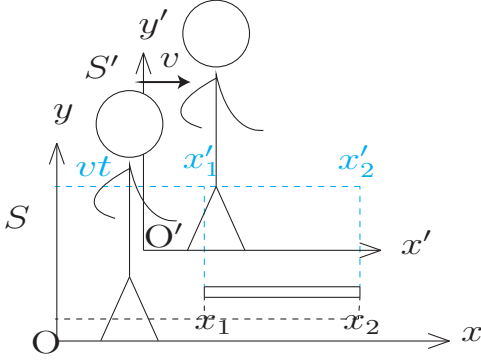


Modern Physics - Lecture Exercises Solutions
Lecture 1
Student Answers by Youssef Samy

Ex. 1 Show that although observers in S and S' measure different coordinates for the ends of a stick at rest in S , they agree on the length of the stick.

Sol. 1



• Known:

System S is stationary (a)

System S' is moving with velocity v relative to S (b)

$$L = x_2 - x_1 \quad (c)$$

$$L' = x'_2 - x'_1 \quad (d)$$

• Showing that $L = L'$ using Galilean Transformation:

$$x'_1 = x_1 - vt \quad (1)$$

$$x'_2 = x_2 - vt \quad (2)$$

$$L' = x'_2 - x'_1 \quad (d)$$

$$L' = (x_2 - vt) - (x_1 - vt)$$

$$L' = x_2 - x_1 \quad (3)$$

$$\therefore L' = L \quad (\text{from (c), (d), and (3)})$$

Ex. 2 Conservation of Linear Momentum Is Covariant Under the Galilean Transformation.

Assume that two masses m_1 and m_2 are moving in the positive x direction with velocities v_1 and v_2 as measured by an observer in S before a collision. After the collision, the two masses stick together and move with a velocity v in S . Show that if an observer in S finds momentum to be conserved, so does an observer in S' .

Sol. 2

We perform Galilean transformation.

• Known:

System S is stationary (a)

System S' is moving with velocity u relative to S (b)

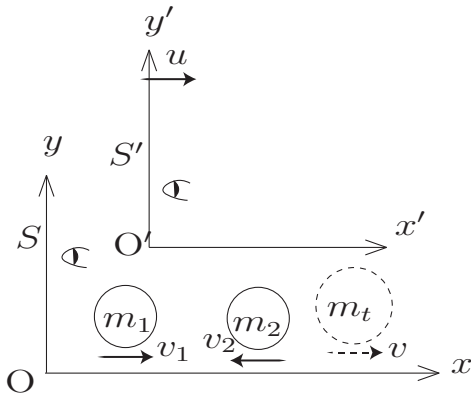
$$m_1 = m'_1, \quad m_2 = m'_2 \quad (\text{invariance of mass}) \quad (c)$$

Motion of observer in S' is in the positive x -direction (d)

Direction of positive velocity $\begin{matrix} \uparrow + \\ \rightarrow + \end{matrix}$ (e)

$$v'_{\text{event}} = v_{\text{event}} - u \quad \text{for all events (covariance of velocity)} \quad (f)$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v \quad (\text{conserved momentum}) \quad (g)$$



• Showing that the momentum is conserved under S' :

$$m'_1 v'_1 + m'_2 v'_2 = (m'_1 + m'_2) v' \quad (\text{to be proven})$$

$$m_1 (v_1 - u) + m_2 (v_2 - u) = (m_1 + m_2) (v - u) \quad (\text{Galilean transformation}) \quad (1)$$

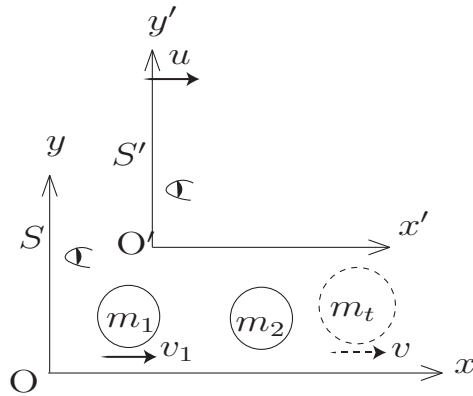
$$m_1 v_1 - \underline{m_1 u} + m_2 v_2 - \underline{m_2 u} = m_1 v + m_2 v - \underline{m_1 u} - \underline{m_2 u} \quad (2)$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v \quad (3)$$

\therefore Momentum is conserved (from (g) and (3))

Ex. 3 A 2000-kg car moving with a speed of 20 m/s collides with and sticks to a 1500-kg car at rest at a stop sign. Show that because momentum is conserved in the rest frame, momentum is also conserved in a reference frame moving with a speed of 10 m/s in the direction of the moving car.

Sol. 3



We perform Galilean transformation.

• Known:

System S is stationary (a)

System S' is moving with velocity $u = 10\text{m/s}$ relative to S (b)

$m_1 = m'_1, \quad m_2 = m'_2$ (invariance of mass) (c)

Motion of observer in S' is in the positive x -direction (d)

Direction of positive velocity (e)

$v'_{\text{event}} = v_{\text{event}} - u$ for all events (covariance of velocity) (f)

$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v$ (conserved momentum) (g)

$m_1 = 2000\text{kg}, m_2 = 1500\text{kg}, v_1 = 20\text{m/s}, v_2 = 0\text{m/s}$ (h)

$2000 \times 20 + 1500 \times 0 = 3500 \times v$ (i)

$v \approx 11.43$ up to the nearest 2^{nd} decimal point (j)

• Showing that the momentum is conserved under S' :

$$m'_1 v'_1 + m'_2 v'_2 = (m'_1 + m'_2) v' \quad (\text{to be proven})$$

$$m_1(v_1 - u) + m_2(v_2 - u) = (m_1 + m_2)(v - u) \quad (\text{Galilean transformation}) \quad (1)$$

$$2000 \times (20 - 10) + 1500 \times (-10) = 3500 \times (11.43 - 10) \quad (2)$$

$$\text{Left hand side} = 5000 \quad (3)$$

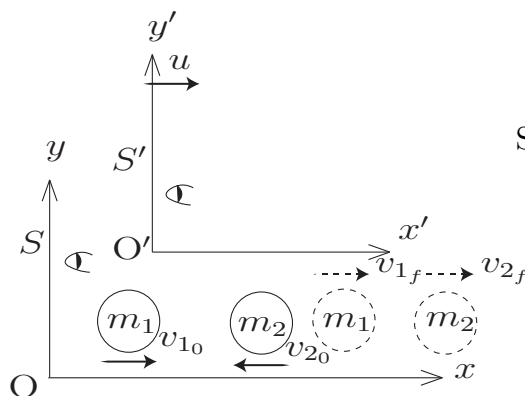
$$\text{Right hand side} = 5005 \quad (4)$$

$$\text{Left hand side} \approx \text{Right hand side} \quad (5)$$

$$\therefore \text{Momentum is conserved} \quad (\text{from (5)})$$

Ex. 4 A billiard ball of mass 0.3 kg moves with a speed of 5 m/s and collides elastically with a ball of mass 0.2 kg moving in the opposite direction with a speed of 3 m/s. Show that because momentum is conserved in the rest frame, it is also conserved in a frame of reference moving with a speed of 2 m/s in the direction of the second ball.

Sol. 4



We perform Galilean transformation.

• Known:

System S is stationary (a)

System S' is moving with velocity $u = 2\text{m/s}$ relative to S (b)

$m_1 = m'_1, \quad m_2 = m'_2$ (invariance of mass) (c)

Motion of observer in S' is in the positive x -direction (d)

Direction of positive velocity (e)

$v'_{\text{event}} = v_{\text{event}} - u$ for all events (covariance of velocity) (f)

$m_1 v_{1_0} + m_2 v_{2_0} = m_1 v_{1_f} + m_2 v_{2_f}$ (conserved momentum) (g)

Solution continued in the following page

- Showing that the momentum is conserved under S' parametrically:

$$m'_1 v'_{1_0} + m'_2 v'_{2_0} = m'_1 v'_{1_f} + m'_2 v'_{2_f} \quad (\text{to be proven})$$

$$m_1(v_{1_0} - u) + m_2(v_{2_0} - u) = m_1(v_{1_f} - u) + m_2(v_{2_f} - u) \quad (\text{Galilean transformation}) \quad (1)$$

$$m_1 v_{1_0} + m_2 v_{2_0} - \underline{u(m_1 + m_2)} = m_1 v_{1_f} + m_2 v_{2_f} - \underline{u(m_1 + m_2)} \quad (2)$$

$$m_1 v_{1_0} + m_2 v_{2_0} = m_1 v_{1_f} + m_2 v_{2_f} \quad (3)$$

\therefore Momentum is conserved (from (3) and (g))

- Alternative solution - to substitute with numbers, assume conservation of kinetic energy due to elastic collision.

In S :

$$\frac{1}{2}m_1 v_{1_0}^2 + \frac{1}{2}m_2 v_{2_0}^2 = \frac{1}{2}m_1 v_{1_f}^2 + \frac{1}{2}m_2 v_{2_f}^2 \quad (\text{conserved kinetic energy}) \quad (h)$$

$$0.3 \times 5 + 0.2 \times -3 = 0.3 v_{1_f} + 0.2 v_{2_f} \quad (\text{substitution in (g)}) \quad (i)$$

$$3v_{1_f} = 9 - 2v_{2_f} \quad (j)$$

$$v_{1_f} = -1.4, \quad v_{2_f} = 6.6 \quad (\text{from substituting with (i) in (h)}) \quad (k)$$

In S' :

$$m'_1 v'_{1_0} + m'_2 v'_{2_0} = m'_1 v'_{1_f} + m'_2 v'_{2_f} \quad (\text{to be proven})$$

$$m_1(v_{1_0} - u) + m_2(v_{2_0} - u) = m_1(v_{1_f} - u) + m_2(v_{2_f} - u) \quad (1)$$

$$0.3 \times (5 - 2) + 0.2 \times (-3 - 2) = 0.3 \times (-1.4 - 2) + 0.2 \times (6.6 - 2) \quad (2)$$

$$\text{Left hand side} = -0.1 \quad (3)$$

$$\text{Right hand side} = -0.1 \quad (4)$$

$$\text{Left hand side} = \text{Right hand side} \quad (5)$$

\therefore Momentum is conserved (from (5))

