ICIC Express Letters ICIC International ©2020 ISSN 1881-803X
Volume **14**, Number **5**, May **2020** pp. 471–476

OPTIMAL TRAVEL ROUTES PROBLEM OF MAXIMIZING TRAVELER'S SATISFACTION BY USING UTILITY FUNCTIONS AND AHP

Jae-Won Noh, Hyun-Kyung Son, Hyun-Soo Choi and Jae-Dong Son*

Department of Industrial and Information Systems Engineering
Soongsil University
511 Sangdo-dong, Dongjak-gu, Seoul 06978, Korea
*Corresponding author: son88@ssu.ac.kr

Received November 2019; accepted February 2020

Abstract. *This paper examines the optimal path problem of selecting a travel route that maximizes individual satisfaction. The factors affecting satisfaction are assumed to be the travel time, transportation price, waiting time for new transportation, and total number of transfers. A mathematical model, LP (linear programming), is formulated using AHP techniques to assess the relative significance of the satisfaction factors introduced in this problem. The constraints of limited time and total cost to reach the destination are applied to the model. This proposed model is validated through empirical analysis with real data.*
Keywords: Linear programming, AHP, Utility function, Travel routes

1. **Introduction.** Travelers currently use transportation applications on their cell phones to search for the fastest way to get their destination. However, such applications cannot currently guarantee the satisfaction of each individual traveler, which consists of a variety of factors. In this study, we take account of the factors of a traveler's satisfaction that might differ between travelers, and present an optimal travel route that maximizes the traveler's satisfaction to the destination. The elements affecting satisfaction include total traveling time, transportation cost, waiting time for new transportation, and the total number of transfers. Each factor of satisfaction can be measured by estimating the utility function, and the analysis of hierarchy process (AHP) is used to estimate each traveler's significance on the factors of satisfaction.

Our paper is part of research into the optimal path based on network optimization theory. Kim et al. [1] presented an optimal path that minimizes the total price of tickets according to the daily ticket price and also determines the order of visits and duration of stay in several cities. Kim et al. [2] and Jo et al. [3] constructed a mathematical model in which the optimal route for fast delivery and reduced transportation cost was found in a network with various origin and destination locations. Seo et al. [4] proposed a network path optimization model that enables efficient traffic routing to avoid congestion associated with the explosive growth of Internet users. Kang et al. [5] verified the validity by applying the optimal network model to the escape route network in the case of a disaster. While these studies above were conducted in terms of network path optimization and do not reflect the significance felt by a decision maker for the various factors that make up the constraints, the following studies have examined methods considering the decision maker's degree of significance for the factors of satisfaction. Ok et al. [6] proposed a shortest path exploration algorithm based on individual preference measured by the index of the heuristic function. Moon et al. [7] used AHP to explore the optimal path for transportation convenience for vulnerable traffic based on the relative importance of

DOI: 10.24507/icicel.14.05.471

each pedestrian obstacle. Kim [8] established a system to help choose the optimal route tailored to the sensitivity of pedestrians.

The remainder of this paper is organized as follows. Section 2 formulates the travel routes problem as a mathematical model, based on which an empirical experiment is conducted in Section 3, and conclusions are summarized in Section 4.

2. **Mathematical Model.** This study investigates the path selection that maximizes individual satisfaction, including time and cost. In order to formulate this problem in terms of linear programming, we assume the following: The departure and arrival of the way point is indicated by (i, j); hence x_{ij} represents a single path from i to j. In addition, the total number of intermediate transit points including the origin and destination is expressed by n. It is assumed that satisfaction considers the following four factors: travel time, price, wait time, and number of transfers. The utility function for the decision maker consists of four components depending on the four elements of satisfaction. By T_{ij}, M_{ij}, and W_{ij}, we respectively denote the decision maker's utility function for the travel time, the cost, and waiting time from i to j, where a utility function of the number of transfers is given by $F(y)$ $(0 \leq y \leq n)$. In addition, by s_t, s_m, s_w, and s_f, we respectively represent the weight value for the travel time, the price, the waiting time, and the number of transfers.

The linear programming of the problem above can be expressed as follows.

$$Max \quad \sum_{i=1}^{n}\sum_{j=1}^{n}\left(s_t T_{ij} + s_m M_{ij} + s_w W_{ij}\right)x_{ij} + s_f F\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n}\sum_{j>i}^{n} c_{ij}x_{ij} < Tc \tag{2}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left(q_{ij} + r_{ij}\right)x_{ij} < Tr \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = \begin{cases} 1, & i = 1 \\ 0, & i \neq 1 \text{ or } i \neq n \\ -1, & i = n \end{cases} \tag{4}$$

$$x_{ij} = 0 \text{ or } 1 \quad (i, j = 1, 2, \ldots, n) \tag{5}$$

Equation (1) represents the objective function that maximizes a decision maker's satisfaction consisting of four factors: travel time, costs, waiting time, and the number of transfers. Equations (2) to (5) are constraints, and the sum of transport costs $c_{ij}x_{ij}$ for the selected route should be less than or equal to the total allowable cost Tc. Equation (3) shall be such that the sum of the travel time q_{ij} and waiting time r_{ij} of the selected path (i, j) should be less than or equal to the total allowable time Tr. Equation (4) guarantees that only one route can be chosen from origin i to destination j. However, the midway point of the route cannot be simultaneously created by two flows. The determinant x_{ij} is defined as 0 or 1, where if it has 1, then the path is selected, otherwise it is not.

3. **Empirical Analysis.** In this section we conduct an empirical analysis based on the mathematical model formulated in the previous section. For this analysis, we consider a network model with 14 nodes and 23 arcs, as shown in Figure 1, where the origin and the destination are respectively set as Soongsil University and Gapyeong Station. With this network we formulate linear programming in which the objective is to maximize a decision maker's satisfaction under constraints regarding total travel time and cost (120 minutes and 13,000 won). A decision maker's satisfaction consists of the following four factors: travel time, transportation fee, waiting time for transfer, and the total transferring number

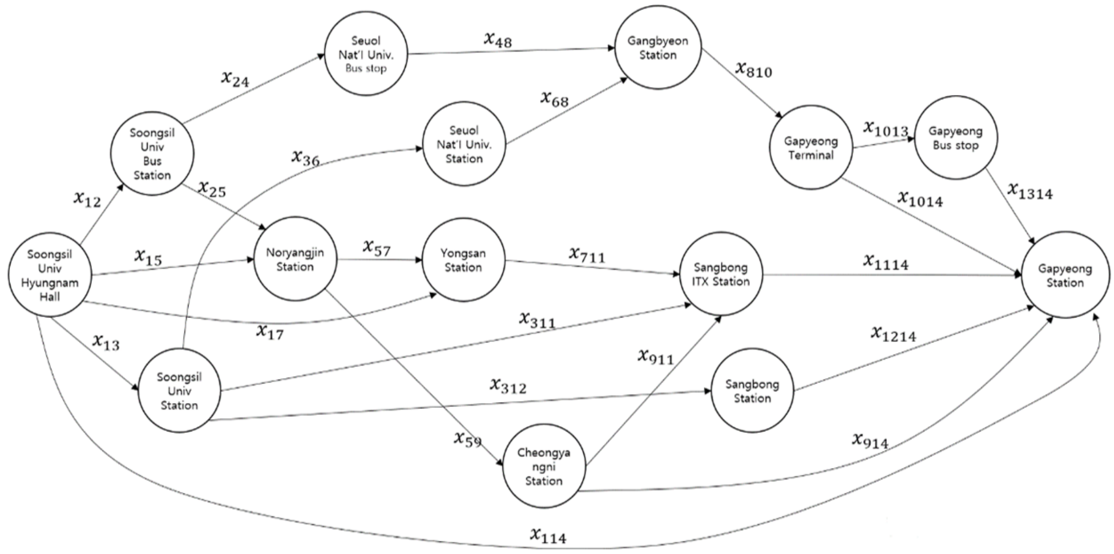

FIGURE 1. Example of travel network model

to the destination. The real values of these factors are acquired from 'Naver Map' at 13:00 on Sep. 30th in 2018.

The optimal path for a decision maker who wants to travel from Soongsil Univ. to Gapyeong using public transportation can be derived through the following three steps: 1) derivation of utility functions (T_{ij}, M_{ij}, W_{ij}, and $F(y)$) by a certainty equivalent method, 2) extraction of weight values (s_t, s_m, s_w, and s_f) for the four factors of satisfaction by AHP, and 3) formulation of LP and solving of LP by LINGO.

Step 1: Utility functions. Here, with the certainty equivalent method, we drive utility functions for the time and price, as well as the number of transfers with respect to each arc. The utility concept involves to quantifying an individual's preference for a particular target. Uncertain revenue opportunities that can result in W from the probability of p or L from the probability of $(1 - p)$.

Step 2: Weight value for satisfaction factors by AHP. It is assumed that a traveler's satisfaction with his/her travel consists of four factors: travel time, transportation fee, waiting time for transfer, and the total transferring number to the destination. Regarding these satisfaction factors, we survey the relative significance, which generates a weight value of each factor. This weight value can be calculated by making a comparison to each other and deriving the importance. The weight value of a participant by AHP is as follows: 0.559 for travel time (s_t), 0.172 for transportation fee (s_m), 0.077 for the wait time (s_w), and 0.192 for the number of transfers (s_f). The consistency index (CI, for short), is less than 0.1 with 0.0386, meaning that there is no contradiction in the questionnaire and thus the weight of this experiment is valid.

Step 3: Optimal path. The value obtained above is multiplied by the utility value and the relevant weight values for four satisfaction factors, which formulates a coefficient for the determinant variables of LP, the optimal solution of which is solved by LINGO.

Figure 2 can largely be divided into the following five parts: ① is the objective function of the LP in Equation (1). ② represents Equation (2) in which the total allowable expenditure is established as a constraint. ③ shows Equation (3), a constraint of total travel time. ④ guarantees that only one arch can be chosen, which is relevant to Equation (4). ⑤ shows that the decision variables must have values of 0 or 1.

From Figure 3, we can say that the optimal satisfaction of the decision maker is 3.27, a value of objective function, when the optimal path is x_{12}, x_{25}, x_{59}, x_{911}, x_{1114} as shown

```
①
max = (0.82318*x12 + 0.78348*x13 + 0.64411*x15 + 0.62593*x17 + 8.82094*x114 + 0.70695*x24 + 0.62145*x25
           + 0.78612*x36 + 0.50626*x311 + 0.51764*x312 + 0.41147*x48 + 0.75495*x57 + 0.55515*x59 + 0.54621*x68
           + 0.55536*x711 + 0.15607*x810 + 0.76071*x911 + 0.35769*x914 + 0.8852*x1013 + 0.79217*x1014
           + 0.48673*x1114 + 0.35173*x1214 + 0.5791*x1314
           + ((x12+x13+x15+x17+x24+x25+x36+x311+x312+x48+x57+x59+x68+x711+x810+x911+x914+x1013+x1014+x1114+x1214+x1314)
               *(-0.1645)+0.9605)*0.191543);

②
0*x12 + 0*x13 + 3900*x15 + 1200*x17 + 65260*x114 + 1200*x24 + 1200*x25
+ 3000*x36 + 1550*x311 + 1550*x312 + 16000*x48 + 1250*x57 + 1350*x59 + 1450*x68
+ 3000*x711 + 6400*x810 + 3000*x911 + 4800*x914 + 0*x1013 + 3000*x1014
+ 4800*x1114 + 2150*x1214 + 1250*x1314 <= 13000;

③
4.5*x12 + 7*x13 + 14*x15 + 19*x17 + 80*x114 + 11*x24 + 17*x25
+ 6*x36 + 39*x311 + 36*x312 + 31*x48 + 8*x57 + 29*x59 + 30*x68
+ 27*x711 + 80*x810 + 7*x911 + 40*x914 + 1*x1013 + 5*x1014
+ 38*x1114 + 57*x1214 + 12*x1314 <= 120;

④
   x12  + x13  + x15  + x17 + x114      = 1;
   x24  + x25  - x12                    = 0;
   x311 + x312 + x36  - x13             = 0;
   x48  - x24                           = 0;
   x57  + x59  - x25  - x15             = 0;
   x68  - x36                           = 0;
   x711 - x57  - x17                    = 0;
   x810 - x48  - x68                    = 0;
   x911 + x914 - x59                    = 0;
   x1013 + x1014 - x810                 = 0;
   x1114 - x711 - x311 -x911            = 0;
   x1214 - x312                         = 0;
   x1314 - x1013                        = 0;
  -x114 -x1314 -x1014 -x1114 - x1214 -x914   = -1;

⑤
@bin(x12);@bin(x13);@bin(x15); @bin(x114);@bin(x24);@bin(x25);@bin(x36);@bin(x311);@bin(x312);@bin(x48);@bin(x57);@bin(x59);
@bin(x68);@bin(x711);@bin(x810);@bin(x911);@bin(x914);@bin(x1013);@bin(x1014);@bin(x1114);@bin(x1214);@bin(x1314);
```

FIGURE 2. LP model for participant A with respect to the travel network model

Variable	Value	Reduced Cost
X12	1.000000	-0.1972500
X13	0.000000	-0.1575500
X15	0.000000	-0.1818000E-01
X17	0.000000	0.000000
X114	0.000000	-8.226519
X24	0.000000	-0.6754412
X25	1.000000	-0.5899412
X36	0.000000	-0.7546112
X311	0.000000	-0.4747512
X312	0.000000	-0.4861312
X48	0.000000	-0.3799612
X57	0.000000	-0.7234412
X59	1.000000	-0.5236412
X68	0.000000	-0.5147012
X711	0.000000	-0.5238512
X810	0.000000	-0.1245612
X911	1.000000	-0.7292012
X914	0.000000	-0.3261812
X1013	0.000000	-0.8536912
X1014	0.000000	-0.7606612
X1114	1.000000	-0.4552212
X1214	0.000000	-0.3202212
X1314	0.000000	-0.5475912

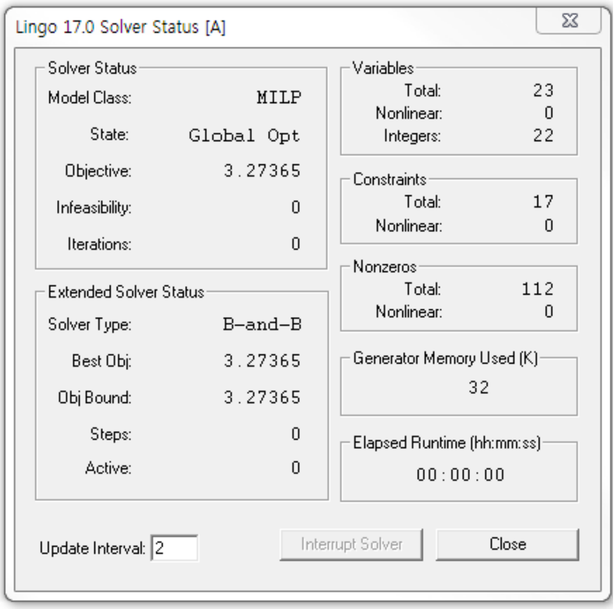

FIGURE 3. Optimal solution by Lingo for participant A

in Figure 4. The optimal path suggests that the decision maker should take a bus from the nearest bus stop around Soongsil Univ., and then transfer to No. 1 subway line to Cheongnyangri Station, where he/she can get ITX to Gapyeong Station. This result increases the participant's satisfaction by 1.2 compared to that with respect to his/her initially planned path, x_{15}, x_{57}, x_{711}, and x_{1114}.

4. **Conclusions.** Our research addressed an optimal path selection problem to maximize individual satisfaction, which has not been considered in previous studies. In order to achieve this, the following three methodologies were used: 1) utility function to quantify

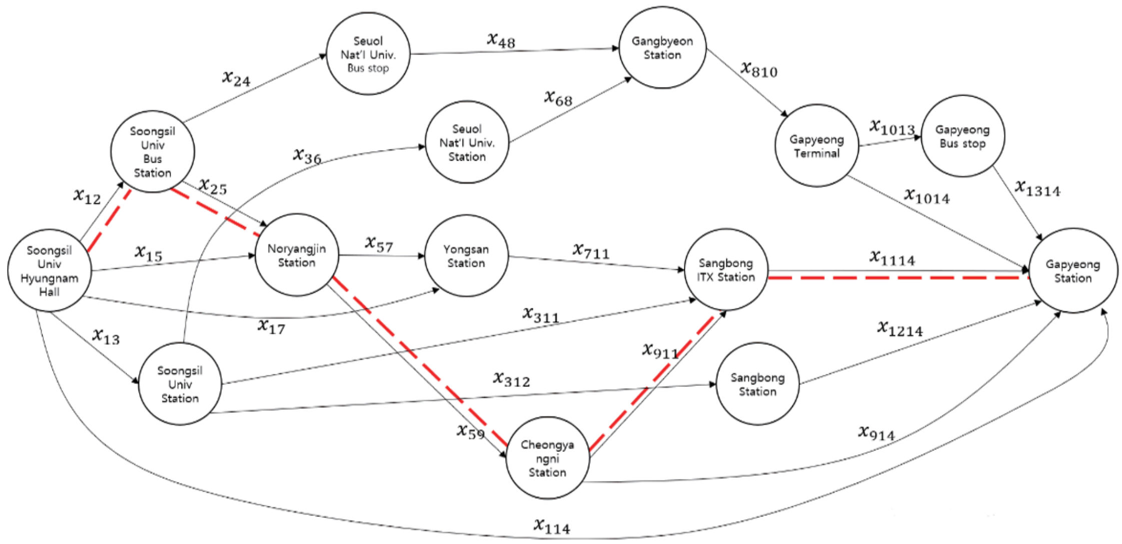

FIGURE 4. Optimal path (dotted line) for participant A

a decision maker's satisfaction for each factor for satisfaction consisting of the travel time, travel cost, waiting time for transfer, and transferring numbers, which denote the relevant arcs in the travel path; 2) AHP to measure the significance felt by the decision maker for each factor; 3) linear programming to formulate our problem that has constraints regarding the total cost and travel time to reach the destination. By solving LP, we could provide the optimal satisfaction path to a decision maker who wants to travel from his departure to the destination within a certain allowable maximum expenditure and a certain travel time limit. Through an experiment we proved that our model can be very useful for recommending a satisfying travel path to a traveler as opposed to simply a faster one.

In order to make our model more realistic, we can consider a transfer discount system that influences the decision maker's satisfaction factor of transportation fee, which may eventually recommend a different optimal path.

REFERENCES

[1] Y. Kim, S. Park, Y. Kwon, S. Lee, W. Choi and C. Lee, Multi-city flight route optimization system using big-data analytics, *Journal of the Korean Institute of Industrial Engineers*, pp.92-101, DOI: 10.7232/JKIIE.2018.44.2.092, 2018.

[2] D.-G. Kim, S.-C. Gang, C.-H. Park and S.-Y. Go, A single allocation hub network design model for intermodal freight transportation, *Journal of Korean Society of Transportation*, vol.27, no.1, pp.129-141, 2009.

[3] Y. Jo, D. Park, H. Park, C. Park and I. Um, A study on network optimization of parcel service industry, *Korea Research Institute for Human Settlements*, vol.72, pp.103-120, 2012.

[4] D.-W. Seo, S. H. Yoon and B.-Y. Chang, Resource and network routing optimization in smart nodes environment, *Journal of the Korea Society for Simulation*, vol.22, no.4, pp.149-156, 2013.

[5] C. Kang, J. Lee, J. Song and K. Jung, Route optimization for emergency evacuation and response in disaster area, *Journal of the Korean Society of Civil Engineers*, vol.34, no.1, pp.517-526, 2014.

[6] S.-H. Ok, J.-H. Ahn, S. Kang and B. Moon, A combined heuristic algorithm for preference-based shortest path search, *Journal of the Institute of Electronics Engineers of Korea*, vol.47, no.8, pp.74-84, 2010.

[7] M. Moon, Y. Lee, K. Yu and J. Kim, Optimized path finding algorithm for walking convenience of the people with reduced mobility, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry*, vol.34, no.3, pp.273-282, 2016.

[8] D. Kim, Route retrieval support system by using of pedestrians' preference data, *Archives of Design Research*, vol.19, no.2, pp.81-90, 2006.

[9] M. Friedman and L. J. Savage, The utility analysis of choices involving risk, *Journal of Political Economy*, vol.56, no.4, pp.279-304, 1948.
[10] G. Harrison, Expected utility theory and the experiments, *Empirical Economics*, pp.223-253, 1994.
[11] P. C. Fishburn, *Utility Theory for Decision Making*, John Wiley & Sons, N.Y., 1970.