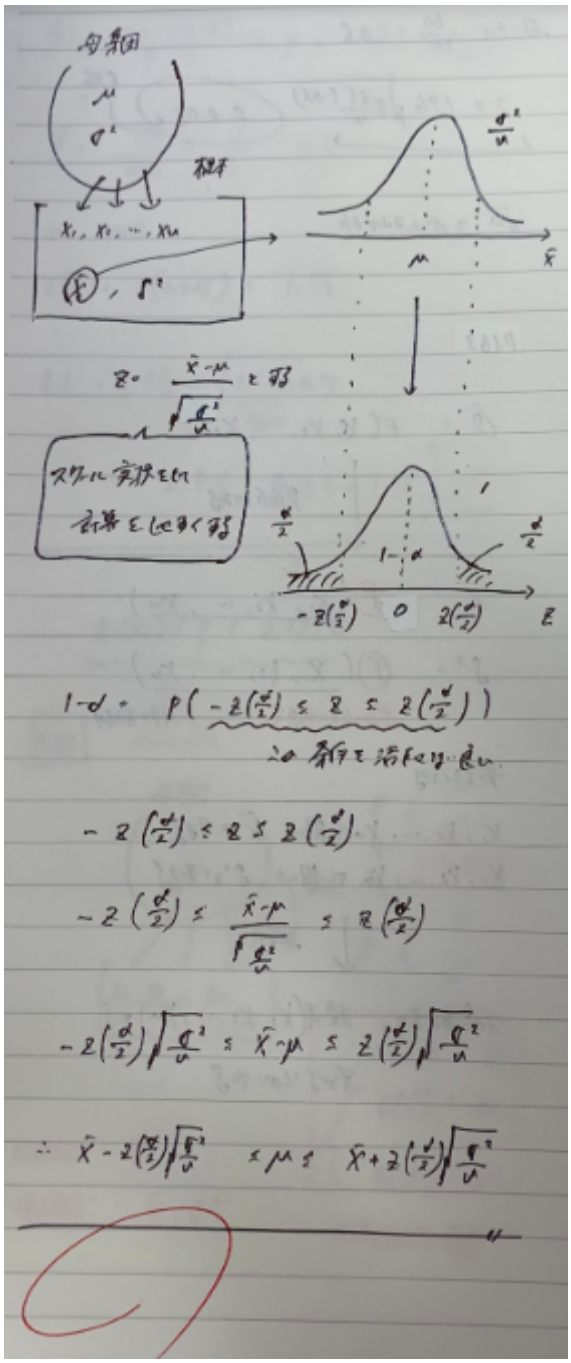
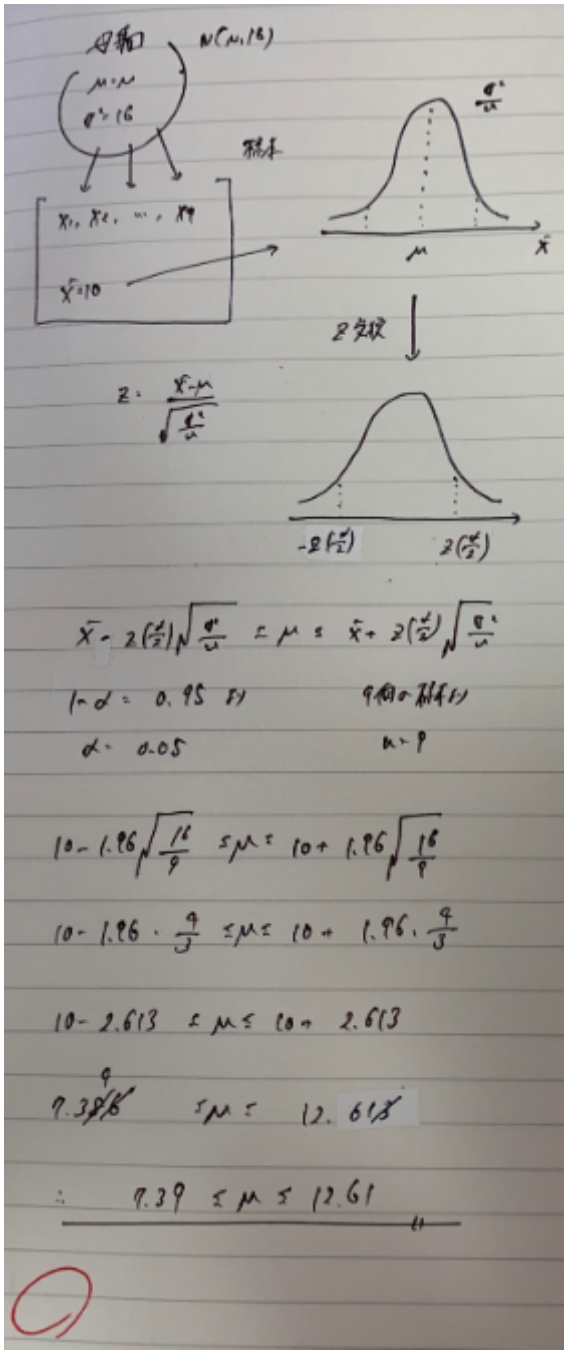
7.2　区間推定

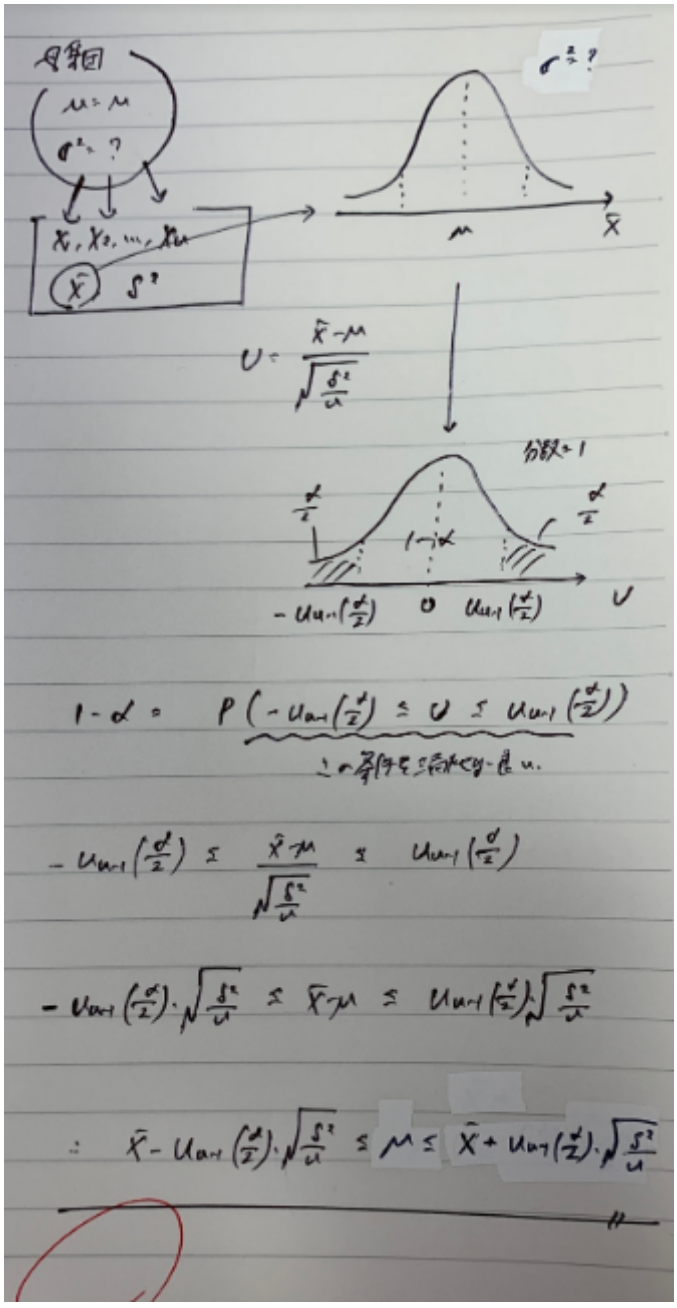
7.2.2　σ^2が既知のときの、母平均μの区間推定



例題 1



7.2.3　σ^2が既知のときの、母平均μの区間推定



7.2.4　母分散σ^2の区間推定

$$U = \frac{\bar{X}-\mu}{\sqrt{\frac{S^2}{n}}} = \frac{\bar{X}-\mu}{\frac{\sqrt{S^2}}{\sqrt{n}}} = \frac{\sqrt{n}(\bar{X}-\mu)}{\sqrt{S^2}} = \frac{\frac{\sqrt{n}(\bar{X}-\mu)}{\sqrt{\sigma^2}}}{\sqrt{\frac{S^2}{\sigma^2}}}$$

$= \frac{\frac{\bar{X}-\mu}{\sqrt{\frac{\sigma^2}{n}}}}{\sqrt{\frac{S^2}{\sigma^2}}}$ ← 標準正規分布

$= \frac{Z}{\sqrt{\frac{V}{n-1}}}$ となる

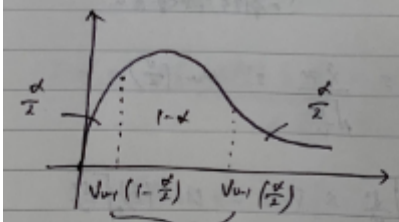
$$Z = \frac{\sqrt{n}(\bar{X}-\mu)}{\sigma}$$

$$V = (n-1)\frac{S^2}{\sigma^2} = (n-1)\cdot\frac{1}{\sigma^2}\cdot\frac{1}{n-1}\sum(X_i-\bar{X})^2$$

$$= \frac{\sum(X_i-\bar{X})^2}{\sigma^2}$$

これが自由度 $n-1$ のカイ2乗分布に従う

これを利用して、母分散の信頼区間を求める



この中に V が入る確率

$$1-\alpha = P\left(V_{n-1}\left(1-\frac{\alpha}{2}\right) \le V \le V_{n-1}\left(\frac{\alpha}{2}\right)\right)$$

$V = (n-1)\frac{S^2}{\sigma^2}$ より

$$V_{n-1}\left(1-\frac{\alpha}{2}\right) \le \frac{(n-1)S^2}{\sigma^2} \le V_{n-1}\left(\frac{\alpha}{2}\right)$$

$$\frac{1}{V_{n-1}\left(\frac{\alpha}{2}\right)} \le \frac{\sigma^2}{(n-1)S^2} \le \frac{1}{V_{n-1}\left(1-\frac{\alpha}{2}\right)}$$

$$\frac{(n-1)S^2}{V_{n-1}\left(\frac{\alpha}{2}\right)} \le \sigma^2 \le \frac{(n-1)S^2}{V_{n-1}\left(1-\frac{\alpha}{2}\right)}$$