# Extra Exercises for Week 2

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Questions are taken from the textbook: A First Course in Probability (9th edition) by Sheldon Ross.

**Problem 1.** A president, treasurer, and secretary, all different, are to be chosen from a club consisting of 10 people. How many different choices of officers are possible if

(a) there are no restrictions?

(b) $A$ and $B$ will not serve together?

(c) $C$ and $D$ will serve together or not at all?

(d) $E$ must be an officer?

(e) F will serve only if he is president?

**Problem 2.** A committee of 6 people is to be chosen from a group consisting of 7 men and 8 women. If the committee must consist of at least 3 women and at least 2 men, how many different committees are possible?

**Problem 3.** How many subsets of size 4 of the set $S = \{1, 2, \ldots, 20\}$ contain at least one of the elements $1, 2, 3, 4, 5$ ?

**Problem 4.** If there are no restrictions on where the digits $(0-9)$ and letters $(A-Z)$ are placed, how many 8-place license plates consisting of 5 letters and 3 digits are possible if no repetitions of letters or digits are allowed? What if the 3 digits must be placed together?

**Problem 5.** From 10 married couples, we want to select a group of 6 people that is not allowed to contain a married couple.

(a) How many choices are there?

(b) How many choices are there if the group must also consist of 3 men and 3 women?

**Problem 6.** The binomial theorem states that, for nonnegative integer $n$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}.$$

What is the value of

$$\binom{n}{0} - \binom{n}{1} + \ldots + (-1)^n \binom{n}{n}?$$

**Problem 7.** If 8 identical blackboards are to be divided among 4 schools, how many divisions are possible? How many if each school must receive at least 1 blackboard?

**Problem 8.** Let $k, n$ be integers such that $0 \leq k \leq n$ and $n \geq 1$. Determine the number of vectors $(x_1, \ldots, x_n)$, such that each $x_i \in \{0, 1\}$ and

$$x_1 + x_2 + \ldots + x_n \geq k.$$

Answers (Let me know if there are any discrepancies):

1(a). $\binom{10}{3} \cdot 3!$

1(b). $\left[\binom{8}{3} + \binom{8}{2} + \binom{8}{2}\right] \cdot 3!$

1(c). $\left[\binom{8}{1} + \binom{8}{3}\right] \cdot 3!$

1(d). $\binom{9}{2} \cdot 3!$

1(e). $\binom{9}{2} \cdot 2! + \binom{9}{3} \cdot 3!$

2. $\binom{7}{3}\binom{8}{3} + \binom{7}{2}\binom{8}{4}$

3. $\binom{20}{4} - \binom{15}{4}$

4. $\binom{26}{5}\binom{10}{3} \cdot 8!$, $\binom{26}{5}\binom{10}{3} \cdot 6! \cdot 3!$

5(a). $\binom{10}{6} \cdot 2^6$

5(b). $\binom{10}{3}\binom{7}{3}$

6. 0

7. $\binom{11}{3}$, $\binom{7}{3}$

8. $\sum_{i=k}^{n} \binom{n}{i}$