Recap for Tutorial 2

MH1101

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1 Fundamental Theorem of Calculus

Theorem 1. Suppose $f(x)$ is continuous on $[a, b]$, then for $x \in (a, b)$, we have

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_a^x f(t)\ \mathrm{d}t = f(x).$$

Theorem 2. If $f(x)$ is continuous on $[a, b]$, then

$$\int_a^b f(x)\ \mathrm{d}x = F(b) - F(a),$$

where $F(x)$ is any antiderivative of $f(x)$.

2 Substitution Rule

Theorem 3. Suppose g' is continuous on $[a, b]$ and f is continuous on the range of $u := g(x)$, then

$$\int_a^b f(g(x))g'(x)\ \mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\ \mathrm{d}u.$$

3 Mean Value Theorem for Integrals

Theorem 4. If f is continuous on $[a, b]$, then there existsa number $c \in [a, b]$, such that

$$f(c) = \frac{1}{b-a} \int_a^b f(x)\ \mathrm{d}x.$$

4 Extra Exercises

Problem 1. If $F(x) = \displaystyle\int_1^x f(t)\ \mathrm{d}t$, where $f(t) = \displaystyle\int_1^{t^2} \frac{\sqrt{1+u^4}}{u}\ \mathrm{d}u$. Find $F''(2)$.

Problem 2. Find a function $f(x)$ and a number a, such that

$$6 + \int_a^x \frac{f(t)}{t^2}\ \mathrm{d}t = 2\sqrt{x} \quad \forall x > 0.$$

Problem 3. Compute the following.

(a) $\displaystyle\int \frac{1+x}{1+x^2}\ \mathrm{d}x.$

(b) $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\ \mathrm{d}x.$

(c) $\displaystyle\int_0^a x\sqrt{a^2 - x^2}\ \mathrm{d}x.$