Recap for Tutorial 4

MH1101

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1 Integration by Parts

Given two functions $u(x)$, $v(x)$, we have

$$\int_a^b u(x)v'(x)\ \mathrm{d}x = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)\ \mathrm{d}x.$$

The purpose of applying integration by parts is to tackle integrals of a product of two different types of functions. $u(x)$ is being differentiated and $v'(x)$ is being integrated. Now, it boils down to choosing our choice of function to differentiate/integrate.

Generally, integration is difficult, it only makes sense for us to integrate by parts if the resultant integral on the right is easy to work with, i.e., $u'(x)v(x)$ is easy to integrate. Hence, we have the rule of thumb of choosing $u(x)$ based on the type of functions, i.e.

(i) Logarithmic functions (e.g. $\ln x$)

(ii) Inverse trigonometric functions

(iii) Algebraic functions (e.g. all polynomials, $\sqrt{x}$, $\frac{1}{x^2}$)

(iv) Trigonometric functions (e.g. $\sin x$, $\cos x$)

(v) Exponential functions (e.g. e^x)

We should prioritise to differentiate the first three types of functions, because their derivatives are algebraic functions and these are easy to integrate. Whereas, differentiating trigonometric functions and exponential functions do not change the type of the functions they are. **However, this rule does not always work!** It comes with a lot of practice to identify $u(x)$ and $v(x)$ quickly and accurately.

2 Extra Exercises

Problem 1. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

(i) $x = 1 + (y-2)^2, \quad x = 2 \quad$ about x-axis.

(ii) $y = 4x - x^2, \quad y = 3 \quad$ about $x = 1$.

Problem 2. Use the method of cylindrical shell to show that the volume of right circular cone (radius $= r$, height $= h$) is $\dfrac{1}{3}\pi r^2 h$.

Problem 3. Solve the following.

- $\displaystyle\int e^{2x}\sin(3x)\ \mathrm{d}x.$
- $\displaystyle\int_1^2 \frac{\ln x}{x^2}\ \mathrm{d}x.$

Problem 4. Let $I_n = \displaystyle\int_0^{\pi/3} \cos^n x\ \mathrm{d}x$. Prove that $I_n = \dfrac{1}{n}\left((n-1)I_{n-2} + \dfrac{\sqrt{3}}{2^n}\right)$. Then, compute I_4.