Extra Exercises for Week 3

MH2500

August 29, 2024

Questions are taken from the textbook: A First Course in Probability (9th edition) by Sheldon Ross.

Problem 1. An ordinary deck of 52 cards is shuffled. What is the probability that the top four cards have

(a) different denominations?

(b) different suits?

Problem 2. For three events E, F, G, prove that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c FG) - P(EF^c G) - P(EFG^c) - 2P(EFG).$$

Problem 3. Answer the following.

(a) A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?

(b) A pair of fair dice is rolled. What is the probability that the second die is at least 2 points higher than the first die.

Problem 4. An urn contains n white and m black balls. The balls are withdrawn one at a time until only those of the same color are left. What is the probability that they are all white?

Problem 5. If there are 12 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?

Problem 6. A 5-card hand is dealt from a well-shuffled deck of 52 playing cards. What is the probability that the hand contains at least one card from each of the four suits?

Problem 7. Given 20 people and 12 months in a year, what is the probability that exactly 4 of these months each have exactly 2 birthdays, and exactly 3 of these months each have exactly 3 birthdays, the remaining months have no birthdays?

Problem 8. An urn contains N balls numbered 1 to N. A person draws n balls one at a time without replacement. Among the n balls drawn, what is the probability that the smallest number is k.

Answers (Let me know if there are any discrepancies):

1(a). $\frac{52 \cdot 48 \cdot 44 \cdot 40}{52 \cdot 51 \cdot 50 \cdot 49}$

1(b). $\frac{52 \cdot 39 \cdot 26 \cdot 13}{52 \cdot 51 \cdot 50 \cdot 49}$

3(a). $\frac{\binom{6}{3}\binom{6}{3}}{\binom{12}{6}}$

3(b). $\frac{5}{18}$

4. $\frac{n}{m+n}$

5. $\frac{12!}{12^{12}}$

6. $\frac{\binom{4}{1}\binom{13}{2}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{5}}$

7. $\frac{\binom{12}{4}\binom{8}{4}\binom{20}{2}\binom{18}{2}\binom{16}{2}\binom{14}{2}\binom{12}{3}\binom{9}{3}\binom{6}{3}}{12^{20}}$

8. $\frac{\binom{N-k}{n-1}}{\binom{N}{n}}$