

Common Mistakes in Test 1

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Here are some common mistakes in Test 1. Not all are penalised, don't worry. Please avoid these in the future.

In Question 1:

- In part (a), you are asked to given the number of possible combinations that the committee is of size k , no need to sum from $k = 1$ to $k = n$. We only want a specific size of committee.
- Suppose we first pick a secretary, then form the committee of size k . After picking one secretary from n students, i.e. $\binom{n}{1}$, you only need to pick $k - 1$ students to form the committee. It is $n\binom{n-1}{k-1}$, not $n\binom{n-1}{k}$.
- It is fine to leave your answer as a combinatorial form, $k\binom{n}{k}$. No need to simplify it as factorials. Some students simplified wrongly, even the combinatorial form was correct.
- $\sum_{k=1}^n \binom{n-1}{k-1} = \sum_{k=0}^{n-1} \binom{n-1}{k}$, beware of the re-indexing. You have to re-index both the starting and the ending index.
- $\sum_{k=0}^n \binom{n}{k}$ is a binomial series, not a geometric series nor a Taylor series. Geometric series takes the form

$$\sum_{i=0}^n ar^i = \frac{a(1-r^{n+1})}{1-r}, \quad \sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}, \text{ for } |r| < 1.$$

Taylor series takes the form

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x-a)^k, \text{ for } |x-a| < \text{radius of convergence}.$$

It expresses a function in terms of a series of derivatives. Although in general, binomial series can be derived from it, but we are not dealing with functions here, it's just a summation of numbers. If you quote Taylor series, you need to provide the relevant function.

In Question 2:

- $P(\text{double 6 does not appear in one throw}) = 1 - P(\text{double 6 appearing in one throw}) = 1 - \left(\frac{1}{6}\right)^2 = \frac{35}{36}$.
- $P(\text{double 6 does not appear in one throw}) \neq \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{25}{36}$ is the probability that number six not appearing at all in any one of the dice, but it is acceptable to get $(1, 6), (6, 2)$, etc.

- $$\left(\frac{35}{36}\right)^n \leq \frac{1}{2} \implies n \ln\left(\frac{35}{36}\right) \leq \ln\left(\frac{1}{2}\right) \implies n \geq \frac{\ln(1/2)}{\ln(35/36)}.$$

Flip the inequality sign because $\ln(35/36) < 0$.

- You are asked how large does n need to be. It is enough to leave your answer as $n \geq 25$. If you write $n = 25$, either you are giving examples or it means that is the only possible n , which is not answering the question.
- If you are asked the minimum n , then you can write $n = 25$, because the minimum number is unique.

In Question 3:

- $P(A|B)$ is the probability of A occurring, given that B occurs, not the other way round.
- Unless stated otherwise, please leave your answer in exact form, not in a bunch of decimals or 3 significant figures or percentage.
- Most people did well.