MH2500 AY21/22

This paper is pretty straightforward, I will just provide the final answer.

Solution 1. (Same as tutorial question)

Solution 2. $e^{-1/2}$.

Solution 3.

$$f(z) = \begin{cases} 2z, & 0 < z < 1, \\ 0, & \text{otherwise}, \end{cases} \qquad \mathbb{E}(Z) = \frac{2}{3}, \qquad Var(Z) = \frac{1}{18}.$$

Solution 4.

(a) $\frac{7}{640}$

(b)

$$f_X(x) = \frac{6}{5}\left(x + \frac{1}{3}\right) \quad 0 \le x \le 1; \qquad f_Y(y) = \frac{6}{5}\left(y^2 + \frac{1}{2}\right) \quad 0 \le y \le 1.$$

Not independent.

(c)

$$f(x|y = 0.3) = \frac{x + 0.09}{0.59}, \quad 0 \le x \le 1.$$

Solution 5.

(a)

$$\mathbb{E}(X) = \frac{10}{3}, \qquad \mathbb{E}(Y) = 5, \qquad \mathbb{E}(XY) = 17, \qquad Cov(X, Y) = \frac{1}{3}.$$

(b)

$$Var(X) = \frac{5}{9}, \qquad Var(Y) = 1, \qquad \rho(X, Y) = \frac{\sqrt{5}}{5}.$$

Solution 6.

(a)

$$P\left(\sum_{i=1}^{20} X_i > 15\right) \le \frac{4}{3}.$$

(b)

$$P\left(\sum_{i=1}^{20} X_i > 15\right) \approx P(Z \ge -1.01) = 0.8438.$$