# Recap for Tutorial 10

MH1101

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## 1 Convergence Tests

**Definition 1** (Absolute Convergence)**.** A series $\sum a_n$ is called absolute convergent if $\sum |a_n|$ converges.

**Definition 2** (Conditional Convergence)**.** A series $\sum a_n$ is called conditionally convergent if it converges but $\sum |a_n|$ diverges.

**Theorem 1.** If $\sum a_n$ converges absolutely, then $\sum a_n$ is convergent.

**Theorem 2** (Alternating Series Test)**.** Suppose $(a_n)_{n\geq 1}$ is a positive, decreasing sequence and $\lim a_n = 0$, then

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{converges.}$$

**Theorem 3** (Ratio Test)**.** Let $(a_n)_{n\geq 1}$ be a sequence and assume that the following limit exists

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

- If $\rho < 1$, then $\sum a_n$ converges absolutely.
- If $\rho > 1$, then $\sum a_n$ diverges.
- If $\rho = 1$, then the test is inconclusive.

**Theorem 4** (Root Test)**.** Let $(a_n)_{n\geq 1}$ be a sequence and assume that the following limit exists

$$L = \lim_{n\to\infty} |a_n|^{1/n}.$$

- If $L < 1$, then $\sum a_n$ converges absolutely.
- If $L > 1$, then $\sum a_n$ diverges.
- If $L = 1$, then the test is inconclusive.

## 2 Extra Exercises

**Problem 1.** Determine whether the following series converges or diverges.

- $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{(n+1)4^{2n+1}}$.
- $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$.
- $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n^2+1}{2n^2+1}\right)^n$.
- $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2} + \sqrt[3]{n(n+1)} + \sqrt[3]{(n+1)^2}}$.
- $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{1+(-1)^n}{n} + \frac{1+(-1)^{n+1}}{n^2}\right)$.