# Extra Exercises for Week 3

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Questions are taken from the textbook: A First Course in Probability (9th edition) by Sheldon Ross.

**Problem 1.** An ordinary deck of 52 cards is shuffled. What is the probability that the top four cards have

(a) different denominations?

(b) different suits?

**Problem 2.** For three events $E, F, G$, prove that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^c FG) - P(EF^c G) - P(EFG^c) - 2P(EFG).$$

**Problem 3.** Answer the following.

(a) A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?

(b) A pair of fair dice is rolled. What is the probability that the second die is at least 2 points higher than the first die.

**Problem 4.** An urn contains $n$ white and $m$ black balls. The balls are withdrawn one at a time until only those of the same color are left. What is the probability that they are all white?

**Problem 5.** If there are 12 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?

**Problem 6.** A 5-card hand is dealt from a well-shuffled deck of 52 playing cards. What is the probability that the hand contains at least one card from each of the four suits?

**Problem 7.** Given 20 people and 12 months in a year, what is the probability that exactly 4 of these months each have exactly 2 birthdays, and exactly another 4 of these months each have exactly 3 birthdays, the remaining months have no birthdays?

**Problem 8.** An urn contains $N$ balls numbered 1 to $N$. A person draws $n$ balls one at a time without replacement. Among the $n$ balls drawn, what is the probability that the smallest number is $k$.

Answers (Let me know if there are any discrepancies):

1(a). $\frac{52 \cdot 48 \cdot 44 \cdot 40}{52 \cdot 51 \cdot 50 \cdot 49}$

1(b). $\frac{52 \cdot 39 \cdot 26 \cdot 13}{52 \cdot 51 \cdot 50 \cdot 49}$

3(a). $\frac{\binom{6}{3}\binom{6}{3}}{\binom{12}{6}}$

3(b). $\frac{5}{18}$

4. $\frac{n}{m+n}$

5. $\frac{12!}{12^{12}}$

6. $\frac{\binom{4}{1}\binom{13}{2}\binom{13}{1}\binom{13}{1}\binom{13}{1}}{\binom{52}{5}}$

7. $\frac{\binom{12}{4}\binom{8}{4}\binom{20}{2}\binom{18}{2}\binom{16}{2}\binom{14}{2}\binom{12}{3}\binom{9}{3}\binom{6}{3}}{12^{20}}$

8. $\frac{\binom{N-k}{n-1}}{\binom{N}{n}}$