Recap for Tutorial 9

MH1101

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1 Convergence Tests

Theorem 1 (Integral Test)**.** Suppose f is a continuous, positive, decreasing function on $[N, \infty)$, and let $a_n = f(n)$.

$$\int_N^\infty f(x)\ \mathrm{d}x \text{ convergent/divergent} \implies \sum_{n=N}^\infty a_n \text{ convergent/divergent} \implies \sum_{n=1}^\infty a_n \text{ convergent/divergent}$$

Theorem 2 (p-series)**.** Consider $\displaystyle\sum_{n=1}^\infty \frac{1}{n^p}$. This series converges if $p > 1$ and diverges otherwise.

Theorem 3 (Comparison Test)**.** Suppose that $\sum a_n$ and $\sum b_n$ are series with **positive** terms.

- If $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ is convergent.
- If $\sum b_n$ diverges and $a_n \ge b_n$ for all n, then $\sum a_n$ diverges.

Theorem 4 (Limit Comparison Test)**.** Suppose that $\sum a_n$ and $\sum b_n$ are series with **positive** terms.

- If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c < \infty$, then either both series converge or both series diverge.
- If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.
- If $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

2 Extra Exercises

Problem 1. Determine whether the following series converges or diverges.

- $\displaystyle\sum_{n=2}^\infty \frac{1}{n \ln n}$.
- $\displaystyle\sum_{n=1}^\infty \frac{4 + 3^n}{2^n}$.
- $\displaystyle\sum_{n=1}^\infty \frac{n^2 - 1}{3n^4 + 1}$.
- $\displaystyle\sum_{n=1}^\infty \frac{n - \sqrt{n}\sin n}{n^2 + n + 1}$.