MH2500 Mini-Quiz

Question 1

Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are

(a) h, t, t (meaning that the first flip results in heads, the second in tails, and the third in tails);

(b) t, h, t.

You may leave your answer uncomputed.

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Solution 1

Given that a total of 6 heads out of 10 throws, this will occur with a probability

$$\binom{10}{6} p^6 (1-p)^4.$$

$$\begin{aligned}
P(\text{h,t,t}|\text{total } 6 \text{ heads}) &= \frac{P(\text{h,t,t} \cap \text{total } 6 \text{ heads})}{\binom{10}{6} p^6 (1-p)^4} \\
&= \frac{P(\text{h,t,t} \cap 5 \text{ heads in the remaining } 7 \text{ throws})}{\binom{10}{6} p^6 (1-p)^4} \\
&= \frac{p(1-p)^2 \cdot \binom{7}{5} p^5 (1-p)^2}{\binom{10}{6} p^6 (1-p)^4} = \frac{\binom{7}{5}}{\binom{10}{6}} = \frac{1}{10}.
\end{aligned}$$

In fact,

$$P(\text{t,h,t}|\text{total } 6 \text{ heads}) = P(\text{h,t,t}|\text{total } 6 \text{ heads}) = \frac{1}{10}.$$

Question 2

A fair coin is flipped until either heads or tails has occurred twice. Find the expected number of flips.

Solution 2

It could be $HH, TT, HTH, THT, THH, HTT$. Let X denote the number of flips. We have

$$P(X=2) = P(HH) + P(TT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

$$P(X=3) = 1 - P(X=2) = \frac{1}{2}.$$

Then,

$$\mathbb{E}(X) = 2 \cdot P(X=2) + 3 \cdot P(X=3) = \frac{5}{2}.$$

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Question 3

A sample of 3 items is selected at random, with replacement from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample. What is the minimum number of items you should select, to ensure you will get a defective item?

Hint: If $X \sim$ Binomial(n, p), then $\mathbb{E}(X) = np$.

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Solution 3

Let X be the number of defective items in the sample of 3 items. X is a binomial random variable, because it consists of $n = 3$ independent trials each with a constant probability of $p = \frac{4}{20}$. Hence,

$$\mathbb{E}(X) = np = \frac{3}{5}.$$

We have 4 defective items out of 20 items, by pigeonhole principle, the answer is 17, to ensure you get a defective item.