

# Extra Exercises for Week 3

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Questions are taken from the textbook: A First Course in Probability (9th edition) by Sheldon Ross.

**Problem 1.** An ordinary deck of 52 cards is shuffled. What is the probability that the top four cards have

- (a) different denominations?
- (b) different suits?

**Problem 2.** For three events  $E, F, G$ , prove that

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E^cFG) - P(EF^cG) - P(EFG^c) - 2P(EFG).$$

**Problem 3.** Answer the following.

- (a) A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?
- (b) A pair of fair dice is rolled. What is the probability that the second die is at least 2 points higher than the first die.

**Problem 4.** An urn contains  $n$  white and  $m$  black balls. The balls are withdrawn one at a time until only those of the same color are left. What is the probability that they are all white?

**Problem 5.** If there are 12 strangers in a room, what is the probability that no two of them celebrate their birthday in the same month?

**Problem 6.** A 5-card hand is dealt from a well-shuffled deck of 52 playing cards. What is the probability that the hand contains at least one card from each of the four suits?

**Problem 7.** Given 20 people, what is the probability that among the 12 months in the year, there are 4 months containing exactly 2 birthdays and 4 containing exactly 3 birthdays?

**Problem 8.** An urn contains  $N$  balls numbered 1 to  $N$ . A person draws  $n$  balls one at a time without replacement. Among the  $n$  balls drawn, what is the probability that the smallest number is  $k$ .

Answers (Let me know if there are any discrepancies):

$$1(a). \frac{52 \cdot 48 \cdot 44 \cdot 40}{52 \cdot 51 \cdot 50 \cdot 49}$$

$$1(b). \frac{52 \cdot 39 \cdot 26 \cdot 13}{52 \cdot 51 \cdot 50 \cdot 49}$$

$$3(a). \frac{\binom{6}{3} \binom{6}{3}}{\binom{12}{6}}$$

$$3(b). \frac{5}{18}$$

$$4. \frac{n}{m+n}$$

$$5. \frac{12!}{12^{12}}$$

$$6. \frac{\binom{4}{1} \binom{13}{2} \binom{13}{1} \binom{13}{1} \binom{13}{1}}{\binom{52}{5}}$$

$$7. \frac{\binom{12}{4} \binom{8}{4} \binom{20}{2} \binom{18}{2} \binom{16}{2} \binom{14}{2} \binom{12}{3} \binom{9}{3} \binom{6}{3}}{12^{20}}$$

$$8. \frac{\binom{N-k}{n-1}}{\binom{N}{n}}$$