Extra Exercises for Week 4

MH2500

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Most questions are taken from the textbook: A First Course in Probability (9th edition) by Sheldon Ross.

Problem 1. For a student who understood the lecture, the probability of giving the correct answer to the quiz question is 0.75. On the other hand, for a student who did not understand the lecture, the probability of giving the correct answer is 0.1. It is estimated that 60% of the students understood the lecture and the remaining 40% did not.
Let U denote the event that a particular student understood the lecture. Let C denote the event that the student answered the question correctly. Find $P(U|C)$.

Problem 2. A parallel system functions whenever at least one of its components works. Consider a parallel system of n components and suppose each component works independently with probability 0.5. Find the conditional probability that component 1 works given that the system is functioning. What value does the probability converge to when n is very large?

Problem 3. Show that if $P(A|B) = 1$, then $P(B^c|A^c) = 1$.

Problem 4. Suppose you have in your pocket a fair coin and a two-headed coin. You select one of the coins at random.

(a) When you flip it once, it shows head. What is the probability that it is the fair coin?

(b) What is the probability that it will show a head on a second flip, given that it shows a head on the first flip?

(c) When you flip the coin a second time, it shows head again (i.e. two successive heads). What is the probability that it is the fair coin?

Problem 5. Show that if $P(A) > 0$, then $P(A \cap B|A) \geq P(A \cap B|A \cup B)$.

Problem 6. Let $S = \{1, 2, \ldots, n\}$ and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including the null set and S itself) of S. Let $N(B)$ denote the number of elements in B. By showing that

$$P(A \subseteq B|N(B) = i) = \frac{2^i}{2^{2n}},$$

and conditioning on $N(B) = i$, for $i = 0, \ldots, n$, compute $P(A \subseteq B)$.

Problem 7. A and B alternate rolling a pair of dice, stopping either when A rolls the sum 9 or when B rolls the sum 6. Assuming that A rolls first, find the probability that the final roll is made by A.

Problem 8. In successive rolls of a pair of fair dice, what is the probability of getting 2 sevens before 6 even numbers?

Problem 9. There is a 60 percent chance that event A will occur. If A does not occur, then there is a 10 percent chance that B will occur. What is the probability that at least one of the events A or B will occur?

Problem 10. An urn contains 4 blue balls and 3 red balls. A fair die is rolled. If the number shown on the die is $i, (i = 1, 2, \ldots, 6)$, then i balls are drawn from the urn uniformly at random without replacement. Given that all balls drawn are red, what is the probability that the remaining balls in the urn are all blue?

Answers (Let me know if there are any discrepancies):

1. $\frac{45}{49}$
2. $\frac{1/2}{1-(1/2)^n}$, $\frac{1}{2}$

4(a). $\frac{1}{3}$

4(b). $\frac{5}{6}$

4(c). $\frac{1}{5}$

6. $\left(\frac{3}{4}\right)^n$
7. $\frac{9}{19}$
8. $\frac{4547}{8192}$
9. $\frac{16}{25}$
10. $\frac{1}{21}$