<u>Part 1: Number Systems and Character Encodings (15 marks)</u>

<table>
<tr><th>Character</th><th>Hex</th><th>Decimal</th><th>7-bit Binary</th><th>Octal</th></tr>
<tr>
<td>Y</td>
<td>19_{16}</td>
<td>$19_{16} = 1 \times 16^1 \ + 9 \ \times 16^0$
$= 25_{10}$</td>
<td><table><tr><td>Base 16</td><td>1</td><td>9</td></tr><tr><td>Base 2</td><td>0001</td><td>1001</td></tr></table>$19_{16} = 001\ 1001_2$</td>
<td>19_{16}
$= 25_{10}$
$= 3 \times 8^1 \ + 1 \ \times 8^0$
$= 31_8$</td>
</tr>
<tr>
<td>a</td>
<td>55_{16}</td>
<td>$55_{16} = 5 \times 16^1 \ + 5 \ \times 16^0$
$= 85_{10}$</td>
<td><table><tr><td>Base 16</td><td>5</td><td>5</td></tr><tr><td>Base 2</td><td>0101</td><td>0101</td></tr></table>$55_{16} = 101\ 0101_2$</td>
<td>55_{16}
$= 85_{10}$
$= 1 \times 8^2 + 2 \times 8^1 \ +$
$5 \ \times 8^0$
$= 125_8$</td>
</tr>
<tr>
<td>n</td>
<td>62_{16}</td>
<td>$62_{16} = 6 \times 16^1 \ + 2 \ \times 16^0$
$= 98_{10}$</td>
<td><table><tr><td>Base 16</td><td>6</td><td>2</td></tr><tr><td>Base 2</td><td>0110</td><td>0010</td></tr></table>$62_{16} = 110\ 0010_2$</td>
<td>62_{16}
$= 98_{10}$
$= 1 \times 8^2 + 4 \times 8^1 \ +$
$4 \ \times 8^0$
$= 142_8$</td>
</tr>
</table>

Task 1.2 Upper case to lower case and lower case to upper case (9 marks)

1.2.1

$Y = 19_{16} = 25_{10}$

$y = 52_{16}$

$= 5 \times 16^1 \ + 2 \times 16^0$

$= 82_{10}$

The difference between Y and y in hexadecimal:

$82_{10} - 25_{10} = 57_{10}$

$57 \div 16 \ = \ 3 \ \ remainder\ 9$

$3 \div 16 \ = \ 0 \ \ remainder\ 3$

$57_{10} = \ 39_{16}$

To convert from upper case to lower case, we need to add 39_{16} to the hexadecimal code based on the encoding table.

To convert from lower case to upper case, we need to deduct 39_{16} from the hexadecimal code based on the encoding table.

1.2.2

Conversion of Y to y:

$19_{16} + 39_{16} = 52_{16}$

$19_{16} \rightarrow 0001\ 1001_2$

$39_{16} \rightarrow 0011\ 1001_2$

$\overline{\underline{0101\ 0010_2}}$

$$
\begin{aligned}
0101\ 0010_2 &= 1 \times 2^6\ + 1\ \times 2^4 + 1 \times 2^1 \\
&= 82_{10} \\
&= 52_{16}
\end{aligned}
$$

$82 \div 16\ =\ 5\ \ remainder\ 2$

$5 \div 16\ =\ 0\ \ remainder\ 5$

Based on the encoding table, 52_{16} represents lower case letter ‘y’.

Conversion of a to A:

$55_{16} - 39_{16} = 55_{16} + (-39_{16})$

$= 1C_{16}$

$55_{16}\ \rightarrow\ 0101\ 0101_2\ \rightarrow 0101\ 0101_2$

$-39_{16}\ \rightarrow -0011\ 1001_2 \rightarrow 1100\ 0111_2$

$\overline{\underline{1\ 0001\ 1100_2}}$

$$
\begin{aligned}
0001\ 1100_2 &= 1 \times 2^4\ + 1\ \times 2^3 + 1 \times 2^2 \\
&= 28_{10} \\
&= 1C_{16}
\end{aligned}
$$

$28 \div 16\ =\ 1\ \ remainder\ 12\ (C)$

$1 \div 16\ =\ 0\ \ remainder\ 1$

Based on the encoding table, $1C_{16}$ represents upper case letter ‘A’.

Part 2: Boolean Algebra (30 marks total)

Task 2.1: Boolean Algebra Expressions (10 marks)

First, there are 2 respective equations for each output. To get the Sum of Product (SOP), we need to refer to rows of the truth table which produced an output of 1. For input, 0 indicates negation to the respective input.

For example,

X1	X2	X3	X4	Z1
0	0	0	0	1

For Z1 output, row 1 in the truth table shows "1" output which is essential to get the SOP. The Boolean algebra for this respective row is $\overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4}$.

The above method is applied throughout the truth table and sum up all the Boolean algebra terms for the rows with "1" output to get SOP for each respective Z1 and Z2 output as follows:

$$Z1 = \overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4} + \overline{X1}\,\overline{X2}\,X3\,\overline{X4} + \overline{X1}\,\overline{X2}\,X3\,X4 + \overline{X1}\,X2\,\overline{X3}\,X4 + \overline{X1}\,X2\,X3\,\overline{X4} + \overline{X1}\,X2\,X3\,X4 +$$
$$X1\,\overline{X2}\,\overline{X3}\,X4 + X1\,\overline{X2}\,X3\,\overline{X4} + X1\,\overline{X2}\,X3\,X4 + X1\,X2\,X3\,\overline{X4}$$

$$Z2 = \overline{X1}\,\overline{X2}\,\overline{X3}\,\overline{X4} + \overline{X1}\,\overline{X2}\,\overline{X3}\,X4 + \overline{X1}\,\overline{X2}\,X3\,X4 + \overline{X1}\,X2\,\overline{X3}\,\overline{X4} + \overline{X1}\,X2\,\overline{X3}\,X4 + \overline{X1}\,X2\,X3\,X4$$
$$+X1\,\overline{X2}\,\overline{X3}\,\overline{X4} + X1\,X2\,X3\,X4$$

Next, there are 2 respective equations for each output. To get the Product of Sum (POS), we need to refer to rows of the truth table which produced an output of 0. For input, 1 indicates negation to the respective input.

For example,

X1	X2	X3	X4	Z1
0	0	0	1	0

For Z1 output, row 2 in the truth table shows "0" output which is essential to get the POS. The Boolean algebra for this respective row is $(X1 + X2 + X3 + \overline{X4})$.

The above method is applied throughout the truth table and multiply all the Boolean algebra terms for the rows with "0" output to get POS for each respective Z1 and Z2 output as follows:

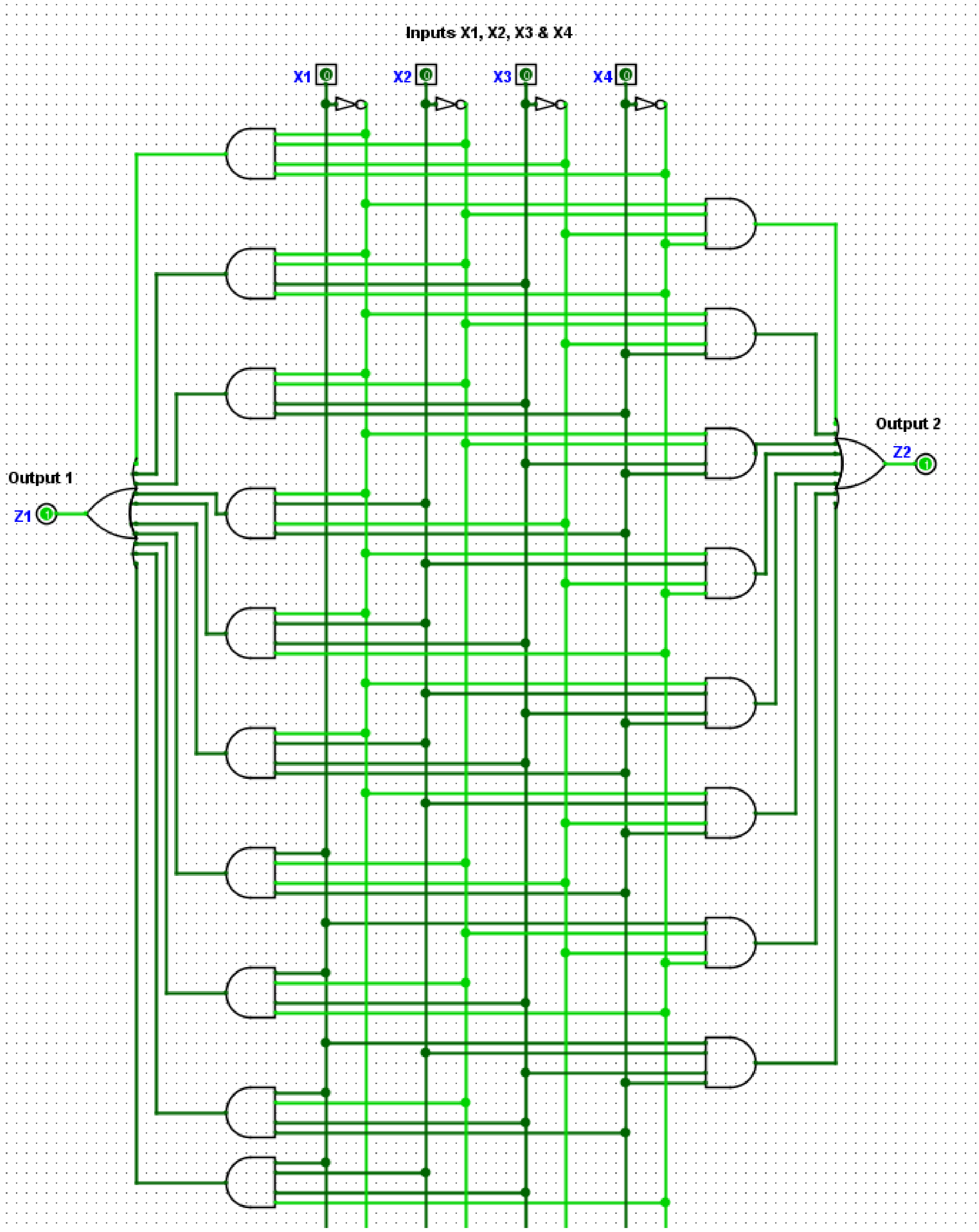
$$Z1 = (X1 + X2 + X3 + \overline{X4}) \cdot (X1 + \overline{X2} + X3 + X4) \cdot (\overline{X1} + X2 + X3 + X4) \cdot (\overline{X1} + \overline{X2} + X3 + X4) \cdot$$
$$(\overline{X1} + \overline{X2} + X3 + \overline{X4}) \cdot (\overline{X1} + \overline{X2} + \overline{X3} + \overline{X4})$$

$$Z2 = (X1 + X2 + \overline{X3} + \overline{X4}) \cdot (X1 + \overline{X2} + \overline{X3} + X4) \cdot (\overline{X1} + X2 + X3 + \overline{X4}) \cdot (\overline{X1} + X2 + \overline{X3} + X4) \cdot$$
$$(\overline{X1} + X2 + \overline{X3} + \overline{X4}) \cdot (\overline{X1} + \overline{X2} + X3 + X4) \cdot (\overline{X1} + \overline{X2} + X3 + \overline{X4}) \cdot (\overline{X1} + \overline{X2} + \overline{X3} + X4)$$

Task 2.2: Logical circuit in Logisim (10 marks)

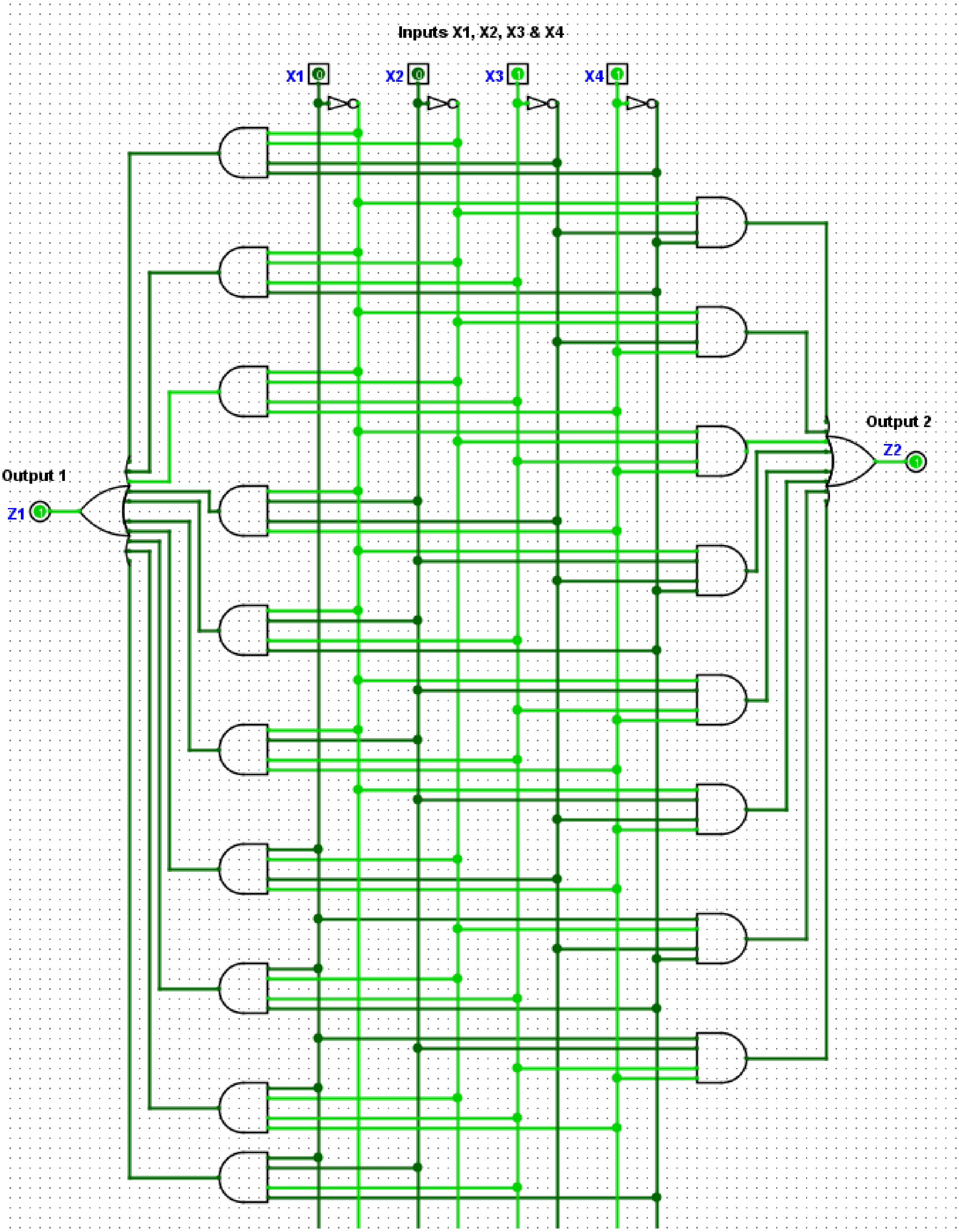
For Z1 Output, there are 4 inputs, and the use of gates are as follows:
NOT gate: 4 gates
AND gate: 10 4-inputs gates
OR gate: 1 10-inputs gate
Total: 15 gates

For Z2 Output, there are 4 inputs, and the use of gates are as follows:
NOT gate: 4 gates
AND gate: 8 4-inputs gates
OR gate: 1 8-inputs gates
Total: 13 gates



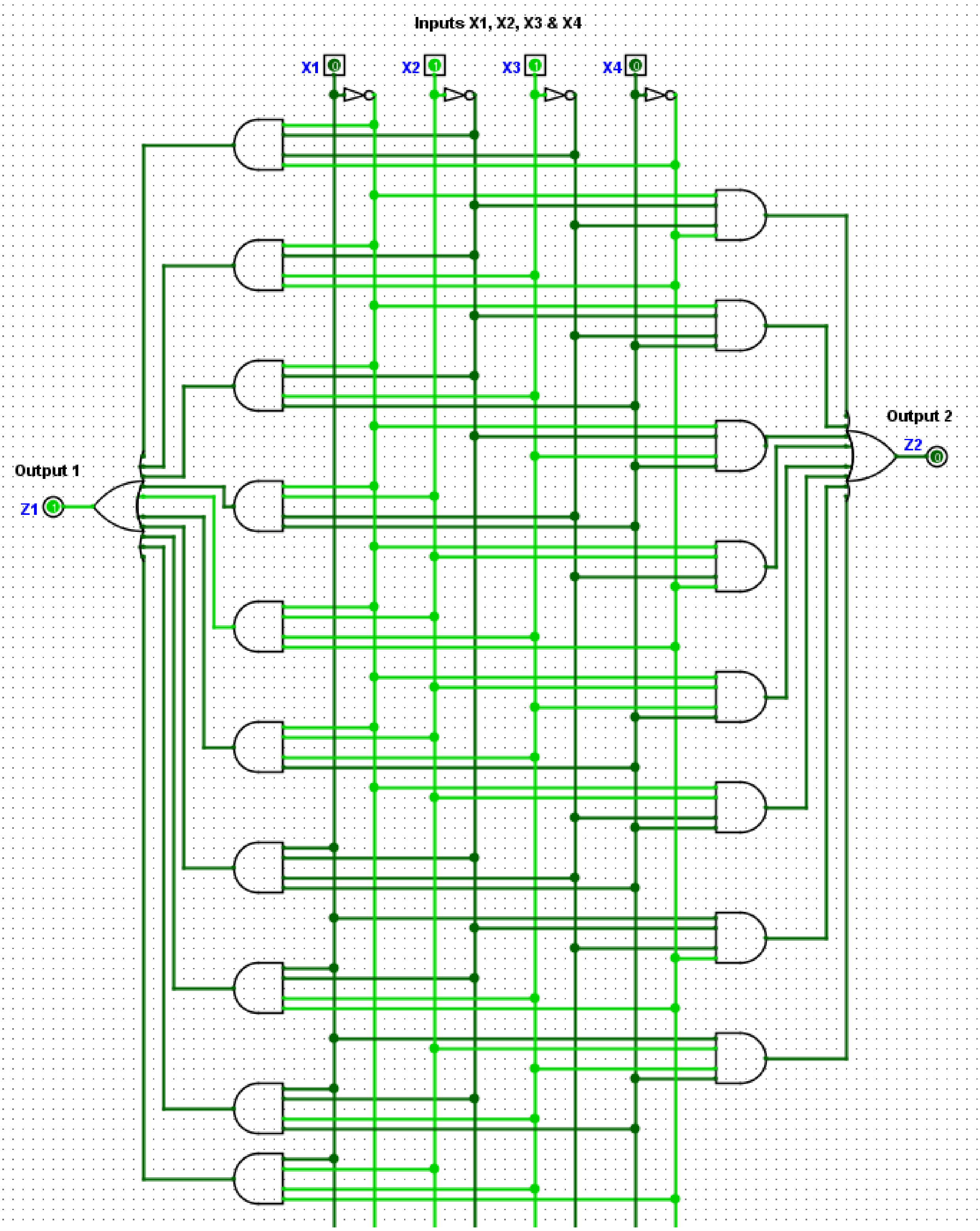
Test Case 1: Row 4

X1	X2	X3	X4	Z1	Z2
0	0	1	1	1	1



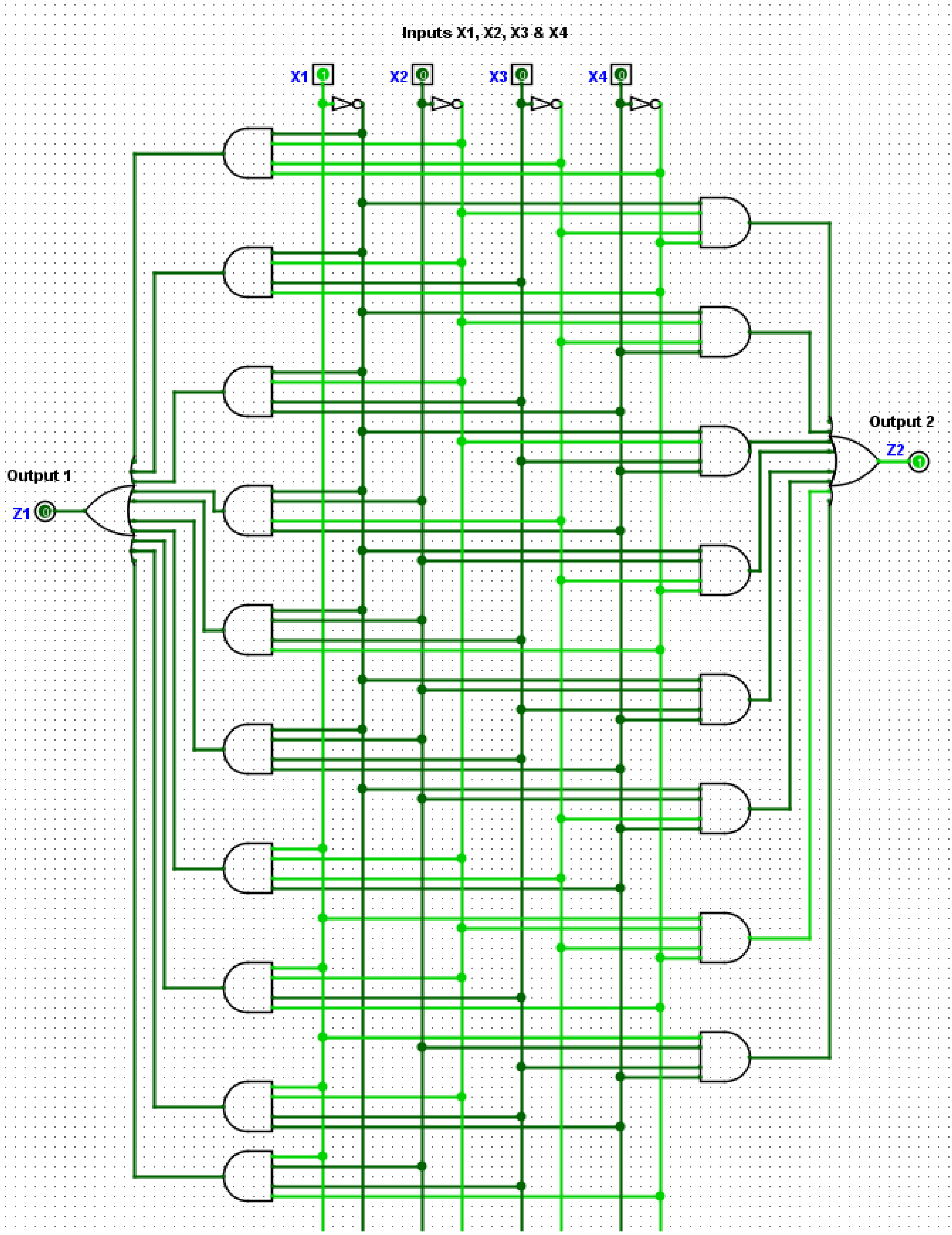
Test Case 2: Row 7

X1	X2	X3	X4	Z1	Z2
0	1	1	0	1	0



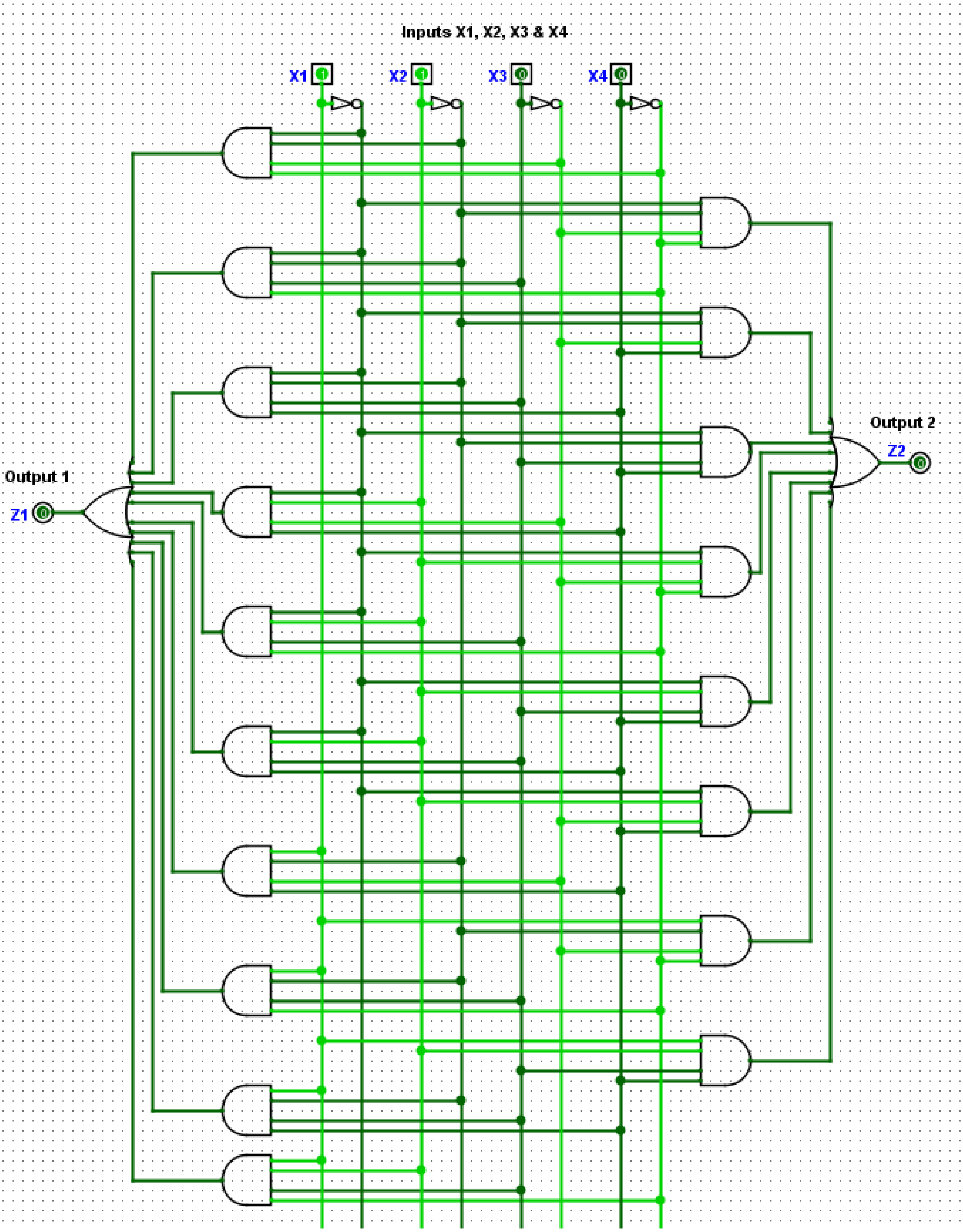
Test Case 3: Row 9

X1	X2	X3	X4	Z1	Z2
1	0	0	0	0	1



Test Case 4: Row 13

X1	X2	X3	X4	Z1	Z2
1	1	0	0	0	0



Task 2.3: Optimised circuit (10 marks)

For Output Z1, the Karnaugh map is constructed as follow:

$X1X2$ \ $X3X4$	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	0	0	0	1
10	0	1	1	1

From the Karnaugh map above, there are a total of 5 groups found in the map labelled with different colours. These are useful in constructing the reduced Boolean function below.

$$FZ1 = \overline{X1}\,X3 + \overline{X1}\,\overline{X2}\,\overline{X4} + \overline{X1}\,X2\,X4 + X1\,\overline{X2}\,X4 + X1\,X3\,\overline{X4}$$

According to the function, there are still 4 inputs, and the use of gates are as follows:
NOT gate: 3 gates
AND gate: 1 2-inputs gate, 4 3-inputs gate
OR gate: 1 5-inputs gate
Total: 9 gates

This indicates that the Boolean function has been optimised as the total number of gates have been reduced from 15 gates to 9 gates (reduce 6 gates).

For Output Z2, the Karnaugh map is constructed as follow:

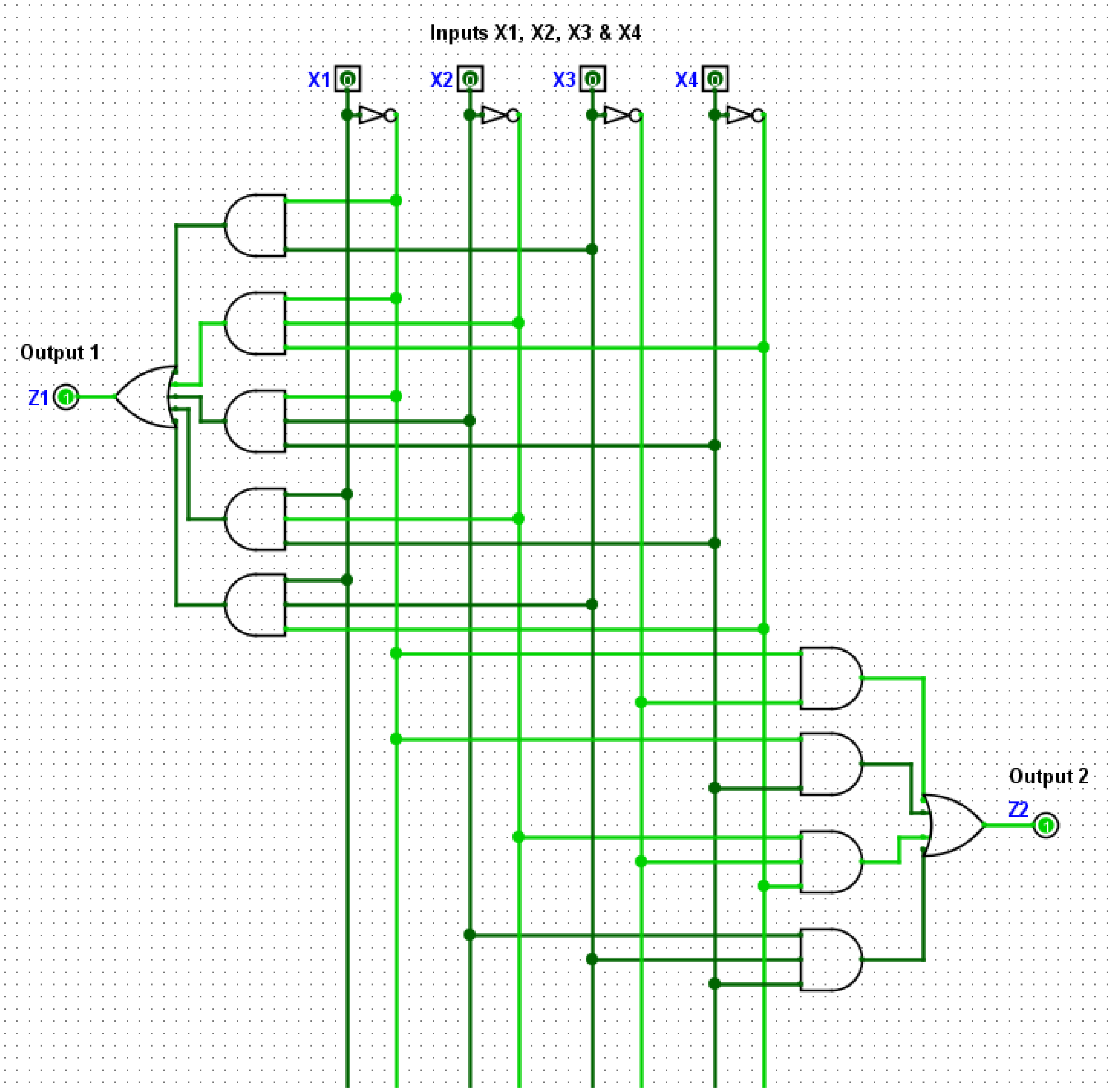
$X1X2$ \ $X3X4$	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	1	0
10	1	0	0	0

From the Karnaugh map above, there are a total of 4 groups found in the map labelled with different colours. These are useful in constructing the reduced Boolean function below.

$$FZ2 = \overline{X1}\,\overline{X3} + \overline{X1}\,X4 + \overline{X2}\,\overline{X3}\,\overline{X4} + X2\,X3\,X4$$

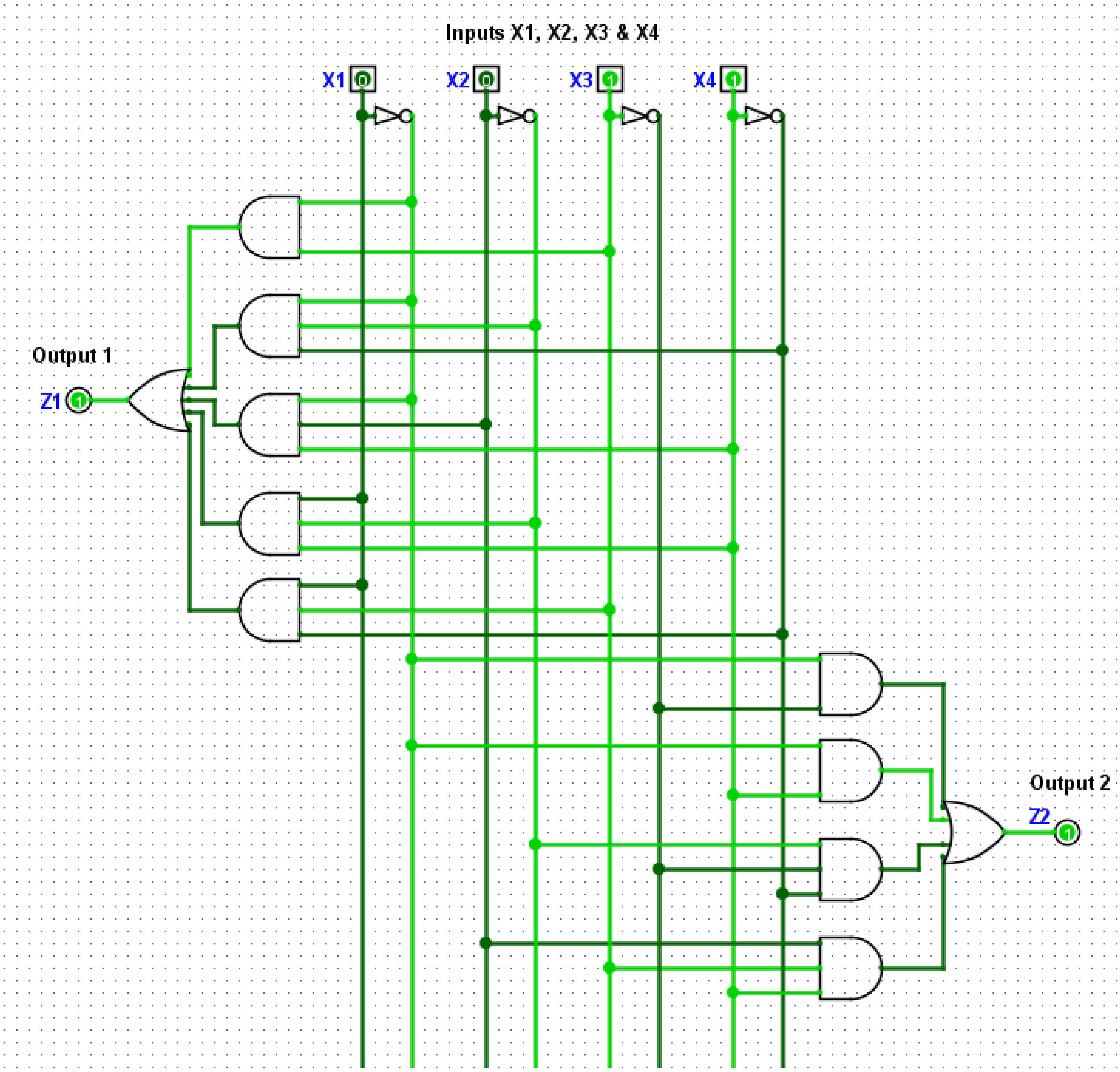
According to the function, there are still 4 inputs, and the use of gates are as follows:
NOT gate: 4 gates
AND gate: 2 2-inputs gate, 2 3-inputs gate
OR gate: 1 4-inputs gate
Total: 9 gates

This indicates that the Boolean function has been optimised as the total number of gates have been reduced from 13 gates to 9 gates (reduce 4 gates).


Inputs X1, X2, X3 & X4
X1
X2
X3
X4
Output 1
Z1
Output 2
Z2

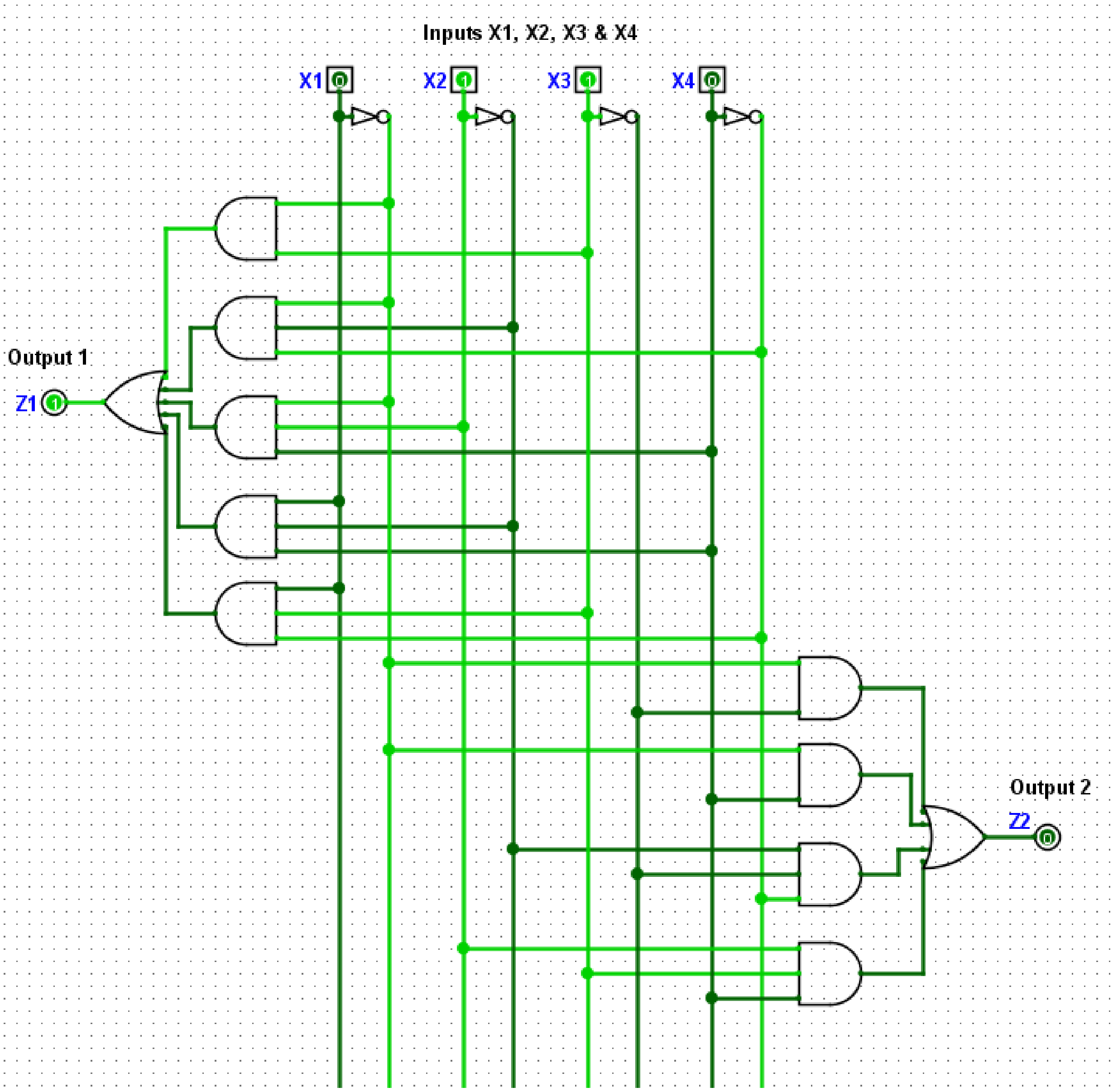
Test Case 1: Row 4

X1	X2	X3	X4	Z1	Z2
0	0	1	1	1	1



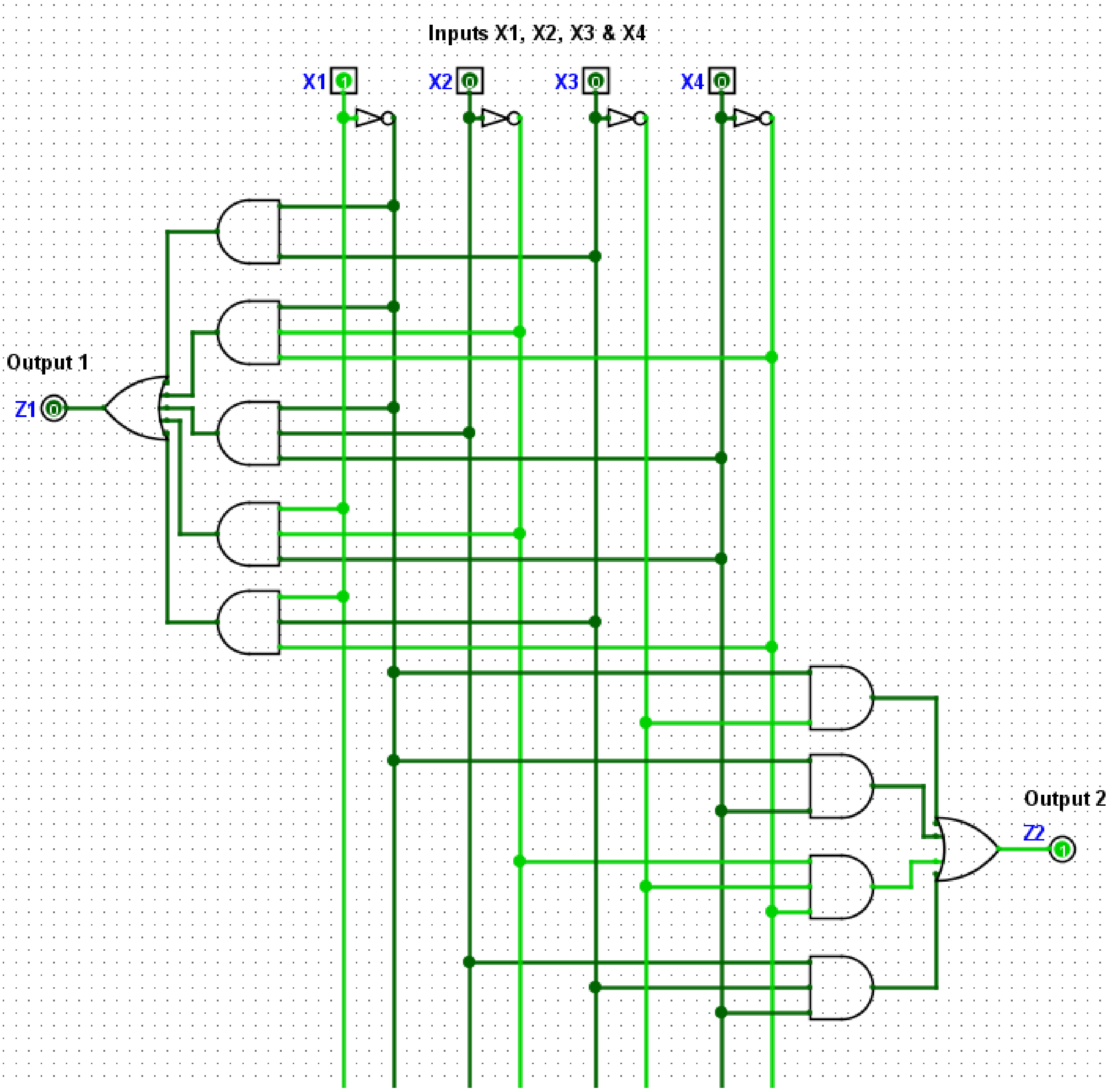
Test Case 2: Row 7

X1	X2	X3	X4	Z1	Z2
0	1	1	0	1	0



Test Case 3: Row 9

X1	X2	X3	X4	Z1	Z2
1	0	0	0	0	1



Test Case 4: Row 13

X1	X2	X3	X4	Z1	Z2
1	1	0	0	0	0