

HOMEWORK 3

COMPUTATIONAL METHODS FOR DATA SCIENCE

FALL SEMESTER 2023

1. **Profit Maximization Problem for Fertilizer Company.** (15 points) A fertilizer company purchases nitrates, phosphates, potash and an inert chalk base at a cost of \$1500, \$500, \$1000 and \$100 per ton, respectively, and produces four fertilizers *A*, *B*, *C*, and *D*. The production cost, selling price, and composition of the four fertilizers are given below.

Fertilizer	Production Cost	Selling Price	Percentage Composition			
			Nitrates	Phosphates	Potash	Chalk
A	100	350	5	10	5	80
B	150	550	5	15	10	70
C	200	450	10	20	10	60
D	250	700	15	5	15	65

During any week, no more than 1000 tons of nitrate, 2000 tons of phosphates, 1500 tons of potash will be available. The company is required to supply a minimum of 5000 tons of fertilizer *A* and 4000 tons of fertilizer *D* per week to its customers, but it is otherwise free to produce the fertilizers in any quantities it pleases. Find the quantity of each fertilizer to be produced by the company to maximize the profit.

2. **Knapsack Problem via GA** (35 points). Suppose you are going to fight for survival in a war. The goal is to maximize your survival points. The following table is a list of weapons you can choose:

Item	Type	Weight	Survival Points
Shadow Daggers	Knife	3.3	7
Huntsman Knife	Knife	3.4	8
Gut Knife	Knife	6.0	13
228 Compact Handgun	Pistols	26.1	29
Night Hawk	Pistols	37.6	48
Desert Eagle Magnum	Pistols	62.5	99
Ingram MAC-10 SMG	Primary	100.2	177
Leone YG1265 Auto Shotgun	Primary	141.1	213
M4A1 Carbine	Primary	119.2	202
AK-47 Rifle	Primary	122.4	210
Krieg 550 Sniper Rifles	Primary	247.6	380
M249 Machine Gun	Primary	352.0	485
Gas Mask	Equipment	24.2	9
Night-Vision Goggle	Equipment	32.1	12
Tactical Shield	Equipment	42.5	15

You can carry at most 529 weight units in your inventory bag. In addition, here are additional requirements and bonus survival points:

- (a) You must carry at least one knife, one pistol and one equipment in your inventory bag.

- (b) If you carry shadow daggers and desert eagle magnum at the same time, add an additional 5 survival points.
 - (c) If you carry 228 compact handgun and either AK-47 rifle or M4A1 carbine at the same time, add an additional 15 survival points.
 - (d) If you carry either Leone YG1265 Auto Shotgun or Krieg 500 Sniper Rifles as primary weapon, plus Desert Eagle Magnum and tactical shield, add an additional 25 survival points.
 - (e) If you carry all three equipments in your inventory, add an additional 70 survival points.
- (a) Write down the statement of the optimization problem from the given information above. State clearly the objective function and the constraints.
 - (b) Calculate the maximum number of possible combinations of inventory bags.
 - (c) Write a program of Genetic Algorithm. We consider the roulette-wheel selection, uniform crossover (crossover probability = 0.1) and multi bit flip mutation (consecutive based on item types) as three operators.
 - (d) Use your GA program to optimize your survival points. The population size is set at 10 and the maximum number of iteration is set at 20 steps. No variation convergence criterion is used for termination.
 - (e) Use the hill climbing algorithm and random walk algorithms with the mutation operator defined by your own on this problem. The maximum number of iteration is set at 200 steps.
 - (f) Draw the progress diagrams (best objective function value vs number of function evaluation) of GA, Hill Climbing and Random Walk. Comment on how their performances are associated with the choice of their operators. (NOTE: think carefully how to compare GA and other two methods on the same plot with number of FE as x-axis).
3. **Optimization of Travel Routes for South Korea Cities.** (50 points) Assume you want to organize a travel trip to visit cities in South Korea. It is obvious that you want to minimize the distance travelled. You arrive at and leave from South Korea via Incheon. Here is the 15 cities you plan to visit. Incheon, Seoul, Busan, Daegu, Daejeon, Gwangju, Suwon-si, Ulsan, Jeonju, Cheongju-si, Changwon, Jeju-si, Chuncheon, Hongsung, Muan.
- (a) Create the distance of location table. In this problem, the table will be symmetric over the diagonal because the distance between City A and City B is fixed without direction difference.
 - (b) Calculate how many evaluation of objective function required if one attempts the exhaustive enumeration.
 - (c) Run a random walk to find the optimal path that passes through these 15 cities with minimum distance. Report the optimal path obtained from the random walk after 100 iterations. Set appropriate values for parameters if needed.
 - (d) Run a hill climbing to find the optimal path that passes through these 15 cities with minimum distance. Record the distances of the best paths in all 100 iterations. Set appropriate values for parameters if needed.
 - (e) Write a program of Tabu Search for traveling salesman problem. Set maximum iteration to be 100, maximum length of tabu list to be 10, aspiration criterion as expectation

improvement, and we swap during the move. Use a random point as the starting point, run tabu search to find the optimal path that passes through these 15 cities (and return to Incheon at the end) with minimum distance.

- (f) Write a program of Simulated Annealing for traveling salesman problem. Set appropriate values for parameters if needed. Assume a linear temperature delay. Use a random point as the starting point, run simulated annealing to find the optimal path that passes through these 15 cities (and return to Incheon at the end) with minimum distance.
- (g) Write a program of Ant Colony optimization for traveling salesman problem. Set appropriate values for parameters if needed. Run the ant colony optimization to find the optimal path that passes through these 15 cities (and return to Incheon at the end) with minimum distance.
- (h) Create the best distance vs iterations plot by plotting the results of Hill Climbing, Random Walk, Simulated Annealing, Tabu Search and Ant Colony Optimization on the same plot. Compare these algorithms and comments on their strength and weakness in this problem.

Hint: Please visit <https://www.distancecalculator.net/country/south-korea> if you do not know where you can obtain the city distance.

4. **More on Optimization of Travel Routes for South Korea Cities.** (*Bonus 10 points*)
In fact, Particle Swarm Optimization can also be used to find the optimal path that passes through 15 cities of South Korea and return to Incheon at the end. Write a program of Particle Swarm Optimization and find the optimal path using this program. Compare the algorithm with the previous 5 algorithms and comments on its strength and weakness in this problem.