Problem Set 2

Write a program that solves the neoclassical growth model with technology shocks through value function iteration.

In the repository you will find some basic code that solves the deterministic model in which a representative household solves

$$\max_{\{(C_t,K_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to the resource constraint $K_{t+1} + C_t = K_t^{\alpha} + (1-\delta) K_t$ and a given K_0.

Modify this program to account for random technology shocks. The production function is now

$$Y_t = A_t K_t^{\alpha}$$

where A_t is random and not known when K_t is chosen. Let's keep things simple by assuming that $\{A_t\}$ follows a Markov process with only two states, A^h and A^l.

We can calibrate the model to the US economy by picking $\delta = 0.025$ (targeting an annual depreciation rate of around 10%), a capital share of income of $.35 = \alpha$, and $\beta = .99$ targeting an interest rate of around 4% per year[1]. For the IES, take the standard value of $\sigma = 2$. Now we need to pick the elements of the transition matrix for A_t denoted $\Pi = \begin{bmatrix} \pi^{hh} & 1-\pi^{hh} \\ 1-\pi^{ll} & \pi^{ll} \end{bmatrix}$ and the values for A^h and A^l. For the transition probabilities we can look at the average frequency and duration of recessions in US data as determined by the NBER. With that information we get $\pi^{hh} = 0.977$ and $\pi^{ll} = 0.926$. We would like to pick the last two parameters to satisfy two conditions: first, we want to normalize the long-run mean of A_t to be 1. Second, we would like the size of the fluctuations in output to roughly match the size of the fluctuations in US GDP, that is we would like the standard deviation of Y_t in the model to be similar to the one in the data. But it is hard to know what that means for the values of A without solving the model first! Therefore, we start with a guess of $A^h = 1.1$ (note that this implies a value of $A^l = 678$ from the invariant distribution of Π).[2] Once we have solved the model and find that the output fluctuations are too large we can still adjust those values – in other words, we calibrate A^h "inside the model".

1. State the dynamic programming problem by writing down the functional equation. Clearly state which ones are the state variable(s) and control variable(s) of the problem.

[1] 4% is actually a bit too high compared to US data, but the smaller we choose β the faster is convergence)

[2] If $\bar{\pi}_h$ and $\bar{\pi}_l = 1 - \bar{\pi}_h$ are the long-run probabilities of the invariant distribution associated with Π then the requirement that the long-run mean of $A_t = 1$ implies $\pi_h A^h + \pi_l A^l = 1$ from which we can back out the value for A^l.

2. Solve the model, and plot the value function over K for each state of A. Is it increasing and concave?

3. Plot the policy function (ie capital K' next period) over K for each state of A. Is it increasing in K and A? Plot savings over K for each A. Is it increasing in K and A?

4. Now, simulate the economy by drawing random values for A_t according the the stochastic process we defined above. Calibrate the values for A^h and A^l such that the standard deviation of output in the model matches the standard deviation of quarterly output in the US of around 1.8%. (To do this, draw only 1 random sequence for $\{A_t\}$ and then use the same sequence to see what happens for different values of A^h.)

5. Extra credit: Solve the problem under uncertainty using EITHER a different programming language OR by using for loops instead of vectorization withing Matlab. Compare the speed differences.