Problem Set 5 – Krusell and Smith

In this problem set you are to compute a version of Krusell/Smith (1998).

Setup The household sector consists of a unit measure of households with preferences given by

$$E\left[\sum_{t=0}^{\infty}\beta^t \log c_t\right].$$

The households' budget constraint is

$$c_t + k_{t+1} = z_t w_t + r_t k_t + (1-\delta)\, k_t.$$

The random variable z_t is binary with $z_t \in \{0,1\}$ so that we can interpret it as a shock determining employment or unemployment. Asset markets are incomplete, so that households have to self-insure against fluctuations (both in z and in A described below) by holding capital which they can rent out to firms at rental rate r_t.

The production side can be represented by a competitive firm with production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}.$$

Aggregate TFP A_t can also take on two values, specifically $A_t \in \left\{A^h, A^l\right\}$.

The exogenous shocks A and z follow a joint Markov process, the transition matrix to which is provided in the repository. (The restrictions on the transition matrix are derived in the following way: First, transition probabilities $\pi_{AA'zz'}$ have to be consistent with a transition matrix Π_A in which the average duration of both good and bad times are 8 quarters, i.e. $\pi_{AA'00} + \pi_{AA'01} = \pi_{AA'11} + \pi_{AA'10} = \pi_{AA'} = 1/8$. Second, K/S assume that the level of unemployment in a good state is always u_g and the level in a bad state is always u_b independent from the aggregate state in the previous period. Third, the average duration of unemployment spells is 1.5 quarters in good times and 2.5 quarters in bad times, and fourth, K/S assume values for the job-finding probabilities depending on the aggregate state.)

Calibration Let $\alpha = 0.36$,$\beta = 0.99$ (quarterly calibration) and $\delta = 0.025$. The values for aggregate TFP are $A^h = 1.01$ and $A^l = 0.99$. You can use $n_K = 10$ using a grid from $K_{min} = 4$ to $K_{max} = 7$, and $n_k = 101$ from $k_{min} = 0$ to $k_{max} = 15$.

Algorithm Solve the model by approximating the distribution over (z,k) through an aggregate state variable K, by following these steps:

1. Conjecture a log-linear functional form for the transitions in aggregate capital,

$$\log \tilde{G}_K(A,K) = \log K' = \begin{cases} a_0^g + a_1^g \log K & \text{if } A = A^g \\ a_0^b + a_1^b \log K & \text{if } A = A^b \end{cases}.$$

As initial guesses for the parameters use $a_0^g = a_0^b = 0$ and $a_1^g = a_1^b = 1$.

2. Using your current guess of parameters $\left(a_0^g, a_0^b, a_1^g, a_1^b\right)$ in the approximate function $\tilde{G}_K$, solve the household's problem using recursive methods to derive the household's policy function. Knowing A determines current labor supply L ($L = 0.96$ in good times and $L = 0.9$ in bad times), and together with current K we can compute w and r today. For taking expectations about the future, you can use linear interpolation to interpolate the value function at the levels of $\tilde{G}_K(A, K) = K'$ between the gridpoints.

3. Simulate the economy for $T = 3500$ periods by drawing a random sequence for $\{A_t\}_{t=1}^T$ according to the transition matrix Π_A and starting with $A_1 = A^h$. (It is important that once you draw the sequence you keep it and use it again for the later iterations of the simulation.) For the initial condition in $t = 1$, $K_1 = K_{ss}$ (which should be around 5.7) and a distribution over (z, k) given by μ_{ss}, where K_{ss} and μ_{ss} are values from a hypothetical steady state in which $A_t = 1\,\forall t$. After the simulation is complete, discard the first 500 periods to avoid dependence on the initial conditions we picked. For each period $t \in \{501, T\}$, compute and store the aggregate capital stock, thereby constructing $\{K_t\}_{t=501}^T$.

4. Use the time series to get parameter estimates $\left(\hat{a}_0^g, \hat{a}_0^b, \hat{a}_1^g, \hat{a}_1^b\right)$ by running a regression of the functional form in step 1. If the parameter estimates are sufficiently close to the current guess you used in step 2, go to step 5. If not, update your parameter guess in the direction of the parameter estimates and go back to step 2.

5. Check the goodness-of-fit of the approximation by reporting the R^2 of the last regression. If the R^2 is not high enough you might have to choose a different approximation function in step 1 – for this you could higher moments of the capital distribution, or choose a different functional form. (In the Krusell/Smith model the R^2 should be high enough, so the function given in step 1 works fine.)