Problem Set 7 – The Particle Filter

The goal of this problem set is to use the particle filter in combination with Metropolis-Hastings to estimate parameters from an observed time series.

The variable X_t evolves according to an ARMA$(2,2)$ process:

$$X_t = \rho_1 X_{t-1} + \rho_2 X_{t-2} + \phi_1 \varepsilon_{t-1} + \phi \varepsilon_{t-2} + \varepsilon_t.$$

Unfortunately we cannot observe X_t directly. Instead, every period we observe only data (A_t, B_t) which is related to X_t in the following way:

$$\begin{aligned} A_t &= \exp\left(X_t + \nu_t^A\right) \\ B_t &= \beta X_t^2 + \nu_t^B \end{aligned}$$

The repository contains the observed data series $\{(A_t, B_t)\}_{t=1}^T$ for a $T = 400$.

1. Write the problem in terms of notation used in class, that is define the transition function g, the observation equation h, the state S_t, observables Y_t and the shocks W_t and V_t to state variables and observables, respectively. Which parameters that we need to estimate does the parameter vector θ contain?

2. Use the particle filter to approximate the likelihood function $p(Y_t|\theta)$ for a given set of parameters θ.

3. Incorporate the routine from 2. into a Metropolis-Hastings algorithm to find posterior distributions for all parameters.